Octal and Hexadecimal Data

CS 350: Computer Organization & Assembler Language Programming

A. Why?

- Octal and hexadecimal numbers are useful for abbreviating long bitstrings.

B. Outcomes

At the end of today, you should:

- Know how octal and hexadecimal numbers work, how to translate to and from binary with them, and how to take their negative (in 2's complement).

C. Converting Between Octal, Hexadecimal, and Binary

- Octal: base 8 (uses digits $0-7$). Hexadecimal (a.k.a. "hex"): base 16 uses 0–9 and A–F as digits for 10–15. Octal and hexadecimal useful for abbreviating bit strings, hex for bitstrings of lengths 4, 8, 12, 16, …, and octal for bitstrings of lengths 3, 6, 9, 12, ….
- Converting Hexadecimal → Binary
    - Replace each hex digit by its 4-bit representation.
        - $0_{16} = 0000_2$, …, $9_{16} = 1001_2$, $A_{16} = 10_{10} = 1010_2$; B = 1011, C = 1100, D = 1101, E = 1110, F = 1111.
    - E.g., $03FC_{16} \rightarrow 0000\ 0011\ 1111\ 1100_2$.
- If we have k hex digits, we end up with $4\,k$ bits unless we're told explicitly that we want 1, 2, or 3 bits fewer than that. E.g., $5D_{16}$ represents the 8 bits 0101 1101 or the 7 bits 101 1101. If we want the decimal number represented by $5D_{16}$, we have to know if it represents a signed or unsigned bitstring; if it represents a signed bitstring, we also need to know how many bits it represents (and what scheme we're using).
    - As an 8-bit unsigned number, $5D_{16} = 0101\ 1101_2 = 93_{10}$.
    - As an 8-bit 2's complement number, $5D_{16} = 0101\ 1101_2 = 93_{10}$.

    - As a 7-bit 2's complement number, $5D_{16} = 101\ 1101_2 = -(010\ 0011_2) = -35_{10}$.
        - We can abbreviate this to "7-bit (2's complement) $5D_{16} = -35_{10}$".
    - As a 7-bit 1's complement number, $5D_{16} = 101\ 1101_2 = -(010\ 0010_2) = -34_{10}$.
    - As a 7-bit sign-magnitude number, $5D_{16} = 101\ 1101_2 = -(01\ 1101_2) = -29_{10}$.
        - (Note $5D_{16}$ can't represent a 5 or 6 bit number.)
- Similarly, if we have k octal digits, we end up with $3\,k$ bits unless told otherwise.
    - E.g., 6-bit unsigned 31_8 and 6-bit 2's complement 31_8 = 011 001 = 25; 5-bit 2's complement 31_8 = 11 001 = –(00 111) = –7; 5-bit 1's complement 31_8 = 11 001 = –(00 110) = –6; 5-bit sign-magnitude 31_8 = 11 001 = –(01001) = –9.
- Converting Binary → Hexadecimal
    - Pad bitstring on left with 0s to get a string of length divisible by 4.
    - Replace each 4-bit sequence by its equivalent hex digit.
    - E.g., $011011_2 \rightarrow 0001\ 1011_2 \rightarrow 1B_{16}$.
    - WRONG: $011011_2 \rightarrow 62_{16}$ (via $011011_2 \rightarrow 0110\ 11_2 \rightarrow 62_{16}$).
- Converting Octal → Binary
    - Replace each octal digit (0–7) by its 3-bit representation.
    - E.g., $106_8 \rightarrow 001\ 000\ 110_2$.
- Converting Binary → Octal
    - Pad bitstring on left with 0s to get a string of length divisible by 3.
    - Replace each 3-bit sequence by its equivalent octal digit.
    - E.g., $11010_2 \rightarrow 011\ \ 010_2 \rightarrow 32_8$. (Not $11010_2 \rightarrow 110\ 10_2 \rightarrow 62_8$.)

D. Converting To/From Decimal and Binary/Octal/Hexadecimal

- We know that converting from base b to decimal involves some multiplication by powers of b. Just in case you haven't seen this, it's possible to factor out the powers of b to get repeated multiplication and addition
  - E.g., $10110_2 = 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2 + 0$
    $= (1 \times 2^3 + 0 \times 2^2 + 1 \times 2 + 1) \times 2 + 0$
    $= ((1 \times 2^2 + 0 \times 2 + 1)\times 2 + 1) \times 2 + 0$
    $= (((1 \times 2 + 0)\times 2 + 1)\times 2 + 1) \times 2 + 0$
- More generally, given a sequence of digits and a base b, to convert to decimal, let r = 0, then for each digit (left-to-right), multiply r by b and add the digit to r. When you run out of digits, r contains the equivalent decimal value.
- Converting a number $n > 0$ from base 10 to base b involves division by b.
  - Start with result string S = the empty string.
  - **while** $n > 0$
    - Divide n by b to get quotient q and remainder r.
    - (If $b > 10$, convert r to the appropriate letter: A for $r = 10$, B for $r = 11$, …)
    - Append character r to the **left** end of result string S.
    - Set $n \leftarrow q$

E. Taking the (1 or 2's Complement) Negative of Bitstrings Represented in Octal/Hexadecimal

- What is the relationship between the octal or hex representations of a bitstring and its 1 or 2's complement negative?
  - **Example**: $3A_{16}$ represents $0011\ 1010_2$, whose 2's complement negative is $1100\ 0110_2$, which is represented by $C6_{16}$. (Its 1's complement negative is $1100\ 0101_2$, which is $C5_{16}$.)
  - **Example**: $FAB_{16} = 1111\ 1010\ 1011_2$, whose 2's complement negative $= 0000\ 0101\ 0101_2 = 055_{16}$.
- More generally, let N be an octal or hex number that represents a bitstring B, and let M be the octal or hex number that represents $-B$. How can we get

from N to M? The obvious way is to take N, convert to B, calculate $-B$, and then convert to M.

- For 1's complement, we can go from N to M directly, digit by digit.
  - Let d be a hex digit, then we get the 1's complement of d by flipping each bit of d; this corresponds to subtracting each bit from 1.
  - When we flip all the bits of d, we're effectively subtracting d from the (unsigned) bitstring of all 1's; the 1's complement of d is 15 − d, also known as the **15's complement** of d. The 15's complement of a hex numeral is the 15's complement of each hex digit. Instead of converting hex $N \mapsto$ binary $B \mapsto$ binary $-B \mapsto$ hex M, we can go directly from N to M via 15's complement.
  - **Examples**: the 15's complement of 3A is C5; the 15's complement of FAB is 054_{16}.
- The 2's complement of hexadecimal N is the 15's complement of N plus 1. We call this the 16's complement of N. So the 16's complement of 3A is C5+1 = C6, and C6 is the hex representation of the bitstring that is the 2's complement negative of the bitstring represented by 3A.

- The negation for octal numbers works similarly
  - The 7's complement of an octal digit d is 7 − d
  - The 7's complement of an octal numeral is the 7's complement of each digit.
  - The 8's complement of an octal numeral is the 7's complement of the numeral, plus 1.
- **Example**: The 7's complement of 7653_8 is 0124_8; its 8's complement is 0125_8. Going through binary, $7653_8 = 111\ 110\ 101\ 011_8$, and $-111\ 110\ 101\ 011_8 = 000\ 001\ 010\ 101_2 = 0125_8$.

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A. Why?

- Octal and hexadecimal numbers are useful for abbreviating long bitstrings.

B. Outcomes

After this activity, you should

- Be able to convert between binary, octal, hexadecimal, and decimal representations of numbers and to take the 1's and 2's complement of octal and hexadecimal numbers (when read as standing for bitstrings).

C. Questions

1. Let B be the bitstring 101101101100.
   a. What is the hex representation of B?
   b. What is the octal representation of B?
   c. What decimal value does B represent as an unsigned binary number?
   d. What decimal value does B represent as a 2's complement binary number?
2. Let B be the the bitstring 1101011101.
   a. What decimal value does B represent as an unsigned binary number?
   b. What decimal value does B represent as a 2's complement binary number?
   c. What is its hex representation if we pad with leading 0's to get to the next multiple of 4 bits?
   d. What is its octal representation if we pad with leading 0's to get to the next multiple of 3 bits?
3. Let bitstring B be the bitstring represented by hexadecimal FAB.
   a. What is B?
   b. What decimal value does B represent as an unsigned integer? (I.e., what is the decimal value of unsigned FAB_{16}?)

c. What is the 2's complement representation of $-B$?

d. Using 2's complement what decimal values do B and $-B$ represent?

e. What is the hex representation of your answer from part (c)?

f. What are the 15's and 16's complement of FAB_{16}? What is the relationship between the 16's complement and the answer from part (e)?

4. Let B be the bitstring represented by 735_8. (a) What is B? (b) Reading B as an unsigned integer, what decimal value does it represent? (c) Reading B as a 2's complement integer, what bitstring is its negative? (d) Continuing, what are B and $-B$ in decimal? (e) What is the octal representation of $-B$? (It should = the 8's complement of 735_8.)

5. The natural binary interpretation of 1601_8 is as a 12-bit string, but we can read 1601_8 as standing for 10-bit string by dropping the two leftmost bits of the 12-bit string. Let B = this 10-bit string. (a) What is B? (b) Using 2's complement, what is $-B$? (c) What are the decimal values of $-B$ and B? (d) What is the octal representation of $-B$? (e) What is the 8's complement of 1601_8? (It should differ from your answer for (d).)

6. Let C = the 12-bit extension of the B from the previous problem. (To get C, we copy the sign bit of B to the left, twice.) (a) What is C? (b) Using 2's complement, what is $-C$? (c) What are the octal representations of $-C$ and C? (d) Is the octal representation of $-C$ indeed the 8's complement of the octal representation of C?

Solution

1. (a) $1011\ 0110\ 1100_2$ = $B6C_{16}$; (b) $101\ 101\ 101\ 100_2 = 5554_8$; (c) $1011\ 0110\ 1100_2 = 2924_{10}$; (d) $1011\ 0110\ 1100_2$ = $-0100\ 1001\ 0100_2$ = -1172
2. (a) $11\ 0101\ 1101_2 = 861_{10}$; (b) $11\ 0101\ 1101_2 = -00\ 1010\ 0011_2 = -163_{10}$; (c) $0011\ 0101\ 1101_2 = 35D_{16}$; (d) $001\ 101\ 011\ 101_2 = 1535_8$
3. (a) B = $1111\ 1010\ 1011_2$; (b) $15 \times 16^2 + 10 \times 16 + 11 = 4011$; (c) $-1111\ 1010\ 1011_2 = 000\ 0101\ 0101_2$; (d) $-B = 85_{10}$ so $B = -85_{10}$. (e) $000\ 0101\ 0101_2 = 055_8$. (f) 054_{16} and 055_{16}. They are equal.
4. (a) $B = 735_8 = 111\ 011\ 101_2$ (b) unsigned, $B = 477_{10}$. (c) $-111\ 011\ 101_2 = 000\ 100\ 011_2$, so (d) $B = -35$ and $-B = 35$; (e) 043_8
5. (a) $B = 1\ 110\ 000\ 001_2$; (b) $-B = 0\ 001\ 111\ 111$; (c) $-B$ represents 127 and B represents -127; (d) The octal for 0 001 111 111 is 0177_8; (e) the 8's complement of 1601_8 is 6177_8.
6. (a) $C = 111\ 110\ 000\ 001_2$; (b) $-C = 000\ 001\ 111\ 111$; (c) $-C = 0177_8$ and $C = 7601_8$. (d) Yes, 0177_8 is the 8's complement of 7601_8.

