Statistics Modeling
Assignment - 03

Q.1 For the following data series, find the first four lags of autocovariance (ɤ0 , ɤ1, ɤ2, ɤ3) and auto correlation function (p0 , p1 , p2, p3) manually.

1	2	3	4	5	6	7	8	9	10
2.34	1.47	1.13	0.62	1.51	0.78	-0.02	1.04	0.80	1.42

```
Q1 = (2.34, 1.47, 1.13, 0.62, 1.51, 0.78, -0.02, 1.04, 0.80, 1.42)
Q1
n = len(Q1)
total = sum(Q1)
mean = total/n
print('number =', n)
print('mean =', mean)
print('total =', total)
```

```
number = 10
mean = 1.1090000000000002
total = 11.090000000000002
```

```
# Calculating autocovariance
ɤ0 = sum([(Q1[i] - mean)**2 for i in range(n)])/n
ɤ1 = sum([(Q1[i] - mean) * (Q1[i-1] - mean) for i in range(1, n)])/n
ɤ2 = sum([(Q1[i] - mean) * (Q1[i-2] - mean) for i in range(2, n)])/n
ɤ3 = sum([(Q1[i] - mean) * (Q1[i-3] - mean) for i in range(3, n)])/n

# Calculating autocorrelation
p0 = 1
p1 = ɤ1/ɤ0
p2 = ɤ2/ɤ0
p3 = ɤ3/ɤ0

print("Autocovariance: ", ɤ0, ɤ1, ɤ2, ɤ3)
print("Autocorrelation: ", p0, p1, p2, p3)
```

<u>Output:</u>

```
Autocovariance:  0.36258899999999994 0.048824900000000025
-0.008400199999999988 -0.018915299999999995
Autocorrelation:  1 0.13465631886240353 -0.023167277551166672
-0.052167329952094395
```

1	2	3	4	5	6	7	8	9	10
2.34	1.47	1.13	0.62	1.51	0.78	-0.02	1.04	0.80	1.42

Auto correlation coefficient $= \rho_k = \dfrac{\gamma_k}{\gamma_0}$

mean $(\bar{y}) = \dfrac{2.34 + 1.47 + 1.13 + 0.62 + 1.51 + 0.78 - 0.02 + 1.04 + 0.80 + 1.42}{10}$

$\bar{y} = 1.109$

$$\gamma_k = \frac{1}{n} \sum_{i=1}^{n-k} (y_i - \bar{y})(y_{i+k} - \bar{y})$$

$$\gamma_0 = \frac{1}{10}\Big[(2.34-1.109)^2 + (1.47-1.109)^2 + (1.13-1.109)^2 + (0.62-1.109)^2 + (1.51-1.109)^2 + (0.78-1.109)^2 + (-0.02-1.109)^2 + (1.04-1.109)^2 + (0.80+1.109)^2 + (1.42-1.109)^2\Big]$$

$$= \frac{1}{10}(3.6258) = 0.363$$

$$\gamma_1 = \frac{1}{10}\Big[(2.34-1.109)(1.47-1.109) + (1.47-1.109)(1.13-1.109) + (0.62-1.109)(1.51-1.109) + (1.13-1.109)(0.62-1.109) + (1.51-1.109)(0.78-1.109) + (0.78-1.109)(-0.02-1.109) + (-0.02-1.109)(1.04-1.109) + (1.04-1.109)(0.80-1.109) + (0.80-1.109)(1.42-1.109)\Big]$$

$$= \frac{1}{10}(0.4882)$$

$$= 0.0488$$

$$\gamma_2 = \frac{1}{10}\Big[(2.34-1.109)(1.13-1.109) + (1.47-1.109)(0.62-1.109) + (1.13-1.109)(1.51-1.109) + (0.62-1.109)(0.78-1.109) + (1.51-1.109)(-0.02-1.109) + (0.78-1.109)(1.04-1.109) + (-0.02-1.109)(0.80-1.109) + (1.04-1.109)(1.42-1.109)\Big]$$

$$= \frac{1}{10}(-0.0842)$$

$$= -0.0084$$

$$\gamma_3 = \frac{1}{10}\Big[(2.34-1.109)(0.62-1.109) + (1.47-1.109)(1.51-1.109) + (1.13-1.109)(0.78-1.109) + (0.62-1.109)(-0.02-1.109) + (1.15-1.109)(1.04-1.109) + (0.78-1.109)(0.80-1.109) + (-0.02-1.109)(1.42-1.109)\Big]$$

$$= \frac{1}{10}(-0.189) = -0.0189$$

Using $\gamma_0, \gamma_1, \gamma_2, \gamma_3$ values,

Calculate the Autocorrelation function (ACF),

$$\rho_0 = 1$$

$$\rho_1 = \frac{\gamma_1}{\gamma_0} = \frac{0.0488}{0.363} = 0.1344$$

$$\rho_2 = \frac{\gamma_2}{\gamma_0} = \frac{-0.0842}{0.363} = -0.0231$$

$$\rho_3 = \frac{\gamma_3}{\gamma_0} = \frac{-0.0189}{0.363} = -0.0521$$

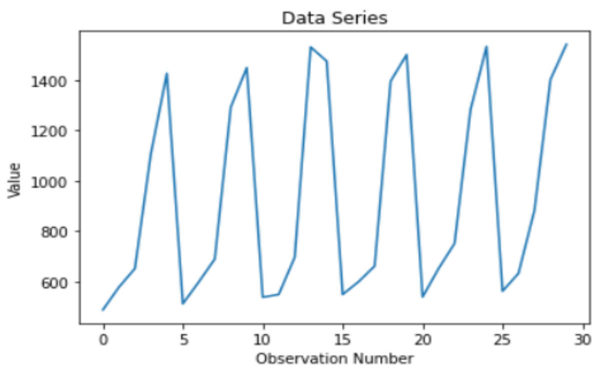
Q.2 For the following data series of 30 observations, plot ACF and PACF and Identify the suitable model and Coefficients (Hint: can use auto_arima method)?

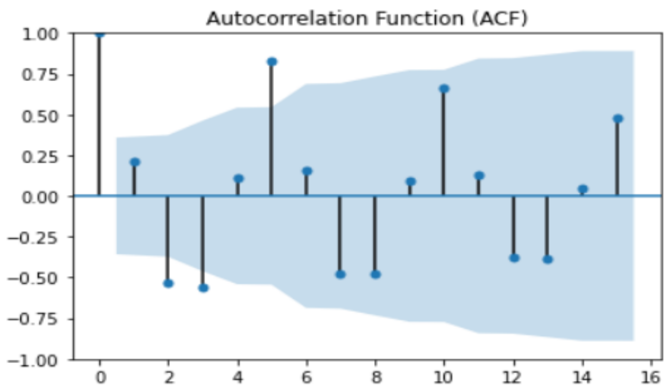
487	577	651	1107	1427
511	598	689	1293	1450
537	548	696	1532	1476
548	599	661	1396	1502
538	651	751	1283	1534
561	632	883	1403	1543

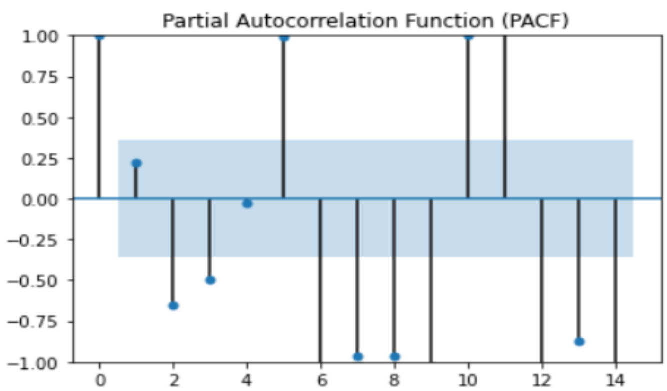
```python
Q2 = pd.DataFrame({'a':[487, 577, 651, 1107, 1427,
                        511, 598, 689, 1293, 1450,
                        537, 548, 696, 1532, 1476,
                        548, 599, 661, 1396, 1502,
                        538, 651, 751, 1283, 1534,
                        561, 632, 883, 1403, 1543]})

x = [487, 577, 651, 1107, 1427,
     511, 598, 689, 1293, 1450,
     537, 548, 696, 1532, 1476,
     548, 599, 661, 1396, 1502,
     538, 651, 751, 1283, 1534,
     561, 632, 883, 1403, 1543]

# Plot the data series
plt.plot(x)
plt.xlabel("Observation Number")
plt.ylabel("Value")
plt.title("Data Series")
plt.show()

# Plot the autocorrelation function
plot_acf(Q2, title="Autocorrelation Function (ACF)")
plt.show()

# Plot the partial autocorrelation function
plot_pacf(Q2,lags = 14, title = "Partial Autocorrelation Function (PACF)")
plt.show()

# Use the auto_arima method to fit the ARIMA model
model = auto_arima(x, suppress_warnings=True, error_action="ignore")
```

Summary:

```python
# Print the ARIMA model coefficients
print("Best ARIMA model:", model.summary())
```







The intercept coefficient of 935.4667 indicates a baseline level of the data, and the non-significant variance suggests that the data is relatively stable.

The tests showed that there is no significant autocorrelation, skewness, kurtosis in the data, indicating that the model is a good fit for the data series.

The ACF and PACF plots showing that there are some significant lags in series.

Q.3 For the following data , Add some dummy dates, do some analysis like Visualization and ADF-test and also list out your observation from data. (submit screenshots as well)

2.34	-0.02	-0.96	2.70	0.79
1.47	1.04	0.29	2.63	1.89
1.13	0.80	2.56	2.44	4.36
0.62	1.42	3.33	1.38	2.23
1.51	1.15	3.74	1.11	2.19
0.78	1.57	2.88	1.10	0.59

```python
import pandas as pd
import matplotlib.pyplot as plt
from statsmodels.tsa.stattools import adfuller

x = [2.34 ,-0.02, -0.96, 2.70 ,0.79 ,
     1.47, 1.04, 0.29, 2.63 ,1.89,
     1.13 ,0.80, 2.56, 2.44, 4.36,
     0.62, 1.42, 3.33, 1.38, 2.23,
     1.51, 1.15, 3.74, 1.11, 2.19,
     0.78, 1.57, 2.88, 1.10, 0.59]

#Q3=pd.DataFrame(x)
dates = pd.date_range("2021-03-03", periods=30, freq="M")
Q3 = pd.DataFrame(x, index=dates, columns=["Value"])
print(Q3)

# Plot the data
plt.figure(figsize=(20,5))
plt.plot(Q3)
plt.xlabel("Date")
plt.ylabel("Value")
plt.title("Data Series")
plt.show()

def adf_test(series,title=''):
    """
    Pass in a time series and an optional title, returns an ADF report
    """
    print(f'Augmented Dickey-Fuller Test: {title}')
    result = adfuller(series.dropna(),autolag='AIC') # .dropna() handles differenced data

    labels = ['ADF test statistic','p-value','# lags used','# observations']
    out = pd.Series(result[0:4],index=labels)
```

```python
    for key,val in result[4].items():
        out[f'critical value ({key})']=val

    print(out.to_string())          # .to_string() removes the line "dtype: float64"

    if result[1] <= 0.05:
        print("Strong evidence against the null hypothesis")
        print("Reject the null hypothesis")
        print("Data has no unit root and is stationary")
    else:
        print("Weak evidence against the null hypothesis")
        print("Fail to reject the null hypothesis")
        print("Data has a unit root and is non-stationary")
```

```
            Value
2021-03-31   2.34
2021-04-30  -0.02
2021-05-31  -0.96
2021-06-30   2.70
2021-07-31   0.79
2021-08-31   1.47
2021-09-30   1.04
2021-10-31   0.29
2021-11-30   2.63
2021-12-31   1.89
2022-01-31   1.13
2022-02-28   0.80
2022-03-31   2.56
2022-04-30   2.44
2022-05-31   4.36
2022-06-30   0.62
2022-07-31   1.42
2022-08-31   3.33
2022-09-30   1.38
2022-10-31   2.23
2022-11-30   1.51
2022-12-31   1.15
2023-01-31   3.74
2023-02-28   1.11
2023-03-31   2.19
2023-04-30   0.78
2023-05-31   1.57
2023-06-30   2.88
2023-07-31   1.10
2023-08-31   0.59
```

```
adf_test(Q3['Value'])
```

```
Augmented Dickey-Fuller Test:
ADF test statistic       -1.805622
p-value                   0.377674
# lags used              6.000000
# observations           23.000000
critical value (1%)      -3.752928
critical value (5%)      -2.998500
critical value (10%)     -2.638967
Weak evidence against the null hypothesis
Fail to reject the null hypothesis
Data has a unit root and is non-stationary
```

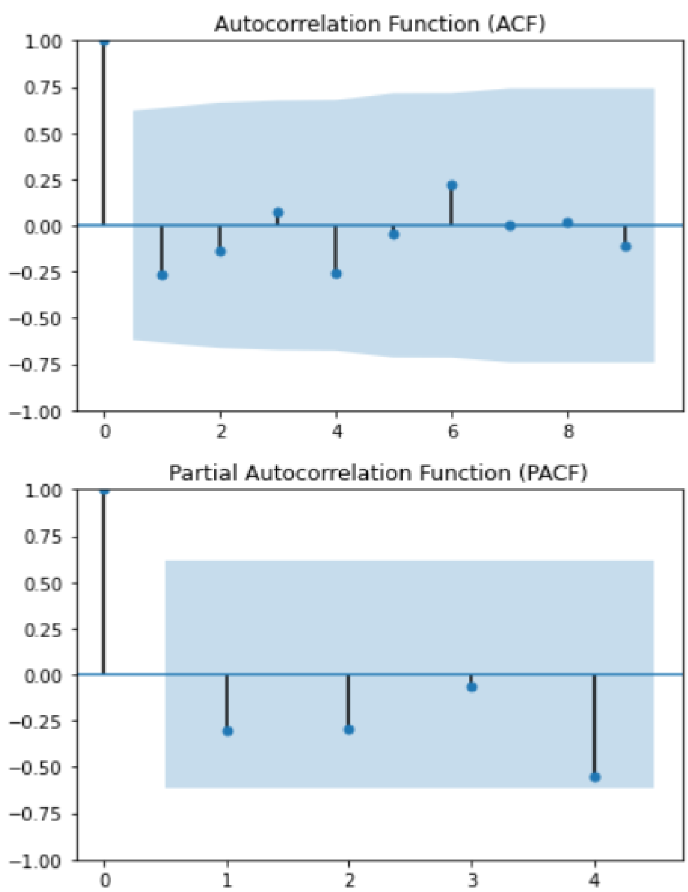
The given data represents a time series of 30 monthly observations from March 2021 to August 2023. We have added dummy dates to represent the time axis. The time series plot shows some variability in the values over time, with some fluctuations around the mean.

The results of the Augmented Dickey-Fuller (ADF) test show that the time series has a p-value of 0.377674, which is greater than the significance level of 0.05. This indicates that we fail to reject the null hypothesis of the ADF test, which suggests that the data has a unit root and is non-stationary. Therefore, the data is not stationary and we should not perform any time series modeling or forecasting directly on the original data.

Q.4 For the given set of data, 3.65, 8.03, 5.72, 4.93, 5.71, 4.79, 4.87, 6.48, 6.40, 6.41 find the order of auto_arima model and check whether it is like ACF and PACF plots observations. If, not same then compare the two models and specify which is the better model and how we decide..

```
y = np.array([3.65, 8.03, 5.72, 4.93, 5.71, 4.79, 4.87, 6.48, 6.40, 6.41])
Q4 =pd.DataFrame({'a':[3.65, 8.03, 5.72, 4.93, 5.71, 4.79, 4.87, 6.48, 6.40, 6.41]})

model = auto_arima(y, suppress_warnings=True, error_action="ignore")
print("Best ARIMA model:", model.summary())

# Plot the autocorrelation function
plot_acf(Q4, title="Autocorrelation Function (ACF)")
plt.show()

# Plot the partial autocorrelation function
plot_pacf(Q4, lags=4, title="Partial Autocorrelation Function (PACF)")
plt.show()
```

```
Best ARIMA model:                               SARIMAX Results
==============================================================================
Dep. Variable:                      y   No. Observations:                   10
Model:                        SARIMAX   Log Likelihood                 -15.632
Date:                Tue, 31 Jan 2023   AIC                             35.264
Time:                        05:48:14   BIC                             35.869
Sample:                             0   HQIC                            34.600
                                 - 10
Covariance Type:                  opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
intercept      5.6990      0.371     15.378      0.000       4.973       6.425
sigma2         1.3344      0.642      2.080      0.038       0.077       2.592
===================================================================================
Ljung-Box (L1) (Q):                   0.98   Jarque-Bera (JB):                 0.10
Prob(Q):                              0.32   Prob(JB):                         0.95
Heteroskedasticity (H):               0.17   Skew:                             0.22
Prob(H) (two-sided):                  0.18   Kurtosis:                         2.78
===================================================================================
```



The ACF and PACF plots of the given data indicate that there is no clear pattern that can be used to identify the order of the ARIMA model. Therefore, the auto_arima function can be useful for selecting the best model.

The selected SARIMAX model giving a good fit for the data, as it has a low AIC value and a low p-value for the Ljung-Box test.
The low p-value indicates that there is no evidence of autocorrelation in the residuals. The model has also passed the normality test.