Recidivism and Adapative expectation

The idea that the punishment imposed on the offender depends on his previous conviction is widely discussed in the economic literature. Previous work has argued for the importance of punishing repeat offenders less severely than first-time or novice offenders ([Polinsky & Rubinfeld, 1991 (https://doi.org/10.1016/0047-2727(91)90009-q)](https://doi.org/10.1016/0047-2727(91)90009-q); [Chu et al., 2000 (https://doi.org/10.1016/s0144-8188(00)00024-7)](https://doi.org/10.1016/s0144-8188(00)00024-7); [Shavell, 2004 (https://doi.org/10.4159/9780674043497)](https://doi.org/10.4159/9780674043497); [Miceli & Bucci, 2005 (https://doi.org/10.2202/1555-5879.1011)](https://doi.org/10.2202/1555-5879.1011); [Müller & Schmitz, 2015 (https://doi.org/10.2202/1555-5879.1011)](https://doi.org/10.2202/1555-5879.1011)). [Shavell, 2004 (https://doi.org/10.4159/9780674043497)](https://doi.org/10.4159/9780674043497) argued that making sanctions dependent on the history of offences can be advantageous even when there are no characteristics to be known. In the same line of reasoning, [Shavell, (p.529) 2004 (https://doi.org/10.4159/9780674043497)](https://doi.org/10.4159/9780674043497) pointed out that when "*the detection of a violation implies not only an immediate sanction, but also a higher sanction for a future violation, an individual will be more deterred from committing a violation now*".

The model we present in this paper is largely based on [Nerlove's (1958) (https://doi.org/10.2307/1880597)](https://doi.org/10.2307/1880597) theory of adaptive anticipation. According to this author, an agent, in a context of imperfect information, carries out adaptive anticipation when he adapts his forecasts by taking into account the information he has on the variables observed in the past and the anticipation errors made on these past values.

The theory of anticipation proposed for use in criminal deterrence assumes that the decision to commit a crime depends on the recidivist's belief in the value of the expected punishment. Indeed, the formulation of anticipation obeys the following mechanism:

- the environment studied is characterised by the value taken by certain variables, known as state variables;
- the decision making process implies, for the recidivist, to know some past, present and/or future values taken by the state variables. When the agent is unaware of one of these values (this is obviously the case for future values), he forms beliefs about it. The decision of each recidivist therefore depends on his beliefs about the state variables.

1. Dynamics of illegal gain

1.1.Solving the adaptive expectation model

To plot the graph of the adaptive expectation model given by $\tilde{\omega}_{k+1}\tilde{\omega}_k = \beta(\omega_k - \tilde{\omega}_k)$, we need to find the explicit expression of this model. To do this, we can write the previous equation in the form :

$$\tilde{\omega}_{k+1} = \tilde{\omega}_k + \beta(\omega_k - \tilde{\omega}_k)$$

This gives a recurrence equation for $tilde\omega_k$, which can be solved explicitly. Indeed, if we pose $$Delta \omega_{k}=\omega_{k}-\tilde{\omega}_{k}$, then the recurrence equation becomes :

$$\tilde{\omega}_{k+1} = \tilde{\omega}_k + \beta\Delta\omega_k$$

This equation can be solved by replacing $tilde\omega_k$ by its previous value, then repeating this process until we obtain an explicit expression for $tilde\omega_k$ as a function of ω_0, β and k. We obtain:

$$\tilde{\omega}_k = \beta^k\tilde{\omega}_0 + \sum_{i=0}^{k-1}\beta^i\Delta\omega_{k-i-1}$$

where $\tilde{\omega}_0$ is the initial value of $\tilde{\omega}$.

The adaptive expectation model of the payoff from illegal activity is used to explain the behaviour of agents who engage in illegal activities such as smuggling, tax evasion, etc. In this model, agents try to make a profit from the illegal activity by using a variety of means. In this model, agents try to maximise their expected gain, but face the risk of arrest and loss of income. The agents therefore use an adaptive expectation strategy, which consists of adjusting their behaviour according to their past experience.

In this model, $\tilde{\omega}_k$ is the expected payoff at step k, $\tilde{\omega}_0$ is the initial expected payoff, $\Delta\omega_k$ is the difference between the actual payoff and the expected payoff at step k, and β is a weighting factor that determines the relative importance of past and recent experiences in the calculation of the expected payoff.

We set the values of β, $\Delta\omega$ and $\tilde{\omega}_0$, and calculated the expected payoff at each step k using the equation above. Then we plotted the model as a function of k. This model suggests that agents who engage in illegal activities are able to adapt and adjust their behaviour according to their past experience. They use an adaptive expectation strategy to maximise their expected gain, while facing the risk of arrest and loss of income. The model quantifies the relative importance of past and recent experience in calculating the expected payoff, as well as the effect of behavioural adjustment on the expected payoff. The resulting graph allows the evolution of the expected gain to be visualised as a function of time, which can be used to better understand the behaviour of agents and the dynamics of illegal activity.

Entrée [1]:

```python
from sympy import *
from matplotlib import pyplot as plt
import numpy as np
import matplotlib
matplotlib.rcParams['text.usetex'] = True
matplotlib.rcParams['text.latex.preamble'] = [r'\usepackage{amsmath}']
init_printing(use_unicode=True)
```

Entrée [2]:

```python
def cobweb_w(beta, omega0, omega_star, num_iterations):
    x = np.linspace(0, 1, 1000)
    fig, ax = plt.subplots(figsize=(8, 6))

    ax.plot(x, beta*(x - omega_star) + omega_star, 'b-', lw=2, alpha=0.6, label='$Anticipation$')
    ax.plot(x, x, 'k-', lw=2, alpha=0.6)

    omega_k = omega0
    for i in range(num_iterations):
        ax.plot([omega_k, omega_k], [omega_k, beta*(omega_k - omega_star) + omega_star], 'r-', lw=1, alpha=0.7)
        ax.plot([omega_k, beta*(omega_k - omega_star) + omega_star], [beta*(omega_k - omega_star) + omega_star, beta*(omega_k - omega
        omega_k = beta*(omega_k - omega_star) + omega_star

    ax.legend(loc='upper left', frameon=False)
    ax.set_xlim([0, 1])
    ax.set_ylim([0, 1])
    ax.set_xlabel(r'$\tilde{\omega}_{k}$')
    ax.set_ylabel(r'$\tilde{\omega}_{k+1}$')
    ax.set_title('Expectation adaptive of gain')
    plt.show()
```

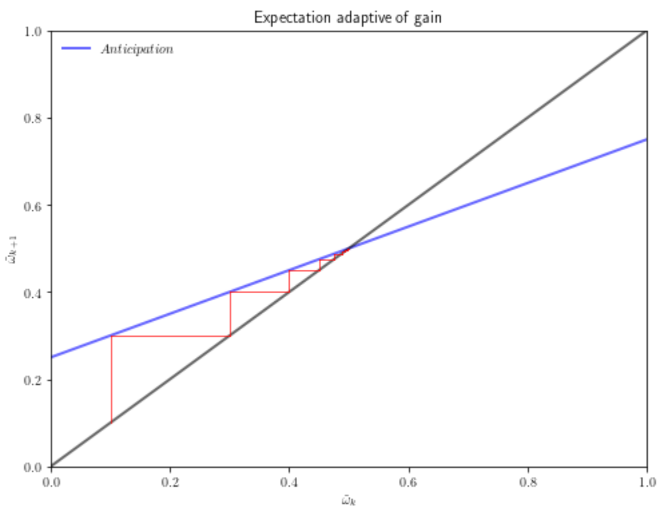
The above code implements a graphical visualisation of the adaptive anticipation dynamics of illegal gain, using the cobweb curve. More precisely, the code uses the cobweb curve to represent the dynamics of the adaptation of the illegal gain, starting from an initial value of illegal activity ω_0 and iterating on the adaptive anticipation formula given by $\tilde{\omega}_{k+1} = \beta(\omega_k - \tilde{\omega}_k) + \tilde{\omega}_k$, where β is the adaptation coefficient, $\tilde{\omega}_k$ is the target value of illegal activity and ω_k is the value of illegal activity at iteration k.

The cobweb curve is used to represent the dynamics of this iteration. The straight line $y = x$ represents the equation $tilde\omega_{k+1} = \tilde{\omega}_k$, which indicates that if the expected value of the gain of illegal activity remains constant, the next value will also be equal to the current value. The blue line represents the adaptive anticipation of the illegal gain, which indicates how the value of the gain from the illegal activity must be adjusted to reach the target or expected value $\tilde{\omega}_k$, given the adaptation coefficient β. The red curve represents the trajectory of the illegal activity value at each iteration.

The parameter '***omega_star***' represents the target value of illegal activity, '***omega_0***' represents the initial value of illegal activity, beta represents the adaptation coefficient and '***num_iterations***' represents the number of iterations to be performed. The function '***cobweb***' plots the cobweb curve for the adaptive anticipation dynamics of illegal gain and displays it using the Matplotlib library. The function takes these parameters as input and displays the cobweb curve.

Entrée [3]:

```python
cobweb_w(0.5, 0.1, 0.5, 10)
```

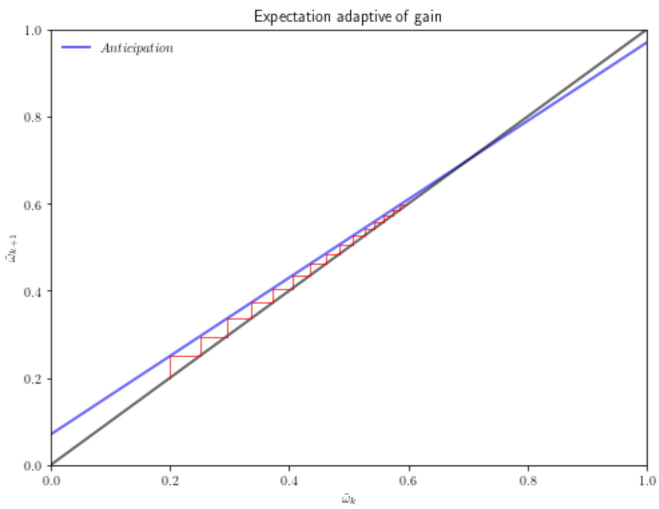


The function **cobweb_w(0.5, 0.1, 0.5, 10)** allows to plot the cobweb curve for an adaptive anticipation model of the gain of the illegal activity with the following parameters:

β = 0.5: coefficient of reduction of the gap between the current observation and the forecast. ω_0 = 0.1: initial observation. $tilde\omega_k$ = 0.5 : initial forecast. num_iterations = 10 : number of iterations for the plot of the cobweb curve. The result is a graph representing the cobweb curve for this model, which makes it possible to visualise the adjustment dynamics of the current observation and the forecast according to the deviation reduction coefficient. This curve can be interpreted as a graphical representation of the convergence of the current observation to the forecast, as a function of the beta value.

Entrée [4]:

```
1 cobweb_w(0.9, 0.2, 0.7, 15)
```



1.2 Interpretation of β expectation coefficient

When β is close to 1, it means that agents attach great importance to their past experience to adjust their behaviour. In other words, they adapt quickly to changes in their environment and are very sensitive to variations in their expected payoff.

This is reflected in the expectation curve, which tends to align with the line y = x. This line represents the situation where the expected value of the next step is equal to the current value, which implies that the agent does not anticipate a significant change in his payoff.

Thus, when β is close to 1, it may indicate that agents have a good knowledge of their environment and tend to adjust their behaviour in an optimal way. It may also imply that the risk of arrest and loss of income is relatively low, as agents tend to adopt a safe and stable strategy.

On the other hand, when β is low or close to zero, it means that agents are less likely to adapt quickly to changes in their environment. They are less sensitive to variations in their expected payoff and may adopt less optimal behaviour.

This is reflected in the expectation curve, which may show significant oscillations around the line y = x. These oscillations may indicate that agents have difficulty adjusting their behaviour in a consistent manner, which may lead to losses in earnings or increased risks of arrest.

In conclusion, the value of β in the adaptive anticipation model of illegal gain can provide valuable information about the behaviour of agents and their adaptation to their environment. A high value of β may indicate fast and efficient adaptation, while a low value may imply adaptation difficulties and increased risks.

2. Dynamics of punishment

The dynamics of the punishment through the cobweb curve can be analysed using the adaptive expectation model of illegal gain presented earlier. The difference is that this time we have a recurrence equation for the expected penalty rather than for the expected illegal gain.

The adaptive penalty expectation model indicates that the penalty expected at time $k+1$, $\tilde{s}_{k+1}$, is obtained by adding to the penalty expected at time k, $\tilde{s}_k$, a correction which is proportional to the difference between the actual penalty at time k, s_k, and the expected penalty at time k, $\tilde{s}_k$, with an expectation coefficient α.

The cobweb curve is a graphical tool that illustrates the dynamics of the model by plotting the adaptive expectation curve of the penalty and the 45 degree line on the same graph. The initial position on the horizontal axis is given by the value of the expected sanction at the initial time. The initial position on the vertical axis is also given by the value of the expected sanction at the initial time.

For each iteration, the vertical position is given by the expected sanction at time $k+1$, $\tilde{s}_{k+1}$, obtained by applying the recurrence equation for the expected sanction. The horizontal position is given by the actual penalty at time k, s_k.

By examining the cobweb curve for different values of α, one can observe the effects of the adaptive expectation of the sanction on the dynamics of the sanction. For example, if $\alpha > 0$, the adaptive sanction expectation curve will tend to align with the 45-degree line more quickly than for $\alpha < 0$ or $\alpha = 0$, which means that the recidivist anticipates an increased severity of the sanction and complies with it more quickly.

Entrée [5]:

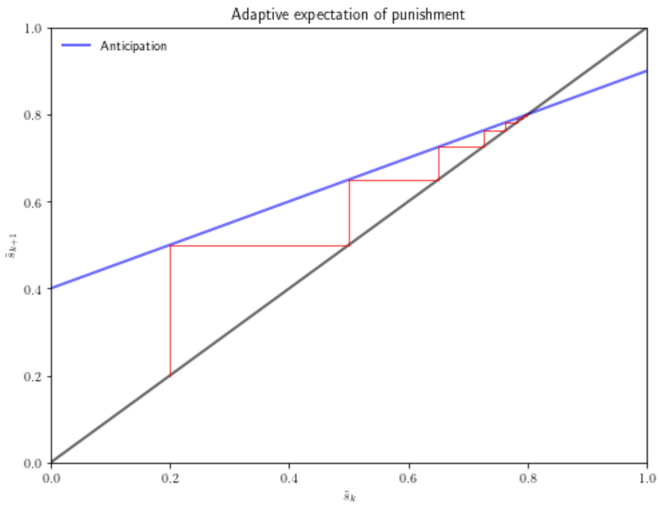
```python
def cobweb_s(alpha, s0, s_star, num_iterations):
    x = np.linspace(0, 1, 1000)
    fig, ax = plt.subplots(figsize=(8, 6))

    ax.plot(x, alpha*(x - s_star) + s_star, 'b-', lw=2, alpha=0.6, label='Anticipation')
    ax.plot(x, x, 'k-', lw=2, alpha=0.6)

    s_k = s0
    for i in range(num_iterations):
        ax.plot([s_k, s_k], [s_k, alpha*(s_k - s_star) + s_star], 'r-', lw=1, alpha=0.7)
        ax.plot([s_k, alpha*(s_k - s_star) + s_star], [alpha*(s_k - s_star) + s_star, alpha*(s_k - s_star) + s_star], 'r-', lw=1, alp
        s_k = alpha*(s_k - s_star) + s_star

    ax.legend(loc='upper left', frameon=False)
    ax.set_xlim([0, 1])
    ax.set_ylim([0, 1])
    ax.set_xlabel(r'$\tilde{s}_{k}$')
    ax.set_ylabel(r'$\tilde{s}_{k+1}$')
    ax.set_title('Adaptive expectation of punishment')
    plt.show()
```

To use the cobweb function, the function is called with the following parameters:

alpha : the expectation coefficient s0 : the initial value of the expected sanction s_star : the actual value of the sanction num_iterations : the number of iterations for the simulation of the cobweb curve For example, to simulate the cobweb curve with an expectation coefficient alpha = 0.5, an initial value of the expected sanction s0 = 0.2, an actual value of the sanction s_star = 0.8 and a number of iterations num_iterations = 10, the function must be called as follows: cobweb_s(0.5, 0.2, 0.8, 10)

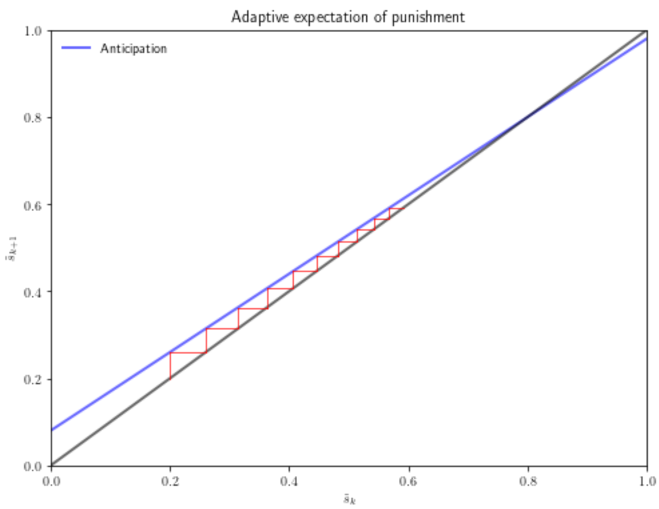
Entrée [6]:

```python
cobweb_s(0.5, 0.2, 0.8, 10)
```



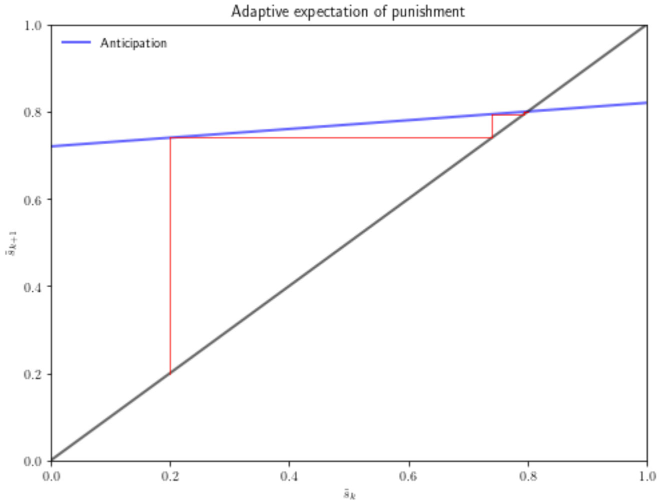
Entrée [7]:

```python
cobweb_s(0.9, 0.2, 0.8, 10)
```



Entrée [8]:

```python
cobweb_s(0.1, 0.2, 0.8, 10)
```



3. Combined model

Entrée [9]:

```python
p = 0.5  # probabilité d'arrestation
beta = 0.6  # coefficient d'anticipation du gain
alpha = 0.4  # coefficient d'anticipation de la sévérité de la sanction
n = 100  # nombre total de périodes

# Conditions initiales
w_tilde_0 = 0.8  # utilité attendue initiale
s_tilde_0 = 0.8  # anticipation de la sévérité de la sanction initiale

# Fonction d'utilité
def u(w_tilde, s_tilde):
    return (1-p)**(t+1)*w_tilde + np.sum([p*(1-p)**k*(w_tilde+s_tilde) for k in range(t+1,n)])

# Itération
w_tilde = np.zeros(n)
s_tilde = np.zeros(n)
for t in range(n):
    w_tilde[t] = w_tilde_0 + beta**t*np.sum([(1-beta)**k*u(w_tilde_0, s_tilde_0) for k in range(t)])
    s_tilde[t] = s_tilde_0 + alpha*(w_tilde[t]-w_tilde_0)

w_margin = (np.max(w_tilde) - np.min(w_tilde)) * 0.1
s_margin = (np.max(s_tilde) - np.min(s_tilde)) * 0.1
# Tracé de la courbe de cobweb
fig, ax = plt.subplots(figsize=(8,6))
ax.set_xlabel('Punishment expected ($\widetilde{s}$)')
ax.set_ylabel('Gain expected ($\widetilde{\omega}$)')
ax.set_title('Cobweb curve')
ax.set_xlim([np.min(s_tilde) - s_margin, np.max(s_tilde) + s_margin])
ax.set_ylim([np.min(w_tilde) - w_margin, np.max(w_tilde) + w_margin])
ax.plot(s_tilde, w_tilde, label='Cobweb curve')
ax.plot(s_tilde, s_tilde, '--', label='Bisecting line')
ax.legend()
plt.show()
```