Applying the accumulating remainder tree algorithm to Kurepa's Conjecture

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The left factorial function

Definition

For positive integers n, Kurepa defined $!n$ as $0! + 1! + \cdots + (n-1)!$ where $0! = 1$.

Kurepa's Left Factorial Conjecture

Kurepa's Left factorial Conjecture

Kurepa's conjecture is that $(!n, n!) = 2$ for all n greater than 1.

Remark

It can be shown that Kurepa's conjecture is equivalent to the statement that $!p \not\equiv 0 \mod p$. This form of the conjecture is the most useful for testing computationally.

Attempted Proofs

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Kurepa claimed that he had a proof in 1992 and R. Bond anoounced a proof too, but neither of these proofs were ever published. Barsky and Benzaghou published a proof in 2004 which was later retracted due to irreparable calcuation errors.

Previous Results

Upper Bound	Year	Author
$p < 3 \cdot 10^5$	1990	Mijajlovic
$p < 10^6$	1991	Gogic
$p < 3 \cdot 10^6$	1998	Malesevic
$p < 2^{23}$	1999	Zivkovic
$p < 1^{26}$	2000	Gallot
$p < 1.44 \cdot 10^8$	2004	Jobling
$p < 2^{34}$	2016	Andrejic

Time Complexity

These algorithms work in $O(p)$ time for each prime p. Combining this with the Prime Number Theorem, the time taken for all primes up to N is $O(N^2/\log(N))$.