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Amrita School of Engineering, Bengaluru
I Sem M. Tech. DS
Computational Linear Algebra-21MA602
Lab exercise-1

1. Consider the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

We can see that columns of A are linearly dependent. Hence any column vector can be expressed in terms of the others. The following code express column b in terms of others using 'pinv' command.

```
a=[1; 1 ;0;0];
b=[0; 1 ;1;0];
c=[0; 0 ;1;1];
d=[1; 0 ;0;1];
A=[a b c d];
B=[ a c d];
  coef= pinv(B)*b
      The output would be
coef =

    1.0000
    1.0000
   -1.0000
```

that is b=1*a+1*b-1*c.

Alter the above code suitably to express the remaining 3 vectors in terms of the other.

2. Find the CR decomposition of

A= $\begin{pmatrix} 1 & 3 & 3 \\ 2 & 6 & 3 \\ 3 & 9 & 2 \end{pmatrix}$ using rref command in MATLAB

3. What is the basis of row and column space of A= $\begin{pmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 6 & 8 \end{pmatrix}$?
4. Program segment to append columns to a given matrix and find its rank.

```
X=[0 9 ]
A=[]
A=[A  randi(X, 3, 1)]
Rank(A)          % ouput is 1
    A=[A randi(X, 3, 1)]
Rank(A)          % ouput is 2
```

.

. . continue…

What do you observe? Why?

Alter the code suitably to append a column each time and find the rank.

5. Program to generate a random integer matrix with given rank :
   R=[0 9]
   A= randi(R, 5, 3)*randi(R, 3, 6) % generates a matrix whose rank is <=3. Mostly 3.
   Alter this to generate a matrix of order 1.
   Use the 'null' command to get the null space of the matrix generated. Verify manually if t is the correct null space of A.

6. If $A = \begin{bmatrix} 1 & 3 & 4 & 7 \\ 2 & 4 & 6 & 10 \\ 3 & 5 & 8 & 13 \\ 4 & 6 & 10 & 16 \end{bmatrix}$

   Is Y= (1, 2, 3, 1)^T in row/ column/left or right null space of A?

Hint: If AY ! = 0 it is not in RNS

IF Y^TA != 0 it is not in LNS

Append Y as last column of A. If rank(A) = 2 then Y is in column space of A.

If rank(A)>2 then it is not.

Append Y as last row of A. If rank(A) = 2 then Y is in row space of A.

If rank(A)>2 then it is not.

Analyze why?

.

```
% Question 01:-

a = [1; 1 ;0;0];
b = [0; 1 ;1;0];
c = [0; 0 ;1;1];
d = [1; 0 ;0;1];
A = [a b c d];
B = [ a c d];
coef = pinv(B)*b
```

```
coef = 3×1
    1.0000
    1.0000
   -1.0000
```

```
coef
```

```
coef = 3×1
    1.0000
    1.0000
   -1.0000
```

```
% Expressing vector 'c' in terms of the other vectors
B = [a b d];
coef_c = pinv(B)*c;
coef_c
```

```
coef_c = 3×1
   -1.0000
    1.0000
    1.0000
```

```
% Expressing vector 'd' in terms of the other vectors
B = [a b c];
coef_d = pinv(B)*d;
coef_d
```

```
coef_d = 3×1
    1.0000
   -1.0000
    1.0000
```

```
% Expressing vector 'a' in terms of the other vectors
B = [b c d];
coef_a = pinv(B)*a;
coef_a
```

```
coef_a = 3×1
    1.0000
   -1.0000
    1.0000
```

```
% The value of coef_c will be [1.0000, -1.0000, 1.0000] and c=1a-1b+1*d
% The value of coef_d will be [1.0000, -1.0000, 1.0000] and d=1a-1b+1*c
% The value of coef_a will be [1.0000, 1.0000, -1.0000] and a=1b+1c-1*d
```

```
clc
A = [1 3 3;2 6 3;3 9 2]
```

```
A = 3×3
     1     3     3
     2     6     3
     3     9     2
```

```
B = rref(A)
```

```
B = 3×3
     1     3     0
     0     0     1
     0     0     0
```

```
x = A(:,1);
y = A(:,2);
z = A(:,3);
calc_Dependent_x_y = rank([x,y])==rank(x)
```

```
calc_Dependent_x_y = logical

   1
```

```
calc_Dependent_x_z = rank([x,z])==rank(x)
```

```
calc_Dependent_x_z = logical

   0
```

```
calc_Dependent_y_z = rank([y,z])==rank(y)
```

```
calc_Dependent_y_z = logical

   0
```

```
if calc_Dependent_x_y == 0
    if calc_Dependent_x_z == 0
        C = [x y z]
    else
        C = [x y]
    end
elseif calc_Dependent_x_z == 0
    C = [x z]
end
```

```
C = 3×2
     1     3
     2     3
     3     2
```

```
r = B(any(B,2),:);
R = transpose(r)
```

```
R = 3×2
     1     0
     3     0
```

```
     0     1
```

```
D = C*r
```

```
D = 3×3
     1     3     3
     2     6     3
     3     9     2
```

```
if D == A
    disp('A = CR, Checked')
else
    disp('wrong answer')
end
```

```
A = CR, Checked
```

```
A = [1 2 3; 3 4 5; 4 6 8];

% Find the reduced row echelon form of A
[R, jb] = rref(A);
R, jb
```

```
R = 3×3
     1     0    -1
     0     1     2
     0     0     0
jb = 1×2
     1     2
```

```
% Find a basis for the row space of A
row_basis = A(find(jb), :);
row_basis
```

```
row_basis = 2×3
     1     2     3
     3     4     5
```

```
% Find a basis for the column space of A
col_basis = A(:, find(jb));
col_basis
```

```
col_basis = 3×2
     1     2
     3     4
     4     6
```

```
%  Find the indices of the free variables
free_vars = setdiff(1:size(A, 2), jb);
% Find a basis for the null space of A
null_basis = A(:, free_vars);
null_basis
```

```
null_basis = 3×1
     3
     5
     8
```

```
% Question 04:-

clc
X=[0 9 ]
```

```
X = 1×2
     0     9
```

```
A=[]
```

```
A =

     []
```

```
for i = 1:X(2)
    A=[A randi(X, 3, 1)]
    disp('The iteration is ')
    disp([i])
    Rank = rank(A)            % ouput is 1
end
```

```
A = 3×1
     7
     7
     2
The iteration is
     1
Rank = 1
A = 3×2
     7     6
     7     6
     2     1
The iteration is
     2
Rank = 2
A = 3×3
     7     6     1
     7     6     4
     2     1     9
The iteration is
     3
Rank = 3
A = 3×4
     7     6     1     3
     7     6     4     5
     2     1     9     2
The iteration is
     4
Rank = 3
A = 3×5
     7     6     1     3     7
     7     6     4     5     2
     2     1     9     2     5
The iteration is
     5
Rank = 3
A = 3×6
     7     6     1     3     7     6
     7     6     4     5     2     8
     2     1     9     2     5     9
```

```
The iteration is
     6
Rank = 3
A = 3×7
     7     6     1     3     7     6     5
     7     6     4     5     2     8     1
     2     1     9     2     5     9     1
The iteration is
     7
Rank = 3
A = 3×8
     7     6     1     3     7     6     5     2
     7     6     4     5     2     8     1     8
     2     1     9     2     5     9     1     2
The iteration is
     8
Rank = 3
A = 3×9
     7     6     1     3     7     6     5     2     8
     7     6     4     5     2     8     1     8     2
     2     1     9     2     5     9     1     2     9
The iteration is
     9
Rank = 3
```

```
% Question 05:-

clc
R=[0 9]
```

```
R = 1×2
     0     9
```

```
A= randi(R, 5, 3)*randi(R, 3, 6)  % generates a matrix whose rank is <=3. Mostly 3.
```

```
A = 5×6
    76    68   104    52    24    40
    41    49    68    37    14    34
    81    54    99    45    27    18
    11    16    21    12     4    12
    25    47    74    43    18    22
```

```
rank(A)
```

```
ans = 3
```

```
B= randi(R, 5, 1)*randi(R, 1, 6)  % generates a matrix whose rank is <=1. Mostly 1.
```

```
B = 5×6
    81    27     9    63    27    18
    81    27     9    63    27    18
    36    12     4    28    12     8
    36    12     4    28    12     8
    27     9     3    21     9     6
```

```
rank(B)
```

```
ans = 1
```

```
n = null(B,'r')
```

```
n = 6×5
   -0.3333   -0.1111   -0.7778   -0.3333   -0.2222
    1.0000         0         0         0         0
         0    1.0000         0         0         0
         0         0    1.0000         0         0
         0         0         0    1.0000         0
         0         0         0         0    1.0000
```

```
A=[1,3,4,7;2,4,6,10;3,5,8,13;4,6,10,16];
y=[1,2,3,1];
transpose(y)
```

```
ans = 4×1
     1
     2
     3
     1
```

```
answer =A*transpose(y)
```

```
answer = 4×1
    26
    38
    50
    62
```

```
if(answer~=0)
    disp('not in RNS');
end
```

```
not in RNS
```

```
ans1=y*A;
if(ans1 ~= 0)
    disp('not in LNS');
end
```

```
not in LNS
```

```
A=[A transpose(y)];
disp(A)
```

```
     1     3     4     7     1
     2     4     6    10     2
     3     5     8    13     3
     4     6    10    16     1
```

```
rank(A)
```

```
ans = 3
```

```
rref(A)
```

```
ans = 4×5
     1     0     1     1     0
     0     1     1     2     0
     0     0     0     0     1
     0     0     0     0     0
```

```
if(rank(A)>2)
    disp('not in column space');
end
```

```
not in column space
```

```
A=[1,3,4,7;2,4,6,10;3,5,8,13;4,6,10,16];
```

```
y=[1,2,3,1] ;
A=[A; y];
disp(A)
```

```
    1     3     4     7
    2     4     6    10
    3     5     8    13
    4     6    10    16
    1     2     3     1
```

```
rank(A)
```

```
ans = 3
```

```
if(rank(A)>2)
    disp('not in row space');
end
```

```
not in row space
```