Examples : numerical.mac

1

```
(%i1) load("/home/jvr/Downloads/numericalPackage/v1/numerical.mac");
(%o1) /home/jvr/Downloads/numericalPackage/v1/numerical.mac
(%i2) ratprint:false;
(%o2) false
```

2

The examples are from the books by :

1. Ward Cheney and David Kincaid :: Numerical Mathematics and Computing
2. Brian Bradie :: A Friendly Introduction to Numerical Analysis

3 Bisection Method

3.1

```
(%i3) bisect(t^3−2·sin(t),    t,    0.50,   2.00,   20,   0.0005);
```

iter	*m*	*ym*	*error*
1	1.25	0.0551557612888276	0.75
2	0.875	−0.8651651294720542	0.375
3	1.0625	−0.5476869797091422	0.1875
4	1.15625	−0.2847914007983883	0.09375
5	1.203125	−0.1247986155094702	0.046875
6	1.2265625	−0.03735980652509796	0.0234375
7	1.23828125	0.00825801590073083	0.01171875
8	1.232421875	−0.01471021624269308	0.005859375
9	1.2353515625	−0.003266014170569153	0.0029296875
10	1.23681640625	0.002486011901918328	0.00146484375
11	1.236083984375	$-3.924970675475148\ 10^{-4}$	$7.32421875\ 10^{-4}$
12	1.2364501953125	0.001046133270412585	$3.662109375\ 10^{-4}$

(%o3) *bisection has converged*

3.2

```
(%i4) bisect(sin(x), x, 2, 4, 20, 0.0005);
```

iter	*m*	*ym*	*error*
1	3	0.1411200080598672	1
2	3.5	−0.3507832276896198	0.5
3	3.25	−0.1081951345301083	0.25
4	3.125	0.0165918922293479	0.125
5	3.1875	−0.04589122327277969	0.0625
6	3.15625	−0.01465682159049232	0.03125
7	3.140625	$9.676534387822795\ 10^{-4}$	0.015625
8	3.1484375	−0.006844792961296519	0.0078125
9	3.14453125	−0.002938592180907726	0.00390625
10	3.142578125	$-9.854712506993688\ 10^{-4}$	0.001953125
11	3.1416015625	$-8.908910206643689\ 10^{-6}$	$9.765625\ 10^{-4}$
12	3.14111328125	$4.793723214334506\ 10^{-4}$	$4.8828125\ 10^{-4}$

(%o4) *bisection has converged*

3.3

(%i5) bisect(x^2−2, x, 1, 2, 20, 0.0005);

iter	*m*	*ym*	*error*
1	1.5	0.25	0.5
2	1.25	−0.4375	0.25
3	1.375	−0.109375	0.125
4	1.4375	0.06640625	0.0625
5	1.40625	−0.0224609375	0.03125
6	1.421875	0.021728515625	0.015625
7	1.4140625	$-4.2724609375\ 10^{-4}$	0.0078125
8	1.41796875	0.0106353759765625	0.00390625
9	1.416015625	0.005100250244140625	0.001953125
10	1.4150390625	0.002335548400878906	$9.765625\ 10^{-4}$
11	1.41455078125	$9.539127349853516\ 10^{-4}$	$4.8828125\ 10^{-4}$

(%o5) *bisection has converged*

3.4

This example was suggested by Professor Barton Willis

```
(%i6) bisect(x^3−2·x^2+x−%pi, x, 2, 3, 10, 0.0001);
```

iter	m	ym	error
1	2.5	2.483407346410207	0.5
2	2.25	0.374032346410206	0.25
3	2.125	−0.4521395285897931	0.125
4	2.1875	−0.05687585671479311	0.0625
5	2.21875	0.1540311257070818	0.03125
6	2.203125	0.04745229880278501	0.015625
7	2.1953125	−0.004991682367869288	0.0078125
8	2.19921875	0.02116015355055722	0.00390625
9	2.197265625	0.008066719276360601	0.001953125
10	2.1962890625	0.001533142169467538	$9.765625\ 10^{-4}$

(%o6) *done*

4 Newton's Method

4.1

```
(%i7) newton(x^3−2·x^2+x−3, x, 3, 0.0001, 0.00001, 8);
```

1	2.4375	2.036865234375
2	2.213032716315109	0.2563633850614177
3	2.175554938721488	0.006463361488812325
4	2.174560100666446	$4.479068049789703\ 10^{-6}$
5	2.174559410293312	$2.156497203031904\ 10^{-12}$

(%o7) *convergence*

4.2

(%i8) newton(sin(x), x, 3.0, 0.0001, 0.00001, 8);

1 3.142546543074278 $-9.538893398264409\ 10^{-4}$

2 3.141592653300477 $2.893162490762184\ 10^{-10}$

3 3.141592653589793 $1.224646799147353\ 10^{-16}$

(%o8) *convergence*

4.3

(%i9) newton(x^2−2, x, 1, 0.0001, 0.00001, 8);

1 1.5 0.25

2 1.416666666666666 0.006944444444444642

3 1.414215686274509 $6.007304882871267\ 10^{-6}$

4 1.414213562374689 $4.510614104447086\ 10^{-12}$

(%o9) *convergence*

(%i10) newton(x^3−2·x^2+x−%pi, x, 3, 0.0001, 0.00001, 8);

1 2.446349540849362 1.975991988443487

2 2.230829413793257 0.237982391674409

3 2.196863381645069 0.005374496729482203

4 2.196060159694346 $2.961172185678151\ 10^{-6}$

5 2.196059716657127 $9.00612917575927\ 10^{-13}$

(%o10) *convergence*

5 Secant Method

5.1

```
(%i11) secant(x^3+2·x^2−3·x−1, x, −2, −3, 0.00001, 10);
```

0	−3	−1
1	−2	5
2	−2.833333333333333	0.8101851851851833
3	−2.907928388746803	0.04629957161572662
4	−2.912449640422374	−0.002380064066290543
5	−2.912228585591192	$6.399876401275151\ 10^{-6}$

(%o11) *convergence*

5.2

```
(%i12) secant(sin(x), x, 1, 4, 0.00001, 10);
```

0	4	−0.7568024953079282
1	1	0.8414709848078965
2	2.579462454848934	0.5329898131328342
3	3.166481028136553	−0.02488580518710502
4	3.140295209333355	0.001297443892426365
5	3.141592780561861	$-1.269720678483891\ 10^{-7}$

(%o12) *convergence*

5.3

```
(%i13) secant(x^2−2, x, 1, 2, 0.00001, 10);
```

0	1	−1
1	2	2
2	1.333333333333333	−0.2222222222222223
3	1.428571428571428	0.04081632653061229
4	1.413793103448275	−0.001189060642093009
5	1.41421143847487	$-6.007286838860537\ 10^{-6}$

(%o13) *convergence*

6 *Regula-Falsi Method*

6.1

(%i14) regula(x^3+2·x^2−3·x−1, x, 1, 2, 20, 0.0005);

iter *m* *ym*
1 1.1 −0.5489999999999999
2 1.221729490022173
0.1436388950901026
3 1.193880682944684 −
0.02924075169187978
4 1.199720665327255
0.006290908737020207
5 1.198472095487697 −
0.001337706025862806
6 1.198737948438891
$2.851619710524389\ 10^{-4}$

(%o14) *regula falsi method has converged*

6.2

(%i15) regula(sin(x), x, 2, 4, 20, 0.0005);

iter *m* *ym*
1 3.091528082734958
0.05004365932452196
2 3.147874957380742 −
0.006282262466726139
3 3.141590357955694
$2.295634098490862\ 10^{-6}$

(%o15) *regula falsi method has converged*

6.3

(%i16) regula(x^2−2, x, 1, 2, 20, 0.0005);

iter *m* *ym*
1 1.333333333333333 −
0.2222222222222218
2 1.428571428571428
0.04081632653061229
3 1.411764705882353 −
0.006920415224913157
4 1.414634146341463
0.001189767995240842
5 1.414141414141414 −
$2.040608101214758\ 10^{-4}$

(%o16) *regula falsi method has converged*

7 LU Factorization

7.1

```
(%i19) A:matrix([1, 2, 3], [3, 5, 7], [4, 6, 9]);
luFactor(A);
luFactor(A)[1];
```

(%o17) $\begin{pmatrix} 1 & 2 & 3 \\ 3 & 5 & 7 \\ 4 & 6 & 9 \end{pmatrix}$

(%o18) $[L=\begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 4 & 2 & 1 \end{pmatrix}, U=\begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & 0 & 1 \end{pmatrix}]$

(%o19) $L=\begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 4 & 2 & 1 \end{pmatrix}$

8 Solving by LU Factorization

8.1

```
(%i22) A:matrix([1, 2, 3], [3, 5, 7], [4, 6, 9]);
b:matrix([1], [2], [3]);
solve_by_lu(A, b);
```

(%o20) $\begin{pmatrix} 1 & 2 & 3 \\ 3 & 5 & 7 \\ 4 & 6 & 9 \end{pmatrix}$

(%o21) $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

(%o22) $[z=\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, x=\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}]$

8.2

```
(%i26) A:matrix([2, 7, 5], [6, 20, 10], [4, 3, 0]);
       b1:matrix([14], [36], [7]);
       b2:matrix([−4], [−16], [−7]);
       b3:matrix([−3], [−12], [6]);
```

(%o23) $\begin{pmatrix} 2 & 7 & 5 \\ 6 & 20 & 10 \\ 4 & 3 & 0 \end{pmatrix}$

(%o24) $\begin{pmatrix} 14 \\ 36 \\ 7 \end{pmatrix}$

(%o25) $\begin{pmatrix} -4 \\ -16 \\ -7 \end{pmatrix}$

(%o26) $\begin{pmatrix} -3 \\ -12 \\ 6 \end{pmatrix}$

```
(%i27) luFactor(A);
```

(%o27) $[L=\begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 11 & 1 \end{pmatrix}, U=\begin{pmatrix} 2 & 7 & 5 \\ 0 & -1 & -5 \\ 0 & 0 & 45 \end{pmatrix}]$

```
(%i28) solve_by_lu(A, b1);
```

(%o28) $[z=\begin{pmatrix} 14 \\ -6 \\ 45 \end{pmatrix}, x=\begin{pmatrix} 1.0 \\ 1.0 \\ 1.0 \end{pmatrix}]$

```
(%i29) solve_by_lu(A, b2);
```

(%o29) $[z=\begin{pmatrix} -4 \\ -4 \\ 45 \end{pmatrix}, x=\begin{pmatrix} -1.0 \\ -1.0 \\ 1.0 \end{pmatrix}]$

```
(%i30) solve_by_lu(A, b3);
```

(%o30) $[z=\begin{pmatrix} -3 \\ -3 \\ 45 \end{pmatrix}, x=\begin{pmatrix} 3.0 \\ -2.0 \\ 1.0 \end{pmatrix}]$

9 Gauss-Jacobi Method

9.1

```
(%i34) A1:matrix([5, 1, 2], [−3, 9, 4], [1, 2, −7]);
       b1:matrix([10], [−14], [−33]);
       start:matrix([0], [0], [0]);
       gauss_jacobi(A1, b1, start, 14);
```

(%o31) $\begin{pmatrix} 5 & 1 & 2 \\ -3 & 9 & 4 \\ 1 & 2 & -7 \end{pmatrix}$

(%o32) $\begin{pmatrix} 10 \\ -14 \\ -33 \end{pmatrix}$

(%o33) $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

1

−−−−

$\begin{pmatrix} 2.0 & -1.555555555555555 & 4.714285714285714 \end{pmatrix}$

2

−−−−

$\begin{pmatrix} 0.4253968253968252 & -2.984126984126984 & 4.555555555555555 \end{pmatrix}$

3

−−−−

$\begin{pmatrix} 0.7746031746031745 & -3.438447971781304 & 3.922448979591836 \end{pmatrix}$

4

−−−−

$\begin{pmatrix} 1.118710002519526 & -3.040665154950868 & 3.842529604434367 \end{pmatrix}$

5

−−−−

$\begin{pmatrix} 1.071121189216427 & -2.890443156686543 & 4.005339956088256 \end{pmatrix}$

6

−−−−

$\begin{pmatrix} 0.9759526489020063 & -2.97866625074486 & 4.041462125120478 \end{pmatrix}$

7

−−−−

$\begin{pmatrix} 0.9791484001007809 & -3.026443394863987 & 4.002660021058898 \end{pmatrix}$

8

−−−−

$\begin{pmatrix} 1.004224670549238 & -3.008132764881471 & 3.989465944338972 \end{pmatrix}$

9

−−−−

$\begin{pmatrix} 1.005840175240705 & -2.993909973967574 & 3.998279877255185 \end{pmatrix}$

10

−−−−

9.2

```
(%i38) A1:matrix([2,  −1,  0], [−1,  4,  2], [0,  2,  6]);
b1:matrix([−1],  [3],  [5]);
start:matrix([0],  [0],  [0]);
gauss_jacobi(A1,  b1,  start,  14);
```

(%o35) $\begin{pmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \\ 0 & 2 & 6 \end{pmatrix}$

(%o36) $\begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix}$

(%o37) $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

1

−−−−

$\begin{pmatrix} -0.5 & 0.75 & 0.8333333333333333 \end{pmatrix}$

2

−−−−

$\begin{pmatrix} -0.125 & 0.2083333333333333 & 0.5833333333333333 \end{pmatrix}$

3

−−−−

$\begin{pmatrix} -0.3958333333333333 & 0.4270833333333333 & 0.7638888888888888 \end{pmatrix}$

4

−−−−

$\begin{pmatrix} -0.2864583333333333 & 0.2690972222222222 & 0.6909722222222221 \end{pmatrix}$

5

−−−−

$\begin{pmatrix} -0.3654513888888888 & 0.3328993055555556 & 0.7436342592592592 \end{pmatrix}$

6

−−−−

$\begin{pmatrix} -0.3335503472222222 & 0.2868200231481482 & 0.722366898148148 \end{pmatrix}$

7

−−−−

$\begin{pmatrix} -0.3565899884259259 & 0.3054289641203704 & 0.7377266589506172 \end{pmatrix}$

8

−−−−

$\begin{pmatrix} -0.3472855179398147 & 0.2919891734182099 & 0.7315236786265431 \end{pmatrix}$

9

−−−−

$\begin{pmatrix} -0.354005413290895 & 0.2974167812017747 & 0.7360036088605966 \end{pmatrix}$

10

−−−−

10 SOR Method

10.1

(%i42)
```
A1:matrix([5, 1, 2], [−3, 9, 4], [1, 2, −7]);
b1:matrix([10], [−14], [−33]);
start:matrix([0], [0], [0]);
sor(A1, b1, 0.9, start, 10);
```

(%o39) $\begin{pmatrix} 5 & 1 & 2 \\ -3 & 9 & 4 \\ 1 & 2 & -7 \end{pmatrix}$

(%o40) $\begin{pmatrix} 10 \\ -14 \\ -33 \end{pmatrix}$

(%o41) $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

1

$\begin{pmatrix} 1.799999999999999 & -0.8599999999999999 & 4.253142857142857 \end{pmatrix}$

2

$\begin{pmatrix} 0.6036685714285714 & -3.006156571428571 & 3.972774269387755 \end{pmatrix}$

3

$\begin{pmatrix} 0.9712763030204081 & -2.998342473991836 & 3.994010601157784 \end{pmatrix}$

4

$\begin{pmatrix} 0.9989854592037691 & -2.997742850101166 & 3.999851029130248 \end{pmatrix}$

5

$\begin{pmatrix} 0.9995458884516973 & -2.999850930126706 & 3.999965049395661 \end{pmatrix}$

6

$\begin{pmatrix} 0.9999403384855388 & -2.999989011225273 & 3.99999165985835 \end{pmatrix}$

7

$\begin{pmatrix} 0.9999950583200967 & -2.999997047569838 & 3.999999289823317 \end{pmatrix}$

8

$\begin{pmatrix} 0.999992300581864 & -2.999999651668854 & 3.999999919560678 \end{pmatrix}$

9

$\begin{pmatrix} 0.999998892643681 & -2.99999966211847 & 3.999999986407011 \end{pmatrix}$

10

```
(%i46) A1:matrix([2, −1, 0], [−1, 4, 2], [0, 2, 6]);
b1:matrix([−1], [3], [5]);
start:matrix([0], [0], [0]);
sor(A1, b1, 0.9, start, 14);
```

(%o43) $\begin{pmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \\ 0 & 2 & 6 \end{pmatrix}$

(%o44) $\begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix}$

(%o45) $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

1

$\begin{pmatrix} -0.45 & 0.5737500000000001 & 0.577875 \end{pmatrix}$

2

$\begin{pmatrix} -0.2368124999999999 & 0.4190484375000001 & 0.68207296875 \end{pmatrix}$

3

$\begin{pmatrix} -0.2851094531249999 & 0.3458223808593751 & 0.7144605826171875 \end{pmatrix}$

4

$\begin{pmatrix} -0.3228908739257812 & 0.3154245292749024 & 0.726818699479248 \end{pmatrix}$

5

$\begin{pmatrix} -0.3403480492188721 & 0.3028957270875825 & 0.7318131518216502 \end{pmatrix}$

6

$\begin{pmatrix} -0.3477317277324751 & 0.2977340156492088 & 0.7338611104874024 \end{pmatrix}$

7

$\begin{pmatrix} -0.3507928657311035 & 0.2956075070560916 & 0.7347038589319128 \end{pmatrix}$

8

$\begin{pmatrix} -0.352055908397869 1 & 0.294731434796727 9 & 0.73505095545417 3 \end{pmatrix}$

9

$\begin{pmatrix} -0.3525764451812593 & 0.2943705133595117 & 0.7351939415375639 \end{pmatrix}$

10

10.2

```
(%i50) A1:matrix([2,   −1,   0], [−1,   4,   2], [0,   2,   6]);
       b1:matrix([−1],   [3],   [5]);
       start:matrix([0],   [0],   [0]);
       sor(A1,   b1,   1.1,   start,   14);
```

(%o47) $\begin{pmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \\ 0 & 2 & 6 \end{pmatrix}$

(%o48) $\begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix}$

(%o49) $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

1
−−−−
$\begin{pmatrix} -0.55 & 0.6737500000000001 & 0.6696249999999999 \end{pmatrix}$
2
−−−−
$\begin{pmatrix} -0.1244374999999999 & 0.3551109375000001 & 0.7194968229166666 \end{pmatrix}$
3
−−−−
$\begin{pmatrix} -0.342245234375 & 0.2996482141927084 & 0.7348459725043401 \end{pmatrix}$
4
−−−−
$\begin{pmatrix} -0.3509689587565104 & 0.2943534300453017 & 0.7352524783996219 \end{pmatrix}$
5
−−−−
$\begin{pmatrix} -0.353008717599433 & 0.2940983965358337 & 0.7353053400968987 \end{pmatrix}$
6
−−−−
$\begin{pmatrix} -0.3529450101453482 & 0.2941123455031516 & 0.7352949393058211 \end{pmatrix}$
7
−−−−
$\begin{pmatrix} -0.3529437089587318 & 0.294117028867832 & 0.7352942621512127 \end{pmatrix}$
8
−−−−
$\begin{pmatrix} -0.3529412632268192 & 0.2941176055426745 & 0.7352941184192313 \end{pmatrix}$
9
−−−−
$\begin{pmatrix} -0.3529411906288471 & 0.2941176468922224 & 0.7352941176309286 \end{pmatrix}$
10
−−−−

11 Gauss-Seidel Method

11.1

```
(%i54) A1:matrix([5, 1, 2], [−3, 9, 4], [1, 2, −7]);
       b1:matrix([10], [−14], [−33]);
       start:matrix([0], [0], [0]);
       gauss_seidel(A1, b1, start, 10);
```

(%o51) $\begin{pmatrix} 5 & 1 & 2 \\ -3 & 9 & 4 \\ 1 & 2 & -7 \end{pmatrix}$

(%o52) $\begin{pmatrix} 10 \\ -14 \\ -33 \end{pmatrix}$

(%o53) $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

1

−−−−

$\begin{pmatrix} 2.0 & -0.8888888888888887 & 4.746031746031746 \end{pmatrix}$

2

−−−−

$\begin{pmatrix} 0.2793650793650792 & -3.571781305114638 & 3.7336860670194 \end{pmatrix}$

3

−−−−

$\begin{pmatrix} 1.220881834215167 & -2.808010973936899 & 4.086408555191624 \end{pmatrix}$

4

−−−−

$\begin{pmatrix} 0.9270387727107303 & -3.062724211403811 & 3.971655764271872 \end{pmatrix}$

5

−−−−

$\begin{pmatrix} 1.023882536572013 & -2.979441716374605 & 4.0092855862604 \end{pmatrix}$

6

−−−−

$\begin{pmatrix} 0.992174108770761 & -3.006735557636591 & 3.996957570499654 \end{pmatrix}$

7

−−−−

$\begin{pmatrix} 1.002564083327456 & -2.997793114668471 & 4.000996836284359 \end{pmatrix}$

8

−−−−

$\begin{pmatrix} 0.9991598884199506 & -3.000723075541953 & 3.999673391048006 \end{pmatrix}$

9

−−−−

$\begin{pmatrix} 1.000275258689188 & -2.99763087569384 & 4.000107011935774 \end{pmatrix}$

10

−−−−

11.2

```
(%i58) A1:matrix([2,  −1,  0], [−1,  4,  2], [0,  2,  6]);
b1:matrix([−1],  [3],  [5]);
start:matrix([0],  [0],  [0]);
gauss_seidel(A1,  b1,  start,  14);
```

(%o55) $\begin{pmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \\ 0 & 2 & 6 \end{pmatrix}$

(%o56) $\begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix}$

(%o57) $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

1

$\begin{pmatrix} -0.5 & 0.625 & 0.6249999999999999 \end{pmatrix}$
2

$\begin{pmatrix} -0.1875 & 0.390625 & 0.7031249999999999 \end{pmatrix}$
3

$\begin{pmatrix} -0.3046875 & 0.322265625 & 0.7259114583333332 \end{pmatrix}$
4

$\begin{pmatrix} -0.3388671875 & 0.3023274739583334 & 0.7325575086805554 \end{pmatrix}$
5

$\begin{pmatrix} -0.3488362630208332 & 0.296512179904514 & 0.7344959400318285 \end{pmatrix}$
6

$\begin{pmatrix} -0.3517439100477429 & 0.29481605247215 & 0.7350613158426166 \end{pmatrix}$
7

$\begin{pmatrix} -0.352591973763925 & 0.2943213486377105 & 0.7352262171207631 \end{pmatrix}$
8

$\begin{pmatrix} -0.3528393256811447 & 0.294177060019332 & 0.735274313326891 \end{pmatrix}$
9

$\begin{pmatrix} -0.3529114699903338 & 0.2941349758389719 & 0.7352883413870093 \end{pmatrix}$
10
----

12 Lagrange Interpolation

12.1

(%i61)
```
xval:[0, 1, -1, 2, -2]$
yval:[-5, -3, -15, 39, -9]$
LP(xval, yval, x);
```

(%o61) $1.625\,(x-1)\,x\,(x+1)\,(x+2)+0.5\,(x-2)\,x\,(x+1)\,(x+2)+1.25\,(1-x)\,(x-2)\,(x+1)\,(x+2)+2.5\,(x-2)\,(x-1)\,x\,(x+2)-0.375\,(x-2)\,(x-1)\,x\,(x+1)$

(%i62)
```
ratsimp(%);
```

(%o62) $3\,x^4+2\,x^3-7\,x^2+4\,x-5$

(%i63)
```
LP(xval, yval, 3);
```

(%o63) 241.0

12.2

(%i66)
```
xval:[-3, 1, 2, 5]$
yval:[-23, -11, -23, 1]$
LP(xval, yval, x);
```

(%o66) $0.01041666666666666\,(x-2)\,(x-1)\,(x+3)+1.533333333333333\,(x-5)\,(x-1)\,(x+3)-0.6875\,(x-5)\,(x-2)\,(x+3)+0.14375\,(x-5)\,(x-2)\,(x-1)$

(%i67)
```
ratsimp(%);
```

(%o67) $x^3-3\,x^2-10\,x+1$

12.3

(%i70)
```
xval:[-1, 0, 1, 2]$
yval:[5, 1, 1, 11]$
LP(xval, yval, x);
```

(%o70) $1.833333333333333\,(x-1)\,x\,(x+1)-0.5\,(x-2)\,x\,(x+1)+0.5\,(x-2)\,(x-1)\,(x+1)-0.8333333333333333\,(x-2)\,(x-1)\,x$

(%i71)
```
ratsimp(%);
```

(%o71) $x^3+2\,x^2-3\,x+1$

13 Newton Interpolation

13.1

13.2

(%i72) dd_table([1, 3/2, 0, 2], [3, 13/4, 3, 5/3]);

(%o72)

$$\begin{pmatrix} 1 & 3 & 0.5 & 0.3333333333333333 & -2.0 \\ 1.5 & 3.25 & 0.1666666666666666 & -1.666666666666666 & A_{2,5} \\ 0 & 3 & -0.6666666666666666 & A_{3,4} & A_{3,5} \\ 2 & 1.666666666666666 & A_{4,3} & A_{4,4} & A_{4,5} \end{pmatrix}$$

13.3

(%i73) dd_table([−1, 0, 1, 2, −2, 3], [5, 1, 1, 11, 5, 35]);

(%o73)

$$\begin{pmatrix} -1 & 5 & -4 & 2 & 1 & -0.08333333333333326 & -1.7347234 \\ 0 & 1 & 0 & 5 & 1.083333333333333 & -0.08333333333333333 & \\ 1 & 1 & 10 & 2.833333333333333 & 0.8333333333333333 & A_{3,6} & \\ 2 & 11 & 1.5 & 4.5 & A_{4,5} & A_{4,6} & \\ -2 & 5 & 6 & A_{5,4} & A_{5,5} & A_{5,6} & \\ 3 & 35 & A_{6,3} & A_{6,4} & A_{6,5} & A_{6,6} & \end{pmatrix}$$

13.4

13.5

(%i76) xval:[−1, 0, 1, 2]$
yval:[5, 1, 1, 11]$
NP(xval, yval, x);

(%o76) $(x-1)\,x\,(x+1)+2\,x\,(x+1)-4\,(x+1)+5$

(%i77) ratsimp(%);

(%o77) $x^3+2\,x^2-3\,x+1$

13.6

```
(%i80) xval:[−1, 0, 1, 2, −2, 3]$
       yval:[5, 1, 1, 11, 5, 35]$
       NP(xval, yval, x);
```

(%o80) $-1.734723475976807\ 10^{-17}(x-2)(x-1)x(x+1)(x+2)-0.08333333333333326(x-2)(x-1)x(x+1)+(x-1)x(x+1)+2x(x+1)-4(x+1)+5$

```
(%i81) NP(xval, yval, 1.5);
```

(%o81) 4.453125

14

```
(%i82) ratprint:false;
```

(%o82) ***false***

14.1 Trapezoidal Rule

Basic Trapezoidal Rule

```
(%i83) T(1/s, s, 1.0, 2.0);
```

(%o83) 0.75

Comparison

```
(%i84) compareT(g, x, a, b):=block(

          [t, v, e, numer:true],
          local(f),

          define(f(x), g),
          t:T(f(x), x, a, b),
          v:integrate(f(x), x, a, b),
          e:abs(v−t),
          print(t, "   ", v, "     ", e)
       );
```

(%o84) compareT(g,x,a,b):=block$\Big($[t,v,e,numer:**true**],local(f),define$(f(x),g)$,t:T(

14.1.1

(%i85) compareT(1/x, x, 1, 2);

0.75 0.6931471805599453 0.05685281944005471

(%o85) 0.05685281944005471

14.1.2

(%i86) compareT(exp(−x), x, 0, 1);

0.6839397205857212 0.6321205588285578
0.05181916175716339

(%o86) 0.05181916175716339

14.2

Composite Trapzoidal Rule

14.2.1

```
(%i87) for k:2 next 2·k thru 150 do block(
           temp:Tc(sin(x), x, 0, %pi, k),
           print(k, "      ", temp)
           );
```

2 1.570796326794896
4 1.896118897937039
8 1.97423160194555
16 1.993570343772339
32 1.998393360970144
64 1.999598388640037
128 1.999899600184203

(%o87) *done*

14.2.2

```
(%i88) for k:2 next 2·k thru 150 do block(
          temp:Tc(sqrt(1+x^3),   x,     0.0,     1.0,   k),
          print(k, "      ", temp)
          );
2        1.133883476483184
4        1.116993293318717
8        1.112830349496382
16        1.111793319381881
32        1.111534292393827
64        1.111469550038714
128        1.111453365349166
(%o88) done
```

15 Simpson Rule

Basic Simpson Rule

15.1

```
(%i89) S(1/u,    u,   1.0,    2.0);
(%o89) 0.6944444444444443
```

15.2

```
(%i90) compareS(g,     x,     a,    b):=block(

          [t,    v,     e,     numer:true],
          local(f),

          define(f(x),    g),
          t:S(f(x),   x,    a,    b),
          v:integrate(f(x),    x,    a,   b),
          e:abs(v−t),
          print(t, "   ",   v,  "      ", e)
       );
(%o90) compareS(g,x,a,b):=block([t,v,e,numer:true],local(f),define(f(x),g),t:S(
```

15.2.1

```
(%i91) compareS(1/x,   x,  1,   2);
       0.6944444444444443     0.6931471805599453
       0.001297263884499022
(%o91) 0.001297263884499022
```

15.2.2

```
(%i92) compareS(exp(−x),   x,  0,   1);
       0.6323336800036626     0.6321205588285578
                           −4
       2.131211751048578 10
                           −4
(%o92) 2.131211751048578 10
```

15.3

Composite Simpson rule

15.3.1

```
(%i93) for k:2 next 2·k thru 150 do block(
             temp:Sc(sin(t),  t,   0,  %pi,  k),
             print(k, "      ", temp)
             );
       2      2.094395102393195
       4      2.00455975498442
       8      2.000269169948387
       16       2.000016591047935
       32       2.000001033369412
       64       2.000000064530001
       128       2.000000004032257
(%o93) done
```

15.3.2

```
(%i94) for k:2 next 2·k thru 150 do block(
          temp:Sc(sqrt(1+x^3),  x,   0,   1,  k),
          print(k, "      ", temp)
          );
2        1.109475708248729
4        1.111363232263895
8        1.111442701555604
16        1.111447642677047
32        1.111447950064475
64        1.111447969253676
128        1.111447970452649
(%o94) done
```

16 Euler Method

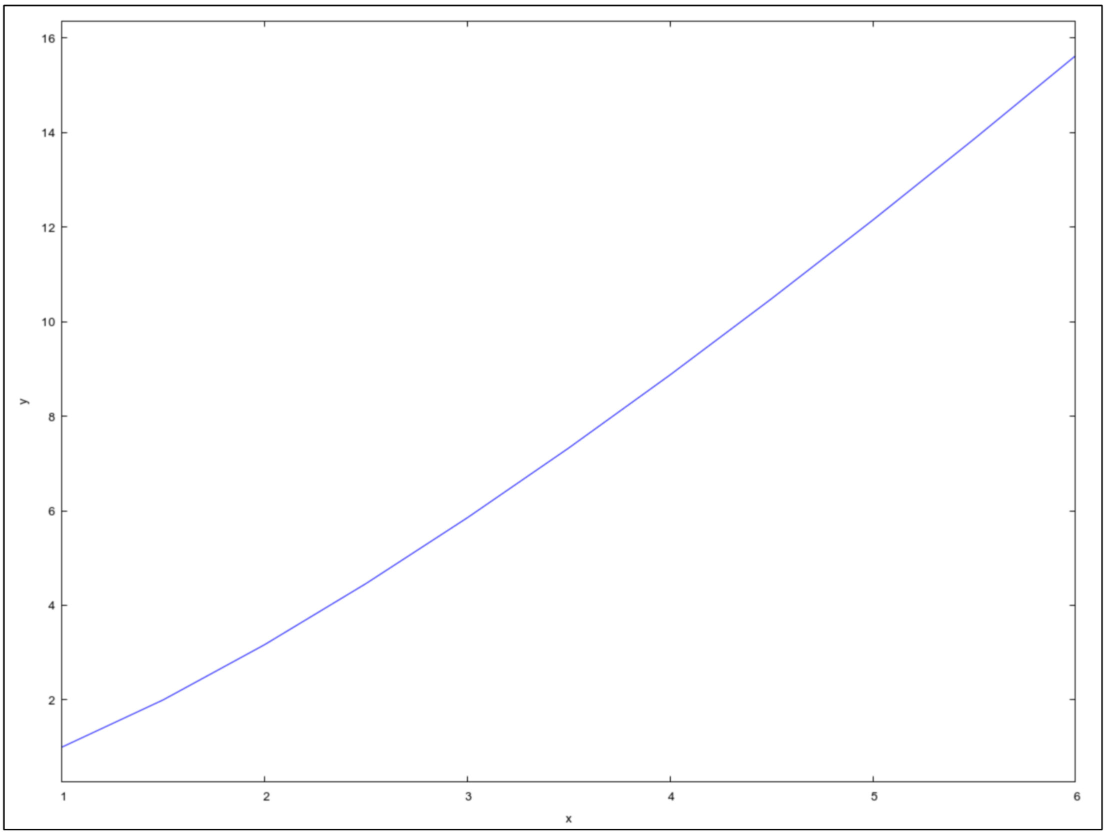
16.1

```
(%i95) ans:eulerm(1+(x/t),   t,   x,   1,  1,  6,   10);
(%o95) [[1,1],[1.5,2.0],[2.0,3.166666666666666],[2.5,
4.458333333333333],[3.0,5.85],[3.5,7.324999999999999],[
4.0,8.87142857142857],[4.5,10.48035714285714],[5.0,
12.14484126984126],[5.5,13.85932539682539],[6.0,
15.61926406926406]]
```

plot of the solution

(%i96) wxplot2d([discrete, ans]);

(%t96)



(%o96)

print the values

(%i97) for k:1 thru 11 do print(ans[k][1],"------------", ans[k][2]);

```
1 ------------ 1
1.5 ------------ 2.0
2.0 ------------ 3.166666666666666
2.5 ------------ 4.458333333333333
3.0 ------------ 5.85
3.5 ------------ 7.324999999999999
4.0 ------------ 8.87142857142857
4.5 ------------ 10.48035714285714
5.0 ------------ 12.14484126984126
5.5 ------------ 13.85932539682539
6.0 ------------ 15.61926406926406
```

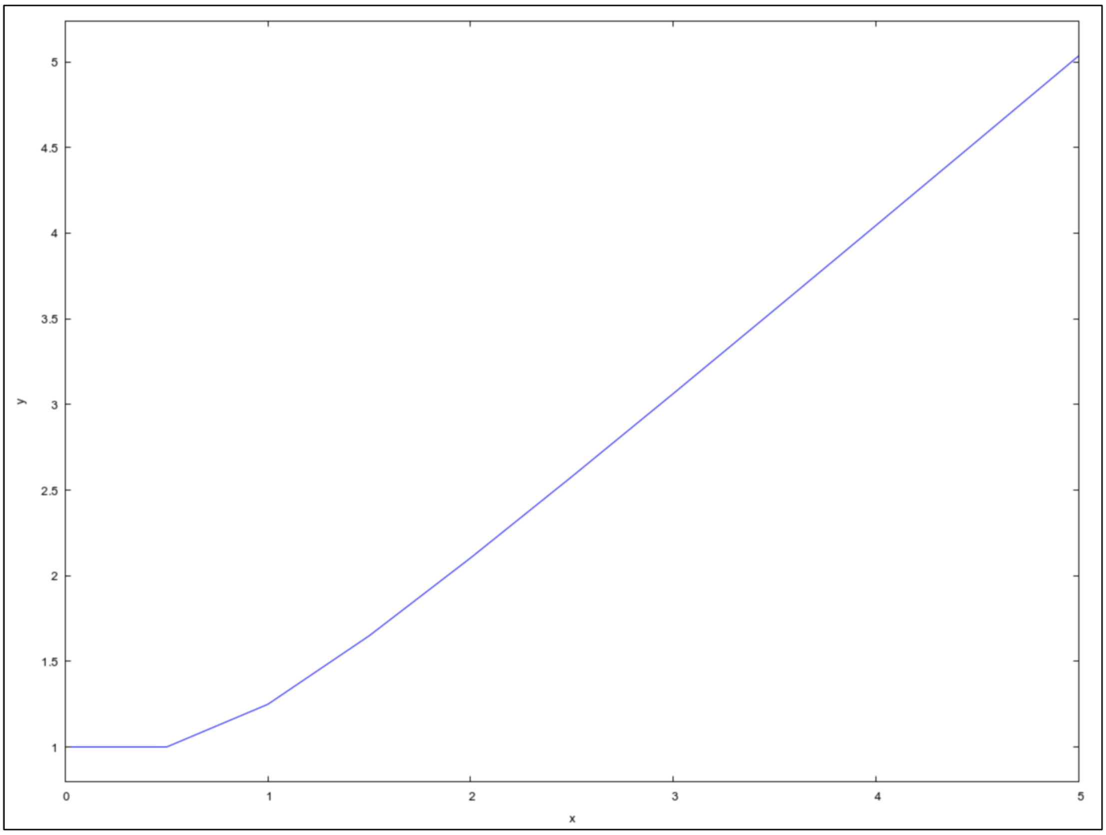
(%o97) *done*

16.2

```
(%i98) ans:eulerm((t/x), t, x, 0, 1, 5, 10);
(%o98) [[0,1],[0.5,1],[1.0,1.25],[1.5,1.65],[2.0,
2.104545454545454],[2.5,2.579707441586491],[3.0,
3.064258511157103],[3.5,3.553773346324708],[4.0,
4.046207677233808],[4.5,4.54049767901822],[5.0,
5.036038071076412]]
```

plot of the solution

```
(%i99) wxplot2d([discrete, ans]);
(%t99)
```




```
(%o99)
```

print the values

```
(%i100) for k:1 thru 11 do print(ans[k][1],"------------", ans[k][2] );
0 ------------ 1
0.5 ------------ 1
1.0 ------------ 1.25
1.5 ------------ 1.65
2.0 ------------ 2.104545454545454
2.5 ------------ 2.579707441586491
3.0 ------------ 3.064258511157103
3.5 ------------ 3.553773346324708
4.0 ------------ 4.046207677233808
4.5 ------------ 4.54049767901822
5.0 ------------ 5.036038071076412
(%o100) done
```

17 Second-Order Runge-Kutta Methods

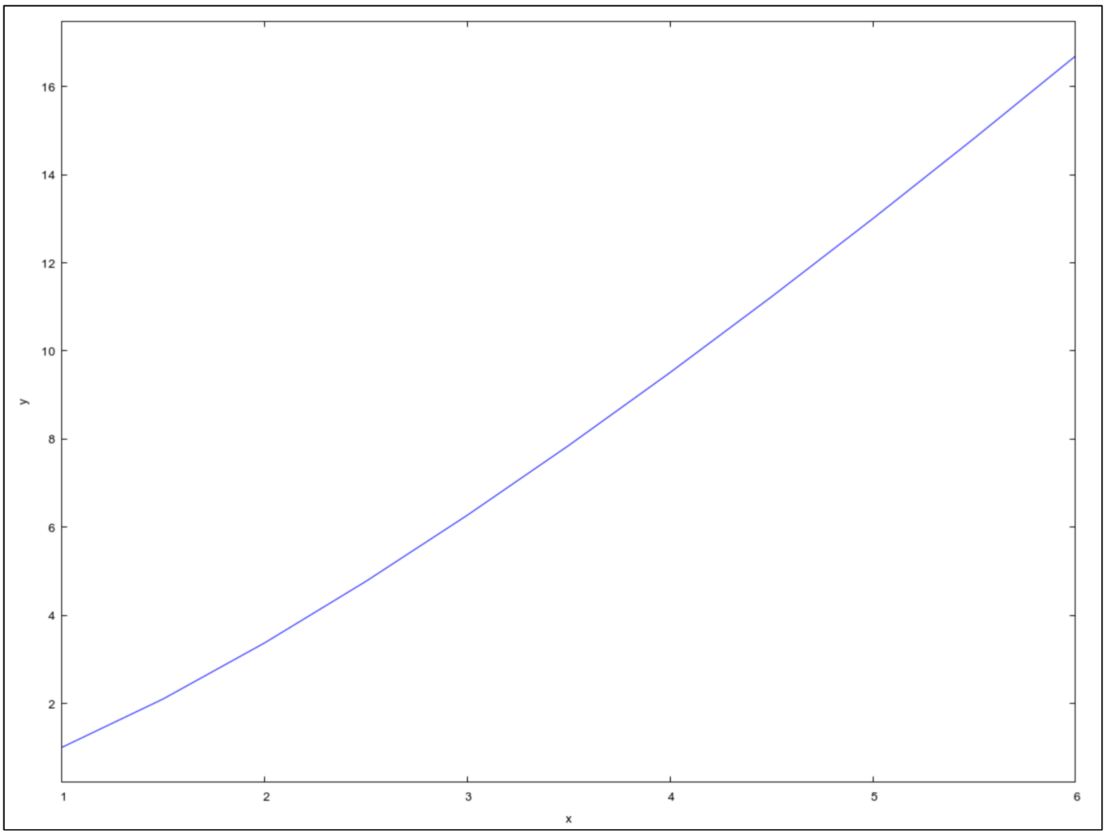
17.1 Modified-Euler Method

17.1.1

```
(%i101) ans:eulermod(1+(x/t),   t,   x,   1.0,    1.0,   6,   10);
(%o101) [[1.0,1.0],[1.5,2.1],[2.0,3.371428571428571],[2.5,
4.76984126984127],[3.0,6.26926406926407],[3.5,
7.852602952602953],[4.0,9.507736707736708],[4.5,
11.22561556090967],[5.0,12.99922196826221],[5.5,
14.82295368889796],[6.0,16.69223406377801]]
```

(%i102) wxplot2d([discrete, ans]);

(%t102)



(%o102)

(%i103) for k:1 thru 11 do print(ans[k][1],"−−−−−−−−−−−−", ans[k][2]);

```
1.0 −−−−−−−−−−−− 1.0
1.5 −−−−−−−−−−−− 2.1
2.0 −−−−−−−−−−−− 3.371428571428571
2.5 −−−−−−−−−−−− 4.76984126984127
3.0 −−−−−−−−−−−− 6.26926406926407
3.5 −−−−−−−−−−−− 7.852602952602953
4.0 −−−−−−−−−−−− 9.507736707736708
4.5 −−−−−−−−−−−− 11.22561556090967
5.0 −−−−−−−−−−−− 12.99922196826221
5.5 −−−−−−−−−−−− 14.82295368889796
6.0 −−−−−−−−−−−− 16.69223406377801
```

(%o103) *done*

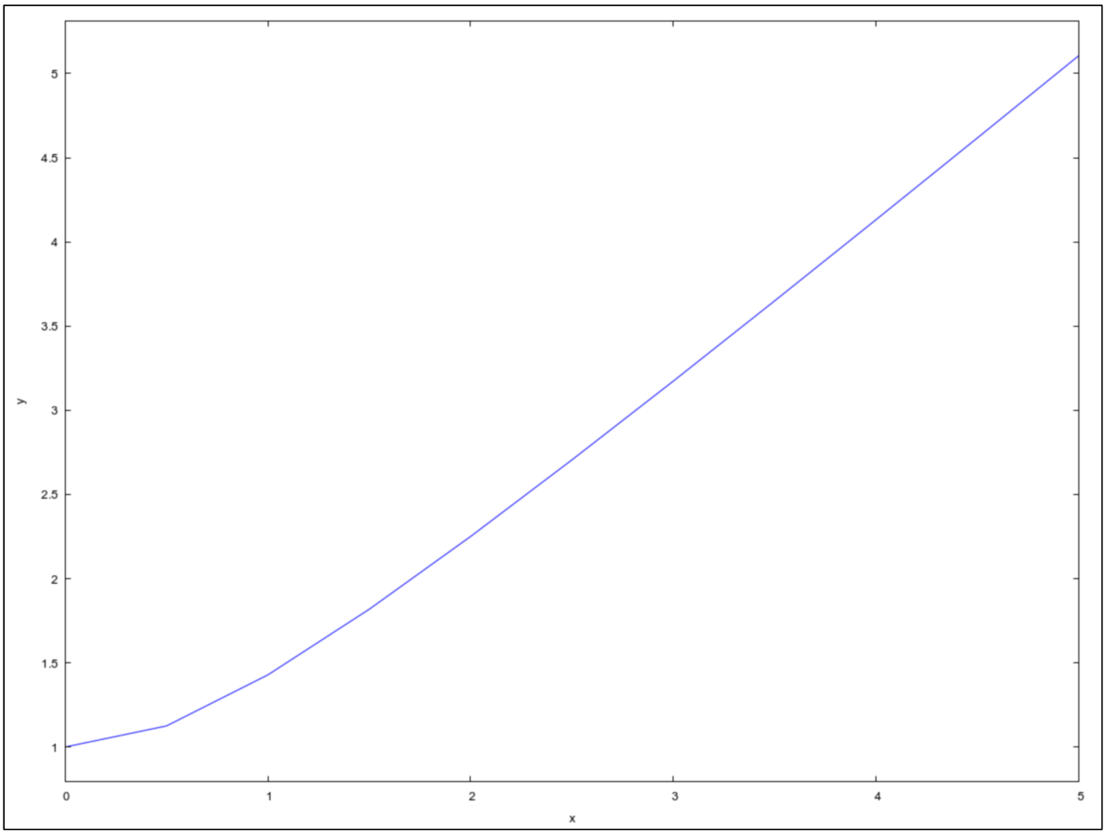
17.1.2

(%i104) ans:eulermod((t/x), t, x, 0, 1, 5, 10);

(%o104) [[0,1],[0.5,1.125],[1.0,1.428370786516854],[1.5,
1.818168594488258],[2.0,2.250391136810608],[2.5,
2.705382439640636],[3.0,3.173642355339167],[3.5,
3.650186992527909],[4.0,4.132204433515342],[4.5,
4.618006883981802],[5.0,5.106527324792223]]

(%i105) wxplot2d([discrete, ans]);

(%t105)



(%o105)

(%i106) for k:1 thru 11 do print(ans[k][1],"−−−−−−−−−−−−", ans[k][2]);

```
0 −−−−−−−−−−−− 1
0.5 −−−−−−−−−−−− 1.125
1.0 −−−−−−−−−−−− 1.428370786516854
1.5 −−−−−−−−−−−− 1.818168594488258
2.0 −−−−−−−−−−−− 2.250391136810608
2.5 −−−−−−−−−−−− 2.705382439640636
3.0 −−−−−−−−−−−− 3.173642355339167
3.5 −−−−−−−−−−−− 3.650186992527909
4.0 −−−−−−−−−−−− 4.132204433515342
4.5 −−−−−−−−−−−− 4.618006883981802
5.0 −−−−−−−−−−−− 5.106527324792223
```

(%o106) *done*

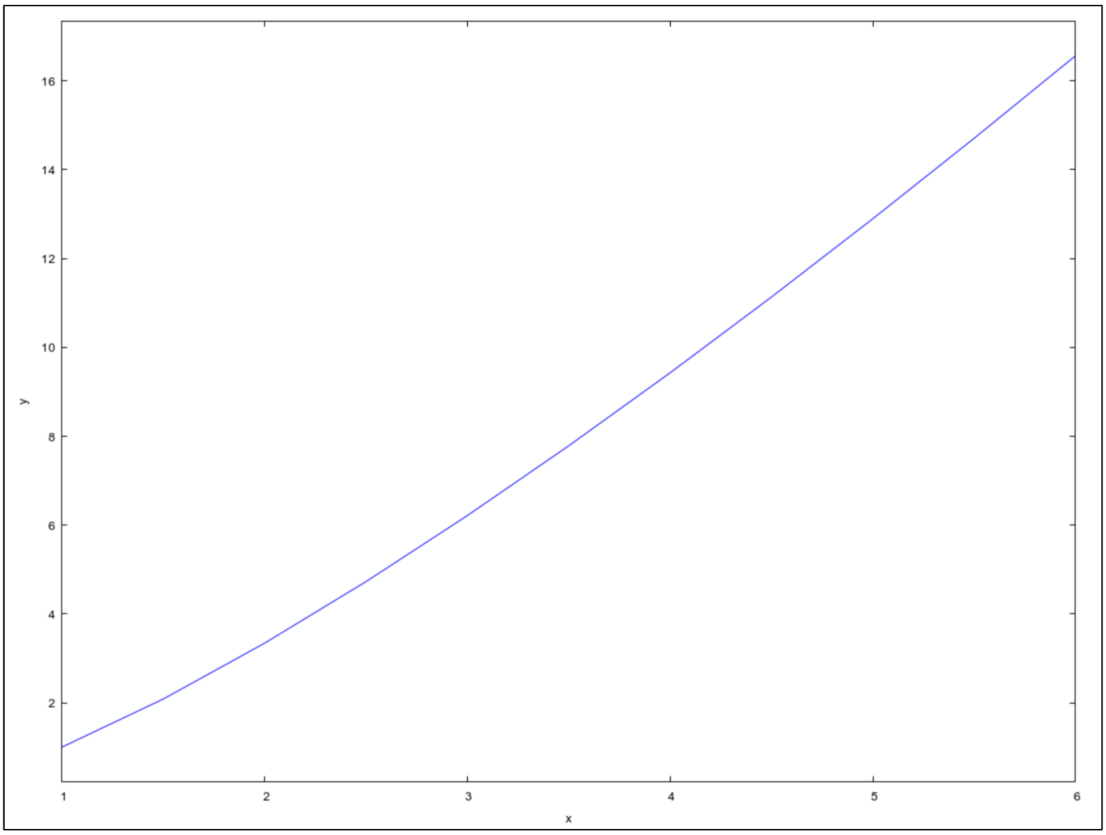
17.2 Heun's Method

17.2.1

(%i109)
```
ans:heun(1+(x/t),   t,   x,   1.0,    1.0,   6,   10);
wxplot2d([discrete,  ans]);
for k:1 thru 11 do print(ans[k][1],"————————————", ans[k][2] );
```

(%o107) [[1.0,1.0],[1.5,2.083333333333333],[2.0,3.340277777777777],[2.5,4.725347222222221],[3.0,6.212083333333332],[3.5,7.78314484126984],[4.0,9.426272675736959],[4.5,11.13233453798185],[5.0,12.89426059775761],[5.5,14.70641393026065],[6.0,16.56419398452677]]

(%t108)



(%o108)

1.0 ———————————— 1.0
1.5 ———————————— 2.083333333333333
2.0 ———————————— 3.340277777777777
2.5 ———————————— 4.725347222222221
3.0 ———————————— 6.212083333333332
3.5 ———————————— 7.78314484126984
4.0 ———————————— 9.426272675736959
4.5 ———————————— 11.13233453798185
5.0 ———————————— 12.89426059775761
5.5 ———————————— 14.70641393026065
6.0 ———————————— 16.56419398452677

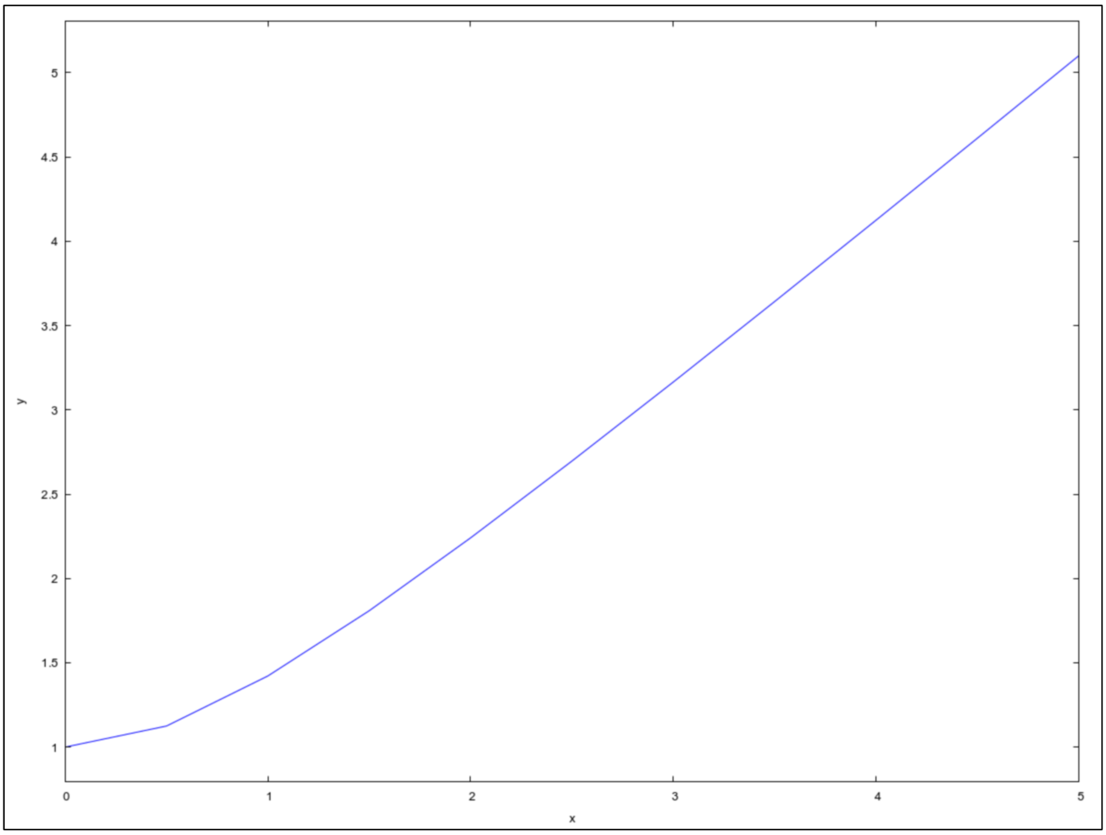
(%o109) *done*

17.2.2

```
(%i112) ans:heun((t/x), t, x, 0, 1, 5, 10);
        wxplot2d([discrete, ans]);
        for k:1 thru 11 do print(ans[k][1],"------------", ans[k][2] );
```

(%o110) [[0,1],[0.5,1.125],[1.0,1.421678121420389],[1.5,1.808987822487826],[2.0,2.241148239963114],[2.5,2.696819448943987],[3.0,3.165891040921445],[3.5,3.643196174669014],[4.0,4.125879763718132],[4.5,4.612253947754017],[5.0,5.101263359939439]]

(%t111)



(%o111)

```
0 ------------ 1
0.5 ------------ 1.125
1.0 ------------ 1.421678121420389
1.5 ------------ 1.808987822487826
2.0 ------------ 2.241148239963114
2.5 ------------ 2.696819448943987
3.0 ------------ 3.165891040921445
3.5 ------------ 3.643196174669014
4.0 ------------ 4.125879763718132
4.5 ------------ 4.612253947754017
5.0 ------------ 5.101263359939439
```

(%o112) *done*

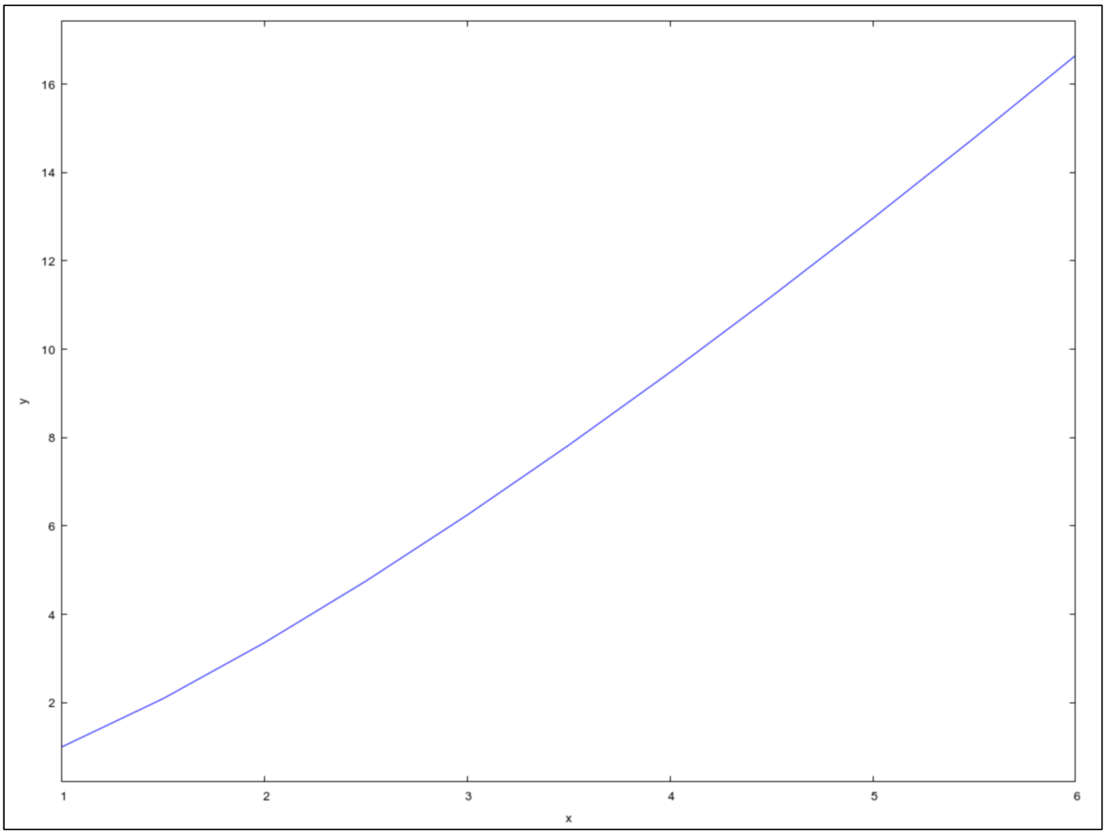
17.3 Optimal RK2 Method

17.3.1

```
(%i115) ans:rk2(1+(x/t),   t,   x,   1.0,    1.0,   6,   10);
        wxplot2d([discrete,  ans]);
        for k:1 thru 11 do print(ans[k][1],"−−−−−−−−−−−−", ans[k][2] );
```

(%o113) [[1.0,1.0],[1.5,2.09375],[2.0,3.359848484848484],[2.5,4.753382034632034],[3.0,6.248176088617265],[3.5,7.827038770053475],[4.0,9.477795861427574],[4.5,11.19136649795217],[5.0,12.96071373335682],[5.5,14.7802226066925],[6.0,16.64530777872948]]

(%t114)



(%o114)

```
1.0 −−−−−−−−−−−− 1.0
1.5 −−−−−−−−−−−− 2.09375
2.0 −−−−−−−−−−−− 3.359848484848484
2.5 −−−−−−−−−−−− 4.753382034632034
3.0 −−−−−−−−−−−− 6.248176088617265
3.5 −−−−−−−−−−−− 7.827038770053475
4.0 −−−−−−−−−−−− 9.477795861427574
4.5 −−−−−−−−−−−− 11.19136649795217
5.0 −−−−−−−−−−−− 12.96071373335682
5.5 −−−−−−−−−−−− 14.7802226066925
6.0 −−−−−−−−−−−− 16.64530777872948
```

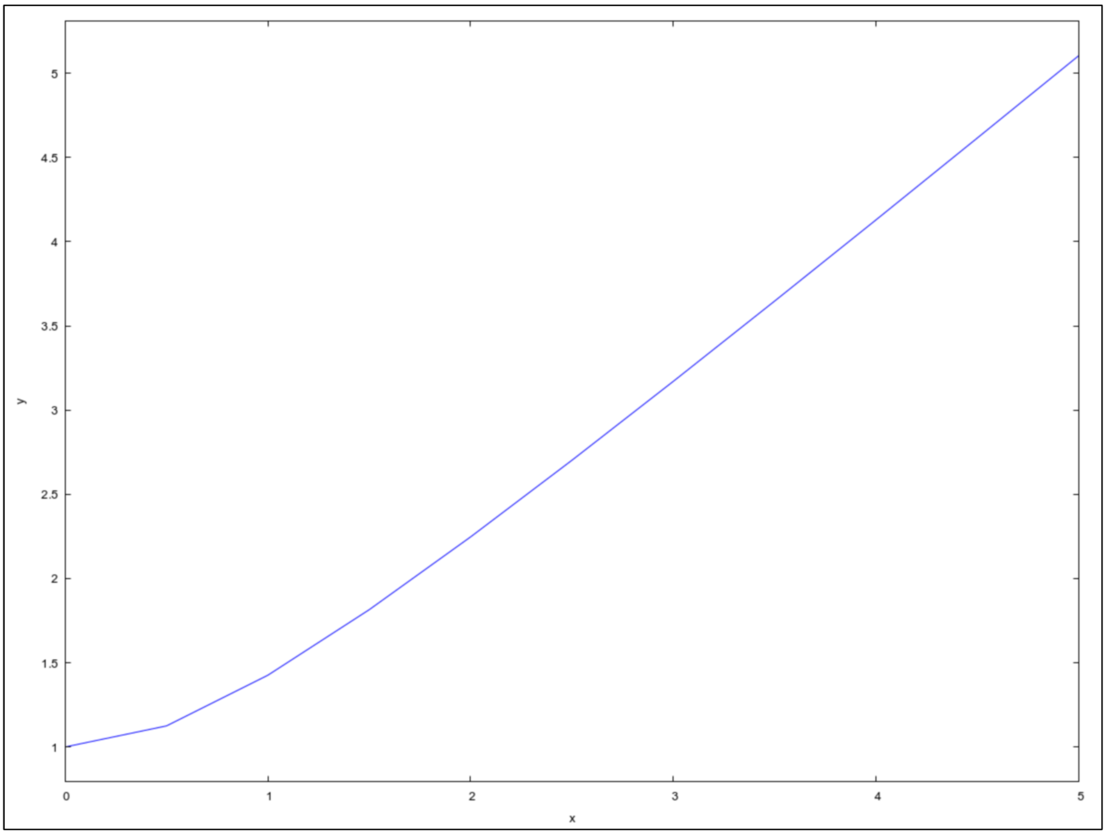
(%o115) *done*

17.3.2

```
(%i118) ans:rk2((t/x), t, x, 0, 1, 5, 10);
wxplot2d([discrete, ans]);
for k:1 thru 11 do print(ans[k][1],"————————————", ans[k][2] );
```

(%o116) [[0,1],[0.5,1.125],[1.0,1.426010101010101],[1.5,1.814915177090553],[2.0,2.24710861815986],[2.5,2.702338151234642],[3.0,3.170885177773465],[3.5,3.64769967070791],[4.0,4.129953831171324],[4.5,4.615959602475442],[5.0,5.104654003183263]]

(%t117)



(%o117)

0 ———————————— 1
0.5 ———————————— 1.125
1.0 ———————————— 1.426010101010101
1.5 ———————————— 1.814915177090553
2.0 ———————————— 2.24710861815986
2.5 ———————————— 2.702338151234642
3.0 ———————————— 3.170885177773465
3.5 ———————————— 3.64769967070791
4.0 ———————————— 4.129953831171324
4.5 ———————————— 4.615959602475442
5.0 ———————————— 5.104654003183263

(%o118) *done*

18 A Check

(%i122)
```
A1:matrix([5, 1, 2], [−3, 9, 4], [1, 2, −7]);
b1:matrix([10], [−14], [−33]);
start:matrix([0], [0], [0]);
gauss_jacobi(A1, b1, start, 14);
```

(%o119) $\begin{pmatrix} 5 & 1 & 2 \\ -3 & 9 & 4 \\ 1 & 2 & -7 \end{pmatrix}$

(%o120) $\begin{pmatrix} 10 \\ -14 \\ -33 \end{pmatrix}$

(%o121) $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

1

−−−−

$\begin{pmatrix} 2.0 & -1.555555555555555 & 4.714285714285714 \end{pmatrix}$

2

−−−−

$\begin{pmatrix} 0.4253968253968252 & -2.984126984126984 & 4.555555555555555 \end{pmatrix}$

3

−−−−

$\begin{pmatrix} 0.7746031746031745 & -3.438447971781304 & 3.922448979591836 \end{pmatrix}$

4

−−−−

$\begin{pmatrix} 1.118710002519526 & -3.040665154950868 & 3.842529604434367 \end{pmatrix}$

5

−−−−

$\begin{pmatrix} 1.071121189216427 & -2.890443156686543 & 4.005339956088256 \end{pmatrix}$

6

−−−−

$\begin{pmatrix} 0.9759526489020063 & -2.97866625074486 & 4.041462125120478 \end{pmatrix}$

7

−−−−

$\begin{pmatrix} 0.9791484001007809 & -3.026443394863987 & 4.002660021058898 \end{pmatrix}$

8

−−−−

$\begin{pmatrix} 1.004224670549238 & -3.008132764881471 & 3.989465944338972 \end{pmatrix}$

9

−−−−

$\begin{pmatrix} 1.005840175240705 & -2.993909973967574 & 3.998279877255185 \end{pmatrix}$

10

−−−−