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I Sem M. Tech. DS
Computational Linear Algebra-21MA602
Lab exercise-2

- Using MATLAB we can find the QR decomposition of a matrix A by: >> [Q,R]=qr(A,0)

- QR Decomposition for finding eigenvalues of matrices numerically :

```
clc;
clear all;
A=randi(10,2,2)
eig(A)
B=A;
for i= 1:10
[Q R]=qr(B,0) % B=QR.
B=R*Q;
```

%Creating a new B=RQ and repeat the computation. B=RQ retains eigenvalues of A (as RQ is similar to A) but approaches to an upper triangular matrix. That is, diagonal elements of B approaches to eigenvalues of A.

```
end
diag(B)
```

%Note that eig(A) and diag(B) are almost the same values.

- Singular Value Decomposition using MATLAB

>>[U,Z,V] = svd(A)

% produces a diagonal matrix Z, of the same dimension as A and with nonnegative diagonal elements in decreasing order, and unitary matrices U and V so that A = U*Z*V'

>>Z = svd(A)

% returns a vector containing the singular values.

- Understanding of SVD through computational experiments using MATLAB

1. $First\ find\ SVD\ \ of\ \ A = \begin{pmatrix} 9 & 10 \\ 10 & 7 \\ 2 & 1 \end{pmatrix}$

>> format bank % display results in 2 decimal places
>>A=[9 10; 10 7; 2 1];
>>[U Z V]=svd(A); % full SVD

2. Verify that U and V are orthogonal by evaluating the following:

>> U'*U
>>U*U'
>> V'*V
>>V* V'

3.
Compute eigenvectors of A^TA and compare with the columns of V.

```
>> [V1, L1]=eig(A'*A)
>>V
```

% Notice that the column vectors of V1 and V are the same, that is the eigenvectors of A^TA.

4. Compute eigenvectors of AA^T and compare with the columns of U.

```
>> [U1, L]=eig(A*A')
>>U
```

% Notice that the column vectors of U1 and U are the same, that is the eigenvectors of AA^T.

5. Compute the eigenvalues of A^TA and AA^T and compare with the singular values of A.

```
>> eig(A'*A)
>>eig(A*A')
>>S=diag(Z) % singular values of A are diagonal elements of Z
>>Ssquare=S.^2 % square of singular values
```

% Notice that the square of singular values of A are same as the non-zero eigenvalues of A^TA and AA^T.

- Find the rank and nullity of the matrix A.

  ```
  >> r=rank(A); m=size(A); nullity=m(1)-r
  ```

- Verify that the third column in U forms a basis for the left nullspace of A.

  ```
  >> A'*U(:,3) .
  ```

  % Notice that this is a zero vector, hence column vectors of A are orthogonal to third column in U and hence it is a basis for left nullspace

- Given the SVD of a matrix is $A=U\Sigma V^T$, the pseudo-inverse of A can be decomposed as $A^+ = V\,\Sigma^+\,U^T$

  ```
  >> A=[1,1,0;0,1,1];[ u,z,v]=svd(A)
  >> pseudoA=pinv(A);
  >> v*pinv(z)*u' % Verify if this is same as pseudoA
  ```

1. Using QR decomposition find the eigenvalues of a random integer symmetric matrix of order 15 and verify the result using direct evaluation of eigenvalues.

2. Generate a random integer 7 by 9 matrix A of rank 6.
(a) Verify the rank of A is 6
(b) Find the SVD of the matrix
(c) Using SVD find an orthonormal basis for all the fundamental subspaces of the matrix A
(d) Verify the results obtained in (c) by checking the orthogonality of RS & NS and CS & LNS.
(e) Find the singular values of A .