

Examples : numerical.mac

1

```
→ load("/home/jvr/Downloads/numericalPackage/numerical.mac");
(%o1) /home/jvr/Downloads/numericalPackage/numerical.mac
→ ratprint:false;
(%o2) false
```

2

The examples are from the books by :

1. Ward Cheney and David Kincaid :: Numerical Mathematics and Computing
2. Brian Bradie :: A Friendly Introduction to Numerical Analysis

3 **Bisection Method**

3.1

```
→ bisect(t^3-2·sin(t), t, 0.50, 2.00, 20, 0.0005);
iter          m          ym          error
1            1.25        0.0551557612888276
2            0.75        -0.8651651294720542
3            0.375       -0.5476869797091422
4            0.1875      -0.2847914007983883
5            0.09375
6            1.203125     -
7            0.1247986155094702 0.046875
8            1.2265625
9            0.03735980652509796 0.0234375
10           1.23828125
11           0.00825801590073083 0.01171875
12           1.232421875
13           0.01471021624269308 0.005859375
14           1.2353515625
15           0.003266014170569153 0.0029296875
16           1.23681640625
17           0.002486011901918328 0.00146484375
18           1.236083984375
19           3.924970675475148 10^-4
20           1.2364501953125
21           0.001046133270412585 3.662109375 10^-4
(%o3) bisection has converged
```

3.2

```
→ bisect(sin(x), x, 2, 4, 20, 0.0005);
iter          m          ym          error
1             3          0.1411200080598672
1
2             3.5         -0.3507832276896198
0.5
3             3.25        -0.1081951345301083
0.25
4             3.125       0.0165918922293479
0.125
5             3.1875      -0.04589122327277969
0.0625
6             3.15625     -
0.01465682159049232   0.03125
7             3.140625    9.676534387822795
-4
10            0.015625
8             3.1484375   -
0.006844792961296519  0.0078125
9             3.14453125  -
0.002938592180907726  0.00390625
10            3.142578125 -
-4
9.854712506993688 10  0.001953125
11            3.1416015625 -
-6
8.908910206643689 10  9.765625 10^-4
12            3.14111328125 -
-4
4.793723214334506 10  4.8828125 10^-4
(%o4) bisection has converged
```

3.3

```
→ bisect(x^2-2, x, 1, 2, 20, 0.0005);
```

iter	m	ym	error
1	1.5	0.25	0.5
2	1.25	-0.4375	0.25
3	1.125	-0.734375	
4	1.0625	-0.87109375	
5	1.03125	-0.9365234375	
6	1.015625	-0.968505859375	
7	1.0078125	-0.98431396484375	
8	1.00390625	-	
9	1.001953125	0.00390625	-
10	1.0009765625	-	
11	1.00048828125	9.765625 10 ⁻⁴	-
	0.9990231990814209	4.8828125 10 ⁻⁴	

(%o5) bisection has converged

3.4

This examples was suggested by Professor Barton Willis

```
→ bisect(x^3-2*x^2+x-%pi, x, 2, 3, 10, 0.0001);
iter          m          ym          error
1            2.5        2.483407346410207
2            0.5        0.3740323464102069
3            0.25       -0.4521395285897931
4            0.125      -0.05687585671479311
5            0.0625     0.1540311257070818
6            0.03125
7            0.015625
8            0.0078125
9            0.00390625
10           0.001953125
10           0.0009765625 10^-4
(%o6) done
```

4 Newton's Method

4.1

```
→ newton(x^3-2*x^2+x-3, x, 3, 0.0001, 0.00001, 8);
1    2.4375      2.036865234375
2    2.213032716315109      0.2563633850614177
3    2.175554938721488      0.006463361488812325

4    2.174560100666446      4.479068049789703
10   2.174559410293312      2.156497203031904
10   2.174559410293312      2.156497203031904
(%o7) convergence
```

4.2

```
→ newton(sin(x), x, 3.0, 0.0001, 0.00001, 8);
  1      3.142546543074278      -9.538893398264409
  10-4
  2      3.141592653300477      2.893162490762184
  10-10
  3      3.141592653589793      1.224646799147353
  10-16
(%o8) convergence
```

4.3

```
→ newton(x^2-2, x, 1, 0.0001, 0.00001, 8);
  1      1.5          0.25
  2      1.416666666666666       0.006944444444444642
  3      1.414215686274509      6.007304882871267
  10-6
  4      1.414213562374689      4.510614104447086
  10-12
(%o9) convergence
```

```
→ newton(x^3-2*x^2+x-%pi, x, 3, 0.0001, 0.00001, 8);
  1      2.446349540849362      1.975991988443487
  2      2.230829413793257      0.237982391674409
  3      2.196863381645069      0.005374496729482203
  4      2.196060159694346      2.961172185678151
  10-6
  5      2.196059716657127      9.00612917575927
  10-13
(%o10) convergence
```

5 Secant Method

5.1

```
→ secant(x^3+2·x^2-3·x-1, x, -2, -3, 0.00001, 10);
 0      -3      -1
 1      -2      5
 2      -2.833333333333333      0.8101851851851833
 3      -2.907928388746803      0.04629957161572662
 4      -2.912449640422374      -0.002380064066290543
 5      -2.912228585591192      6.399876401275151 10-6
```

(%o11) convergence

5.2

```
→ secant(sin(x), x, 1, 4, 0.00001, 10);
 0      4      -0.7568024953079282
 1      1      0.8414709848078965
 2      2.579462454848934      0.5329898131328342
 3      3.166481028136553      -0.02488580518710502
 4      3.140295209333355      0.001297443892426365
 5      3.141592780561861      -1.269720678483891 10-7
```

(%o12) convergence

5.3

```
→ secant(x^2-2, x, 1, 2, 0.00001, 10);
 0      1      -1
 1      2      2
 2      1.333333333333333      -0.222222222222223
 3      1.428571428571428      0.04081632653061229
 4      1.413793103448275      -0.001189060642093009
 5      1.41421143847487      -6.007286838860537 10-6
```

(%o13) convergence

6 Regula-Falsi Method

6.1

```
→ regula(x^3+2·x^2-3·x-1, x, 1, 2, 20, 0.0005);
iter          m                      ym
1            1.1           -0.5489999999999999
2            1.221729490022173
0.1436388950901026
3            1.193880682944684
0.02924075169187978
4            1.199720665327255
0.006290908737020207
5            1.198472095487697
0.001337706025862806
6            1.198737948438891
2.851619710524389 10-4
(%o14) regula falsi method has converged
```

6.2

```
→ regula(sin(x), x, 2, 4, 20, 0.0005);
iter          m                      ym
1            3.091528082734958
0.05004365932452196
2            3.147874957380742
0.006282262466726139
3            3.141590357955694
2.295634098490862 10-6
(%o15) regula falsi method has converged
```

6.3

```
→ regula(x^2-2, x, 1, 2, 20, 0.0005);
iter          m                      ym
1            1.3333333333333333
0.2222222222222218
2            1.428571428571428
0.04081632653061229
3            1.411764705882353
0.006920415224913157
4            1.414634146341463
0.001189767995240842
5            1.414141414141414
2.040608101214758 10-4
(%o16) regula falsi method has converged
```

7 LU Factorization

7.1

```
→ A:matrix([1, 2, 3], [3, 5, 7], [4, 6, 9]);
luFactor(A);
luFactor(A)[1];
(%o17) ⎛1 2 3⎞
      ⎝3 5 7⎠
      ⎛4 6 9⎠
(%o18) ⎣L= ⎛1 0 0⎞, U= ⎛1 2 3⎞⎦
          ⎝3 1 0⎠           ⎝0 -1 -2⎠
          ⎝4 2 1⎠
(%o19) L= ⎛1 0 0⎞
          ⎝3 1 0⎠
          ⎝4 2 1⎠
```

8 Solving by LU Factorization

8.1

```
→ A:matrix([1, 2, 3], [3, 5, 7], [4, 6, 9]);
b:matrix([1], [2], [3]);
solve_by_lu(A, b);
(%o20) ⎛1 2 3⎞
      ⎝3 5 7⎠
      ⎝4 6 9⎠
(%o21) ⎛1
      ⎝2
      ⎝3
(%o22) ⎣z= ⎛1
          ⎝-1
          ⎝1
          , x= ⎛0
          ⎝-1
          ⎝1⎦]
```

8.2

→ A:matrix([2, 7, 5], [6, 20, 10], [4, 3, 0]);
 b1:matrix([14], [36], [7]);
 b2:matrix([-4], [-16], [-7]);
 b3:matrix([-3], [-12], [6]);

(%o23)
$$\begin{pmatrix} 2 & 7 & 5 \\ 6 & 20 & 10 \\ 4 & 3 & 0 \end{pmatrix}$$

(%o24)
$$\begin{pmatrix} 14 \\ 36 \\ 7 \end{pmatrix}$$

(%o25)
$$\begin{pmatrix} -4 \\ -16 \\ -7 \end{pmatrix}$$

(%o26)
$$\begin{pmatrix} -3 \\ -12 \\ 6 \end{pmatrix}$$

→ luFactor(A);

(%o27)
$$L = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 11 & 1 \end{pmatrix}, U = \begin{pmatrix} 2 & 7 & 5 \\ 0 & -1 & -5 \\ 0 & 0 & 45 \end{pmatrix}$$

→ solve_by_lu(A, b1);

(%o28)
$$z = \begin{pmatrix} 14 \\ -6 \\ 45 \end{pmatrix}, x = \begin{pmatrix} 1.0 \\ 1.0 \\ 1.0 \end{pmatrix}$$

→ solve_by_lu(A, b2);

(%o29)
$$z = \begin{pmatrix} -4 \\ -4 \\ 45 \end{pmatrix}, x = \begin{pmatrix} -1.0 \\ -1.0 \\ 1.0 \end{pmatrix}$$

→ solve_by_lu(A, b3);

(%o30)
$$z = \begin{pmatrix} -3 \\ -3 \\ 45 \end{pmatrix}, x = \begin{pmatrix} 3.0 \\ -2.0 \\ 1.0 \end{pmatrix}$$

9 Gauss-Jacobi Method

9.1

```
→ A1:=matrix([5, 1, 2], [-3, 9, 4], [1, 2, -7]);  
b1:=matrix([10], [-14], [-33]);  
start:=matrix([0], [0], [0]);  
gauss_jacobi(A1, b1, start, 14);  
  
(%o31) 
$$\begin{pmatrix} 5 & 1 & 2 \\ -3 & 9 & 4 \\ 1 & 2 & -7 \end{pmatrix}$$
  
(%o32) 
$$\begin{pmatrix} 10 \\ -14 \\ -33 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$
  
-----  
(%o33) 
$$\begin{pmatrix} 2.0 & -1.55555555555555 & 4.714285714285714 \\ 0.4253968253968252 & -2.984126984126984 & 4.55555555555555 \\ 0.7746031746031745 & -3.438447971781304 & 3.922448979591836 \\ 1.118710002519526 & -3.040665154950868 & 3.842529604434367 \\ 1.071121189216427 & -2.890443156686543 & 4.005339956088256 \\ 0.9759526489020063 & -2.97866625074486 & 4.041462125120478 \\ 0.9791484001007809 & -3.026443394863987 & 4.002660021058898 \\ 1.004224670549238 & -3.008132764881471 & 3.989465944338972 \\ 1.005840175240705 & -2.993909973967574 & 3.998279877255185 \\ 0.9994700438914408 & -2.997288775922069 & 4.002574318186508 \end{pmatrix}$$

```

9.2

```
→ A1:matrix([2, -1, 0], [-1, 4, 2], [0, 2, 6]);
b1:matrix([-1], [3], [5]);
start:matrix([0], [0], [0]);
gauss_jacobi(A1, b1, start, 14);

(%o35) ⎛ 2 -1 0 ⎞
      ⎜ -1 4 2 ⎟
      ⎜ 0 2 6 ⎟
      ⎜ -1 ⎟
(%o36) ⎜ 3 ⎟
      ⎜ 5 ⎟
      ⎜ 0 ⎟
      ⎜ 0 ⎟
      ⎜ 0 ⎟
      ⎜ 1 ⎟
      -----
      ⎛ -0.5 0.75 0.8333333333333333 ⎞
      ⎜ 2 ⎠
      -----
      ⎛ -0.125 0.2083333333333333 0.5833333333333333 ⎞
      ⎜ 3 ⎠
      -----
      ⎛ -0.3958333333333333 0.4270833333333333 0.7638888888888888 ⎞
      ⎜ 4 ⎠
      -----
      ⎛ -0.2864583333333333 0.2690972222222222 0.6909722222222221 ⎞
      ⎜ 5 ⎠
      -----
      ⎛ -0.3654513888888888 0.3328993055555556 0.7436342592592592 ⎞
      ⎜ 6 ⎠
      -----
      ⎛ -0.3335503472222222 0.2868200231481482 0.722366898148148 ⎞
      ⎜ 7 ⎠
      -----
      ⎛ -0.3565899884259259 0.3054289641203704 0.7377266589506172 ⎞
      ⎜ 8 ⎠
      -----
      ⎛ -0.3472855179398147 0.2919891734182099 0.7315236786265431 ⎞
      ⎜ 9 ⎠
      -----
      ⎛ -0.354005413290895 0.2974167812017747 0.7360036088605966 ⎞
      ⎜ 10 ⎠
      -----
      ⎛ -0.3512916093991126 0.293496842246978 0.734194406266075 ⎞
      ⎜ 11 ⎠
```

10 SOR Method

10.1

```
→ A1:matrix([5, 1, 2], [-3, 9, 4], [1, 2, -7]);  
b1:matrix([10], [-14], [-33]);  
start:matrix([0], [0], [0]);  
sor(A1, b1, 0.9, start, 10);  
  
(%o39) 
$$\begin{pmatrix} 5 & 1 & 2 \\ -3 & 9 & 4 \\ 1 & 2 & -7 \end{pmatrix}$$
  
  
(%o40) 
$$\begin{pmatrix} 10 \\ -14 \\ -33 \end{pmatrix}$$
  
  
(%o41) 
$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$
  
-----  

$$\begin{pmatrix} 1.799999999999999 & -0.859999999999999 & 4.253142857142857 \end{pmatrix}$$
  
2  
-----  

$$\begin{pmatrix} 0.6036685714285714 & -3.006156571428571 & 3.972774269387755 \end{pmatrix}$$
  
3  
-----  

$$\begin{pmatrix} 0.9712763030204081 & -2.998342473991836 & 3.994010601157784 \end{pmatrix}$$
  
4  
-----  

$$\begin{pmatrix} 0.9989854592037691 & -2.997742850101166 & 3.999851029130248 \end{pmatrix}$$
  
5  
-----  

$$\begin{pmatrix} 0.9995458884516973 & -2.999850930126706 & 3.999965049395661 \end{pmatrix}$$
  
6  
-----  

$$\begin{pmatrix} 0.9999403384855388 & -2.999989011225273 & 3.99999165985835 \end{pmatrix}$$
  
7  
-----  

$$\begin{pmatrix} 0.9999950583200967 & -2.999997047569838 & 3.999999289823317 \end{pmatrix}$$
  
8  
-----  

$$\begin{pmatrix} 0.9999992300581864 & -2.999999651668854 & 3.999999919560678 \end{pmatrix}$$
  
9  
-----  

$$\begin{pmatrix} 0.9999998892643681 & -2.999999966211847 & 3.999999986407011 \end{pmatrix}$$
  
10  
-----  

$$\begin{pmatrix} 0.999999987738045 & -2.99999994862575 & 3.99999998385215 \end{pmatrix}$$
  
(%o42) done
```

```
→ A1:matrix([2, -1, 0], [-1, 4, 2], [0, 2, 6]);
b1:matrix([-1], [3], [5]);
start:matrix([0], [0], [0]);
sor(A1, b1, 0.9, start, 14);

(%o43) ⎛ 2 -1 0 ⎞
      ⎜ -1 4 2 ⎟
      ⎜ 0 2 6 ⎟
      ⎜ -1 ⎟
(%o44) ⎜ 3 ⎟
      ⎜ 5 ⎟
      ⎜ 0 ⎟
      ⎜ 0 ⎟
      ⎜ 0 ⎟
      ⎜ 1 ⎟
      -----
      ⎛ -0.45 0.5737500000000001 0.577875 ⎞
      2
      -----
      ⎛ -0.2368124999999999 0.4190484375000001 0.68207296875 ⎞
      3
      -----
      ⎛ -0.2851094531249999 0.3458223808593751 0.7144605826171875 ⎞
      4
      -----
      ⎛ -0.3228908739257812 0.3154245292749024 0.726818699479248 ⎞
      5
      -----
      ⎛ -0.3403480492188721 0.3028957270875825 0.7318131518216502 ⎞
      6
      -----
      ⎛ -0.3477317277324751 0.2977340156492088 0.7338611104874024 ⎞
      7
      -----
      ⎛ -0.3507928657311035 0.2956075070560916 0.7347038589319128 ⎞
      8
      -----
      ⎛ -0.3520559083978691 0.2947314347967279 0.735050955454173 ⎞
      9
      -----
      ⎛ -0.3525764451812593 0.2943705133595117 0.7351939415375639 ⎞
      10
      -----
      ⎛ -0.3527909135063456 0.2942218221051197 0.7352528475222205 ⎞
      11
```

10.2

```
→ A1:matrix([2, -1, 0], [-1, 4, 2], [0, 2, 6]);
b1:matrix([-1], [3], [5]);
start:matrix([0], [0], [0]);
sor(A1, b1, 1.1, start, 14);

(%o47) ⎛ 2 -1 0 ⎞
      ⎜ -1 4 2 ⎟
      ⎜ 0 2 6 ⎟
      ⎜ -1 ⎟
(%o48) ⎜ 3 ⎟
      ⎜ 5 ⎟
      ⎜ 0 ⎟
      ⎜ 0 ⎟
      ⎜ 0 ⎟
      ⎜ 1 ⎟
      -----
      ⎛ -0.55 0.6737500000000001 0.669624999999999 ⎞
      2
      -----
      ⎛ -0.1244374999999999 0.3551109375000001 0.7194968229166666 ⎞
      3
      -----
      ⎛ -0.342245234375 0.2996482141927084 0.7348459725043401 ⎞
      4
      -----
      ⎛ -0.3509689587565104 0.2943534300453017 0.7352524783996219 ⎞
      5
      -----
      ⎛ -0.353008717599433 0.2940983965358337 0.7353053400968987 ⎞
      6
      -----
      ⎛ -0.3529450101453482 0.2941123455031516 0.7352949393058211 ⎞
      7
      -----
      ⎛ -0.3529437089587318 0.294117028867832 0.7352942621512127 ⎞
      8
      -----
      ⎛ -0.3529412632268192 0.2941176055426745 0.7352941184192313 ⎞
      9
      -----
      ⎛ -0.3529411906288471 0.2941176468922224 0.7352941176309286 ⎞
      10
      -----
      ⎛ -0.352941175146393 0.294117647448509 0.7352941175057871 ⎞
      11
```

11 Gauss-Seidel Method

11.1

```
→ A1:=matrix([5, 1, 2], [-3, 9, 4], [1, 2, -7]);  
b1:=matrix([10], [-14], [-33]);  
start:=matrix([0], [0], [0]);  
gauss_seidel(A1, b1, start, 10);  
  
(%o51) 
$$\begin{pmatrix} 5 & 1 & 2 \\ -3 & 9 & 4 \\ 1 & 2 & -7 \end{pmatrix}$$
  
(%o52) 
$$\begin{pmatrix} 10 \\ -14 \\ -33 \end{pmatrix}$$
  
(%o53) 
$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$
  
-----  
(%o54) 
$$\begin{pmatrix} 2.0 & -0.8888888888888887 & 4.746031746031746 \end{pmatrix}$$
  
2  
-----  
(%o55) 
$$\begin{pmatrix} 0.2793650793650792 & -3.571781305114638 & 3.7336860670194 \end{pmatrix}$$
  
3  
-----  
(%o56) 
$$\begin{pmatrix} 1.220881834215167 & -2.808010973936899 & 4.086408555191624 \end{pmatrix}$$
  
4  
-----  
(%o57) 
$$\begin{pmatrix} 0.9270387727107303 & -3.062724211403811 & 3.971655764271872 \end{pmatrix}$$
  
5  
-----  
(%o58) 
$$\begin{pmatrix} 1.023882536572013 & -2.979441716374605 & 4.0092855862604 \end{pmatrix}$$
  
6  
-----  
(%o59) 
$$\begin{pmatrix} 0.992174108770761 & -3.006735557636591 & 3.996957570499654 \end{pmatrix}$$
  
7  
-----  
(%o60) 
$$\begin{pmatrix} 1.002564083327456 & -2.997793114668471 & 4.000996836284359 \end{pmatrix}$$
  
8  
-----  
(%o61) 
$$\begin{pmatrix} 0.9991598884199506 & -3.000723075541953 & 3.999673391048006 \end{pmatrix}$$
  
9  
-----  
(%o62) 
$$\begin{pmatrix} 1.000275258689188 & -2.999763087569384 & 4.000107011935774 \end{pmatrix}$$
  
10  
-----  
(%o63) 
$$\begin{pmatrix} 0.9999098127395674 & -3.000077623280488 & 3.999964938025513 \end{pmatrix}$$
  
(%o64) done
```

11.2

```
→ A1:matrix([2, -1, 0], [-1, 4, 2], [0, 2, 6]);
b1:matrix([-1], [3], [5]);
start:matrix([0], [0], [0]);
gauss_seidel(A1, b1, start, 14);

(%o55) ⎛ 2 -1 0 ⎞
      ⎜ -1 4 2 ⎟
      ⎜ 0 2 6 ⎟
      ⎜ -1 ⎟
(%o56) ⎜ 3 ⎟
      ⎜ 5 ⎟
      ⎜ 0 ⎟
      ⎜ 0 ⎟
      ⎜ 0 ⎟
      ⎜ 1 ⎟
      -----
      ⎛ -0.5 0.625 0.6249999999999999 ⎞
      ⎜ 2 ⎟
      -----
      ⎛ -0.1875 0.390625 0.7031249999999999 ⎞
      ⎜ 3 ⎟
      -----
      ⎛ -0.3046875 0.322265625 0.7259114583333332 ⎞
      ⎜ 4 ⎟
      -----
      ⎛ -0.3388671875 0.3023274739583334 0.7325575086805554 ⎞
      ⎜ 5 ⎟
      -----
      ⎛ -0.3488362630208332 0.296512179904514 0.7344959400318285 ⎞
      ⎜ 6 ⎟
      -----
      ⎛ -0.3517439100477429 0.29481605247215 0.7350613158426166 ⎞
      ⎜ 7 ⎟
      -----
      ⎛ -0.352591973763925 0.2943213486377105 0.7352262171207631 ⎞
      ⎜ 8 ⎟
      -----
      ⎛ -0.3528393256811447 0.2941770600193322 0.7352743133268891 ⎞
      ⎜ 9 ⎟
      -----
      ⎛ -0.3529114699903338 0.2941349758389719 0.7352883413870093 ⎞
      ⎜ 10 ⎟
      -----
      ⎛ -0.352932512080514 0.2941227012863668 0.7352924329045443 ⎞
      ⎜ 11 ⎟
```

12 Lagrange Interpolation

12.1

```

→      xval:[0, 1, -1, 2, -2]$
      yval:[-5, -3, -15, 39, -9]$
      LP(xval, yval, x);

(%o61) 1.625 (x-1) x (x+1) (x+2)+0.5 (x-2) x (x+1) (x+2)
         +1.25 (1-x) (x-2) (x+1) (x+2)+2.5 (x-2) (x-1) x (x+2)-
         0.375 (x-2) (x-1) x (x+1)

→      ratsimp(%);

(%o62) 3 x4+2 x3-7 x2+4 x-5

→      LP(xval, yval, 3);

(%o63) 241.0

```

12.2

```

→      xval:[-3, 1, 2, 5]$
      yval:[-23, -11, -23, 1]$
      LP(xval, yval, x);

(%o66) 0.0104166666666666 (x-2) (x-1) (x+3) +
         1.53333333333333 (x-5) (x-1) (x+3)-0.6875 (x-5) (x-2)
         (x+3)+0.14375 (x-5) (x-2) (x-1)

→      ratsimp(%);

(%o67) x3-3 x2-10 x+1

```

12.3

```

→      xval:[-1, 0, 1, 2]$
      yval:[5, 1, 1, 11]$
      LP(xval, yval, x);

(%o70) 1.83333333333333 (x-1) x (x+1)-0.5 (x-2) x (x+1)
         +0.5 (x-2) (x-1) (x+1)-0.83333333333333 (x-2) (x-1)
         x

→      ratsimp(%);

(%o71) x3+2 x2-3 x+1

```

13 Newton Interpolation

13.1

13.2

→ dd_table([1, 3/2, 0, 2], [3, 13/4, 3, 5/3]);

(%o72)

1	3	0.5	0.3333333333333333	-2.0
1.5	3.25	0.1666666666666666	-1.666666666666666	$A_{2,5}$
0	3	-0.6666666666666666		$A_{3,4}$
2	1.6666666666666666	$A_{4,3}$	$A_{4,4}$	$A_{4,5}$

13.3

→ dd_table([-1, 0, 1, 2, -2, 3], [5, 1, 1, 11, 5, 35]);

(%o73)

-1	5	-4	2	1	-0.0833333333333326	-1.7347234
0	1	0	5	1.083333333333333	-0.0833333333333333	
1	1	10	2.833333333333333	0.833333333333333		$A_{3,6}$
2	11	1.5	4.5	$A_{4,5}$		$A_{4,6}$
-2	5	6	$A_{5,4}$	$A_{5,5}$		$A_{5,6}$
3	35	$A_{6,3}$	$A_{6,4}$	$A_{6,5}$		$A_{6,6}$

13.4

13.5

→ xval: [-1, 0, 1, 2]\$
yval: [5, 1, 1, 11]\$
NP(xval, yval, x);

(%o76) $(x-1)x(x+1)+2x(x+1)-4(x+1)+5$

→ ratsimp(%);

(%o77) $x^3 + 2x^2 - 3x + 1$

13.6

```

→ xval: [-1, 0, 1, 2, -2, 3]$  

yval: [5, 1, 1, 11, 5, 35]$  

NP(xval, yval, x);  

(%o80) 
$$\frac{-1.734723475976807 \cdot 10^{-17} (x-2)(x-1)x(x+1)}{(x+2)-0.083333333333326 (x-2)(x-1)x(x+1)+(x-1)x}$$
  


$$(x+1)+2x(x+1)-4(x+1)+5$$
  

→ NP(xval, yval, 1.5);  

(%o81) 4.453125

```

14

```

→ ratprint:false;  

(%o82) false

```

14.1 Trapezoidal Rule

Basic Trapezoidal Rule

```

→ T(1/s, s, 1.0, 2.0);  

(%o83) 0.75

```

Comparison

```

→ compareT(g, x, a, b):=block(  

[t, v, e, numer:true],  

local(f),  

define(f(x), g),  

t:T(f(x), x, a, b),  

v:integrate(f(x), x, a, b),  

e:abs(v-t),  

print(t, " ", v, " ", e)  

);

```

```

(%o84) compareT(g,x,a,b):=block( $[t, v, e, numer:true], local(f), define(f(x), g), t:T(f(x), x, a, b), v:integrate(f(x), x, a, b), e:abs(v-t)$ )

```

14.1.1

```
→ compareT(1/x, x, 1, 2);
  0.75   0.6931471805599453      0.05685281944005471
(%o85) 0.05685281944005471
```

14.1.2

```
→ compareT(exp(-x), x, 0, 1);
  0.6839397205857212   0.6321205588285578
  0.05181916175716339
(%o86) 0.05181916175716339
```

14.2

Composite Trapzoidal Rule

14.2.1

```
→ for k:2 next 2·k thru 150 do block(
    temp:Tc(sin(x), x, 0, %pi, k),
    print(k, " ", temp)
);
  2   1.570796326794896
  4   1.896118897937039
  8   1.97423160194555
 16   1.993570343772339
 32   1.998393360970144
 64   1.999598388640037
128   1.999899600184203
(%o87) done
```

14.2.2

```
→ for k:2 next 2·k thru 150 do block(
    temp:Tc(sqrt(1+x^3), x, 0.0, 1.0, k),
    print(k, " ", temp)
);
  2   1.133883476483184
  4   1.116993293318717
  8   1.112830349496382
 16   1.111793319381881
 32   1.111534292393827
 64   1.111469550038714
128   1.111453365349166
(%o88) done
```

15 Simpson Rule

Basic Simpson Rule

15.1

```
→ S(1/u, u, 1.0, 2.0);
(%o89) 0.6944444444444443
```

15.2

```
→ compareS(g, x, a, b):=block(
  [t, v, e, numer:true],
  local(f),
  define(f(x), g),
  t:S(f(x), x, a, b),
  v:integrate(f(x), x, a, b),
  e:abs(v-t),
  print(t, " ", v, " ", e)
);
```

(%o90) $\text{compareS}(g, x, a, b) := \text{block}\left([t, v, e, \text{numer: true}], \text{local}(f), \text{define}(f(x), g), t : S(f(x), x, a, b), v : \int f(x) dx, e : |v - t|, \text{print}(t, " ", v, " ", e)\right)$

15.2.1

```
→ compareS(1/x, x, 1, 2);
0.6944444444444443 0.6931471805599453
0.001297263884499022
(%o91) 0.001297263884499022
```

15.2.2

```
→ compareS(exp(-x), x, 0, 1);
0.6323336800036626 0.6321205588285578
2.131211751048578  $10^{-4}$ 
(%o92) 2.131211751048578  $10^{-4}$ 
```

15.3

Composite Simpson rule

15.3.1

```
→ for k:2 next 2·k thru 150 do block(
    temp:Sc(sin(t), t, 0, %pi, k),
    print(k, " ", temp)
);
2      2.094395102393195
4      2.00455975498442
8      2.000269169948387
16     2.000016591047935
32     2.000001033369412
64     2.000000064530001
128    2.000000004032257
(%o93) done
```

15.3.2

```
→ for k:2 next 2·k thru 150 do block(
    temp:Sc(sqrt(1+x^3), x, 0, 1, k),
    print(k, " ", temp)
);
2      1.109475708248729
4      1.111363232263895
8      1.111442701555604
16     1.111447642677047
32     1.111447950064475
64     1.111447969253676
128    1.111447970452649
(%o94) done
```

16 Euler Method**16.1**

```
→ eulerm(1+(x/t), t, x, 1.0, 1.0, 6, 10);  
1.0 ----- 1.0  
1.5 ----- 2.0  
2.0 ----- 3.166666666666666  
2.5 ----- 4.458333333333333  
3.0 ----- 5.85  
3.5 ----- 7.324999999999999  
4.0 ----- 8.87142857142857  
4.5 ----- 10.48035714285714  
5.0 ----- 12.14484126984126  
5.5 ----- 13.85932539682539  
6.0 ----- 15.61926406926406  
(%o95) done
```

16.2

```
→ eulerm((t/x), t, x, 0, 1, 5, 10);  
0 ----- 1  
0.5 ----- 1  
1.0 ----- 1.25  
1.5 ----- 1.65  
2.0 ----- 2.104545454545454  
2.5 ----- 2.579707441586491  
3.0 ----- 3.064258511157103  
3.5 ----- 3.553773346324708  
4.0 ----- 4.046207677233808  
4.5 ----- 4.54049767901822  
5.0 ----- 5.036038071076412  
(%o96) done
```

17 Second-Order Runge-Kutta Methods

17.1 Modified-Euler Method

17.1.1

```
→ eulermod(1+(x/t), t, x, 1.0, 1.0, 6, 10);
1.0 ---- 1.0
1.5 ---- 2.1
2.0 ---- 3.371428571428571
2.5 ---- 4.76984126984127
3.0 ---- 6.26926406926407
3.5 ---- 7.852602952602953
4.0 ---- 9.507736707736708
4.5 ---- 11.22561556090967
5.0 ---- 12.99922196826221
5.5 ---- 14.82295368889796
6.0 ---- 16.69223406377801
(%o97) done
```

17.1.2

```
→ eulermod((t/x), t, x, 0, 1, 5, 10);
0 ---- 1
0.5 ---- 1.125
1.0 ---- 1.428370786516854
1.5 ---- 1.818168594488258
2.0 ---- 2.250391136810608
2.5 ---- 2.705382439640636
3.0 ---- 3.173642355339167
3.5 ---- 3.650186992527909
4.0 ---- 4.132204433515342
4.5 ---- 4.618006883981802
5.0 ---- 5.106527324792223
(%o98) done
```

17.2 Heun's Method

17.2.1

```
→ heun(1+(x/t), t, x, 1.0, 1.0, 6, 10);  
1.0 ----- 1.0  
1.5 ----- 2.08333333333333  
2.0 ----- 3.34027777777777  
2.5 ----- 4.72534722222221  
3.0 ----- 6.21208333333332  
3.5 ----- 7.78314484126984  
4.0 ----- 9.426272675736959  
4.5 ----- 11.13233453798185  
5.0 ----- 12.89426059775761  
5.5 ----- 14.70641393026065  
6.0 ----- 16.56419398452677  
(%o99) done
```

17.2.2

```
→ heun((t/x), t, x, 0, 1, 5, 10);  
0 ----- 1  
0.5 ----- 1.125  
1.0 ----- 1.421678121420389  
1.5 ----- 1.808987822487826  
2.0 ----- 2.241148239963114  
2.5 ----- 2.696819448943987  
3.0 ----- 3.165891040921445  
3.5 ----- 3.643196174669014  
4.0 ----- 4.125879763718132  
4.5 ----- 4.612253947754017  
5.0 ----- 5.101263359939439  
(%o100) done
```

17.3 Optimal RK2 Method

17.3.1

```
→ rk2(1+(x/t), t, x, 1.0, 1.0, 6, 10);  
1.0 ----- 1.0  
1.5 ----- 2.09375  
2.0 ----- 3.359848484848484  
2.5 ----- 4.753382034632034  
3.0 ----- 6.248176088617265  
3.5 ----- 7.827038770053475  
4.0 ----- 9.477795861427574  
4.5 ----- 11.19136649795217  
5.0 ----- 12.96071373335682  
5.5 ----- 14.7802226066925  
6.0 ----- 16.64530777872948  
(%o101) done
```

17.3.2

```
→ rk2((t/x), t, x, 0, 1, 5, 10);  
0 ----- 1  
0.5 ----- 1.125  
1.0 ----- 1.426010101010101  
1.5 ----- 1.814915177090553  
2.0 ----- 2.24710861815986  
2.5 ----- 2.702338151234642  
3.0 ----- 3.170885177773465  
3.5 ----- 3.64769967070791  
4.0 ----- 4.129953831171324  
4.5 ----- 4.615959602475442  
5.0 ----- 5.104654003183263  
(%o102) done
```

18 A Check

```
→ A1:matrix([5, 1, 2], [-3, 9, 4], [1, 2, -7]);  
b1:matrix([10], [-14], [-33]);  
start:matrix([0], [0], [0]);  
gauss_jacobi(A1, b1, start, 14);  
  
(%o103) 
$$\begin{pmatrix} 5 & 1 & 2 \\ -3 & 9 & 4 \\ 1 & 2 & -7 \end{pmatrix}$$
  
(%o104) 
$$\begin{pmatrix} 10 \\ -14 \\ -33 \end{pmatrix}$$
  
(%o105) 
$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$
  
-----  
(%o106) 
$$\begin{pmatrix} 2.0 & -1.55555555555555 & 4.714285714285714 \end{pmatrix}$$
  
2  
-----  
(%o107) 
$$\begin{pmatrix} 0.4253968253968252 & -2.984126984126984 & 4.555555555555555 \end{pmatrix}$$
  
3  
-----  
(%o108) 
$$\begin{pmatrix} 0.7746031746031745 & -3.438447971781304 & 3.922448979591836 \end{pmatrix}$$
  
4  
-----  
(%o109) 
$$\begin{pmatrix} 1.118710002519526 & -3.040665154950868 & 3.842529604434367 \end{pmatrix}$$
  
5  
-----  
(%o110) 
$$\begin{pmatrix} 1.071121189216427 & -2.890443156686543 & 4.005339956088256 \end{pmatrix}$$
  
6  
-----  
(%o111) 
$$\begin{pmatrix} 0.9759526489020063 & -2.97866625074486 & 4.041462125120478 \end{pmatrix}$$
  
7  
-----  
(%o112) 
$$\begin{pmatrix} 0.9791484001007809 & -3.026443394863987 & 4.002660021058898 \end{pmatrix}$$
  
8  
-----  
(%o113) 
$$\begin{pmatrix} 1.004224670549238 & -3.008132764881471 & 3.989465944338972 \end{pmatrix}$$
  
9  
-----  
(%o114) 
$$\begin{pmatrix} 1.005840175240705 & -2.993909973967574 & 3.998279877255185 \end{pmatrix}$$
  
10  
-----  
(%o115) 
$$\begin{pmatrix} 0.9994700438914408 & -2.997288775922069 & 4.002574318186508 \end{pmatrix}$$
  
11
```