21DS636-Statistical Modelling

Course Introduction
Basics of Probability Theory

Dr. Deepa Gupta, Professor
Human Language Technology Lab
Department of Computer Science & Engineering
ASE-Bangalore

Course Pre-requisites

Calculus, algebra, matrix algebra, and programming statistical package Python3 and the math-friendly documentation language Latex.

Syllabus

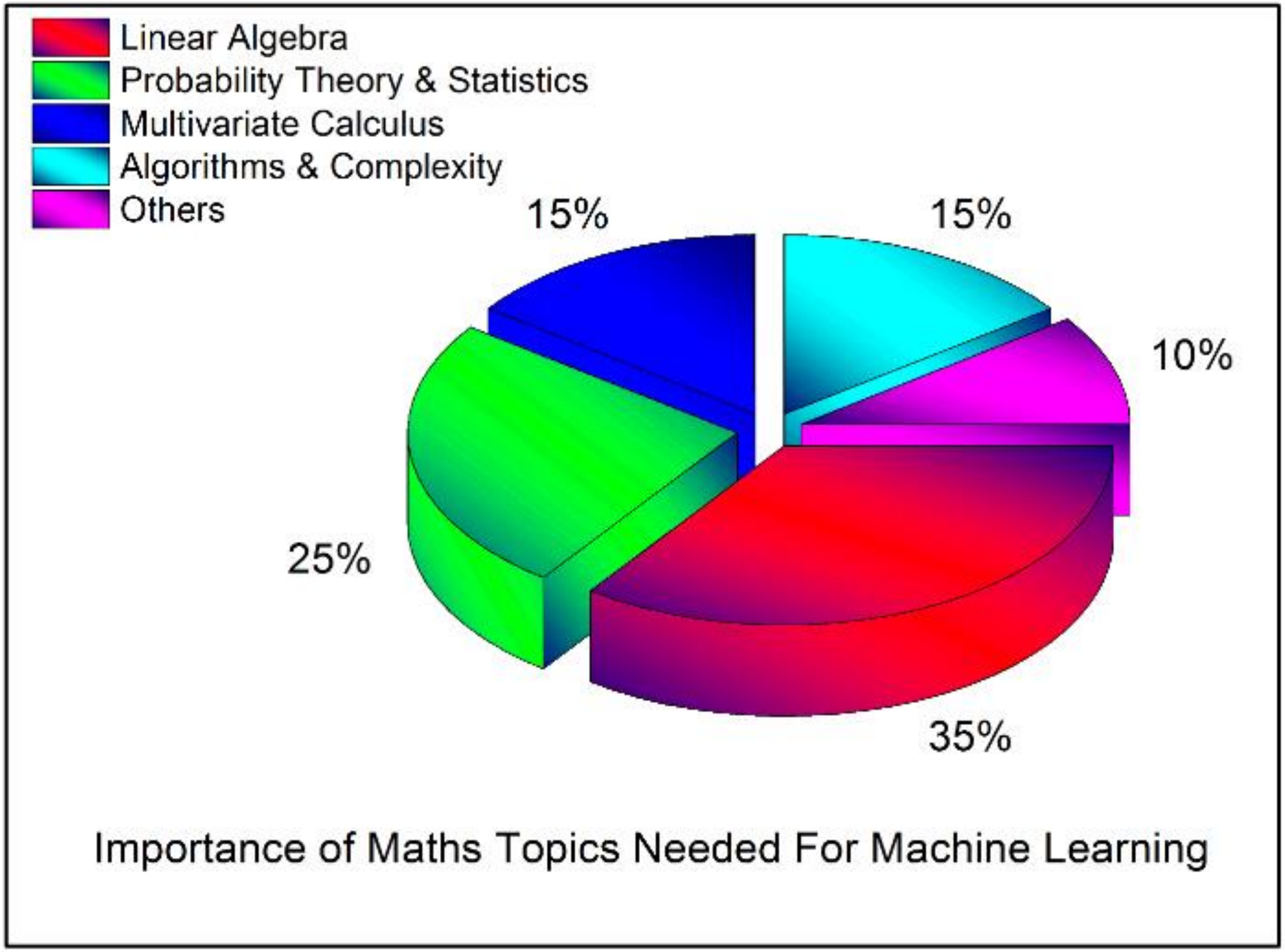
Probability, Random Variables & Probability Distributions. Sampling, analysis of sample data-Empirical Distributions, Sampling from a Population Estimation, confidence intervals, point estimation--Maximum Likelihood, Probability mass functions, Modeling distributions, Relationships between variables and Estimation, Hypothesis testing- Z, t, Chi-Square & F-test. ANOVA & Designs of Experiments - Single, Two factor ANOVA, Factorials ANOVA models.

Linear least squares, Correlation & Regression Models-linear regression methods, Ridge regression, LASSO, univariate and Multivariate Linear Regression, probabilistic interpretation, Regularization, Logistic regression, locally weighted regression.

Exploratory data analysis, Time series analysis, Analytical methods – ARIMA and SARIMA.

Evaluation Pattern

Assessment component	Type of Assessment	Minimum Number of Assessments	Weightage (%)
Internal	Quizzes	2	20
	Lab Test +Assignments	1+2	30
	Presentation &voluntary work	15 (2papers) + 5	20
External	Term Project	1	20



https://towardsdatascience.com/the-mathematics-of-machine-learning-894f046c568

Probability Theory

Probability – Models for random phenomena

The foundation of Statistics

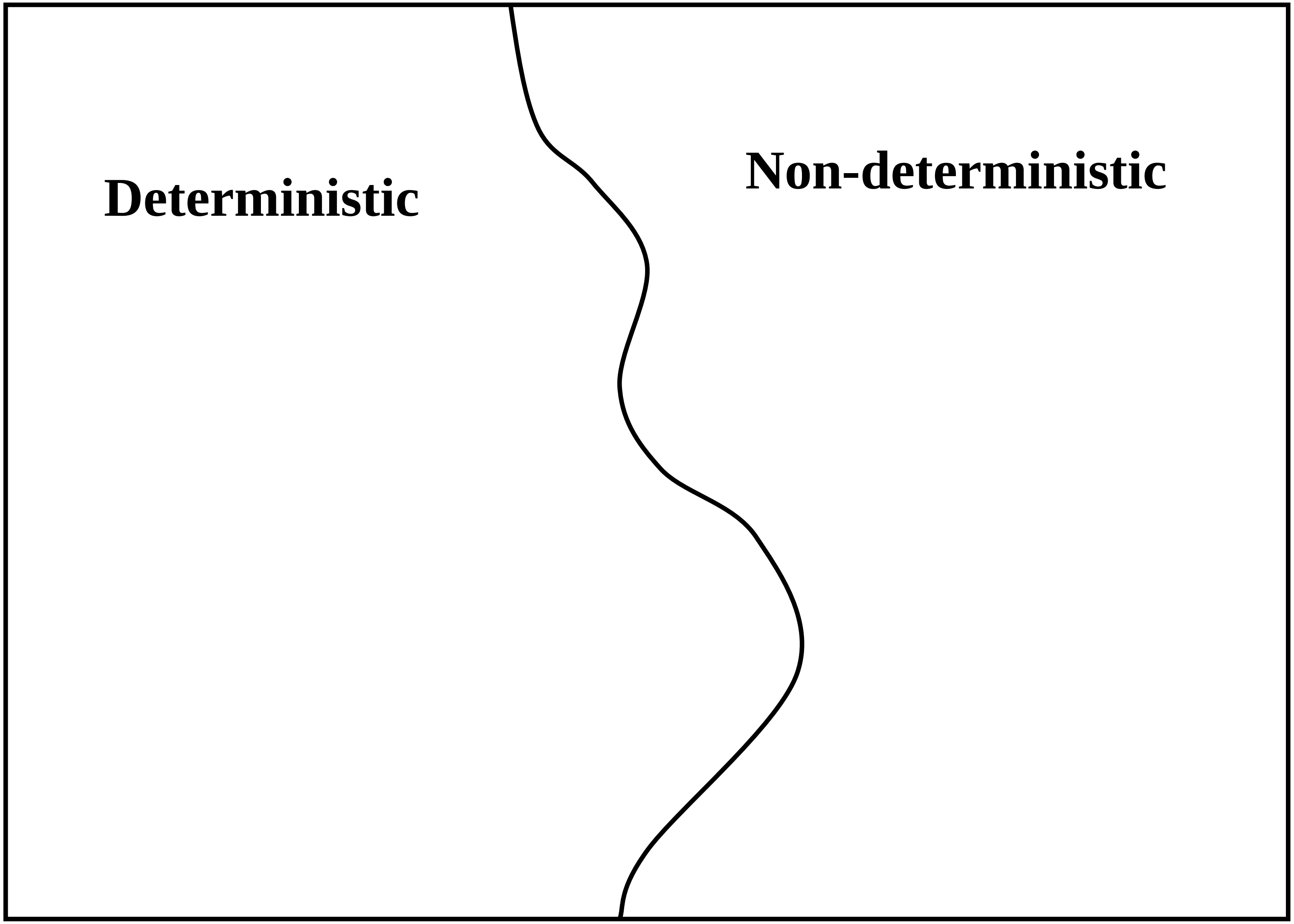
Some slides updated from Vicki Borlaug and web

Probability Theory

Statistics and Probability theory constitutes a branch of mathematics for dealing with **uncertainty**

Probability theory provides a basis for the science of **statistical inference** from data

Phenomena



Deterministic Phenomena

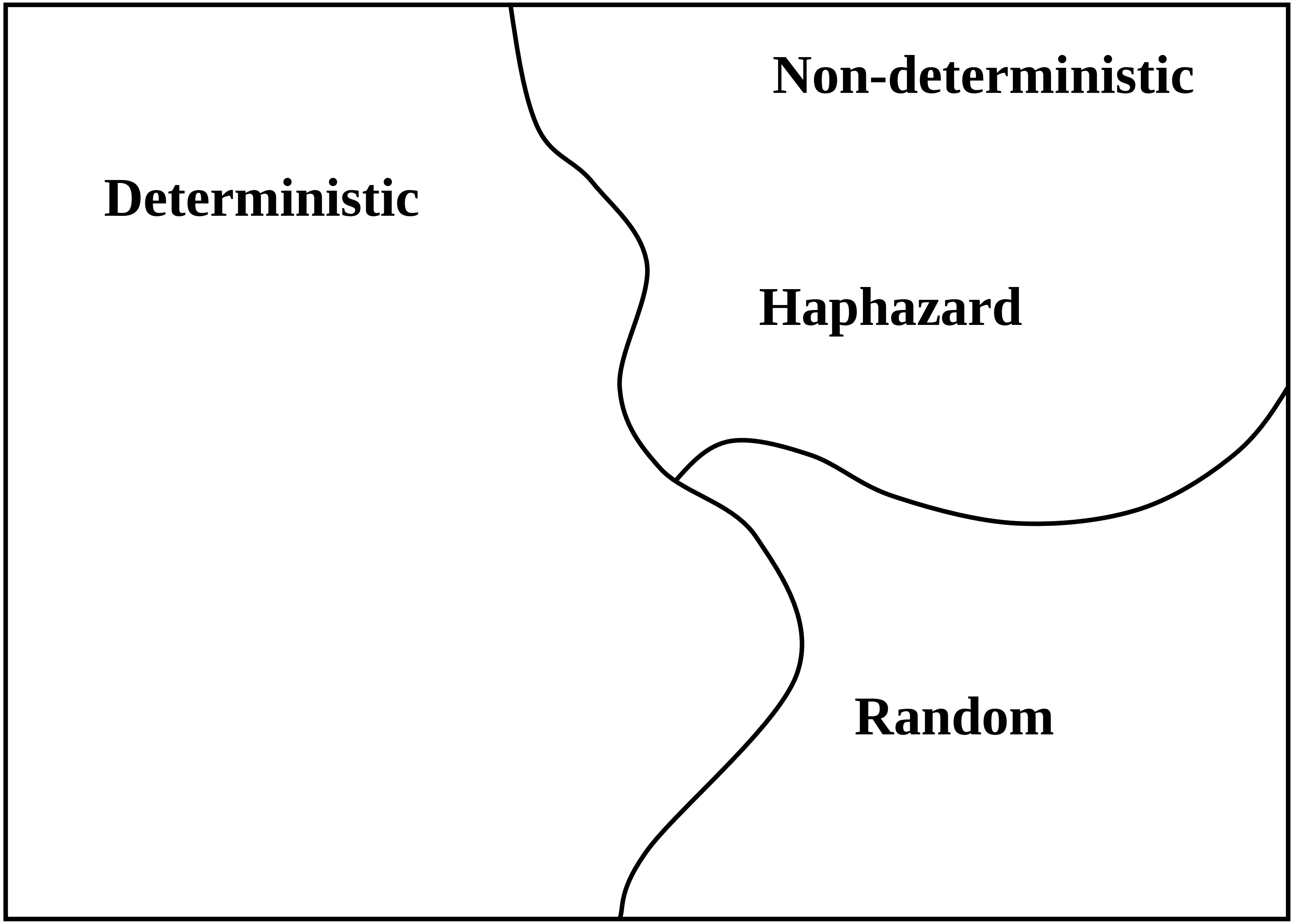
- There exists a mathematical model that allows "***perfect***" prediction the phenomena's outcome.

- Many examples exist in Physics, Chemistry (the exact sciences).

Non-deterministic Phenomena

- **No** mathematical model exists that allows "***perfect*** prediction the phenomena's outcome.

Phenomena



Non-deterministic Phenomena

- may be divided into two groups.

1. **Random phenomena**
   - Unable to predict the outcomes, but in the long-run, the outcomes exhibit **statistical regularity**.

2. **Haphazard phenomena**
   - unpredictable outcomes, but no long-run, exhibition of statistical regularity in the outcomes.

Random phenomena

– Unable to predict the outcomes, but in the long-run, the outcomes exhibit statistical regularity.

Examples

1. Tossing a coin – outcomes S ={**Head, Tail**}

Unable to predict on each toss whether is Head or Tail.

In the long run can predict that 50% of the time heads will occur and 50% of the time tails will occur

2. Rolling a die – outcomes

$$S = \{⚀, ⚁, ⚂, ⚃, ⚄, ⚅\}$$

- Unable to predict outcome but in the long run can one can determine that each outcome will occur 1/6 of the time.

- **Use symmetry:** Each side is the same. One side should not occur more frequently than another side in the long run. If the die is not balanced this may not be true.

Definitions

Random Experiment

- A statistical experiment is a chance mechanism satisfying the following:
  - All possible distinct outcomes are known in advance
  - Any performance of that experiment results in an outcome that is not known in advance
  - It can be repeated under identical conditions

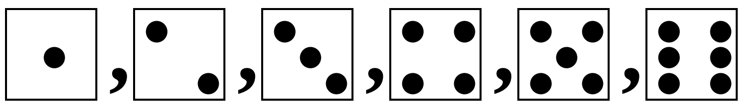
Sample Space

The **sample space**, *S*, for a random phenomena is the set of all possible distinct outcomes.

Examples

1. Tossing a coin – outcomes $S=\{\textbf{Head, Tail}\}$

2. Rolling a die – outcomes

$S=\{$  $\}$

$$S=\{1, 2, 3, 4, 5, 6\}$$

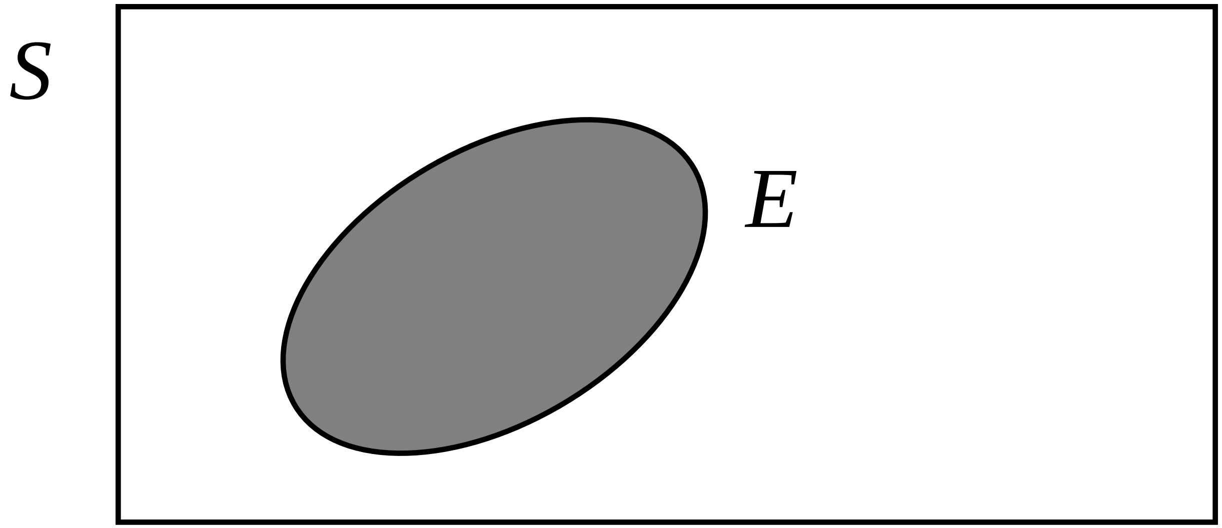
3. Lifetime of a battery-outcomes

$S=\{\ x \mid 0 \le x < \infty\} \rightarrow$ continuous sample space

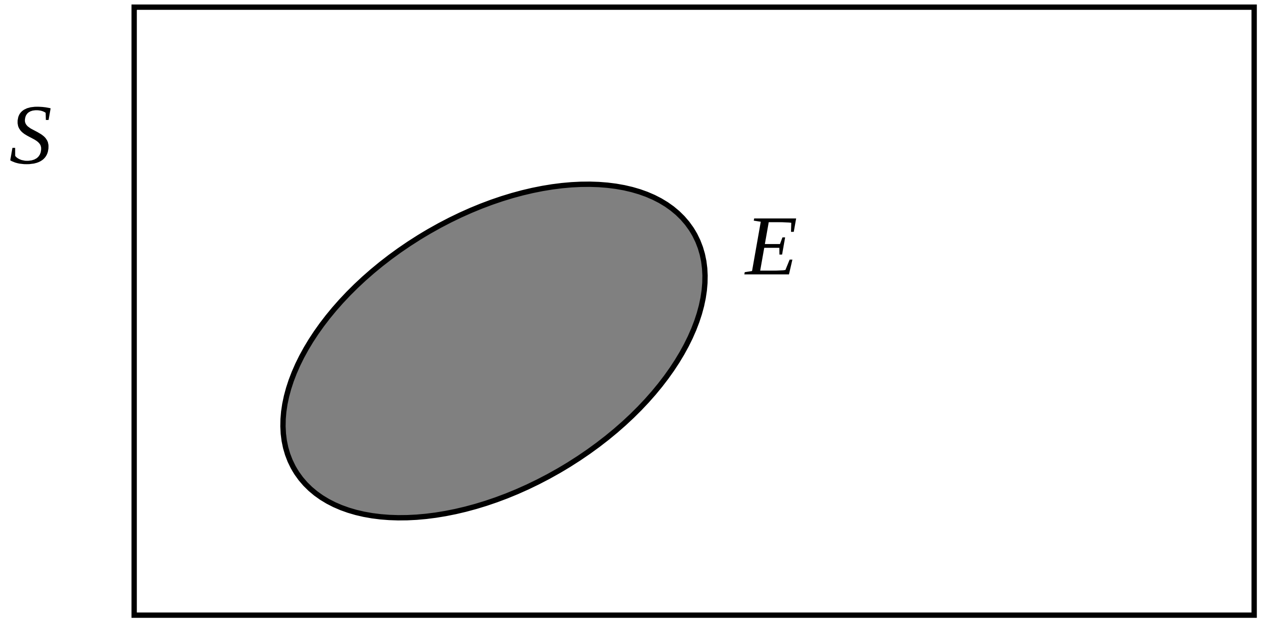
An Event , *E*

The **event**, *E*, is any subset of the **sample space**, *S*. i.e. any set of outcomes (not necessarily all outcomes) of the random phenomena

Venn diagram



The **event**, *E*, is said to **have occurred** if after the outcome has been observed the outcome lies in *E*.



Examples

1. Rolling a die – outcomes

$S = \{$ ⚀, ⚁, ⚂, ⚃, ⚄, ⚅ $\}$

$= \{1, 2, 3, 4, 5, 6\}$

E = the event that an even number is rolled

$= \{2, 4, 6\}$

$= \{$ ⚁, ⚃, ⚅ $\}$

Special Events

The Null Event, The empty event - ϕ

$\phi = \{ \ \} =$ the event that contains no outcomes

The Sure/Entire Event, The Sample Space - S

$S =$ the event that contains all outcomes

The empty event, ϕ, never occurs.

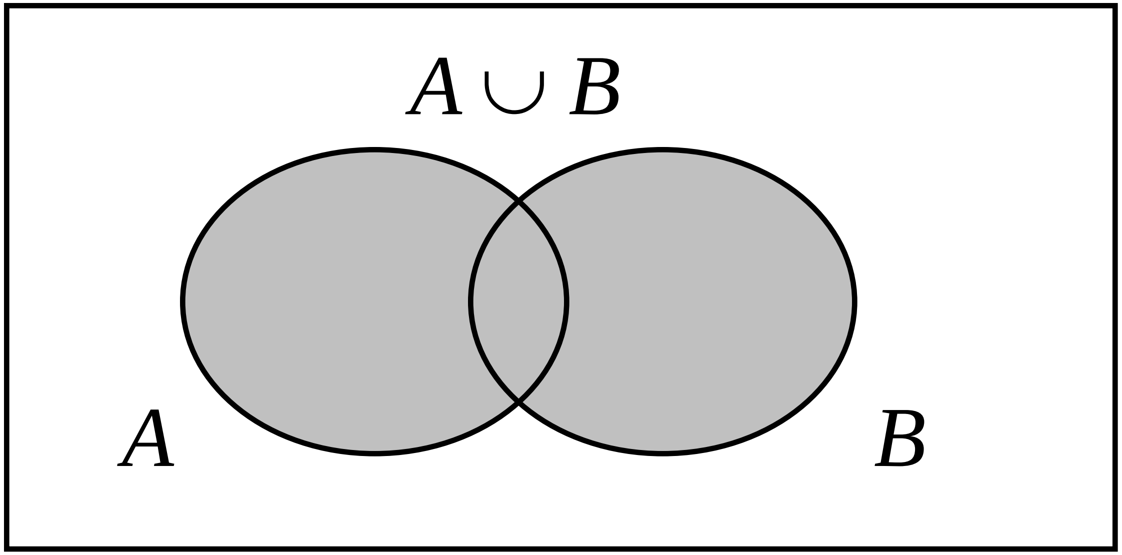
The entire event, S, always occurs.

Set operations on Events

Union

Let A and B be two events, then the **union** of A and B is the event (denoted by $A \cup B$) defined by:

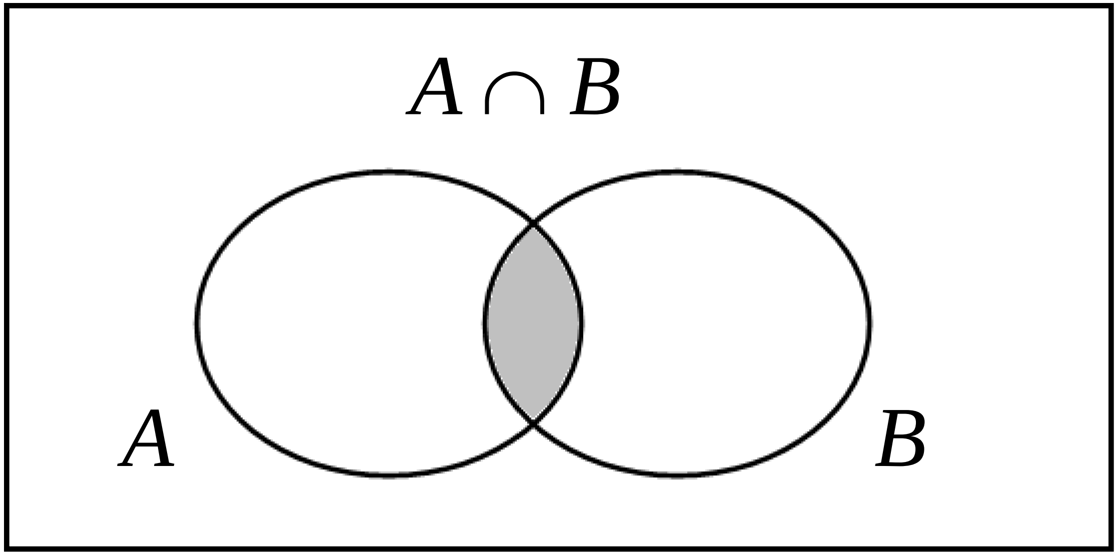
$$A \cup B = \{e \mid e \text{ belongs to } A \textbf{ or } e \text{ belongs to } B\}$$



Intersection

Let A and B be two events, then the **intersection** of A and B is the event (denoted by $A \cap B$) defined by:

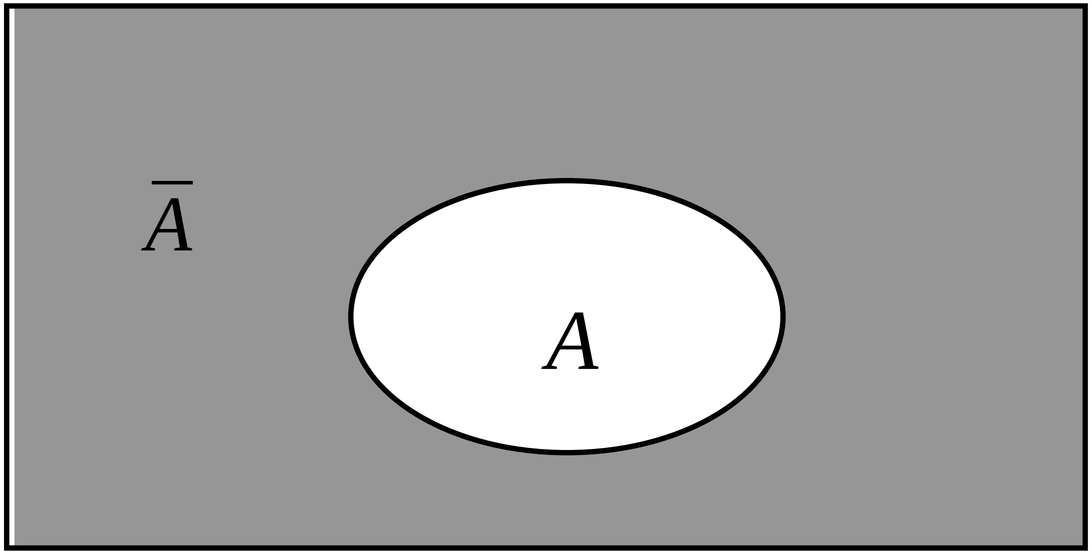
$$A \cap B = \{e \mid e \text{ belongs to } A \textbf{ and } e \text{ belongs to } B\}$$



Complement

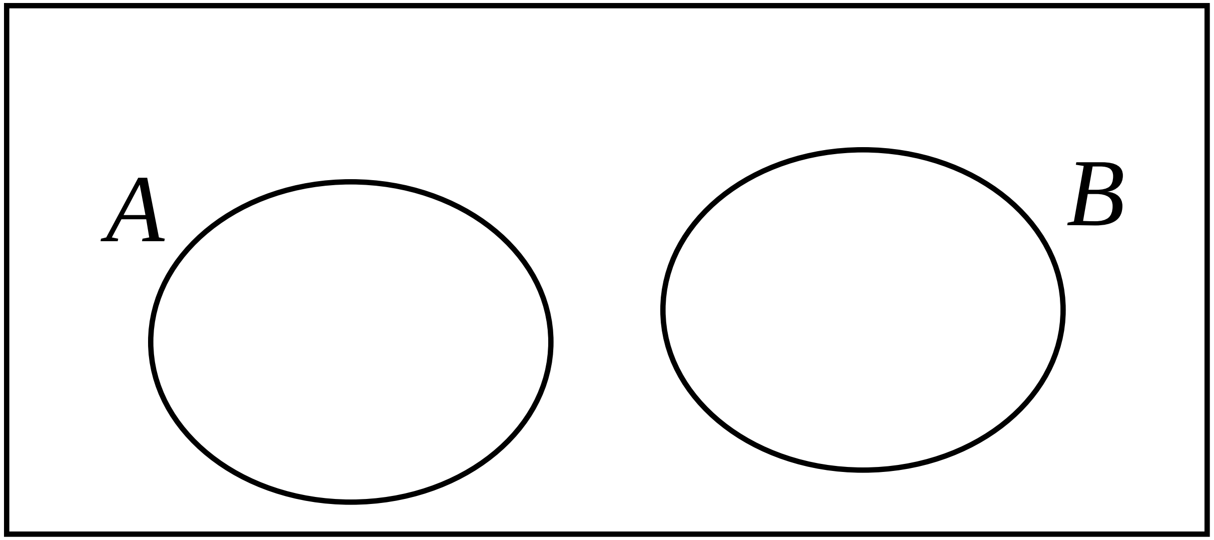
Let A be any event, then the **complement** of A (denoted by $\bar{A}$) defined by:

$$\bar{A} = \{e \mid e \textbf{ does not } \text{belong to } A\}$$



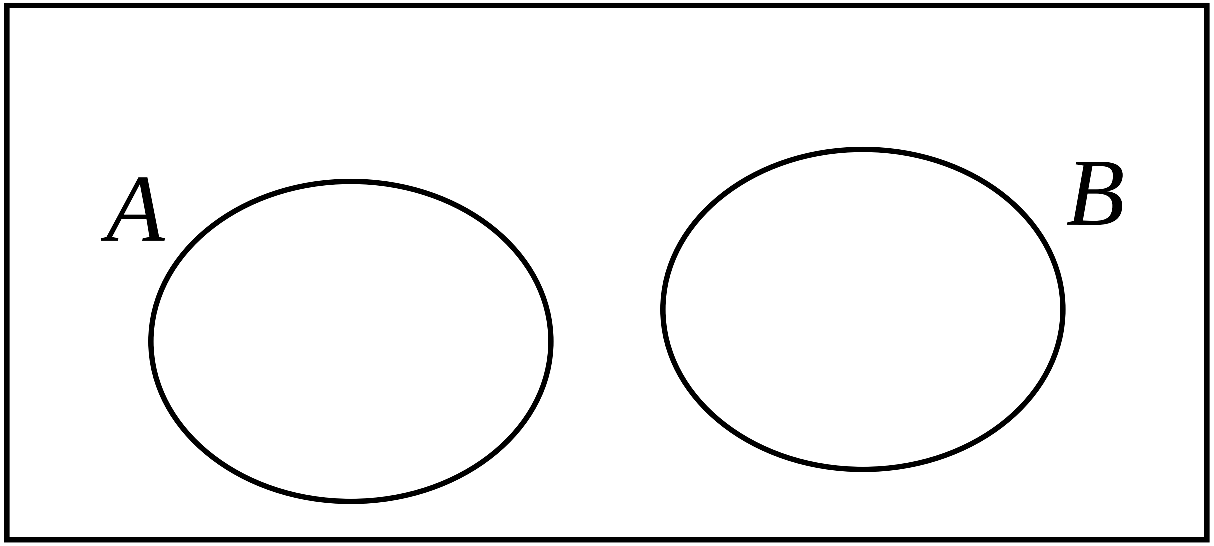
Definition: Mutually Exclusive Event

Two events A and B are called **mutually exclusive** if: $A \cap B = \phi$



If two events *A* and *B* are are **mutually exclusive,** then:

1. They have no outcomes in common. They can’t occur at the same time.
2. The outcome of the random experiment can not belong to both *A* and *B*.



In problems you will recognize that you are working with:

1. **Union** if you see the word **or**,
2. **Intersection** if you see the word **and**,
3. **Complement** if you see the word **not**.

Example

Probability

Definition: Probability of an Event *E*.

Laplace's Classical Definition: Suppose that the sample space $S = \{o_1, o_2, o_3, \ldots o_N\}$ has a finite number, N, of outcomes.

Also each of the outcomes is equally likely (because of symmetry).

Then for any event E

$$P[E] = \frac{n(E)}{n(S)} = \frac{n(E)}{N} = \frac{\text{no. of outcomes in } E}{\text{total no. of outcomes}}$$

Note: the symbol $n(A) =$ no. of elements of A

- Consider a box with *n* white and *m* red balls. In this case, there are two elementary outcomes: white ball or red ball. Probability of "selecting a white ball" $= \dfrac{n}{n+m}$.

Applies only to the special case when

1. The sample space has a finite no. of outcomes, and
2. Each outcome is equi-probable

Note: If this is not true a more general definition of probability is required.

Relative Frequency Definition: The probability of an event A is defined as

$$P(A) = \lim_{n \to \infty} \frac{n_A}{n}$$

- where n_A is the number of occurrences of A and n is the total number of trials.
- Example: Filp a coin, 10000 times, suppose we get approximately 4980 heads, the probability of heads is =4980/10000=0.498≈0.5

Subjective probability: degree of belief in a proportion

- Example: based on the weather condition rain prediction -chance of rain 70%-personal belief

Axioms of Probability

The axiomatic approach to probability, due to **Kolmogorov (1933),** developed through a set of axioms is generally recognized as superior to the above definitions, as it provides a solid foundation for complicated applications.

For any event A, we assign a number $P(A)$, called the probability of the event A. This number satisfies the following three conditions that act the axioms of probability.

$$\text{(i)} \quad P(A) \geq 0 \quad \text{(Probability is a nonnegative number)}$$

$$\text{(ii)} \quad P(\Omega) = 1 \quad \text{(Probability of the whole set is unity)}$$

$$\text{(iii)} \quad \text{If } A \cap B = \phi, \quad \text{then} \quad P(A \cup B) = P(A) + P(B).$$

(Note that (iii) states that if *A* and *B* are mutually exclusive (M.E.) events, the probability of their union is the sum of their probabilities.)

The following conclusions follow from these axioms:

a. Since $A \cup \overline{A} = \Omega$, we have using (ii)

$$\mathrm{P}(A \cup \overline{A}) = P(\Omega) = 1.$$

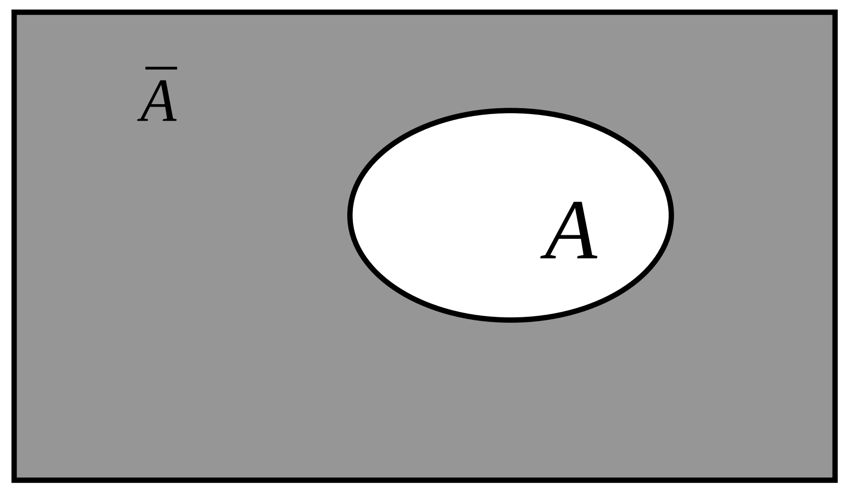
But $A \cap \overline{A} \in \phi$, and using (iii),

$$\mathrm{P}(A \cup \overline{A}) = P(A) + \mathrm{P}(\overline{A}) = 1 \quad \text{or} \quad \mathrm{P}(\overline{A}) = 1 - P(A).$$

Rule for complements

$$P[\text{not } A] = 1 - P[A]$$

2. $$P[\overline{A}] = 1 - P[A]$$



The following conclusions follow from these axioms:

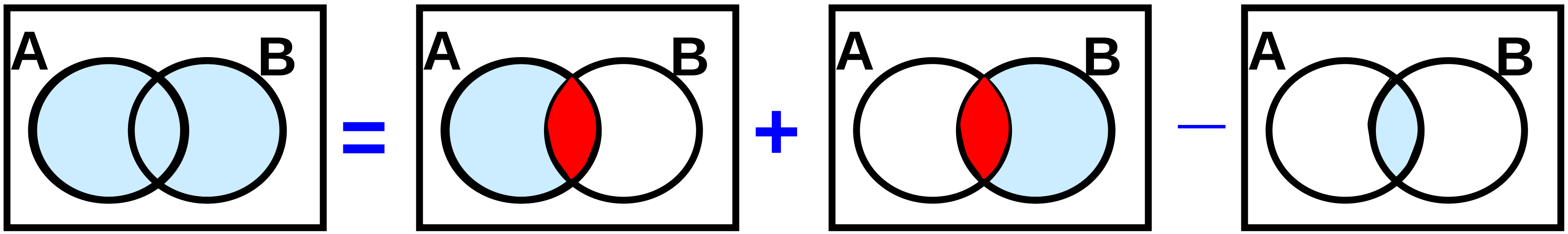
a $\quad \mathrm{P}(A \cup \overline{A}) = P(A) + \mathrm{P}(\overline{A}) = 1 \quad \text{or} \quad \mathrm{P}(\overline{A}) = 1 - P(A).$

b. Suppose *A* and *B* are *not* mutually exclusive (M.E.)?

How does one compute $\quad P(A \cup B) = ?$

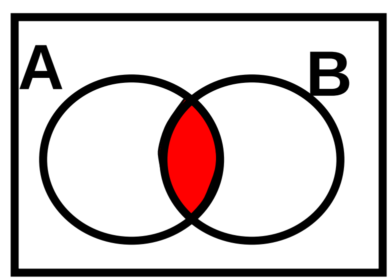
Rules of Probability

The Addition Rule for Probability



P(A <u>or</u> B) = P(A) + P(B) - P(A <u>and</u> B)

But we have added this piece twice! That is one extra time!



We need to subtract off the extra time!

<u>Example #1</u>)

Given the following probabilities:

P(A)=0.8 P(B)=0.3 P(A <u>and</u> B)=0.2

Find the P(A <u>or</u> B).

This can be solved two ways.

1. Using Venn Diagrams
2. Using the formula

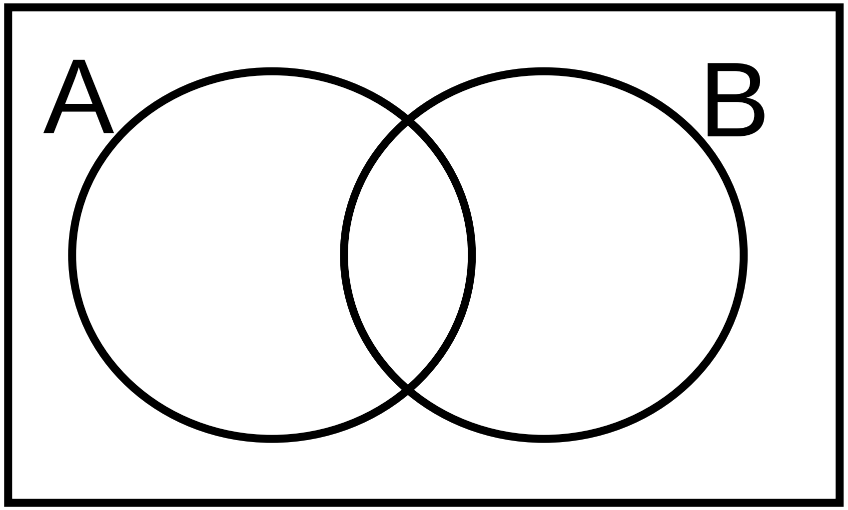
We will solve it both ways.

Example #1 (continued)

P(A)=0.8   P(B)=0.3   P(A and B)=0.2

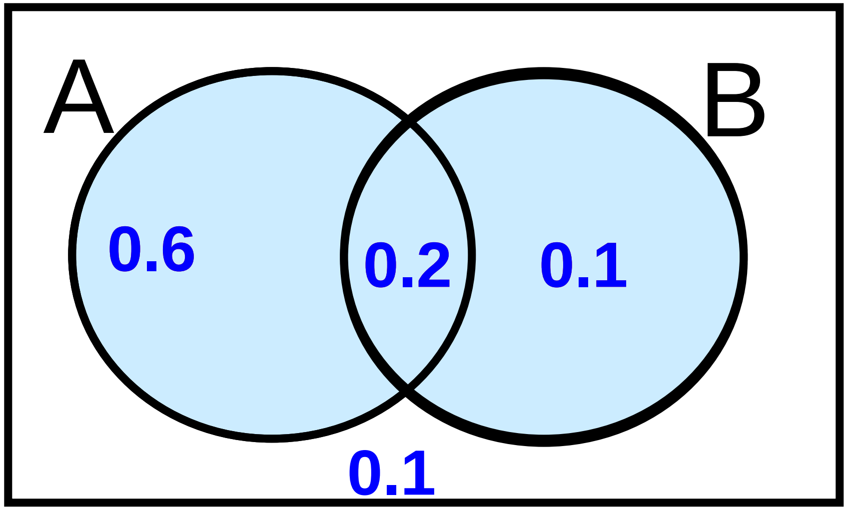
Find the P(A or B).

Solution using Venn Diagrams:



In this example we will fill up the Venn Diagram with probabilities.

Then find the probability of A <u>or</u> B.



I will start by shading **A <u>or</u> B**.

Then I will add up the probabilities in the shaded area.

$$P(A \text{ or } B) = 0.6 + 0.2 + 0.1$$

$$= \boxed{0.9}$$

Example #1 (continued)

P(A)=0.8 P(B)=0.3 P(A and B)=0.2

Find the P(A or B).

Solution using the formula:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$= 0.8 + 0.3 - 0.2$$

$$= \boxed{0.9}$$

Example #2.)

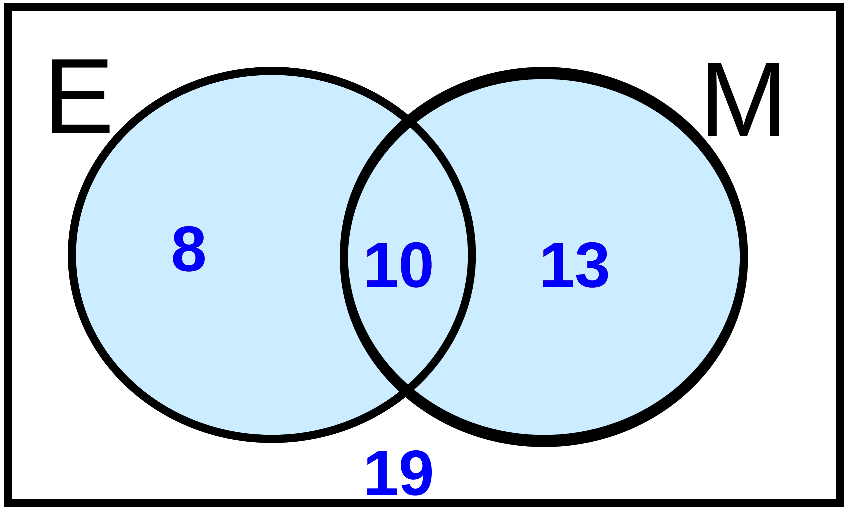
There are 50 students. 18 are taking English. 23 are taking Math. 10 are taking English and Math.

If one is selected at random, find the probability that the student is taking English or Math.

E = taking English

M = taking Math

Then find the probability of English or Math.



I will start by shading **E or M**.

Then I will find the probability in the shaded area.

$$P(E \text{ or } M) = \frac{8+10+13}{50}$$

$$= \boxed{0.62}$$

<u>Example #2</u> (continued) There are 50 students. 18 are taking English. 23 are taking Math. 10 are taking English <u>and</u> Math.

If one is selected at random, find the probability that the student is taking English <u>or</u> Math.

<u>Solution using the formula</u>:

P(E <u>or</u> M) = P(E) + P(M) - P(E <u>and</u> M)

$$= \frac{18}{50} + \frac{23}{50} - \frac{10}{50}$$

$$= \boxed{\mathbf{0.62}}$$

Question 1#)

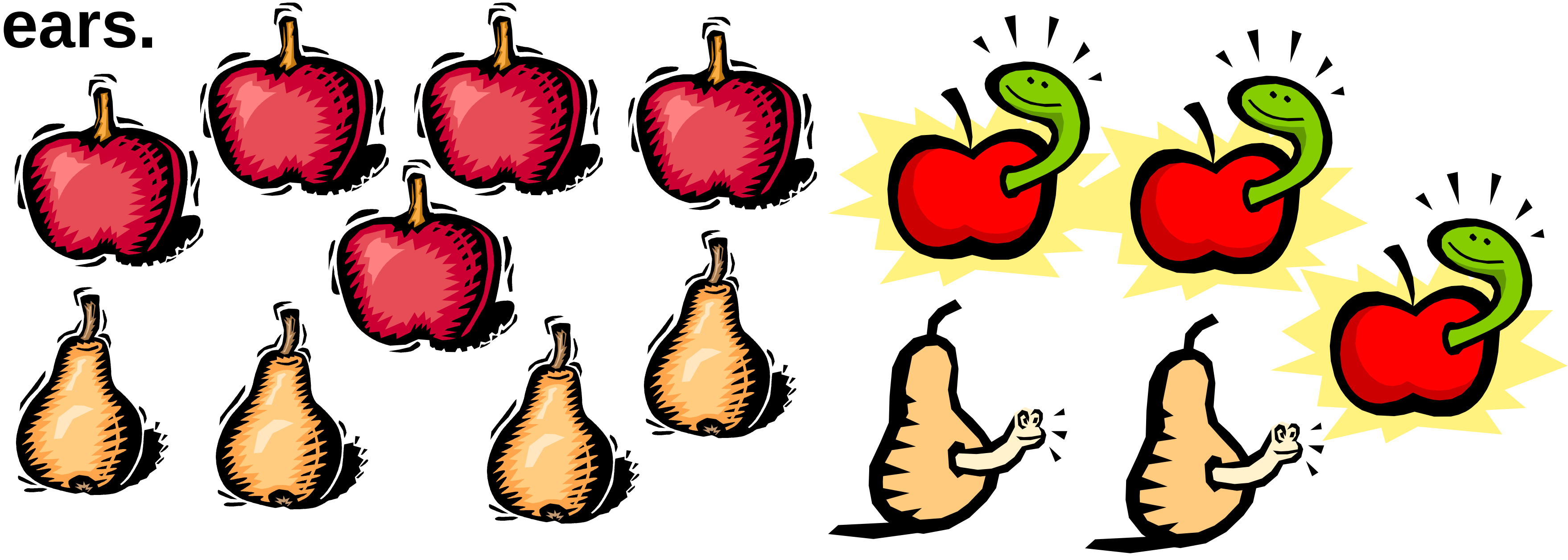
There are 1580 people in an amusement park. 570 of these people ride the rollercoaster. 700 of these people ride the merry-go-round. 220 of these people ride the roller coaster **and** merry-go-round.

If one person is selected at random, find the probability that that person rides the roller coaster **or** the merry-go-round.

a.) Solve using Venn Diagrams.

b.) Solve using the formula for the Addition Rule for Probability.

Example #3) Population of apples and pears.



Each member of this population can be described in two ways.

1. Type of fruit
2. Whether it has a worm or not

We will make a table to organize this data.

Ex. #3 (continued)

	no worm	worm	
apple	5	3	8
pear	4	2	6
	9	5	**grand total** 14

Experiment: One is selected at random.

Find the probability that . . .

a.) . . . it is a pear and has a worm.

b.) . . . it is a pear or has a worm.

Ex. #3 (continued)

	no worm	worm	
apple	5	3	8
pear	4	2	6
	9	5	**grand total** 14

Solution to #3a.)

$$P(\text{pear } \underline{\text{and}} \text{ worm}) = \frac{2}{14} = \boxed{0.1429}$$

Ex. #3 (continued)

	no worm	worm	
apple	5	3	8
pear	4	2	6
	9	5	**grand total** 14

Solution to #3b.)

$$P(\text{pear } \underline{\text{or}} \text{ worm}) = \frac{4+2+3}{14} = \boxed{0.6429}$$

Ex. #3 (continued)

	no worm	worm	
apple	5	3	8
pear	4	2	6
	9	5	grand total 14

Alternate Solution to #3b.)

P(**pear or worm**)= P(**pear**) + P(**worm**) – P (**pear and worm**)

$$= \frac{6}{14} + \frac{5}{14} - \frac{2}{14}$$

$$= \boxed{0.6429}$$

$$\boxed{P[A \cup B] = P[A] + P[B] - P[A \cap B]}$$

Example: Chicago and New York are two of the cities competing for the World university games. (There are also many others). The organizers are narrowing the competition to the **final 5 cities.**

There is a 20% chance that Chicago will be amongst the **final 5**. There is a 35% chance that New York will be amongst the **final 5** and an 8% chance that both Chicago and New York will be amongst the **final 5**.

What is the probability that Chicago or New York will be amongst the **final 5.**

Solution:

Let C= the event that Chicago is amongst the **final 5**.

Let N= the event that New York is amongst the **final 5**.

Given $P[C] = 0.20$, $P[N] = 0.35$, and $P[C \cap N] = 0.08$

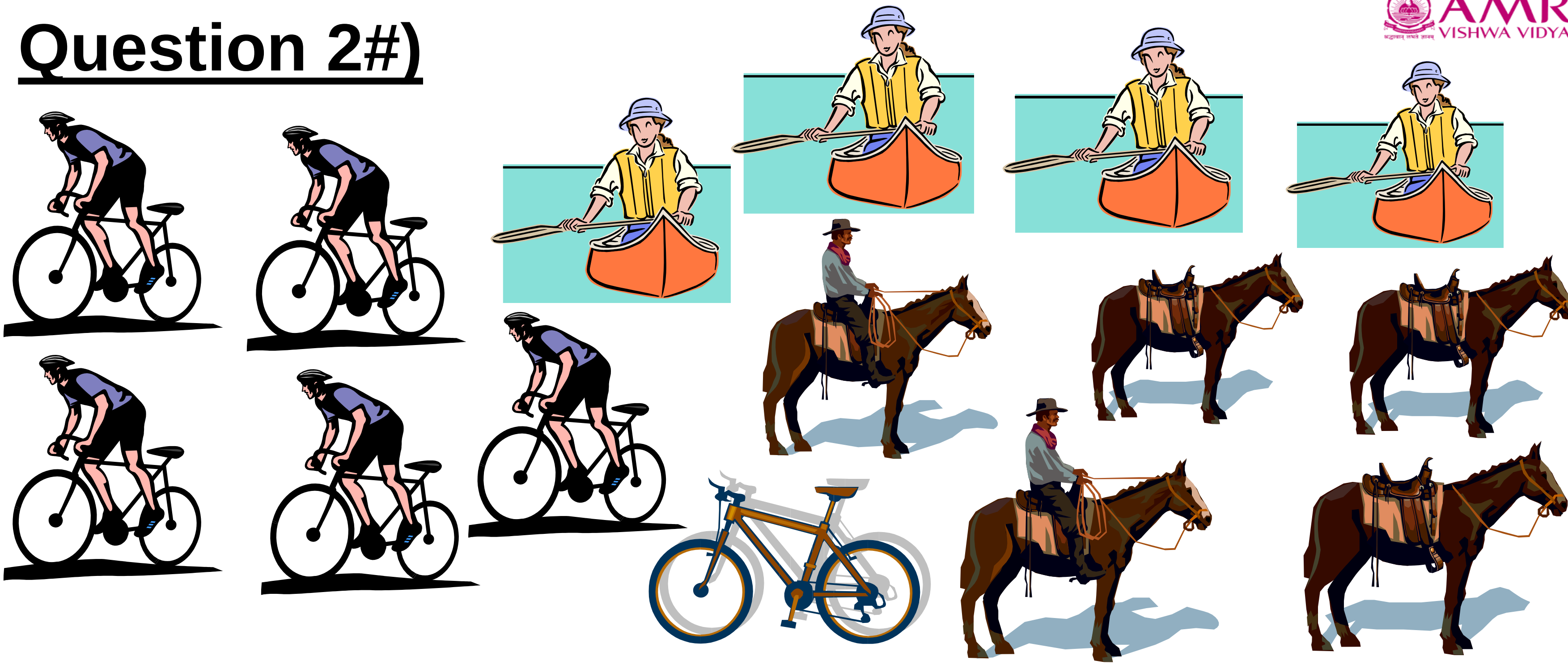
What is $P[C \cup N]$?

Note: "and" $\equiv \cap$**,** "or" $\equiv \cup$.

$$P[C \cup N] = P[C]+P[N]- P[C \cap N]$$

$$= 0.20 + 0.35 - 0.08 = 0.47$$

Question 2#)

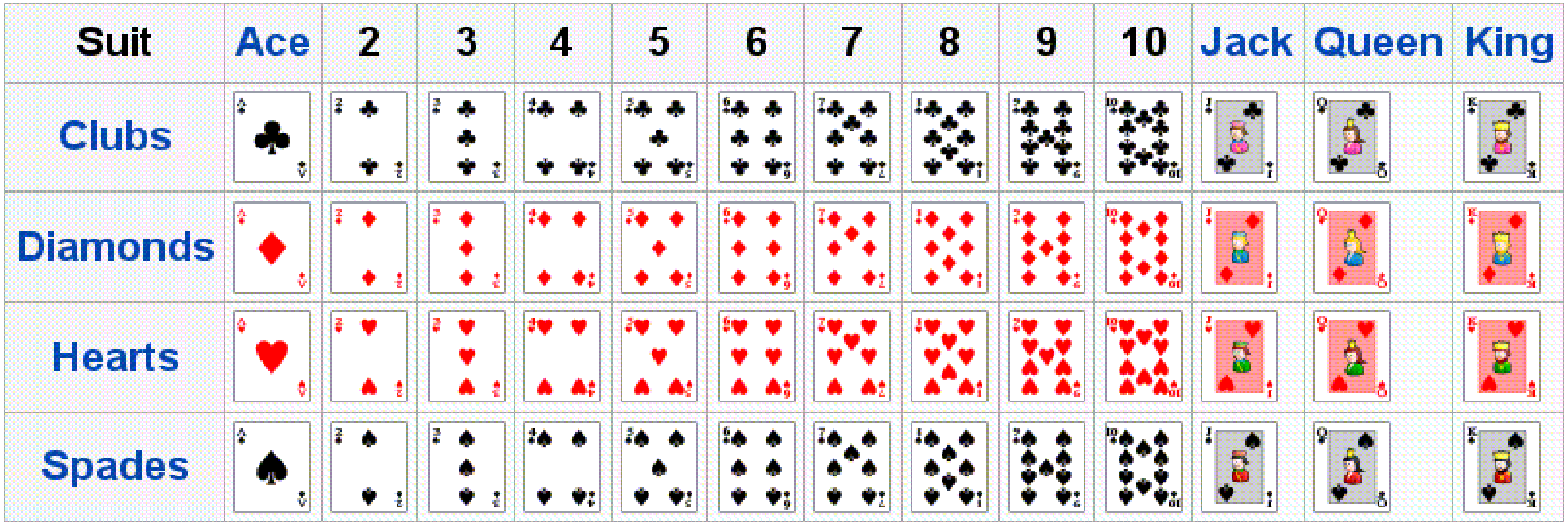


There are our modes of transportation – horse, bike, & canoe. Each has a person or does not have a person.

1.) Make a table to represent this data.
2.) If one is selected at random find the following:
   a.) P(horse <u>or</u> has a person)
   b.) P(horse <u>and</u> has a person)
   c.) P(bike <u>or</u> does not have a person)

- **Example:** A card is selected at random from a type of 52 cards. Find the probability that
    1. It is a king or an Ace
    2. It is a 6 or a diamond
- **Solution:**

Example set of 52 poker playing cards



- **Example:** A store accepts either VISA or master card. 50% of the store customers have VISA card 30% have master card and 10% have both.

1. What % of state customer have a credit card the store accepts.
2. What % of store customers have exactly one of the credit cards the store accepts
3. What % of the store customer does not have card the store accepts

- **Solution:**

Multiplication Rule of Probability

- Dependent events
- Conditional probability
- Independent events

Dependent Events……

- When the outcome or occurrence of the first event affects the outcome or occurrence of the second event in such a way that the probability is changed.

Examples of Dependent Events……

1. Draw a card from a deck. Do not replace it and draw another card.
2. Having high grades and getting a scholarship
3. Parking in a no parking zone and getting a ticket

Multiplication Rule – Dependent Events……

- When 2 events are dependent, the probability of both occurring is

$$\boxed{P(A\, and\, B) = P(A) \cdot P(B|A)}$$

- The **slash reads**:

  “The probability that B occurs given that A has already occurred.”

Example……

- 53% of residents had homeowner's insurance. Of these, 27% also had car insurance. If a resident is selected at random, find the prob. that the resident has both homeowner's ***and*** car insurance.

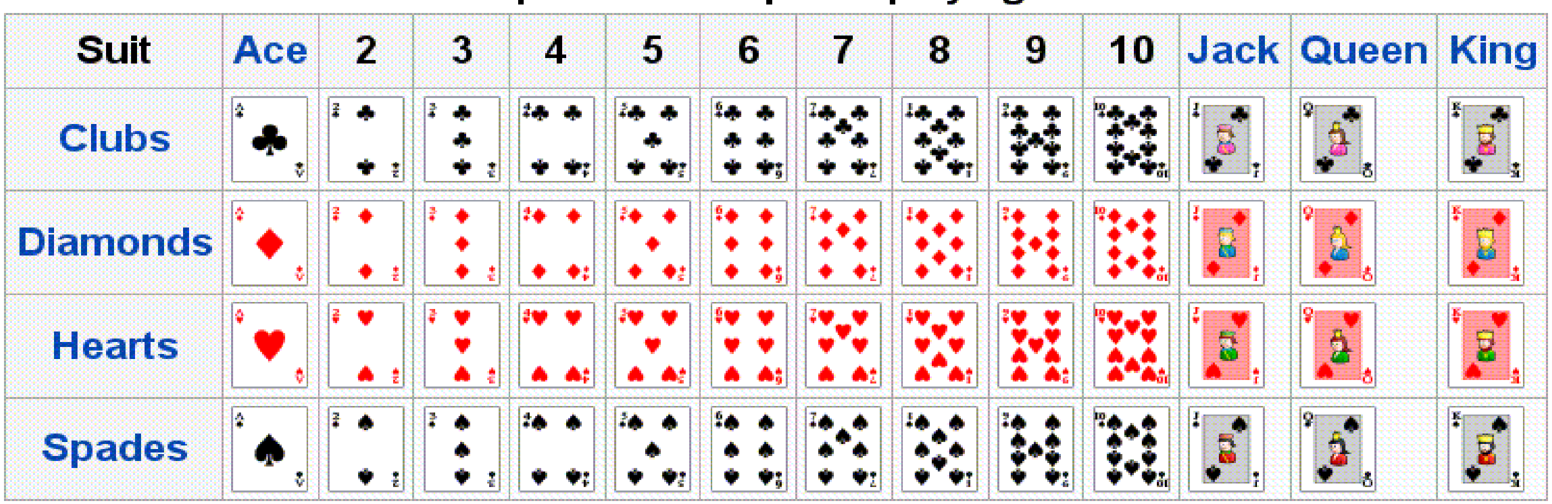
- Answer:

$$P(H\ and\ C) = P(H)\cdot P(C|H)$$

$$P(H\ and\ C) = (.53)(.27) = .1431$$

- **Example:** Three cards are drawn from a deck and **NOT** replaced. Find the following probabilities.
  a. Getting 3 jacks
  b. Getting an ace, king, and queen
  c. Getting a club, spade, and heart
  d. Getting 3 clubs.

Example set of 52 poker playing cards



a. Getting 3 jacks……

$$P(J \ and \ J \ and \ J) = \frac{4}{52} \cdot \frac{3}{51} \cdot \frac{2}{50} = \frac{1}{5525} = .000181$$

b. Getting an ace, king, queen……

$$P(A\, and\, K\, and\, Q) = \frac{4}{52} \cdot \frac{4}{51} \cdot \frac{4}{50} = \frac{8}{16575} = .000483$$

c. Getting a club, spade, and heart……

$$P(C\ and\ S\ and\ H) = \frac{13}{52} \cdot \frac{13}{51} \cdot \frac{13}{50} = \frac{169}{10200} = .017$$

d. Getting 3 clubs……

$$P(C\,and\,C\,and\,C)=\frac{13}{52}\cdot\frac{12}{51}\cdot\frac{11}{50}=\frac{11}{850}\,or\,.013$$

Conditional Probability

Conditional Probability

$$\boxed{P(A\,and\,B) = P(A) \cdot P\left(B/A\right)}$$

- **Algebraically change this so that it is now in the form……**

Definition: Conditional Probability

Suppose that we are interested in computing the probability of event A and we have been told event B has occurred.

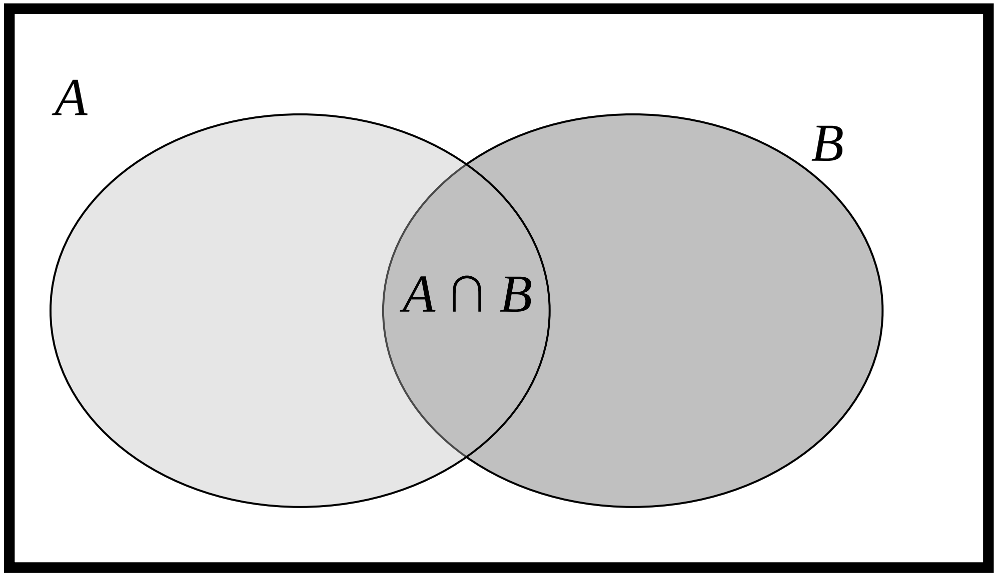
Then the conditional probability of ***A* given *B*** is defined to be:

$$P\left[A \middle| B\right] = \frac{P\left[A \cap B\right]}{P\left[B\right]} \quad \text{if} \quad P\left[B\right] \neq 0$$

Rationale:

- If we're told that event B has occurred then the sample space is restricted to B.---***Very Important point***
- The probability within B has to be normalized, This is achieved by dividing by $P[B]$
- The event A can now only occur if the outcome is in of $A \cap B$. Hence the new probability of A is:

$$P\left[A|B\right]=\frac{P\left[A\cap B\right]}{P\left[B\right]}$$



An Example

The academy awards is soon to be shown on TV.

For a specific married couple the probability that the husband watches the show is 80%, the probability that his wife watches the show is 65%, while the probability that they both watch the show is 60%.

If the husband is watching the show, what is the probability that his wife is also watching the show.

Solution:

- **Example:** A box contains 24 toasters and 3 of which are defective if two toasters are selected and tested. Find the probability that both are defective.
- **Solution:**

Example: A recent survey asked 100 people if they thought women in the armed forces should be permitted to participate in combat. The results are shown in the table.

	Yes	No	Total
Male	32	18	50
Female	8	42	50
Total	40	60	100

Solution:

1. Find the probability that they were male, *given* that they answered no.

2. Find the probability that they answered yes, *given* that they were female.

General Multiplication Law of Probability

- $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)}$$

$$\Rightarrow P(A \cap B) = P(B)P(A \mid B) = P(A)P(B \mid A)$$

- $P(C \mid A \cap B) = \dfrac{P(A \cap B \cap C)}{P(A \cap B)}$

$$\Rightarrow P(A \cap B \cap C) = P(A \cap B)P(C \mid A \cap B)) = P(A)P(B \mid A)P(C \mid A \cap B)$$

- **Probabilities of Event Intersections:** The probability of the **intersection of a series of events** $A_1, A_2, \ldots, A_n$ **can** be calculated from the expression

$$P(A_1 \cap \cdots \cap A_n) = P(A_1)P(A_2 \mid A_1) \cdots P(A_n \mid A_1 \cap \cdots \cap A_{n-1})$$

Independence

Definition: Independent Events

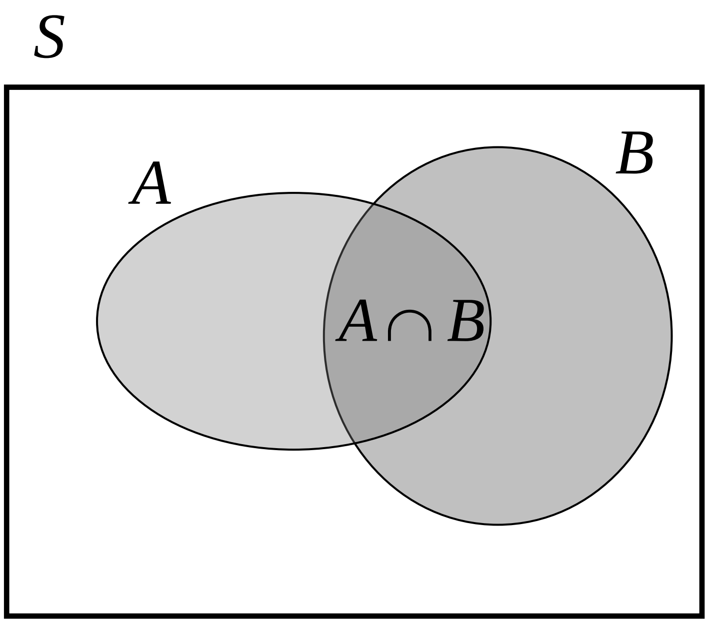
Two events A and B are called **independent** if

$$P[A \cap B] = P[A]P[B]$$

$P(A) = P(A/S)$

$P(A \cap B) = P(A).P(B/A)$

$P(B/A) = P(B/S)$



The ratio of the probability of the set A within B is the same as the ratio of the probability of the set A within the entire sample S.

Note if $P[B] \neq 0$ and $P[A] \neq 0$ then

$$P[A|B] = \frac{P[A \cap B]}{P[B]} = \frac{P[A]P[B]}{P[B]} = P[A]$$

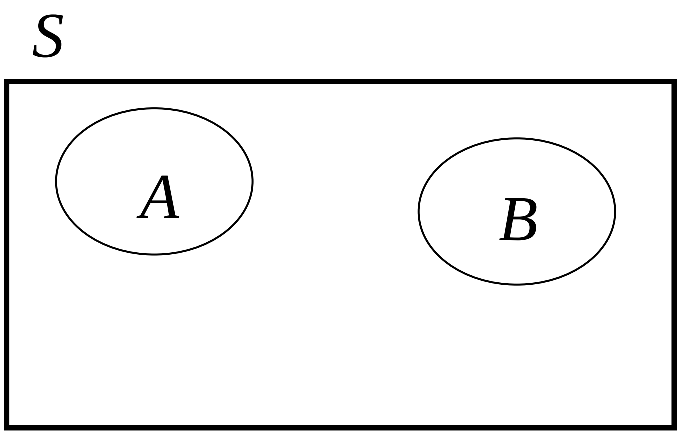
and $$P[B|A] = \frac{P[A \cap B]}{P[A]} = \frac{P[A]P[B]}{P[A]} = P[B]$$

Note: In the case of independence, the conditional probability of an event is not affected by the knowledge of the other event

Difference between Independence and Mutually Exclusive

Two mutually exclusive events are independent only in the special case where

$P[A]=0$ ~~and~~ or $P[B]=0$. (also $P[A\cap B]=0$



I.E.

$P(A\cap B) = P(A).P(B)$

$P(A\cap B) = 0 \quad \because A\cap B = \emptyset$

A and B are M.E.E

The multiplicative Rule of Probability

- **Dependent Events**

$$P[A \cap B] = \begin{cases} P[A]P\left[B|A\right] & \text{if } P[A] \neq 0 \\ P[B]P\left[A|B\right] & \text{if } P[B] \neq 0 \end{cases}$$

- Two events A and B are said to be **independent events** if one of the following holds:

$$P(A \mid B) = P(A), P(B \mid A) = P(B), \text{ and } P(A \cap B) = P(A)P(B)$$

Any one of these conditions implies the other two.

- **Intersections of Independent Events:** The probability of the intersection of a series of **independent events** is simply given by $A_1, A_2, \ldots, A_n$

$$P(A_1 \cap \cdots \cap A_n) = P(A_1)P(A_2)\cdots P(A_n)$$

Example……

- A poll found that 46% of Americans say they suffer from stress. If 3 people are selected at random, find the probability that **all three** will say they suffer from stress.

- **Answer:** Let S = selected Americans having from stress

Given $P(S) = 0.46$

$P(S) \cdot P(S|S) \cdot P(S|S \cap S)$

$P(SSS) = P(S) \cdot P(S) \cdot P(S)$

$= 0.46 \times 0.46 \times 0.46$

$$P(S \, and \, S \, and \, S) = P(S) \cdot P(S) \cdot P(S) \Rightarrow P(Stress) = (0.46)^3 = 0.097$$

Example: Given two fair dices, what is the probability that two dices sum to 8? What is the probability that two dices sum to 8 when the first dice is 3?

Solution:

$S = \{(i,j) \mid i,j = 1(1)6\}$

$= \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6)\}$

$|S| = 36$

Let E = sum is 8

$E = \{(3,5), (4,4), (5,3), (2,6), (6,2)\}$

D_1 = first dice shows no 3 = $\{(3,1), (3,2), \ldots, (3,6)\}$

1) $P(E) = \frac{5}{36}$

2) $P(E/D_1) = \frac{P(E \cap D_1)}{P(D_1)} = \frac{1/36}{6/36} = \frac{1}{6}$

Example: In a group of kids, if one is selected at random the probability that he/she likes oranges is 0.6, the probability that he/she likes oranges AND apples is 0.3. If a kid, who likes oranges, is selected at random, what is the probability that he/she also likes apples?

Solution:

- **Example:** A box contains **2 red**, **3 green** and **5 blue** balls. A ball is selected at random, and its color is noted. Then it is replaced, and another ball is selected, and its color is noted. Find the probability of

  1. Selecting **2 blue** balls.
  2. Selecting **1 blue** and then **a red**
  3. Selecting **1 green** and **a blue**

B G R

Total Balls = 10

- **Solution:**

1) $P(B,B) = P(\text{Two Blue Balls}) = P(B).P(B) = \frac{5}{10} \cdot \frac{5}{10}$

2) $P(B \cap R) = P(B).P(R|B) = P(B).P(R) = \frac{5}{10} \cdot \frac{2}{10}$

3) $P(G,B) = P(G).P(B) = \frac{3}{10} \cdot \frac{5}{10}$

$= P(GB \text{ or } BG) = P(GB) + P(BG) = 2 \cdot \frac{3}{10} \cdot \frac{5}{10}$

- **Example:** A die is tossed 3 times
  1) find the probability of getting three 6's
  2) find the probability of two 6's and one 5
- **Solution:** $|S| = 6^3 = 6 \times 6 \times 6$

1) $P(666) = \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6}$

2) $P(665 \text{ or } 566 \text{ or } 656)$

$= P(665) + P(566) + P(656)$

$= \left(\frac{1}{6}\right)^3 + \left(\frac{1}{6}\right)^3 + \left(\frac{1}{6}\right)^3 = 3\left(\frac{1}{6}\right)^3$

- **Example:** In studying the causes of power failures these data have been gathered. 5% are due to transformer damage; 80% are due to line damage, 1% due to both the problems. Based on these information what is the probability that a given power failure involves
  1. Line damage given that there is a transformer damage
  2. Transformer given that there is a line damage
  3. Transformer damage but not line damage
  4. Transformer damage given that there is no line damage

$\left[P(A') = 1 - P(A) \Rightarrow P(A'|S) = 1 - P(A|S) \right]$

- **Solution:**

Let T = Power failure due to transformer damages

L = " " " " line "

Given $P(T) = 0.05$ $P(L) = 0.8$ $P(L \cap T) = 0.01$

1) $P(L|T) = \frac{P(L \cap T)}{P(T)} = \frac{0.01}{0.05} = 1/5$

2) $P(T|L) = P(T \cap L)/P(L) = \frac{0.01}{0.8} = \frac{1}{80}$

3) $P(T \cap L') = P(T) \cdot P(L'|T) = P(T)\left[1 - P(L|T)\right] = 0.05 \times (1 - 1/5)$

$P(L') \cdot P(T|L') = (1 - P(L)) \cdot (1 - P(\cancel{T}|L'))$

4) $P(T|L') = \frac{P(T \cap L')}{P(L')} = \frac{0.05 \times ...}{1 - 0.8}$

- **Example:** **One box** contains 6 Red, 4 Green balls and **second box** contains 7 Red, 3 Green balls. A ball is randomly chosen from the first box and placed in the second box then a ball is randomly selected from the second box and placed in the first box.
  1. What is the probability that a red ball is selected from 1st box and red ball is selected from second box.
  2. As the conclusion of the selection process what is the probability that the number of red and green balls in the first box are identical to the no. at the beginning.

6R
4G
B1

7R
3G
B2

R_1 $R_2 \rightarrow$ - - - -

Reed ball is selected from Box 1

- **Solution:**

1) $P(R_1 \cap R_2) = P(R_1) \cdot P(R_2 | R_1)$

$= \frac{6}{10} \cdot \frac{8}{11}$

2) $P(R_1 R_2 \text{ OR } G_1 G_2) = P(R_1 R_2) + P(G_1 G_2)$

$= \frac{6}{10} \cdot \frac{8}{11} + \frac{4}{10} \cdot \frac{4}{11}$

$= \boxed{}$

- https://towardsdatascience.com/12-probability-practice-questions-for-data-science-interviews-2ec5230304d9

Thank You