

Practical 9 : Lagrange Interpolation

1

Lagrange form of the interpolation polynomial

Figure 1:

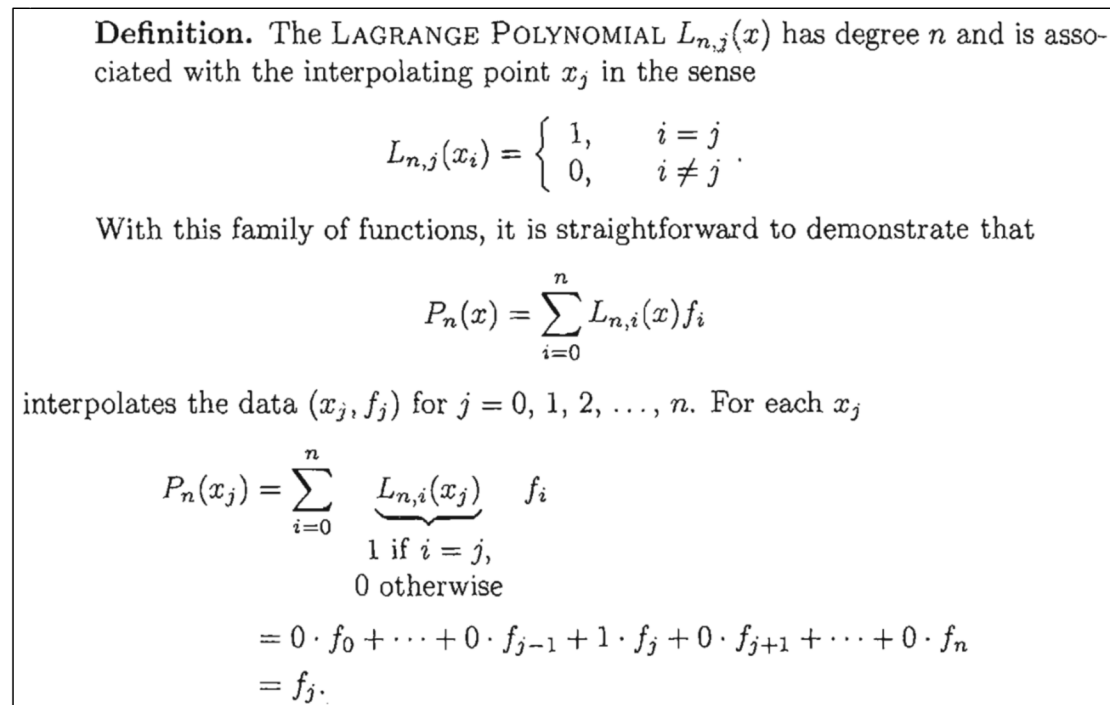


Figure 2:

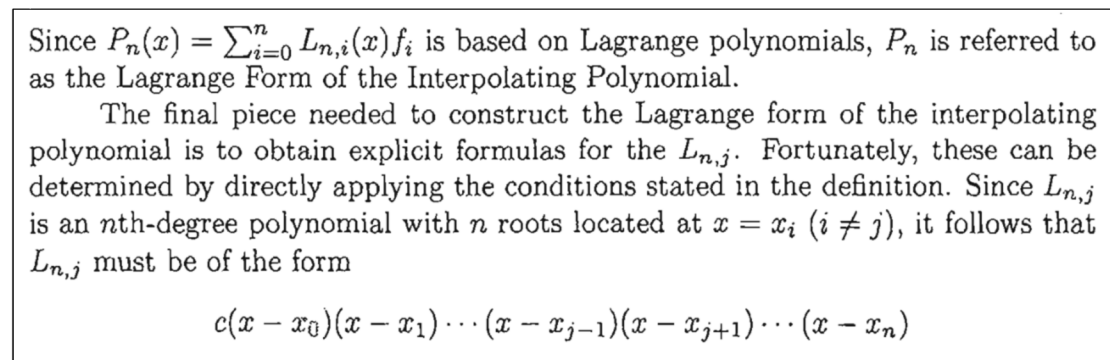


Figure 3:

for some constant c . The final condition of the definition, $L_{n,j}(x_j) = 1$, determines the value of c :

$$c = \frac{1}{(x_j - x_0)(x_j - x_1) \cdots (x_j - x_{j-1})(x_j - x_{j+1}) \cdots (x_j - x_n)}.$$

Therefore,

$$\begin{aligned} L_{n,j}(x) &= \frac{(x - x_0)(x - x_1) \cdots (x - x_{j-1})(x - x_{j+1}) \cdots (x - x_n)}{(x_j - x_0)(x_j - x_1) \cdots (x_j - x_{j-1})(x_j - x_{j+1}) \cdots (x_j - x_n)} \\ &= \prod_{i=0, i \neq j}^n \frac{x - x_i}{x_j - x_i}. \end{aligned}$$

2

function for Lagrange interpolating polynomial

```
(%i1) kill(all);
(%o0) done

(%i1) P(a, f, x):=block
(
  [n, i, temp, s, j, k],
  n:length(a),
  L(i, x):=block
  (
    temp:1,
    for j:1 thru n do ( if(j#i) then temp:temp*(x-a[j])/(a[i]-a[j]) ),
    return(temp)
  ),
  s:0,
  for k:1 thru n do s:s+(L(k, x)*f[k]),
  return(s)
)
;

(%o1) P(a,f,x):=block([n,i,temp,s,j,k],n:length(a),L(i,x)
:=block(temp:1,for j thru n do if j#i then temp:=
temp*(x-a_j)/(a_i-a_j),return(temp)),s:0,for k thru n do s:s+L(k,x)f_k,return(s))
```

3

```
(%i2) P([1/3, 1/4, 1], [2, -1, 7], x);
```

```
(%o2) 14 \left(x - \frac{1}{3}\right) \left(x - \frac{1}{4}\right) - 36 (x-1) \left(x - \frac{1}{4}\right) - 16 (x-1) \left(x - \frac{1}{3}\right)
```

```
(%i3) P([1/3, 1/4, 1], [2, -1, 7], 1);
```

```
(%o3) 7
```

```
(%i4) P([0, 1, -1, 2, -2], [-5, -3, -15, 39, -9], x);
```

```
(%o4) \frac{13 (x-1) x (x+1) (x+2)}{8} + \frac{(x-2) x (x+1) (x+2)}{2} +
```

$$\frac{5 (1-x) (x-2) (x+1) (x+2)}{4} + \frac{5 (x-2) (x-1) x (x+2)}{2} -$$

$$\frac{3 (x-2) (x-1) x (x+1)}{8}$$

```
(%i5) P([0, 1, -1, 2, -2], [-5, -3, -15, 39, -9], 3);
```

```
(%o5) 241
```

```
(%i6) P([0, 1, -1, 2, -2], [-5, -3, -15, 39, -9], 0);
```

```
(%o6) -5
```

3.1 Q7

Figure 4:

7. Consider the data set

x	-1	0	1	2
y	5	1	1	11

(a) Show that the polynomials $f(x) = x^3 + 2x^2 - 3x + 1$ and $g(x) = \frac{1}{8}x^4 + \frac{3}{4}x^3 + \frac{15}{8}x^2 - \frac{11}{4}x + 1$ both interpolate all of the data.

```
(%i7) P([-1, 0, 1, 2], [5, 1, 1, 11], x);
```

```
(%o7) \frac{11 (x-1) x (x+1)}{6} - \frac{(x-2) x (x+1)}{2} + \frac{(x-2) (x-1) (x+1)}{2}
```

$$- \frac{5 (x-2) (x-1) x}{6}$$

```
(%i8) ratsimp(P([-1, 0, 1, 2], [5, 1, 1, 11], x));
```

```
(%o8) x^3 + 2 x^2 - 3 x + 1
```

3.2 Q8

Figure 5:

8. Consider the data set

x	-3	1	2	5
y	-23	-11	-23	1

(a) Show that the polynomials $f(x) = x^3 - 3x^2 - 10x + 1$ and $g(x) = -23 + 3(x-3) - 3(x+3)(x-1) + (x+3)(x-1)(x-2)$ both interpolate all of the data.

```
(%i9) P([-3, 1, 2, 5], [-23, -11, -23, 1], x);
```

```
(%o9) (x-2)(x-1)(x+3) / 96 + 23(x-5)(x-1)(x+3) / 15 -
      11(x-5)(x-2)(x+3) / 16 + 23(x-5)(x-2)(x-1) / 160
```

```
(%i10) ratsimp(P([-3, 1, 2, 5], [-23, -11, -23, 1], x));
```

```
(%o10) x^3 - 3x^2 - 10x + 1
```

4

Exercise

4.1

Figure 6:

4. Consider the function $f(x) = \ln x$.

- Construct the Lagrange form of the interpolating polynomial for f passing through the points $(1, \ln 1)$, $(2, \ln 2)$, and $(3, \ln 3)$.
- Plot the polynomial obtained in part (a) on the same set of axes as $f(x) = \ln x$. Use an x range of $[1, 3]$. Next, generate a plot of the difference between the polynomial obtained in part (a) and $f(x) = \ln x$.
- Use the polynomial obtained in part (a) to estimate both $\ln(1.5)$ and $\ln(2.4)$. What is the error in each approximation?

4.2

Figure 7:

5. Consider the function $f(x) = \sin x$.

- Construct the Lagrange form of the interpolating polynomial for f passing through the points $(0, \sin 0)$, $(\pi/4, \sin \pi/4)$, and $(\pi/2, \sin \pi/2)$.
- Plot the polynomial obtained in part (a) on the same set of axes as $f(x) = \sin x$. Use an x range of $[0, \pi/2]$. Next, generate a plot of the difference between the polynomial obtained in part (a) and $f(x) = \sin x$.
- Use the polynomial obtained in part (a) to estimate both $\sin(\pi/3)$ and $\sin(\pi/6)$. What is the error in each approximation?

4.3

Figure 8:

6. Consider the function $f(x) = e^x$.
- (a) Construct the Lagrange form of the interpolating polynomial for f passing through the points $(-1, e^{-1})$, $(0, e^0)$, and $(1, e^1)$.
 - (b) Plot the polynomial obtained in part (a) on the same set of axes as $f(x) = e^x$. Use an x range of $[-1, 1]$. Next, generate a plot of the difference between the polynomial obtained in part (a) and $f(x) = e^x$.
 - (c) Use the polynomial obtained in part (a) to estimate both \sqrt{e} and $e^{-1/3}$. What is the error in each approximation?