Karnaugh Maps, Part II

CS 350: Computer Organization & Assembler Language Programming

A. Why?

- Simplification of logical expressions can improve their readability and implementability.
- Karnaugh maps are one way to simplify logical expressions.

B. Outcomes

After this lecture, you should

- Know how to simplify boolean expressions using Karnaugh maps.

C. 4-Variable Karnaugh Maps

- For a 4-variable map, say for V, X, Y, and Z, we can use four 2-variable maps. Each row is a 2-variable map for Y and Z, given some particular values for V and X. Similarly, each column is a map for $V\,X$ given some particular values for Y and Z.
- The row headers for $V\,X$ are in 00, 01, 11, 10 order and the column headers for $Y\,Z$ are in 00, 01, 11, 10 order, so we can look for pairs going up-and-down or left-and-right; here's an example:

		YZ			
		00	01	11	10
	00	1	0	1	1
VX	01	1	0	0	0
	11	0	1	1	1
	10	0	1	0	0

$$\overline{V}\;\overline{Y}\;\overline{Z} + V\,\overline{Y}\,Z + \overline{V}\;\overline{X}\;Y + V\,X\,Y$$

- The pairs can also wrap around the edges from top-to-bottom or left-to-right; some examples:

VX \ ***YZ***	00	01	11	10
00	1	1	0	1
01	0	0	0	0
11	1	0	0	1
10	0	1	0	1

$$\overline{X}\,\overline{Y}\,Z + \overline{X}\,Y\,\overline{Z} + \overline{V}\,\overline{X}\,\overline{Z} + V\,X\,\overline{Z}$$

- In general, we can look for single elements, pairs, 2×2 squares, 4×1 and 1×4 rectangles, and 2×4 and 4×2 rectangles. (The 4×4 square is for the constant T.)
- The 2×2 squares can be embedded within the map or can wrap around the edges; some examples:

VX \ ***YZ***	00	01	11	10
00	0	0	1	1
01	0	1	1	1
11	1	1	1	0
10	1	1	0	0

$$V\,\overline{Y} + X\,Z + \overline{V}\,Y$$

VX \ ***YZ***	00	01	11	10
00	1	0	1	1
01	1	0	0	1
11	0	0	0	0
10	0	0	1	1

$$\overline{X}\,Y + \overline{V}\,\overline{Z}$$

		YZ			
		00	01	11	10
	00	1	1	1	0
VX	01	1	1	0	0
	11	0	0	1	1
	10	0	1	1	1

$\overline{V}\ \overline{Y} + V\,Y + \overline{X}\,Z$

- There is one extremely weird square, the four-corner square $\overline{X}\ \overline{Z}$, which is wrapped around top-to-bottom and left-to-right:

		YZ			
		00	01	11	10
	00	1	0	0	1
VX	01	0	0	0	0
	11	0	0	0	0
	10	1	0	0	1

The 4-Corner Square $\overline{X}\ \overline{Z}$

- This version may help you see the four-corner wraparound square more easily; it joins 4 copies of the same map to make their corners adjacent.

VX \ ***YZ***	00	01	11	10	00	01	11	10
00	1	0	0	1	1	0	0	1
01	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0
10	1	0	0	1	1	0	0	1
00	1	0	0	1	1	0	0	1
01	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0
10	1	0	0	1	1	0	0	1

The 4-Corner Square $\overline{X}\,\overline{Z}$

- The examples below show a 4×1 column, a 4×2 rectangle, and two wrap around 4×2 rectangles.

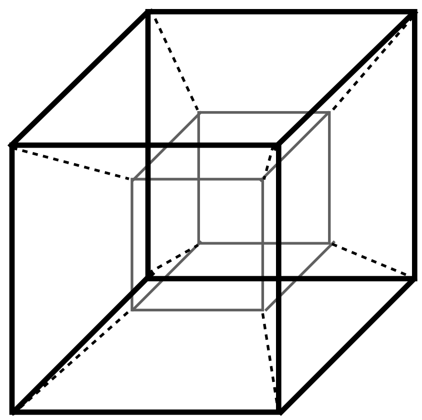
VX \ ***YZ***	00	01	11	10
00	0	0	0	1
01	1	1	1	1
11	1	1	1	1
10	0	0	0	1

$X + Y\,\overline{Z}$

VX \ ***YZ***	00	01	11	10
00	1	1	1	1
01	1	0	0	1
11	1	0	0	1
10	1	1	1	1

$\overline{X} + \overline{Z}$

- A 4-variable map can also be drawn as $2 \times 2 \times 2 \times 2$ hypercube, the 4-dimensional analogy of a 3-dimensional cube, typically drawn as a tesseract[1]. Below, the other cube describes V, X, and Y given $Z = 0$; the inner cube describes V, X, and Y given $Z = 1$. The dotted lines connect the $Z = 0$ and $Z = 1$ points for a given V, X, Y combination (0000 and 0001, 0010 and 0011, etc).



Tesseract Joining Two 3-Variable Karnaugh Map Cubes

D. 5-Variable Karnaugh Maps

- We aren't doing these (scared you!), but just to describe them, you draw two 4-variable maps, say above each other. Each coordinate of the top map is logically above the same coordinate of the bottom map, so they form a pair. (So the the 0000s in the two maps form a pair, the 0001s form a pair, and so on.) In addition to the flat implicants, there are solids like the $2 \times 2 \times 4$ parallelepiped. The weirdest implicant is the 8-corner wraparound cube.

[1](A Wikipedia link to hypercubes.)

Karnaugh Maps II

CS 350: Computer Organization & Assembler Language Programming

A. Why?

- Karnaugh maps can help us simplify boolean expressions.

B. Outcomes

After this activity, you should be able to

- Use Karnaugh maps to simplify boolean expressions with 3 or 4 variables.

C. Questions

For Questions 1 – 4, use a Karnaugh map to simplify the given expression. Feel free to algebraically simplify the expression into something more readable before creating the truth table or Karnaugh map. Also, you can omit the truth table for the expression if you feel don't need it.

1. $X + X\,Y\,Z + X\,\overline{Z} + Y\,\overline{Z}$
2. $V + X\,Y + \overline{X}\;\overline{Y}\;\overline{Z} + V\,X\,Y\,Z$
3. $(X + Y)(X + Z) + \neg(\overline{X}\;\overline{Z})$
4. $\neg(X\,Z) + V\,(X\,Y\,Z + \overline{X}\;\overline{Z})$

Solution

1. $X + XYZ + X\,\overline{Z} + Y\,\overline{Z}$ and its simplification, $X + Y\,\overline{Z}$, are shown below. ($XYZ + X\,\overline{Z}$ are both redundant because they are contained within X.)

YZ

		00	01	11	10
X	0	0	0	0	1
	1	1	1	1	1

$X + XYZ + X\,\overline{Z} + Y\,\overline{Z}$

YZ

		00	01	11	10
X	0	0	0	0	1
	1	1	1	1	1

$X + + Y\,\overline{Z}$

2. $V + X\,Y + \overline{X}\;\overline{Y}\;\overline{Z} + V\,X\,Y\,Z$ and its simplification, $V + X\,Y + \overline{X}\;\overline{Y}\;\overline{Z}$, are shown below.

YZ

		00	01	11	10
	00	1	0	0	0
VX	01	0	0	1	1
	11	1	1	1	1
	10	1	1	1	1

$V + X\,Y + \overline{X}\;\overline{Y}\;\overline{Z} + V\,X\,Y\,Z$

YZ

		00	01	11	10
	00	1	0	0	0
VX	01	0	0	1	1
	11	1	1	1	1
	10	1	1	1	1

$V + X\,Y + \overline{X}\;\overline{Y}\;\overline{Z}$

3. $(X + Y)(X + Z) + \neg(\overline{X}\,\overline{Z}) = XX + XZ + XY + YZ + (X + Z) = X + Z$

X \ YZ	00	01	11	10
0	0	1	1	0
1	1	1	1	1

$X + Z$

4. $\neg(X\,Z) + V\,(X\,Y\,Z + \overline{X}\,\overline{Z}) = \overline{X} + \overline{Z} + VXYZ + V\,\overline{X}\,\overline{Z}$, which simplifies to $\overline{X} + \overline{Z} + VY$

VX \ YZ	00	01	11	10
00	1	1	1	1
01	1			1
11	1		1	1
10	1	1	1	1

$\overline{X} + \overline{Z} + VXYZ + V\,\overline{X}\,\overline{Z}$

VX \ YZ	00	01	11	10
00	1	0	0	0
01	0	0	1	1
11	1	1	1	1
10	1	1	1	1

$\overline{X} + \overline{Z} + VY$