Vectors (Mathematics)

Raman Ghimire
(Class of 2018, Pulchowk Campus,
Founder kaaphal.com)

December 22, 2020

Direction Cosines and Vector

If a Vector $\vec{a}$ makes an angle of α, β *and* γ with x, y and z coordinate axis then:

- $\cos\alpha = \dfrac{\vec{a}.\hat{i}}{|\vec{a}|}$
- $\cos\beta = \dfrac{\vec{a}.\hat{j}}{|\vec{a}|}$
- $\cos\gamma = \dfrac{\vec{a}.\hat{k}}{|\vec{a}|}$

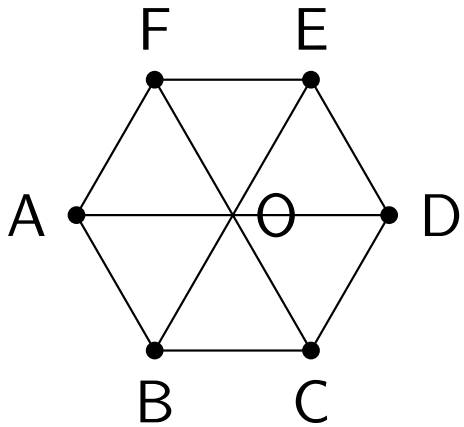
Question -1

If a Vector $\vec{a}$ makes an angle of α, β *and* γ with x, y and z coordinate axis then what is the value of $\sin^2\theta + \sin^2\beta + \sin^2\gamma =$?

a) 1
b) 0
c) 2
d) 3

Question -2

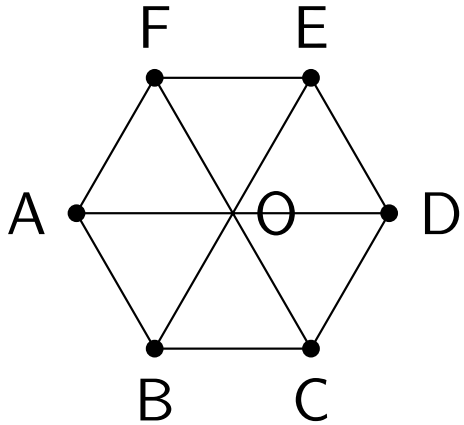
If ABCDEF is a regular hexagon what is the value of $\vec{AB} + \vec{AC} + \vec{AD} + \vec{AE} + \vec{AF}$



a) $\vec{AO}$
b) $2\vec{AO}$
c) $4\vec{AO}$
d) $6\vec{AO}$

Question -2

If ABCDEF is a regular hexagon what is the value of
$\vec{AB} + \vec{AC} + \vec{AD} + \vec{AE} + \vec{AF}$



$\vec{OB} - \vec{OA} + \vec{OC} - \vec{OA} + \vec{OD} - \vec{OA} + \vec{OE} - \vec{OA} + \vec{OF} - \vec{OA}$
$\vec{OA} + \vec{OB} + \vec{OB} + \vec{OB} + \vec{OB} + \vec{OB} - 5\vec{OA} - \vec{OA} - 6\vec{OA}$ a) $\vec{AO}$

b) $2\vec{AO}$
c) $4\vec{AO}$
d) $6\vec{AO}$

Question -3

The vectors from origin to the points A and B are $A = 3\hat{i} - 6\hat{j} + 2\hat{k}$ and $B = 2\hat{i} + 1\hat{j} - 2\hat{k}$respectively. The area of the triangle OAB be

a) $\frac{5}{2}\sqrt{12}$
b) $\frac{2}{3}\sqrt{12}$
c) $\frac{3}{5}\sqrt{12}$
d) $\frac{5}{3}\sqrt{12}$

Question -3

The vectors from origin to the points A and B are $a = 3\hat{i} - 6\hat{j} + 2\hat{k}$ and $b = 2\hat{i} + 1\hat{j} - 2\hat{k}$respectively. The area of the triangle OAB be

Area of triangle $= 1/2\vec{a} \times \vec{b}$

a) $\frac{5}{2}\sqrt{12}$

b) $\frac{2}{3}\sqrt{12}$

c) $\frac{3}{5}\sqrt{12}$

d) $\frac{5}{3}\sqrt{12}$

Question -4

If $\vec{r}.\hat{i} = \vec{r}.\hat{j} = \vec{r}.\hat{k}$ and $|\vec{r}| = 3$ then, $\vec{r} =$?

a) $\pm 3(\hat{i} + \hat{j} + \hat{k})$
b) $\pm \frac{1}{3}(\hat{i} + \hat{j} + \hat{k})$
c) $\pm \frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$
d) $\pm \sqrt{3}(\hat{i} + \hat{j} + \hat{k})$

Question -4

If $\vec{r}.\hat{i} = \vec{r}.\hat{j} = \vec{r}.\hat{k}$ and $|\vec{r}| = 3$ then, $\vec{r} =$?

Here, $|\vec{r}|^2 = r_x^2 + r_y^2 + r_z^2$
$r_x^2 = \frac{9}{3}$
$r_x = \pm\sqrt{3}$

a) $\pm 3(\hat{i} + \hat{j} + \hat{k})$
b)$\pm\frac{1}{3}(\hat{i} + \hat{j} + \hat{k})$
c) $\pm\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$
d)$\pm\sqrt{3}(\hat{i} + \hat{j} + \hat{k})$

Question -5

If $\overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c}$ are non-zero vectors such that $\overrightarrow{a}.\overrightarrow{b} = \overrightarrow{a}.\overrightarrow{c}$, then which statement is true

a) $\vec{b} = \vec{c}$
b)$\vec{a} \perp (\vec{b} - \vec{c})$
c) both a and b
d) none

Question -5

If $\overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c}$ are non-zero vectors such that $\overrightarrow{a}.\overrightarrow{b} = \overrightarrow{a}.\overrightarrow{c}$, then which statement is true

$\vec{a}.\vec{b} - \vec{a}.\vec{c} = 0$
$\vec{a}(\vec{b} - \vec{c}) = 0$

a) $\vec{b} = \vec{c}$
b)$\vec{a} \perp (\vec{b} - \vec{c})$
c)both a and b
d) none

Question -6

If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors of equal magnitudes, then the angle between the vectors $\vec{a}$ and $\vec{a} + \vec{b} + \vec{c}$ is

a) $\pi/3$
b)$\pi/6$
c) $\cos^{-1} \frac{1}{\sqrt{3}}$
d) $\pi/2$

Question -6

If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors of equal magnitudes, then the angle between the vectors $\vec{a}$ and $\vec{a} + \vec{b} + \vec{c}$ is

Best way is to substitute a well known three mutually perpendicular vector :
$\hat{i}, \hat{j}, \hat{k}$
we know,
angle made by $\hat{i} + \hat{j} + \hat{k}$ with $\hat{i}$
$\cos\theta = \frac{\hat{i}.(\hat{i}+\hat{j}+\hat{k})}{\sqrt{3}}$

a) $\pi/3$
b)$\pi/6$
c) $\cos^{-1}\frac{1}{\sqrt{3}}$
d) $\pi/2$

Question -7 DIY

If $|\vec{a}+\vec{b}| = |\vec{c}|$ and $\vec{a}+\vec{b} = \vec{c}$ then the angle between $\vec{a}$ and $\vec{b}$ is:

Question -8

If $\vec{a}, \vec{b}, \vec{c}$ are three vector such that $\vec{a} = \vec{b} + \vec{c}$ and the angle between $\vec{b}$ and $\vec{c}$ is $\pi/2$ then

a) $a^2 = b^2 + c^2$
b) $b^2 = a^2 + c^2$
c) $c^2 = b^2 + a^2$
d) $2a^2 = b^2 + c^2$

Question -8

If $\vec{a}, \vec{b}, \vec{c}$ are three vector such that $\vec{a} = \vec{b} + \vec{c}$ and the angle between $\vec{b}$ and $\vec{c}$ is $\pi/2$ then

Instead of solving this problem using vector algebra lets visualize the problem.
Since $\vec{a} = \vec{b} + \vec{c}$ it forms a triangle
also, angle between $\vec{b}$ and $\vec{c}$ is $\pi/2$, it is a right angle one is hypotenuse = a

a) $a^2 = b^2 + c^2$
b) $b^2 = a^2 + c^2$
c) $c^2 = b^2 + a^2$
d) $2a^2 = b^2 + c^2$

Question -8

If $\vec{a}, \vec{b}, \vec{c}$ are three vector such that $\vec{a} = \vec{b} + \vec{c}$ and the angle between $\vec{b}$ and $\vec{c}$ is $\pi/2$ then

$\vec{a}.\vec{a} = \vec{a}.\vec{b} + \vec{a}.\vec{c}$
$\vec{b}.\vec{a} = \vec{b}.\vec{b} + \vec{b}.\vec{c}$
$\vec{c}.\vec{a} = \vec{c}.\vec{b} + \vec{c}.\vec{c}$
Add above euations to get the answer.

a) $a^2 = b^2 + c^2$
b)$b^2 = a^2 + c^2$
c)$c^2 = b^2 + a^2$
d) $2a^2 = b^2 + c^2$

Question - 9

If $\vec{r}$ be position vector of any point on a sphere and $\vec{a}$, $\vec{b}$ are respectively position vectors of the extremities of a diameter, then

a) $\vec{r}.(\vec{a}-\vec{b})=0$
b)$\vec{r}.(\vec{a}+\vec{b})=0$
c)$(\vec{r}+\vec{a}).(\vec{r}+\vec{b})=0$
d) $(\vec{r}-\vec{a}).(\vec{r}-\vec{b})=0$

Question - 9

If $\vec{r}$ be position vector of any point on a sphere and $\vec{a}$, $\vec{b}$ are respectively position vectors of the extremities of a diameter, then

Any diameter subtends and angle of 90 in the circle

a) $\vec{r}.(\vec{a}-\vec{b})=0$
b)$\vec{r}.(\vec{a}+\vec{b})=0$
c)$(\vec{r}+\vec{a}).(\vec{r}+\vec{b})=0$
d) $(\vec{r}-\vec{a}).(\vec{r}-\vec{b})=0$

Question - 10

If $\vec{a}$ and $\vec{b}$ are unit vectors such that $\vec{a} \times \vec{b}$ is also a unit vector, then the angle between a and b is

a) 0
b)$\pi/2$
c)$\pi/3$
d)π

Question - 10

If $\vec{a}$ and $\vec{b}$ are unit vectors such that $\vec{a} \times \vec{b}$ is also a unit vector, then the angle between a and b is

$|\vec{a} \times \vec{b}| = 1$
$|\vec{a}||\vec{b}| \sin\theta = 1$
$\sin\theta = 1$
$\theta = \pi/2$

a) 0
b)$\pi/2$
c)$\pi/3$
d)π

Question - 11

$(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b})$

a) $2(\vec{a} \times \vec{b})$
b) $(\vec{a} \times \vec{b})$
c) $2(\vec{b} \times \vec{a})$
d)None

Question - 11

If $(\vec{a} + \vec{b} + \vec{c}) = 0$ then which is correct ?

a) $\vec{a} = \vec{b} = \vec{c}$
b) $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a}$
c) $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$
d)None

Question - 11

If $(\vec{a} + \vec{b} + \vec{c}) = 0$ then which is correct ?

$\vec{a} \times \vec{b}$
$\vec{a} \times (-\vec{a} - \vec{c})$
$-\vec{a} \times \vec{c}$
$\vec{c} \times \vec{a}$
and so on.

a) $\vec{a} = \vec{b} = \vec{c}$
b) $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a}$
c) $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$
d)None

Question - 12

If $(\vec{a} \times \vec{b} = \vec{b} \times \vec{c} \neq 0)$ and k is a scalar then which is correct ?

a) $\vec{a} + \vec{c} = k\vec{b}$
b) $\vec{b} + \vec{c} = k\vec{a}$
c) $\vec{b} + \vec{a} = k\vec{c}$
d)None

Question - 12

If $(\vec{a} \times \vec{b} = \vec{b} \times \vec{c} \neq 0)$ and k is a scalar then which is correct ?

$\vec{a} \times \vec{b} - \vec{b} \times \vec{c} = 0$
$\vec{a} \times \vec{b} + \vec{c} \times \vec{b} = 0$
$(\vec{a} + \vec{c}) \times \vec{b} = 0$
so $\vec{a} + \vec{c}$ is parallel to $\vec{b}$

a)$\vec{a} + \vec{c} = k\vec{b}$
b) $\vec{b} + \vec{c} = k\vec{a}$
c) $\vec{b} + \vec{a} = k\vec{c}$
d)None

Question - 13

If the value of dot products of two vector as well as the value of cross product is zero than?

a) two vectors are parallel
b) two vectors are perpendicular
c) Either one of them must be a zero vector
d)None

Question - 14

If $\vec{a} + \vec{b}$ bisects the angle between $\vec{a}$ and $\vec{b}$ then?

a) two vectors are perpendicular
b) two vectors are unlike vectors
c) both of them have same magnitude
d) angle between the two vector is 120 degree

Question - 14

The magnitudes of mutually perpendicular forces a, b and c are 2, 10 and 11 respectively. Then the magnitude of its resultant is

a) 12
b) 15
c) 9
d) 10

Question - 15

The point having position vectors $2i+3j+4k$,$3i+4j+2k$, $4i+2j+3k$ are the vertices of

a) right angled triangle
b) isosceles triangle
c) equilateral triangle
d) co linear

Question - 15

The point having position vectors 2i+3j+4k,3i+4j+2k, 4i+2j+3k are the vertices of

$\vec{AB} = \vec{OB} - \vec{OA} = i + j - 2k$
$\vec{BC} = \vec{OC} - \vec{OB} = i - 2j + k$
$\vec{CA} = \vec{OA} - \vec{OC} = -2i + j + k$
They all have same magnitude

a) right angled triangle
b) isosceles triangle
c) equilateral triangle
d) co linear