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Joint Optimization of Plant, Controller, and Sensor/Actuator Design

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***Abstract*— This paper presents a Linear Matrix Inequality (LMI) framework for selecting sensor and actuator precision jointly with the determination of control law and free structure parameters. The sub-optimal solution of this non-convex system design problem is found by iterating over an approximated convex problem through the use of a convexifying potential function which enables convergence to a local minimum. The authors believe this is the first time an integrated approach combining structure design, control law, and information architecture is developed to further advance the theory of system-level design optimization.**

I. INTRODUCTION

It is well known that the design of a structure and selection of its controller are not two independent problems. They should be integrated such that the dynamics of the controller and the structure cooperate to meet performance objectives. Similarly, controller design (control algorithm) and signal processing (actuator and sensor precision) problems should be integrated to determine the required precision of sensors and actuators to guarantee desired output covariance bound. Research is needed to produce system designs where all components of the system are cooperatively designed to yield a specified system performance. The work of Li *et al.* [1] integrated control design and selection of information architecture (actuator and sensor precision) to meet specified performance requirement (output covariance upper bound) formulating the constraints in Linear Matrix Inequalities (LMIs) [2]. It is proven that integration of information architecture and control design is a convex problem for a linear plant with full-state feedback or full-order output feedback. Li *et al.* [1] also provides an ad hoc algorithm to reduce the set of required sensors or actuators. This is accomplished by repeatedly deleting the sensors or actuators with the least precision required until the design requirements cannot be met. Radhika *et al.* [3] made advancements in the information architecture theory by adding model uncertainty. More recently, various researchers have looked at the problem of finding a smaller set of sensors from a larger admissible set if precision is given a priori [4], [5].

Some researchers provided a framework to solve the instrument and control design problem [6] while others discussed the problem of integrated structure and control design assuming sensor and actuator precisions are known [7]–[9]. Grigoriadis *et al.* [9] provided a two-step solution to the simultaneous design problem of structure and control by iterating over two convex sub-problems. First, the structure parameters were fixed and a controller was designed to satisfy the specified output covariance upper bound. In the second step, both the structure parameters and the controller are optimized such that the state covariance from the previous step was preserved. The algorithm iterates between these two steps to converge to a local minimum. Lu *et al.* [7] considered the more general structure parameterization and used mixed H_2/H_∞ performance criteria. However, this approach also constrained the closed loop state covariance matrix to be preserved in the second step. These approaches find a solution in reduced domain space, which may not necessarily be an optimal solution to the combined problem. Another approach to solving the integrated problem is by convexifying LMIs methods [10], [11]. In this approach, the authors first formulate a nonlinear matrix inequality to satisfy the performance requirement and then add another nonlinear matrix inequality to finally generate an LMI. There are some conditions on the added nonlinear matrix inequality (convexifying potential function) which guarantees that the solution will reach a stationary point.

The authors believe that this is the first time one integrated domain formulation is developed to solve for all three decision variables i.e. control design, information architecture, and structure design simultaneously. The formulation of this paper is as follows: first, we describe a linear system in descriptor state-space form with plant matrices affine in structure parameters. We assume noisy actuators and measurement sensors but the intensity of those noises are optimization variables (specifically we define the inverse of noise to be the precision and solve for

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the precision). The cost of actuators and sensors is assumed to be directly proportional to their precision to add a budget constraint in the problem. Then, we write matrix inequalities to satisfy output and control covariance constraints [2]. The simultaneous design of a structure, information architecture and controller results in nonlinear matrix inequalities even for linear systems. As the domain set is nonlinear inequalities (not proven to be non-convex), we add a convexifying potential function to the original problem to create a convex subproblem, on which we iterate to solve for the stationary optimum solution.

II. Dynamic Compensation Problem

A. Problem Statement

A continuous linear time-invariant system is described by the following *descriptor* state-space representation:

$$E(\alpha)\dot{x} = A(\alpha)x + Bu + D_p(\alpha)w_p + D_a(\alpha)w_a, \quad (1)$$

$$y = C_y(\alpha)x, \quad \text{(output)} \quad (2)$$

$$z = C_z x + D_s w_s, \quad \text{(measurement)} \quad (3)$$

where $x \in \mathbb{R}^n$ is the state of the system, $u \in \mathbb{R}^m$ is the control vector, $y \in \mathbb{R}^p$ is the output of the system, $z \in \mathbb{R}^\ell$ denotes the measurement vector, and w_i for $i = a, s, p$ are noisy inputs to the system. The vector α consists of generalized variable structure parameters that can be treated as decision variables in the system design problem. It is assumed that the following matrices are affine in the parameters α: $A(\alpha), E(\alpha), D_p(\alpha), D_a(\alpha)$ and $C_y(\alpha)$. Note that in a typical second order system, it is paramount to adopt the descriptor representation in order to preserve the affine property of the system mass matrix. The matrix $E(\alpha)$ is also assumed to be full rank.

In the above model (1-3), the actuator noise is defined by w_a, sensor noise by w_s, and ambient process noise by w_p. These vectors are modeled as independent zero mean white noises with intensities W_a, W_s and W_p, respectively, i.e.:

$$\mathbb{E}_\infty(w_i) = 0, \quad (4)$$

$$\mathbb{E}_\infty(w_i w_i^T) = W_i \delta(t-\tau), \quad (5)$$

where $i = a, s, p$, and $\mathbb{E}_\infty(x) = \lim_{t\to\infty} \mathbb{E}(x)$ which denotes the asymptotic expected value of the random variable x. We assume the process noise intensity W_p to be known and fixed. The actuator and sensor precisions are defined to be inversely proportional to the respective noise intensities.

$$\Gamma_a \triangleq W_a^{-1}, \quad \Gamma_s \triangleq W_s^{-1}. \quad (6)$$

We also define the vectors γ_a and γ_s such that:

$$\Gamma_a \triangleq \text{diag}(\gamma_a), \quad \Gamma_s \triangleq \text{diag}(\gamma_s). \quad (7)$$

As defined in [1], we associate a price to each actuator/sensor that is inversely proportional to the noise intensity associated with that instrument. In this work, prices associated with the structure parameters are also considered. Therefore, the total design price can be expressed as:

$$\$ = p_a^T\gamma_a + p_s^T\gamma_s + p_\alpha^T\alpha, \quad (8)$$

where p_a, p_s and p_α are vectors containing the price per unit of actuator precision, sensor precision and price per unit of structure parameter, respectively. Now, the problem to be solved is defined as:

Design a dynamic compensator of the form:

$$\begin{aligned} \dot{x}_c &= A_c x_c + B_c z, \\ u &= C_c x_c + D_c z, \end{aligned} \quad (9)$$

and simultaneously select the structure parameter values, appropriate actuator and sensor precisions such that the following constraints are satisfied:

$$\begin{aligned} \$ < \bar{\$}, \quad \gamma_a < \bar{\gamma}_a, \quad \gamma_s < \bar{\gamma}_s, \quad \bar{\alpha}_L < \alpha < \bar{\alpha}_U \\ \mathbb{E}_\infty(uu^T) < \bar{U}, \quad \mathbb{E}_\infty(yy^T) < \bar{Y} \end{aligned} \quad (10)$$

for given $\bar{\$}$, $\bar{U}$, $\bar{Y}$, $\bar{\gamma}_a$, $\bar{\gamma}_s$, $\bar{\alpha}_L$, and $\bar{\alpha}_U$.

B. Solution to the Dynamic Compensation Problem

Theorem II.1. *Let a continuous time-invariant linear system be described by the descriptor state space equation (1), the output equation (2) and the measurement equation (3). There exist controller matrices A_c, B_c, C_c and structure parameters α such that the cost and performance constraints (10) are satisfied, if and only if for some constant matrix G, there exists a symmetric matrix Q, vectors γ_a, γ_s and α such that the following LMIs are satisfied:*

$$p_a^T\gamma_a + p_s^T\gamma_s + p_\alpha^T\alpha < \bar{\$} \quad (11)$$

$$\gamma_a < \bar{\gamma}_a, \quad \gamma_s < \bar{\gamma}_s \quad (12)$$

$$\bar{\alpha}_L < \alpha < \bar{\alpha}_U \quad (13)$$

$$\begin{bmatrix} \bar{U} & M_{cl} \\ M_{cl}^T & Q \end{bmatrix} > 0, \quad \begin{bmatrix} \bar{Y} & C_{cl} \\ C_{cl}^T & Q \end{bmatrix} > 0 \quad (14)$$

$$\begin{bmatrix} (\star) & B_{cl} & A_{cl} & E_{cl} \\ B_{cl}^T & -W^{-1} & 0 & 0 \\ A_{cl}^T & 0 & -Q & 0 \\ E_{cl}^T & 0 & 0 & -Q \end{bmatrix} < 0 \quad (15)$$

where

$$(\star) = -(A_{cl} - E_{cl})G^T - G(A_{cl} - E_{cl})^T + GQG^T,$$

$$W = \begin{bmatrix} W_p & 0 & 0 \\ 0 & W_a & 0 \\ 0 & 0 & W_s \end{bmatrix}, \quad A_{cl} = \begin{bmatrix} A(\alpha) & BC_c \\ B_cC_z & A_c \end{bmatrix},$$

$$E_{cl} = \begin{bmatrix} E(\alpha) & 0 \\ 0 & I_n \end{bmatrix}, B_{cl} = \begin{bmatrix} D_p(\alpha) & D_a(\alpha) & 0 \\ 0 & 0 & B_cD_s \end{bmatrix},$$

$$C_{cl} = \begin{bmatrix} C_y(\alpha) & 0 \end{bmatrix}, \quad M_{cl} = \begin{bmatrix} 0 & C_c \end{bmatrix},$$

and I_n is a $n \times n$ identity matrix.

Proof. Let us define the augmented vector $\tilde{x}$ and w as:

$$\tilde{x}^T = \begin{bmatrix} x^T & x_c^T \end{bmatrix}, \quad w^T = \begin{bmatrix} w_p^T & w_a^T & w_s^T \end{bmatrix}. \quad (16)$$

The closed loop dynamics for the state $\tilde{x}$ can be written in the descriptor representation form along with the output and control equations as:

$$E_{cl}\dot{\tilde{x}} = A_{cl}\tilde{x} + B_{cl}w \quad (17)$$

$$y = C_{cl}\tilde{x} \quad (18)$$

$$u = M_{cl}\tilde{x} + F_{cl}w \quad (19)$$

where all the close loop matrices can easily be obtained from the system equations (1-3) and controller equations (9).

Defining $\bar{A}_{cl} = E_{cl}^{-1}A_{cl}$ and $\bar{B}_{cl} = E_{cl}^{-1}B_{cl}$ and rearranging equation (17) gives:

$$\dot{\tilde{x}} = \bar{A}_{cl}x + \bar{B}_{cl}w. \quad (20)$$

It is a standard result that the above closed loop system is stable if and only if there exists a positive definite symmetric matrix X such that:

$$\bar{A}_{cl}X + X\bar{A}_{cl}^T + \bar{B}_{cl}W\bar{B}_{cl}^T < 0. \quad (21)$$

Multiplying the inequality (21) from left by E_{cl} and from right by E_{cl}^T yields:

$$A_{cl}XE_{cl}^T + E_{cl}XA_{cl}^T + B_{cl}WB_{cl}^T < 0. \quad (22)$$

Applying Schur's complement on (22) gives:

$$\begin{bmatrix} A_{cl}XE_{cl}^T + E_{cl}XA_{cl}^T & B_{cl} \\ B_{cl}^T & -W^{-1} \end{bmatrix} < 0. \quad (23)$$

It can be shown that after substitution of E_{cl}, A_{cl} and B_{cl}, inequality (23) does not form an LMI since it is not affine in the decision variables A_c, B_c, α, etc. On completing the squares, the inequality (23) can be rewritten as:

$$\begin{bmatrix} A_{cl}XA_{cl}^T + E_{cl}XE_{cl}^T & \\ -(A_{cl} - E_{cl})X(A_{cl} - E_{cl})^T & B_{cl} \\ B_{cl}^T & -W^{-1} \end{bmatrix} < 0. \quad (24)$$

Defining $\delta \triangleq (A_c, B_c, C_c, \gamma_a, \gamma_s, \alpha, Q)$, $Q \triangleq X^{-1}$, and applying Schur's complement, we can write the inequality (24) as:

$$\mathbb{F}(\delta) \triangleq \begin{bmatrix} (\bullet) & B_{cl} & A_{cl} & E_{cl} \\ B_{cl}^T & -W^{-1} & 0 & 0 \\ A_{cl}^T & 0 & -Q & 0 \\ E_{cl}^T & 0 & 0 & -Q \end{bmatrix} < 0, \quad (25)$$

where $(\bullet) = -(A_{cl} - E_{cl})X(A_{cl} - E_{cl})^T$. Note that $\mathbb{F}(\delta)$ is not an LMI. Let us introduce the convexifying algorithm Lemma to write a new LMI.

Lemma II.1. Convexifying Algorithm Lemma. Let δ, η belong to a convex set ϕ, and $\mathbb{F}(\delta)$ be a first order differentiable non-convex matrix function. A convexifying potential function is a first order differentiable function $\mathbb{G}(\delta, \eta)$ such that the function $\mathbb{F}(\delta) + \mathbb{G}(\delta, \eta)$ is convex in δ for all $\delta, \eta \in \phi$. Thus, if $\mathbb{F}(\delta)$ satisfies certain conditions, a stationary point of the non-convex optimization problem

$$\bar{\delta} = \arg\min_{\delta \in \Omega} f(\delta), \quad \Omega = \{\delta \in \phi | \mathbb{F}(\delta) < 0\} \quad (26)$$

can be obtained by iterating over a sequence of convex subproblems given by

$$\bar{\delta}_{k+1} = \arg\min_{\delta \in \Omega_k} f(\delta), \Omega_k = \{\delta \in \phi | \mathbb{F}(\delta) + \mathbb{G}(\delta, \delta_k) < 0\}. \quad (27)$$

To ensure that the optimality conditions of both optimization problems (26) and (27) are identical, the potential function $\mathbb{G}$ should be non-negative definite with $\mathbb{G}(\delta, \eta) = 0$ if and only if $\delta = \eta$.

Remark II.1. The previous Lemma is proven and discussed in further detail in [11]. Although the Convexifying Algorithm will converge to a stationary point, a global solution is not guaranteed.

To use the previous Lemma, let us define the matrix G as:

$$G(\eta) \triangleq (A_{cl} - E_{cl})X \quad (28)$$

and the convexifying potential function as:

$$\mathbb{G}(\delta, \eta) \triangleq \begin{bmatrix} (*) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \quad (29)$$

$$(*) = (A_{cl} - E_{cl} - G(\eta)Q)X(A_{cl} - E_{cl} - G(\eta)Q)^T$$

$$\mathbb{G}(\delta, \eta) \geq 0 \quad (30)$$

The matrix function $\mathbb{F}(\delta) + \mathbb{G}(\delta, \eta)$:

$$\begin{bmatrix} (\star) & B_{cl} & A_{cl} & E_{cl} \\ B_{cl}^T & -W^{-1} & 0 & 0 \\ A_{cl}^T & 0 & -Q & 0 \\ E_{cl}^T & 0 & 0 & -Q \end{bmatrix} < 0, \quad (31)$$

where $(\star) = -(A_{cl} - E_{cl})G^T - G(A_{cl} - E_{cl})^T + GQG^T$, is convex, where the dependency of the matrix G on η is omitted for brevity. The function $\mathbb{G}(\delta, \eta)$ satisfies the convexifying assumptions since it is positive semidefinite and $\mathbb{G}(\delta, \eta) = 0$ if and only if $\delta = \eta$. Furthermore, using Lemma (II.1), it can be shown that any solution to (31) will also satisfy (25) [11].

The second constraint of the constraint set (10) can be evaluated by substituting in the expression for the control law (19) given by:

$$\mathbb{E}_\infty(uu^T) = \mathbb{E}_\infty((M_{cl}\tilde{x})(M_{cl}\tilde{x})^T) + \mathbb{E}_\infty((F_{cl}\tilde{w})(F_{cl}\tilde{w})^T) \quad (32)$$

$$\mathbb{E}_\infty(uu^T) < \bar{U}. \quad (33)$$

$$M_{cl}XM_{cl}^T + F_{cl}WF_{cl}^T < \bar{U} \quad (34)$$

The second term can grow unbounded if $F_{cl} \neq 0$. Hence, substituting for $F_{cl} = 0$ in the above equation gives:

$$M_{cl}XM_{cl}^T < \bar{U} \text{ and } D_c = 0 \ (D_s \text{ is full rank}). \quad (35)$$

and applying Schur's complement to first term results in left inequality of equation (14). It is then straightforward to show that the last constraint of (10) is satisfied if and only if

$$C_{cl}XC_{cl}^T < \bar{Y}. \quad (36)$$

Applying Schur's complement to this inequality results in (14). Finally, first four constraints in (10) are first four inequalities of Theorem II.1. □

Remark II.2. Assume that $\bar{\gamma}_a$ and $\bar{\gamma}_s$ are dictated by the marketplace. Let four parameters out of the set ($\bar{\alpha}_L$, $\bar{\alpha}_U$,$\bar{\$}$, $\bar{U}$, $\bar{Y}$) be hard constraints and let the fifth parameter, denoted $\bar{z}$, be any value for which the LMIs of Theorem II.1 are feasible. The following iterative algorithm takes advantage of Lemma II.1 to find an extrema for $\bar{z}$ (a minimum if $\bar{z} = \bar{\alpha}_U, \bar{\$}, \bar{U}, \bar{Y}$ or a maximum if $\bar{z} = \bar{\alpha}_L$).

Extrema-Finding Algorithm using the Convexifying Potential Function

- Set fixed nominal values for $\bar{z}_0$ and α_0. Compute controller matrices $A_{c,0}$, $B_{c,0}$, $C_{c,0}$, precision vectors $\gamma_{a,0}$, $\gamma_{s,0}$ and inverse covariance matrix Q_0 according to [1] or some alternative method. Set ϵ to some prescribed tolerance and $k = 0$
- **Repeat:** Set $G_k \leftarrow (A_{cl}(\alpha_k) - E_{cl}(\alpha_k))Q_k^{-1}$
  -For fixed $G = G_k$, find the extrema of $\bar{z}$ for which the LMIs of Theorem II.1 are feasible
  -Denote the solution ($\bar{z}_{k+1}$, α_{k+1}, $A_{c,k+1}$, $B_{c,k+1}$, $C_{c,k+1}$,$\gamma_{a,k+1}$, $\gamma_{s,k+1}$,Q_{k+1})
  -Set k = k+1
- **Until:** $\|\bar{z}_k - \bar{z}_{k-1}\| < \epsilon$

III. State Feedback Problem

A. Problem Statement

Consider the situation where full-state feedback is available for measurement. The system can now be described as:

$$E(\alpha)\dot{x} = A(\alpha)x + Bu + D_p(\alpha)w_p + D_a(\alpha)w_a, \quad (37)$$

$$y = C_y(\alpha)x \ \text{(output)} \quad (38)$$

As there is no measurement noise, the total design price is expressed as:

$$\$ = p_a^T\gamma_a + p_\alpha^T\alpha. \quad (39)$$

The state feedback problem can now be defined as:

Design a state feedback controller $u = -Kx$ and simultaneously select the structure parameters and the actuator precisions such that the following constraints are satisfied:

$$\begin{aligned} &\$ < \bar{\$}, \gamma_a < \bar{\gamma}_a, \bar{\alpha}_L < \alpha < \bar{\alpha}_U \\ &\mathbb{E}_\infty(uu^T) < \bar{U}, \quad \mathbb{E}_\infty(yy^T) < \bar{Y} \end{aligned} \quad (40)$$

for given $\bar{\$}$, $\bar{U}$, $\bar{Y}$, $\bar{\gamma}_a$, $\bar{\alpha}_L$, and $\bar{\alpha}_U$.

B. Solution to the State Feedback Problem

Lemma III.1. Let a continuous time-invariant linear system be described by the descriptor state space equation (37) and the output equation (38). There exists a controller gain K and structure parameters α such that the cost and performance constraints (40) are satisfied if and only if for some constant matrix G there exists a symmetric matrix Q and vectors γ_a and α such that the following LMIs are satisfied:

$$p_a^T\gamma_a + p_\alpha^T\alpha < \bar{\$} \quad (41)$$

$$\gamma_a < \bar{\gamma}_a, \quad \bar{\alpha}_L < \alpha < \bar{\alpha}_U \quad (42)$$

$$\begin{bmatrix} \bar{U} & K \\ K^T & Q \end{bmatrix} > 0, \ \begin{bmatrix} \bar{Y} & C_y(\alpha) \\ C_y(\alpha)^T & Q \end{bmatrix} > 0 \quad (43)$$

$$\begin{bmatrix} (\star) & B_{cl} & A_{cl} & E(\alpha) \\ B_{cl}^T & -W^{-1} & 0 & 0 \\ A_{cl}^T & 0 & -Q & 0 \\ E^T(\alpha) & 0 & 0 & -Q \end{bmatrix} < 0 \quad (44)$$

where

$$(\star) = -(A_{cl} - E(\alpha))G^T - G(A_{cl} - E(\alpha))^T + GQG^T$$

$$W = \begin{bmatrix} W_p & 0 \\ 0 & W_a \end{bmatrix} \quad A_{cl} = A(\alpha) - BK$$

$$B_{cl} = \begin{bmatrix} D_p(\alpha) & D_a(\alpha) \end{bmatrix}.$$

Remark III.1. The proof is excluded as it follows very closely with Theorem II.1.

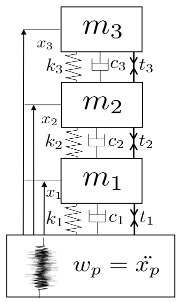

Fig. 1. 3-Story Building Model

IV. Numerical Example

To illustrate the proposed concept of simultaneous optimization of structure, information architecture, and control, we provide an example of civil engineering structure. The idea is to design a three-storey building to sustain an earthquake of given intensity with active or passive control. The building is modeled as a spring-mass-damper system with all springs, masses, and dampers as variable structure parameters as shown in Fig. 1. The prestressed cables provide the actuation force on three masses. The dynamics of the system can be written as:

$$\begin{bmatrix} I & 0 \\ 0 & \mathbb{M} \end{bmatrix} \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -\mathbb{K} & -\mathbb{C} \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ B_u \end{bmatrix} u + \begin{bmatrix} 0 \\ M \end{bmatrix} w_p + \begin{bmatrix} 0 \\ B_u \end{bmatrix} w_a. \quad (45)$$

where

$$\mathbb{K} = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 \\ -k_2 & k_2 + k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix}, B_u = \begin{bmatrix} -1 \ 1 \ 0 \\ 0 \ -1 \ 1 \\ 0 \ 0 \ -1 \end{bmatrix},$$

$\mathbb{M} = diag(m_1, m_2, m_3)$ and $\mathbb{C}$ can be written identical to $\mathbb{K}$. The control input is a vector of tensions in the cables $u = [t_1 \ t_2 \ t_3]^T$ and the disturbance due to earthquake (at acceleration level) is modeled as a zero-mean white noise with intensity $W_p = 1$ m^2/s^4, which multiplied by $M = [m_1 \ 0 \ 0]^T$ gives disturbance force transferred to the first floor due to earthquake. The output and measurement equations can be written as:

$$y = \begin{bmatrix} x_1 & x_2 - x_1 & x_3 - x_2 & \dot{x_1} & \dot{x_2} - \dot{x_1} & \dot{x_3} - \dot{x_2} \end{bmatrix}^T$$
$$y = c_y \begin{bmatrix} x \\ \dot{x} \end{bmatrix}, \ z = \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + I w_s. \quad (46)$$

where output to be minimized is the relative displacement and relative velocity between consecutive floors. We assume all the states are available for measurement with noisy sensors. Constraints used for this example are $\bar{\alpha}_L = 0.2\alpha_0, \bar{\alpha}_U = 3\alpha_0, \bar{\gamma}_a = 1e4, \bar{\gamma}_s = 1e4, p_a = p_s = p_\alpha = 20$.

A. Active to Passive Control

First, we show that minimum control required to bound the output covariance with a given budget constraint constantly decreases while optimizing structure parameters. The output covariance constraint is assumed to be $\mathbb{E}_\infty(y_i^2) = 0.01$ m^2 for $i = 1, 2, 3$ and $\mathbb{E}_\infty(y_i^2) = 0.1$ m^2/s^2 for $i = 4, 5, 6$. Fig. 2 shows the variation in structural parameters, when started from nominal values, and decrease in active control during optimization iterations. The required active control decreases from $\bar{U} = 4.5e4$ N^2 to $\bar{U} = 3.5e4$ N^2 when only springs and dampers are considered as optimization variable.

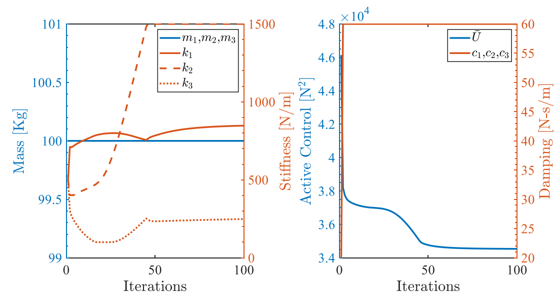

Fig. 2. Variation in structure parameters with constant mass and required active control. ($\bar{Y} = diag(.01, .01, .01, .1, .1, .1), \bar{\$} = 1e5$).

Second, we assume all the structure parameters to be variables. Fig. 3 shows that control required goes to zero with the optimized structure parameters. The structure can now sustain the earthquake with new passive design i.e. without any control.

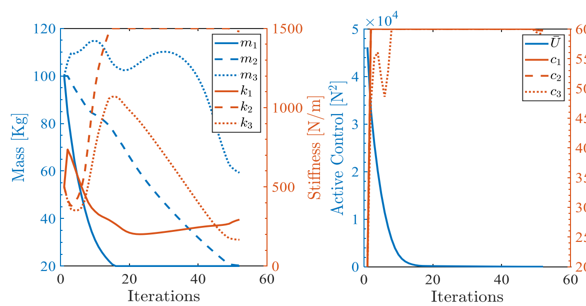

Fig. 3. Variation in all structure parameters and transition from active to passive control. ($\bar{Y} = diag(.01, .01, .01, .1, .1, .1), \bar{\$} = 1e5$).

B. Trade-off Analysis

The minimum cost required to achieve given output and control covariance is calculated along with the optimized structural parameters. Fig. 4 shows the variation in cost and structure parameters value during optimization iterations. As there are prices associated with all the structure parameters, their values go to their respective minimum bound to reduce dollar value. Basically, the algorithm fills up the upper bound on performance and control constraints by reducing the parameters and precisions values which in turn reduces the total cost.

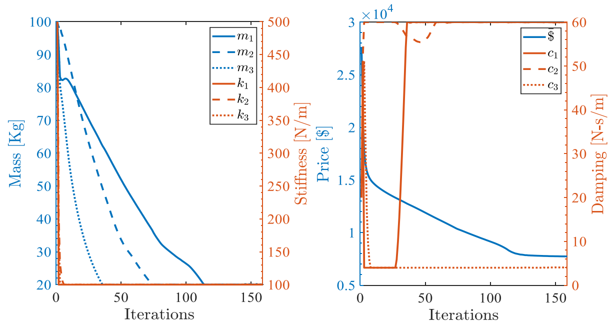

Fig. 4. Variation in all structure parameters and price value. ($\bar{U} = 1e5, \bar{Y} = diag(.01, .01, .01, .1, .1, .1)$).

Finally, Fig. 5 shows the decrease in required cost as we relax the output covariance bound for a fixed control requirement. The sensor and actuator precision also monotonically decreases by relaxing the performance constraints. Also, note that more precise measurements are needed for displacement and velocity of the first floor as compared to other floors. A similar decreasing trend in dollar value was observed while relaxing the control covariance bound and fixing the output constraint.

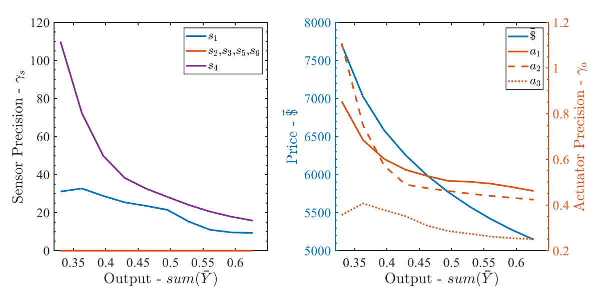

Fig. 5. Variation in sensor and actuator precision and price value. ($\bar{U} = 1e5, \bar{\$} = 1e5$).

V. Conclusion

This paper developed a novel system-level design approach by simultaneous selection of control law, instrument precision, and structure parameters. The system dynamics are linear and the free structural parameters also appear linearly. The problem is set as a feasibility problem, where matrix upper bounds are specified for the covariance of selected outputs and the covariance of the control signals. Specified upper bounds on the available precision of sensors/actuators and structural parameters are also given. The problem is not proven convex with the addition of structural parameters in the previous work of integrating control and information architecture. The sub-optimal solution is found by iterating over constraint convex functions generated by adding a convexifying potential function. Results are provided for both dynamic compensation and full state feedback controller design. This paper may be used to design passive structures by reducing the level set of control covariance (to zero) while holding the output covariance constraint as shown in the example. Trade-off analysis provided between cost vs. control energy and performance requirement showed that as performance and control energy constraints are relaxed, tighter budget constraints are achievable.

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