Practical 1 : Bisection Method

Solutions

1

```
(%i1) kill(all);
(%o0) done

(%i1) bisect(g, a, b, kmax, e):=block([ya, yb, ym, err, iter, k],
  define(f(x), g),
   ya:f(a),
   yb:f(b),
   if(signum(ya)=signum(yb)) then ( print("fn has same sign at end points"))
   else
   (
      disp("iter                m              a              b       ym                    err
      err:(b-a),
      for k:1 thru kmax do
      (
         err:err/2,
         m:a+err,
         ym:f(m),
         iter:k,
         print(iter, "              ", m, "              ", a, "              ", b, "              ", fl
         if(abs(err)<e) then (return("bisection has converged") ),
         if(signum(ym) # signum(ya)) then ( b:m, yb:ym )
         else (a:m,  ya:ym)
      )

   )
)
;
(%o1) bisect(g,a,b,kmax,e):=block([ya,yb,ym,err,iter,k],
define(f(x),g),ya:f(a),yb:f(b),if signum(ya)=signum(yb)
then print(fn has same sign at end points) else (disp(
iter                m              a              b       ym                    error
),err:b-a,for k thru kmax do (err:err/2,m:a+err,ym:f(m),
iter:k,print(iter,              ,m,              ,a,              ,b,
              ,float(ym),              ,err),if |err|<e then
return(bisection has converged) ,if signum(ym)#signum(ya)
then (b:m,yb:ym) else (a:m,ya:ym))))
```

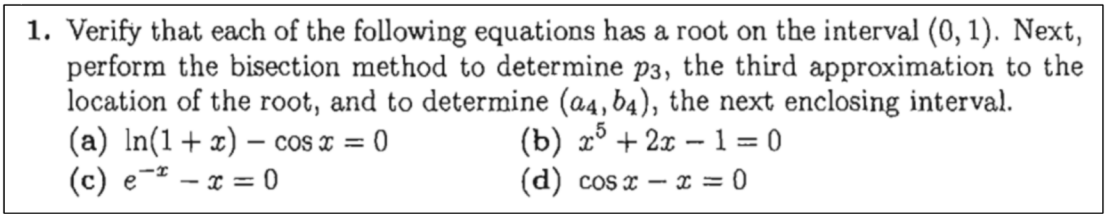
2

Exercise

Also plot the curves

2.1

Figure 1:



1. Verify that each of the following equations has a root on the interval $(0, 1)$. Next, perform the bisection method to determine p_3, the third approximation to the location of the root, and to determine (a_4, b_4), the next enclosing interval.
   (a) $\ln(1 + x) - \cos x = 0$ (b) $x^5 + 2x - 1 = 0$
   (c) $e^{-x} - x = 0$ (d) $\cos x - x = 0$

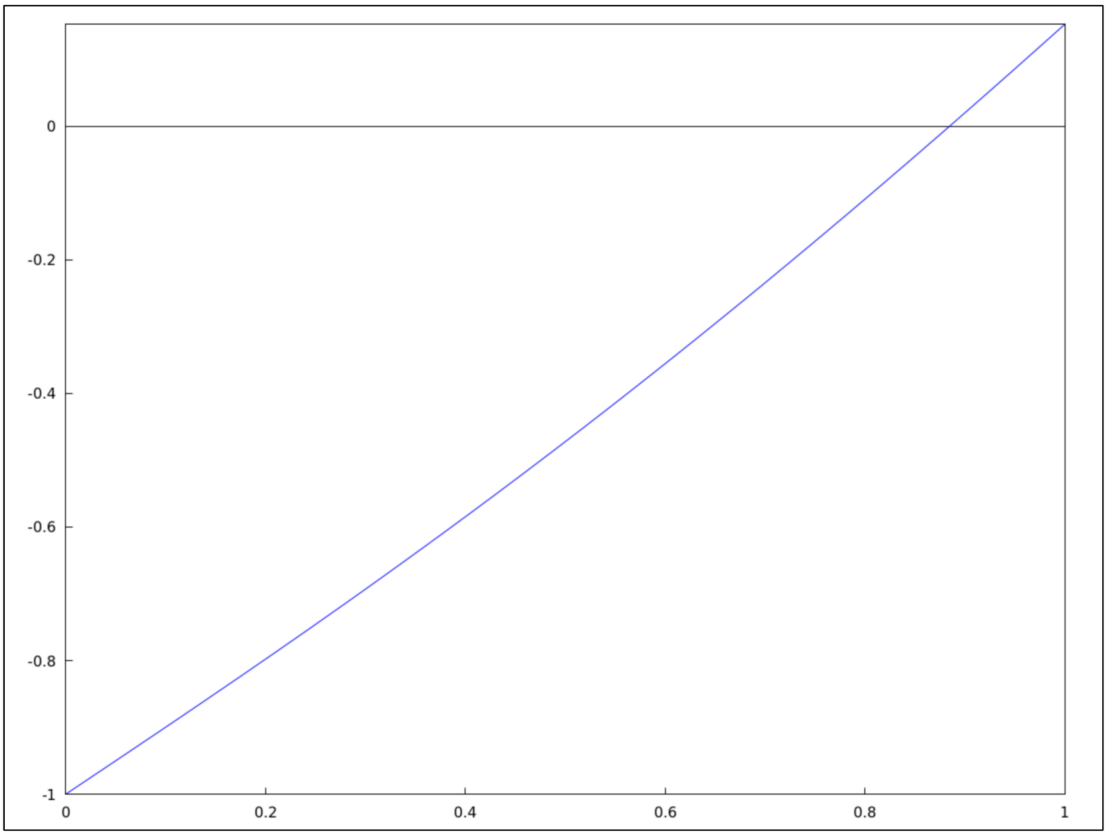
2.1.1 (a)

```
(%i2) h(x):=log(1+x)-cos(x);
(%o2) h(x):=log(1+x)-cos(x)

(%i3) h(0.0);
(%o3) -1.0

(%i4) h(1.0);
(%o4) 0.1528448746918055
```

plot

```
(%i5)  wxdraw2d(
         xaxis = true, xaxis_type = solid,
         yaxis = true, yaxis_type = solid,

         explicit(h(x),    x,   0,   1)
       );
```

(%t5)



(%o5)

(%i6) bisect(h(x), 0.0, 1.0, 20, 0.0005);

iter	*m*	*a*	*b*	*ym*	*error*
1	0.5	0.0	1.0	−0.4721174537822084	0.5
2	0.75	0.5	1.0	−0.1720730809383982	0.25
3	0.875	0.75	1.0	−0.01238819874095098	0.125
4	0.9375	0.875	1.0	0.06959340715288753	0.0625
5	0.90625	0.875	0.9375	0.02843589519467271	0.03125
6	0.890625	0.875	0.90625	0.007981228405989916	0.015625
7	0.8828125	0.875	0.890625	−0.002214254407558069	0.0078125
8	0.88671875	0.8828125	0.890625	0.002880808814339719	0.00390625
9	0.884765625	0.8828125	0.88671875	$3.32605881430581\ 10^{-4}$	0.001953125
10	0.8837890625	0.8828125	0.884765625	$-9.409923146403987\ 10^{-4}$	$9.765625\ 10^{-4}$
11	0.88427734375	0.8837890625	0.884765625	$-3.042352019027028\ 10^{-4}$	$4.8828125\ 10^{-4}$

(%o6) *bisection has converged*

2.1.2 (b)

(%i7) h1(x):=x^5+2·x−1;

(%o7) $h1(x):=x^5+2\,x-1$

(%i8) h1(0.0);

(%o8) −1.0

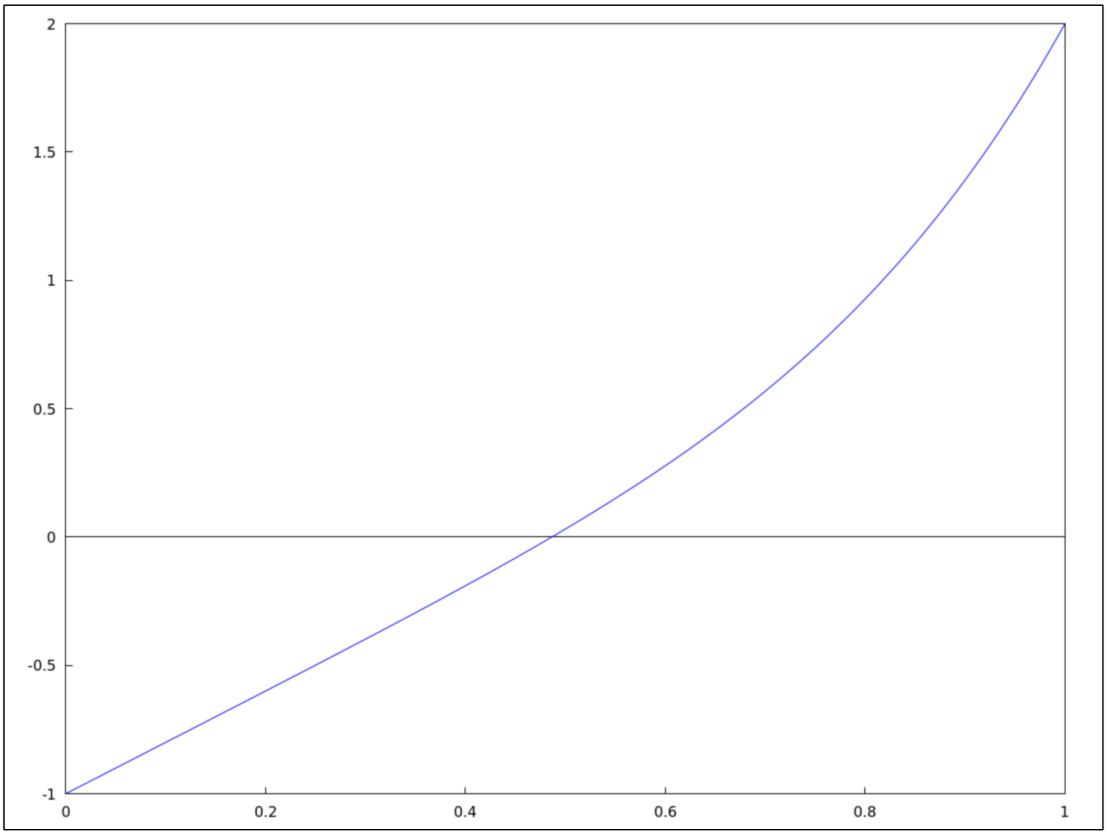
(%i9) h1(1.0);

(%o9) 2.0

plot

```
(%i10) wxdraw2d(
         xaxis = true, xaxis_type = solid,
         yaxis = true, yaxis_type = solid,

         explicit(h1(x),   x,   0,   1)
       );
```

(%t10)



(%o10)

```
(%i11) bisect(h1(x), 0.0, 1.0, 20, 0.0005);
```

iter	*m*	*a*	*b*	*ym*	*error*
1	0.5	0.0	1.0	0.03125	0.5
2	0.25	0.0	0.5	−0.4990234375	0.25
3	0.375	0.25	0.5	−0.242584228515625	0.125
4	0.4375	0.375	0.5	−0.1089715957641601	0.0625
5	0.46875	0.4375	0.5	−0.03986886143684387	0.03125
6	0.484375	0.46875	0.5	−0.004587025381624699	0.015625
7	0.4921875	0.484375	0.5	0.0132587048865389	0.0078125
8	0.48828125	0.484375	0.4921875	0.004318075615628913	0.00390625
9	0.486328125	0.484375	0.48828125	$-1.38862732654843\ 10^{-4}$	0.001953125
10	0.4873046875	0.486328125	0.48828125	0.002088502864270758	$9.765625\ 10^{-4}$
11	0.48681640625	0.486328125	0.4873046875	$9.745450004287337\ 10^{-4}$	$4.8828125\ 10^{-4}$

```
(%o11) bisection has converged
```

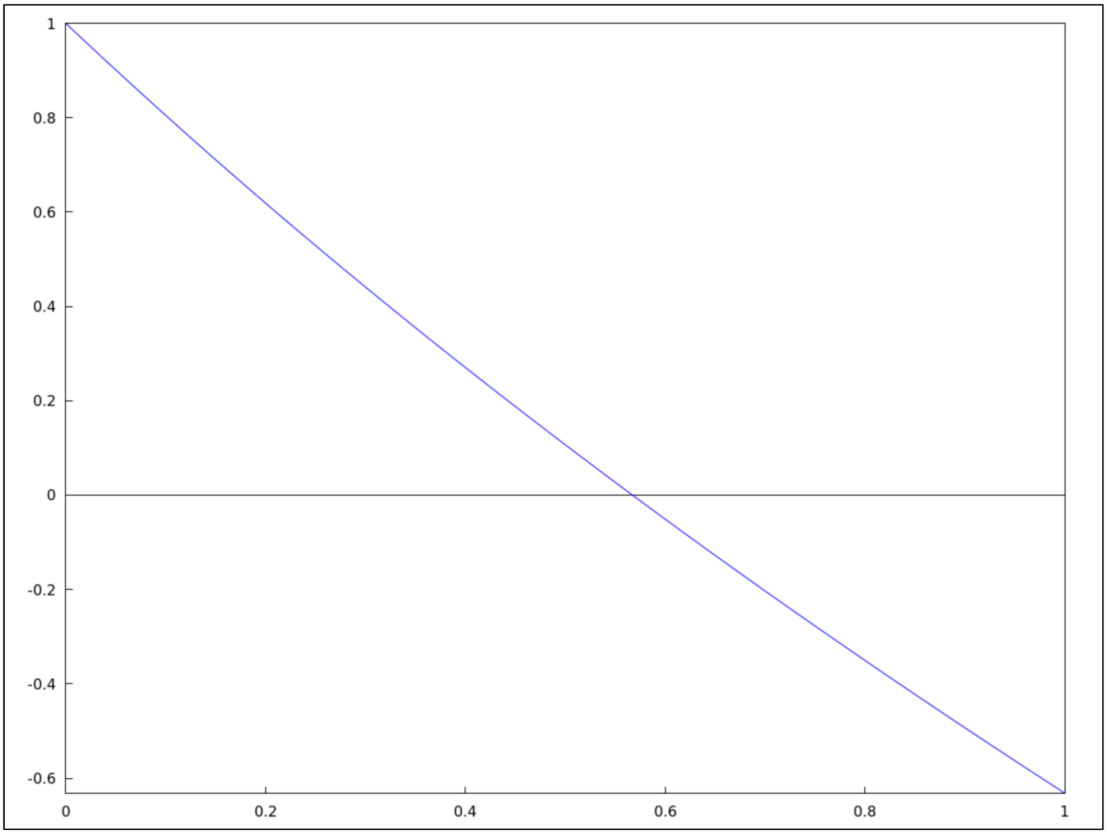
2.1.3 (c)

```
(%i12) h2(x):=exp(-x)-x;
(%o12) h2(x):=exp(-x)-x

(%i13) h2(0.0);
(%o13) 1.0

(%i14) h2(1.0);
(%o14) -0.6321205588285577
```

plot

```
(%i15) wxdraw2d(
         xaxis = true, xaxis_type = solid,
         yaxis = true, yaxis_type = solid,

         explicit(h2(x),   x,   0,   1)
       );
```

(%t15)



(%o15)

(%i16) bisect(h2(x), 0.0, 1.0, 20, 0.0005);

iter	*m*	*a*	*b*	*ym*	*error*
1	0.5	0.0	1.0	0.1065306597126334	0.5
2	0.75	0.5	1.0	−0.2776334472589853	0.25
3	0.625	0.5	0.75	−0.08973857148100972	0.125
4	0.5625	0.5	0.625	0.00728282473092301	0.0625
5	0.59375	0.5625	0.625	−0.04149754983697962	0.03125
6	0.578125	0.5625	0.59375	−0.01717583918552923	0.015625
7	0.5703125	0.5625	0.578125	−0.004963760389385841	0.0078125
8	0.56640625	0.5625	0.5703125	0.001155202015024392	0.00390625
9	0.568359375	0.56640625	0.5703125	−0.001905359612816015	0.001953125
10	0.5673828125	0.56640625	0.568359375	$-3.75349169144834\ 10^{-4}$	$9.765625\ 10^{-4}$
11	0.56689453125	0.56640625	0.5673828125	$3.898587973693024\ 10^{-4}$	$4.8828125\ 10^{-4}$

(%o16) *bisection has converged*

2.1.4 (a)

(%i17) h3(x):=cos(x)−x;

(%o17) $h3(x):=\cos(x)-x$

(%i18) h3(0.0);

(%o18) 1.0

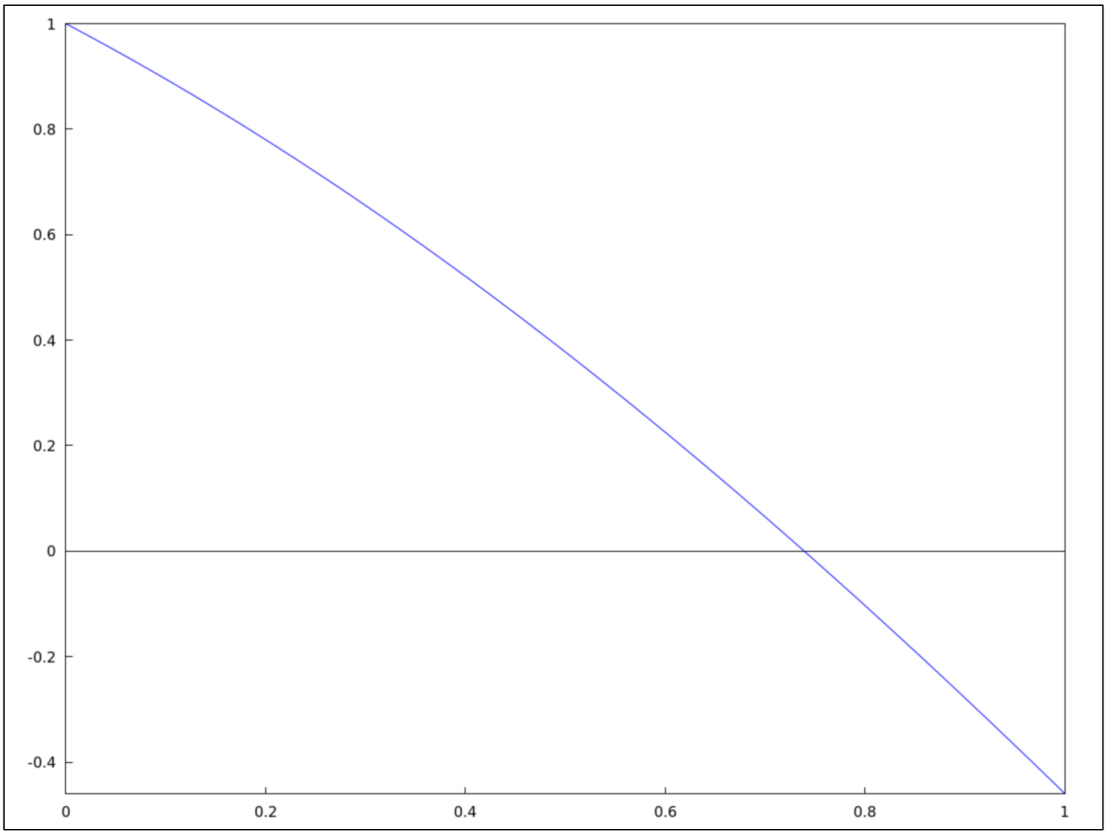
(%i19) h3(1.0);

(%o19) −0.4596976941318602

plot

```
(%i20) wxdraw2d(
         xaxis = true, xaxis_type = solid,
         yaxis = true, yaxis_type = solid,

         explicit(h3(x),   x,   0,   1)
       );
```

(%t20)



(%o20)

(%i21) `bisect(h3(x), 0.0, 1.0, 20, 0.0005);`

iter	*m*	*a*	*b*	*ym*	*error*
1	0.5	0.0	1.0	0.3775825618903727	0.5
2	0.75	0.5	1.0	−0.0183111311261791	0.25
3	0.625	0.5	0.75	0.1859631195052179	0.125
4	0.6875	0.625	0.75	0.0853349461524715	0.0625
5	0.71875	0.6875	0.75	0.03387937241806649	0.03125
6	0.734375	0.71875	0.75	0.00787472545850132	0.015625
7	0.7421875	0.734375	0.75	−0.005195711743759213	0.0078125
8	0.73828125	0.734375	0.7421875	0.001345149751805108	0.00390625
9	0.740234375	0.73828125	0.7421875	−0.001923872780897673	0.001953125
10	0.7392578125	0.73828125	0.740234375	$-2.890091467900868\cdot10^{-4}$	$9.765625\cdot10^{-4}$
11	0.73876953125	0.73828125	0.7392578125	$5.281584336581657\cdot10^{-4}$	$4.8828125\cdot10^{-4}$

(%o21) *bisection has converged*

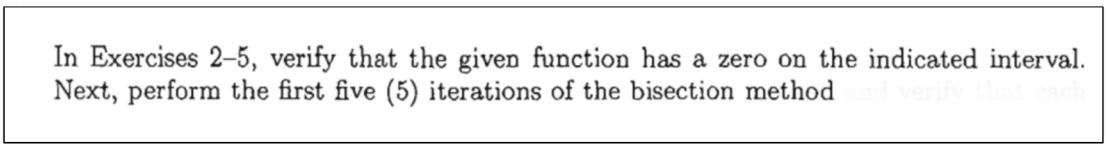
(%i22) `bisect(h3(x), 1.0, 2.0, 20, 0.0005);`

fn has same sign at end points

(%o22) *fn has same sign at end points*

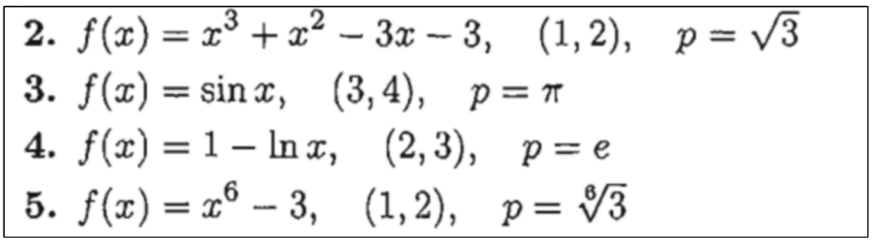
2.2

Figure 2:



In Exercises 2–5, verify that the given function has a zero on the indicated interval. Next, perform the first five (5) iterations of the bisection method and verify that each

Figure 3:



2.2.1

(%i23) k1(x):=x^3+x^2-3·x-3;

(%o23) $k1(x) := x^3 + x^2 + (-3)\,x - 3$

(%i24) k1(1.0);

(%o24) −4.0

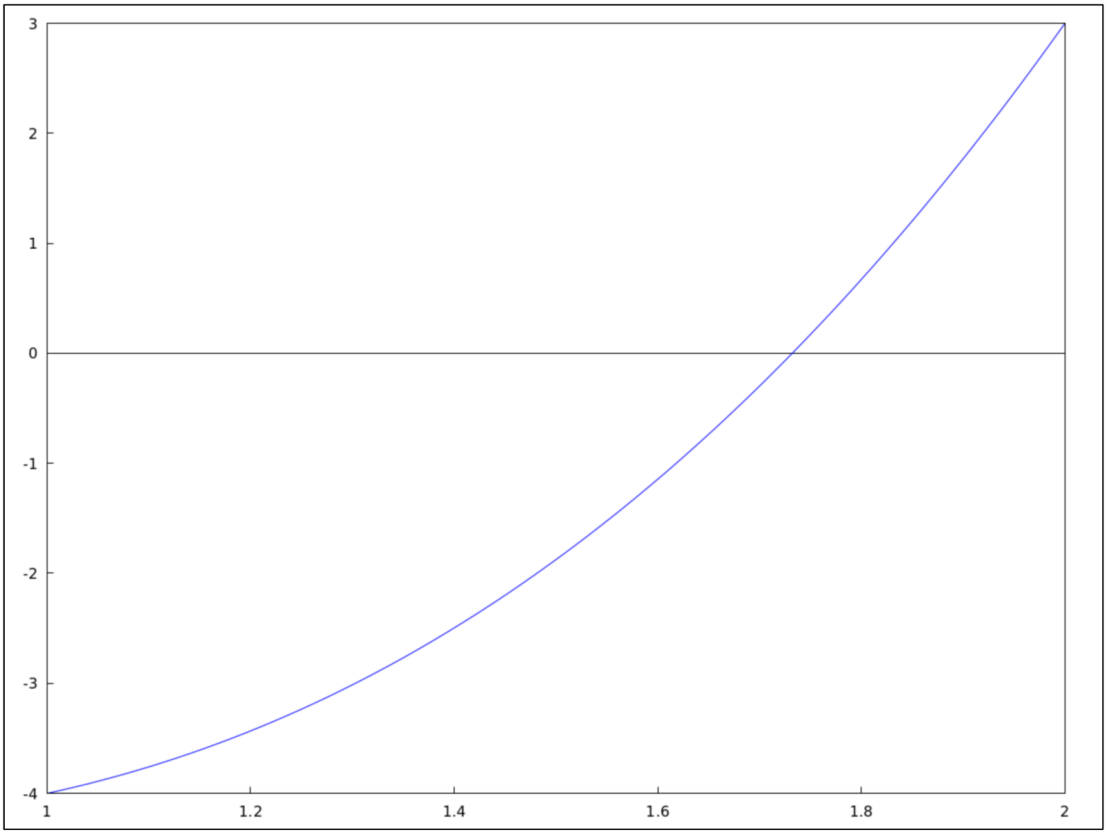
(%i25) k1(2.0);

(%o25) 3.0

plot

```
(%i26) wxdraw2d(
         xaxis = true, xaxis_type = solid,
         yaxis = true, yaxis_type = solid,

         explicit(k1(x),   x,   1,   2)
       );
```

(%t26)



(%o26)

(%i27) bisect(k1(x), 1.0, 2.0, 20, 0.0005);

iter	*m*	*a*	*b*	*ym*	*error*
1	1.5	1.0	2.0	−1.875	0.5
2	1.75	1.5	2.0	0.171875	0.25
3	1.625	1.5	1.75	−0.943359375	0.125
4	1.6875	1.625	1.75	−0.409423828125	0.0625
5	1.71875	1.6875	1.75	−0.124786376953125	0.03125
6	1.734375	1.71875	1.75	0.022029876708984375	0.015625
7	1.7265625	1.71875	1.734375	−0.05175542831420898	0.0078125
8	1.73046875	1.7265625	1.734375	−0.014957249164581299	0.00390625
9	1.732421875	1.73046875	1.734375	0.003512673079967499	0.001953125
10	1.7314453125	1.73046875	1.732421875	-0.005728195421397686	$9.765625\,10^{-4}$
11	1.73193359375	1.7314453125	1.732421875	-0.001109238364733755	$4.8828125\,10^{-4}$

(%o27) *bisection has converged*

2.2.2

(%i28) k2(x):=sin(x);

(%o28) $k2(x) := \sin(x)$

(%i29) k2(3.0);

(%o29) 0.1411200080598672

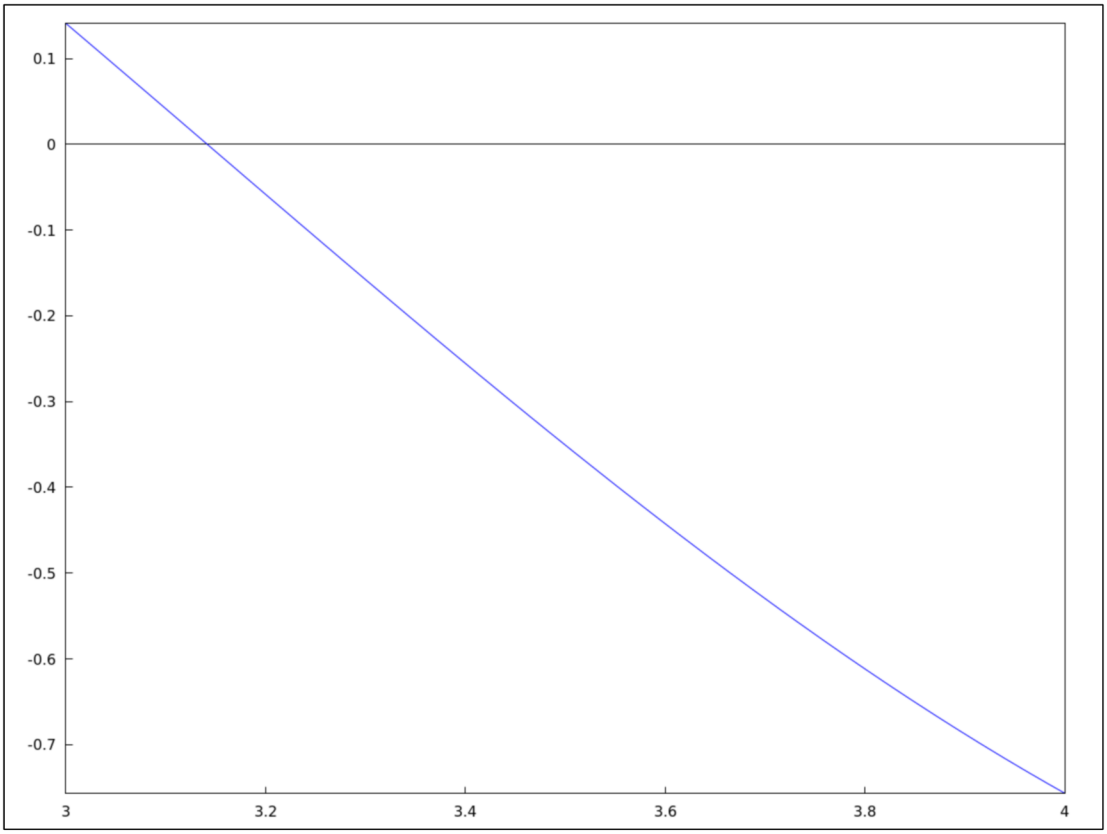
(%i30) k2(4.0);

(%o30) −0.7568024953079282

plot

```
(%i31) wxdraw2d(
         xaxis = true, xaxis_type = solid,
         yaxis = true, yaxis_type = solid,

         explicit(k2(x),   x,   3,   4)
       );
```

(%t31)



(%o31)

(%i32) bisect(k2(x), 3.0, 4.0, 20, 0.0005);

iter	*m*	*a*	*b*	*ym*	*error*
1	3.5	3.0	4.0	−0.3507832276896198	0.5
2	3.25	3.0	3.5	−0.1081951345301083	0.25
3	3.125	3.0	3.25	0.0165918922293479	0.125
4	3.1875	3.125	3.25	−0.04589122327277969	0.0625
5	3.15625	3.125	3.1875	−0.01465682159049232	0.03125
6	3.140625	3.125	3.15625	$9.676534387822795\ 10^{-4}$	0.015625
7	3.1484375	3.140625	3.15625	−0.006844792961296519	0.0078125
8	3.14453125	3.140625	3.1484375	−0.002938592180907726	0.00390625
9	3.142578125	3.140625	3.14453125	$-9.854712506993688\ 10^{-4}$	0.001953125
10	3.1416015625	3.140625	3.142578125	$-8.908910206643689\ 10^{-6}$	$9.765625\ 10^{-4}$
11	3.14111328125	3.140625	3.1416015625	$4.793723214334506\ 10^{-4}$	$4.8828125\ 10^{-4}$

(%o32) *bisection has converged*

2.2.3

(%i34) k3(x):=1−log(x);

(%o34) $k3(x) := 1 - \log(x)$

(%i35) k3(2.0);

(%o35) 0.3068528194400547

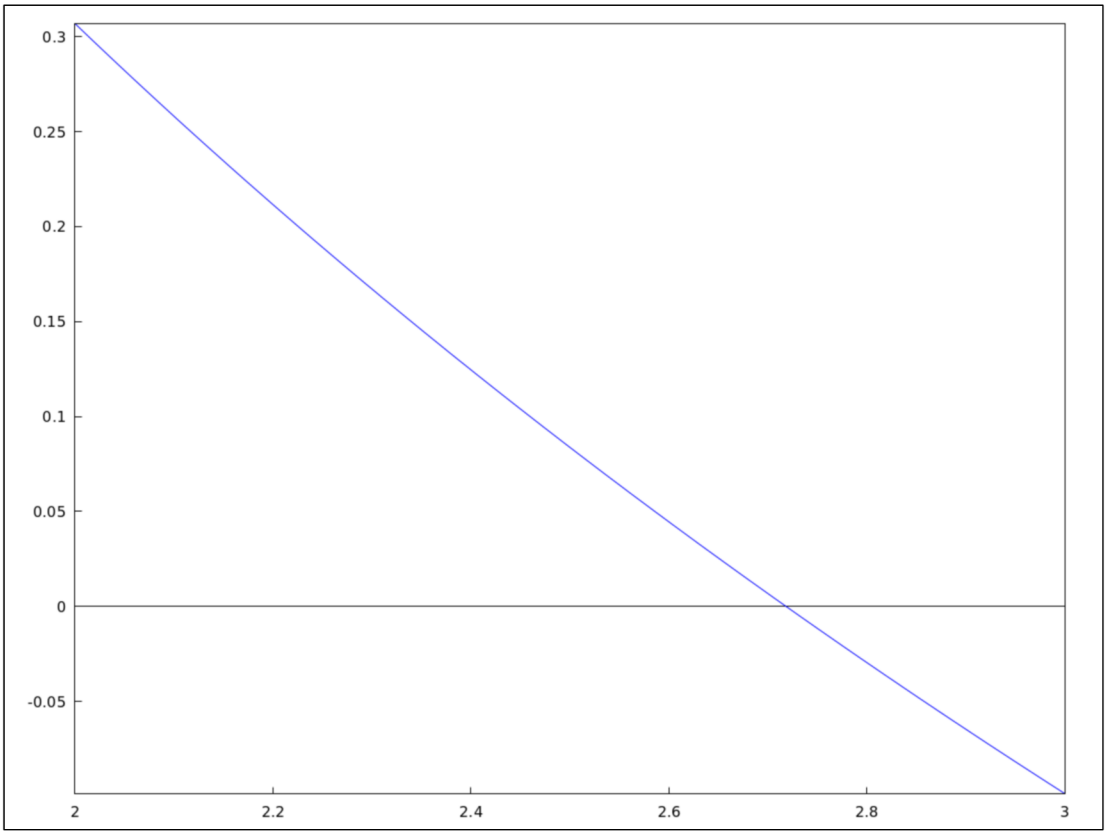
(%i36) k3(3.0);

(%o36) −0.09861228866810978

plot

```
(%i37) wxdraw2d(
         xaxis = true, xaxis_type = solid,
         yaxis = true, yaxis_type = solid,

         explicit(k3(x),   x,   2,   3)
       );
```

(%t37)



(%o37)

(%i38) bisect(k3(x), 2.0, 3.0, 20, 0.0005);

iter	*m*	*a*	*b*	*ym*	*error*
1	2.5	2.0	3.0	0.0837092681258449	0.5
2	2.75	2.5	3.0	−0.01160091167847987	0.25
3	2.625	2.5	2.75	0.03491910395641295	0.125
4	2.6875	2.625	2.75	0.01138860654621876	0.0625
5	2.71875	2.6875	2.75	$-1.722158548571606\ 10^{-4}$	0.03125
6	2.703125	2.6875	2.71875	0.005591488861892868	0.015625
7	2.7109375	2.703125	2.71875	0.002705483972757938	0.0078125
8	2.71484375	2.7109375	2.71875	0.001265598914770294	0.00390625
9	2.716796875	2.71484375	2.71875	$5.464331160648372\ 10^{-4}$	0.001953125
10	2.7177734375	2.716796875	2.71875	$1.870440735622924\ 10^{-4}$	$9.765625\ 10^{-4}$
11	2.71826171875	2.7177734375	2.71875	$7.397975890666153\ 10^{-6}$	$4.8828125\ 10^{-4}$

(%o38) *bisection has converged*

2.2.4

(%i40) k4(x):=x^6−3;

(%o40) $k4(x):=x^6-3$

(%i41) k4(1.0);

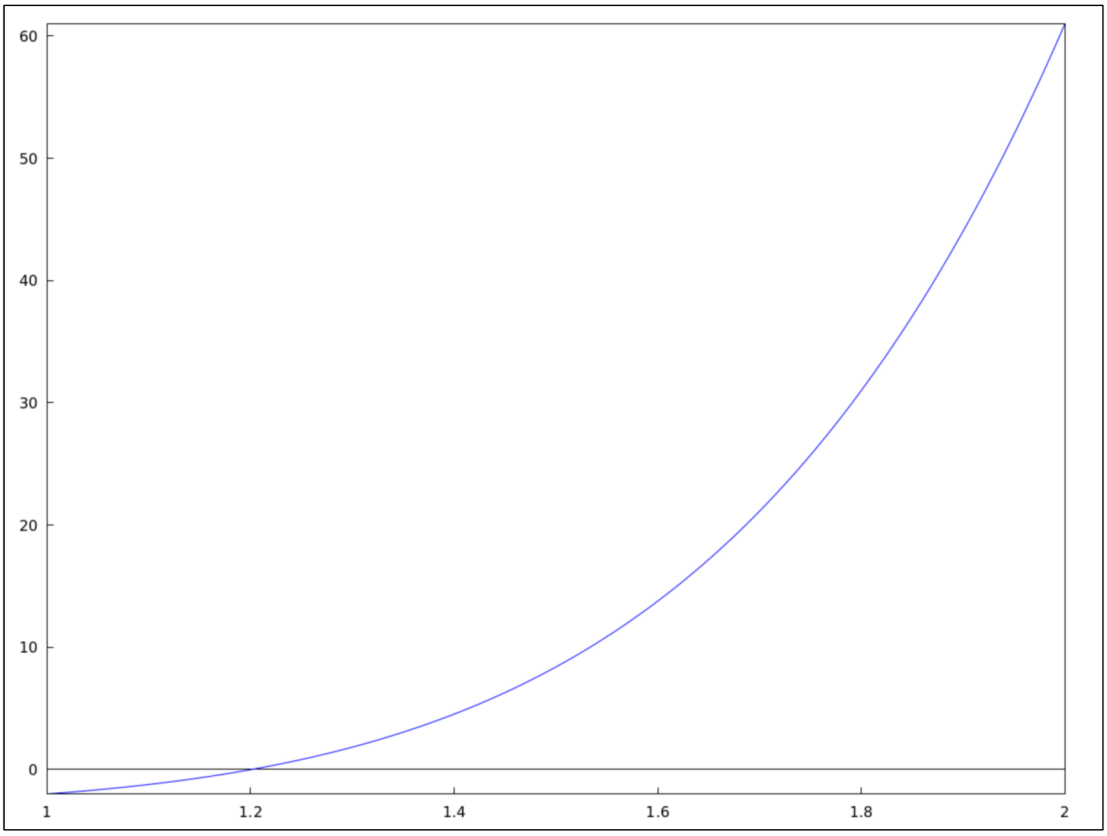
(%o41) −2.0

```
(%i42) k4(2.0);
(%o42) 61.0
```

plot

```
(%i43) wxdraw2d(
         xaxis = true, xaxis_type = solid,
         yaxis = true, yaxis_type = solid,

         explicit(k4(x),   x,   1,   2)
       );
```

(%t43)



(%o43)

(%i44) `bisect(k4(x), 1.0, 2.0, 20, 0.0005);`

iter	*m*	*a*	*b*	*ym*	*error*
1	1.5	1.0	2.0	8.390625	0.5
2	1.25	1.0	1.5	0.814697265625	0.25
3	1.125	1.0	1.25	−0.9727134704589844	0.125
4	1.1875	1.125	1.25	−0.1958469748497009	0.0625
5	1.21875	1.1875	1.25	0.277085498906672	0.03125
6	1.203125	1.1875	1.21875	0.03294480674958322	0.015625
7	1.1953125	1.1875	1.203125	−0.08332011165816766	0.0078125
8	1.19921875	1.1953125	1.203125	−0.02566103209644765	0.00390625
9	1.201171875	1.19921875	1.203125	0.003522770502788042	0.001953125
10	1.2001953125	1.19921875	1.201171875	$-$ 0.01109881321903882	$9.765625\,10^{-4}$
11	1.20068359375	1.2001953125	1.201171875	$-$ 0.003795454043200585	$4.8828125\,10^{-4}$

(%o44) *bisection has converged*

2.3

Figure 4:

16. For each of the functions given below, use the bisection method to approximate all real zeros. Use an absolute tolerance of 10^{-6} as a stopping criterion.

(a) $f(x) = e^x + x^2 - x - 4$

(b) $f(x) = x^3 - x^2 - 10x + 7$

(c) $f(x) = 1.05 - 1.04x + \ln x$

2.3.1

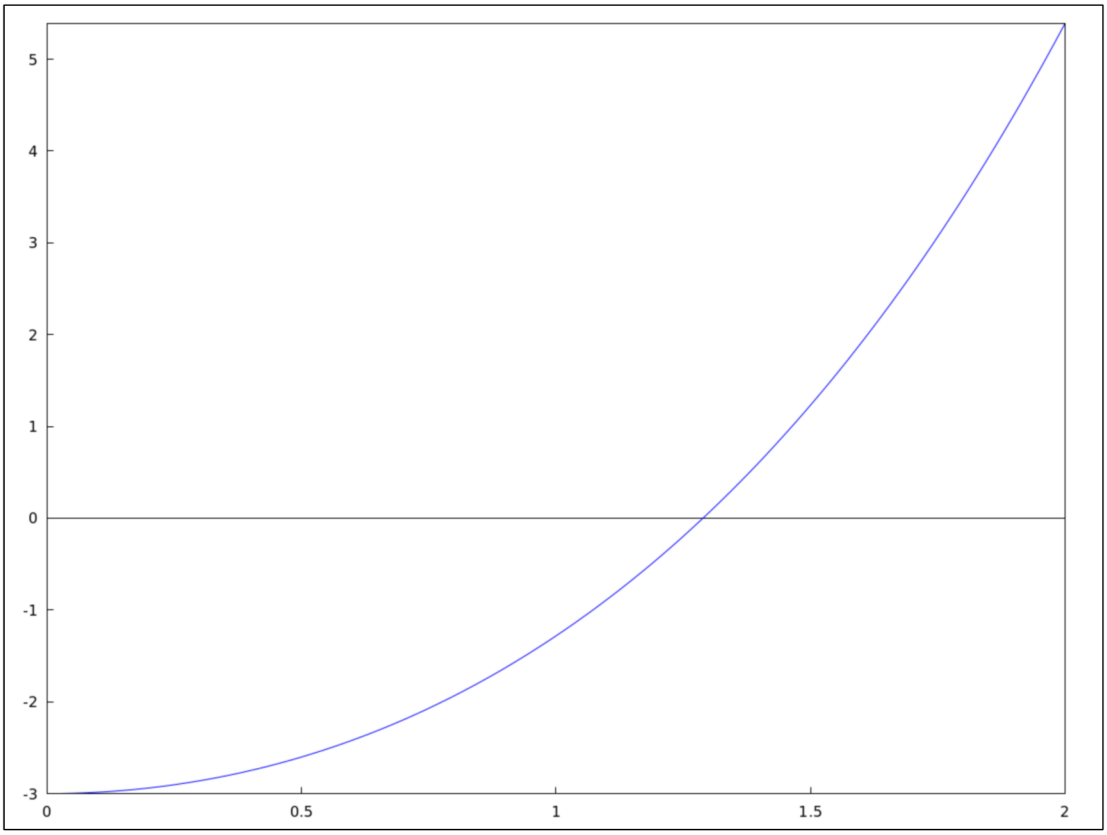
```
(%i45) j1(x):=exp(x)+x^2-x-4;
```

(%o45) $j1(x):=\exp(x)+x^2-x-4$

plot

```
(%i46) wxdraw2d(
         xaxis = true, xaxis_type = solid,
         yaxis = true, yaxis_type = solid,

         explicit(j1(x),   x,   0,   2)
       );
```

(%t46)



(%o46)

```
(%i47) j1(0.0);
```

(%o47) -3.0

```
(%i48) j1(2.0);
```

(%o48) 5.38905609893065

plot

(%i51) bisect(j1(x), 0.0, 2.0, 30, 0.000001);

iter	*m*	*a*	*b*	*ym*	*error*
1	1.0	0.0	2.0	−1.281718171540955	1.0
2	1.5	1.0	2.0	1.231689070338064	0.5
3	1.25	1.0	1.5	−0.1971570425381585	0.25
4	1.375	1.25	1.5	0.4707017229205772	0.125
5	1.3125	1.25	1.375	0.1256069879411039	0.0625
6	1.28125	1.25	1.3125	−0.03851010122782439	0.03125
7	1.296875	1.28125	1.3125	0.04285777899713228	0.015625
8	1.2890625	1.28125	1.296875	0.002002043203674919	0.0078125
9	1.28515625	1.28125	1.2890625	−0.01829686987351131	0.00390625
10	1.287109375	1.28515625	1.2890625	−0.008158137024658175	0.001953125
11	1.2880859375	1.287109375	1.2890625	−0.003080729520104075	$9.765625\,10^{-4}$
12	1.28857421875	1.2880859375	1.2890625	$-5.400140216949012\,10^{-4}$	$4.8828125\,10^{-4}$
13	1.288818359375	1.28857421875	1.2890625	$7.308468487239317\,10^{-4}$	$2.44140625\,10^{-4}$
14	1.2886962890625	1.28857421875	1.288818359375	$9.537448124774528\,10^{-5}$	$1.220703125\,10^{-4}$
15	1.28863525390625	1.28857421875	1.2886962890625	$-2.223302528778781\,10^{-4}$	$6.103515625\,10^{-5}$
16	1.288665771484375	1.28863525390625	1.2886962890625	$-6.348050653004478\,10^{-5}$	$3.0517578125\,10^{-5}$
17	1.288681030273437				

2.3.2

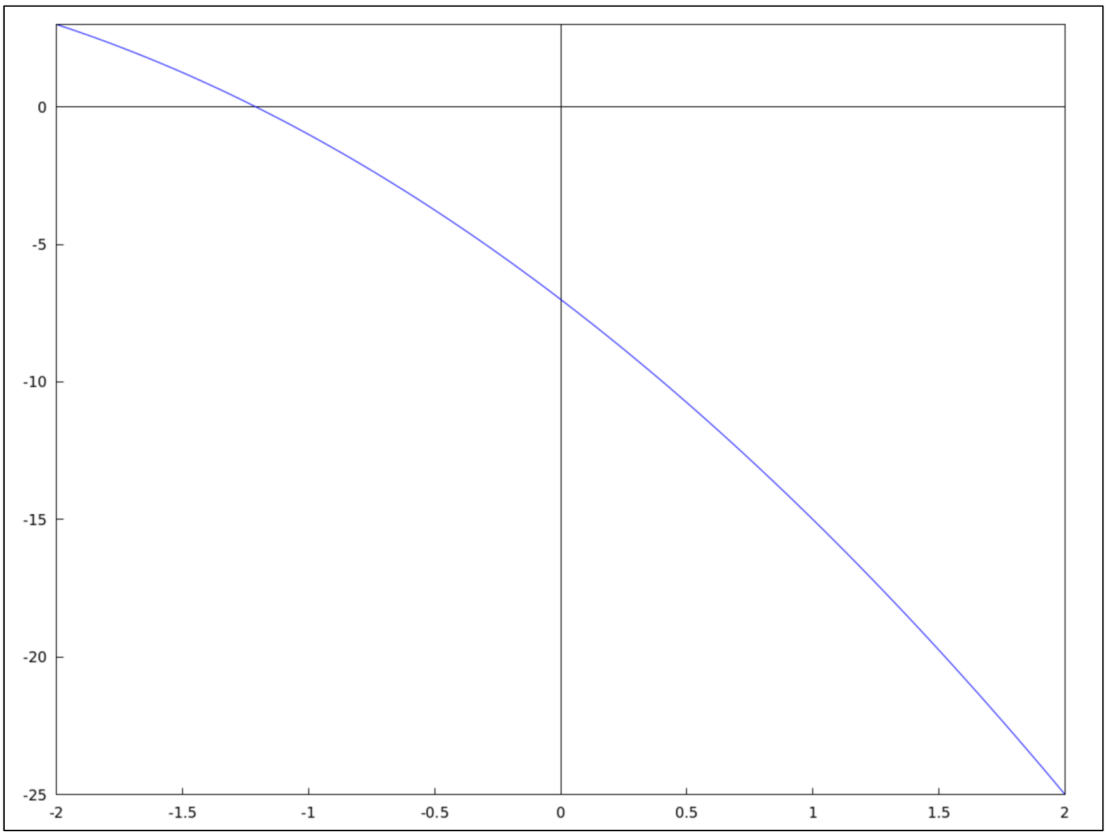
```
(%i52) j2(x):=x·3−x^2−10·x−7;
```

$$(\%o52)\quad j2(x):=x\,3-x^{2}+(-10)\,x-7$$

plot

```
(%i54) wxdraw2d(
         xaxis = true, xaxis_type = solid,
         yaxis = true, yaxis_type = solid,

         explicit(j2(x),   x,   −2,   2)
       );
```

(%t54)



(%o54)

```
(%i55) j2(−2.0);
```

(%o55) 3.0

```
(%i56) j2(0.0);
```

(%o56) −7.0

plot

(%i57) bisect(j2(x), −2.0, 0.0, 30, 0.000001);

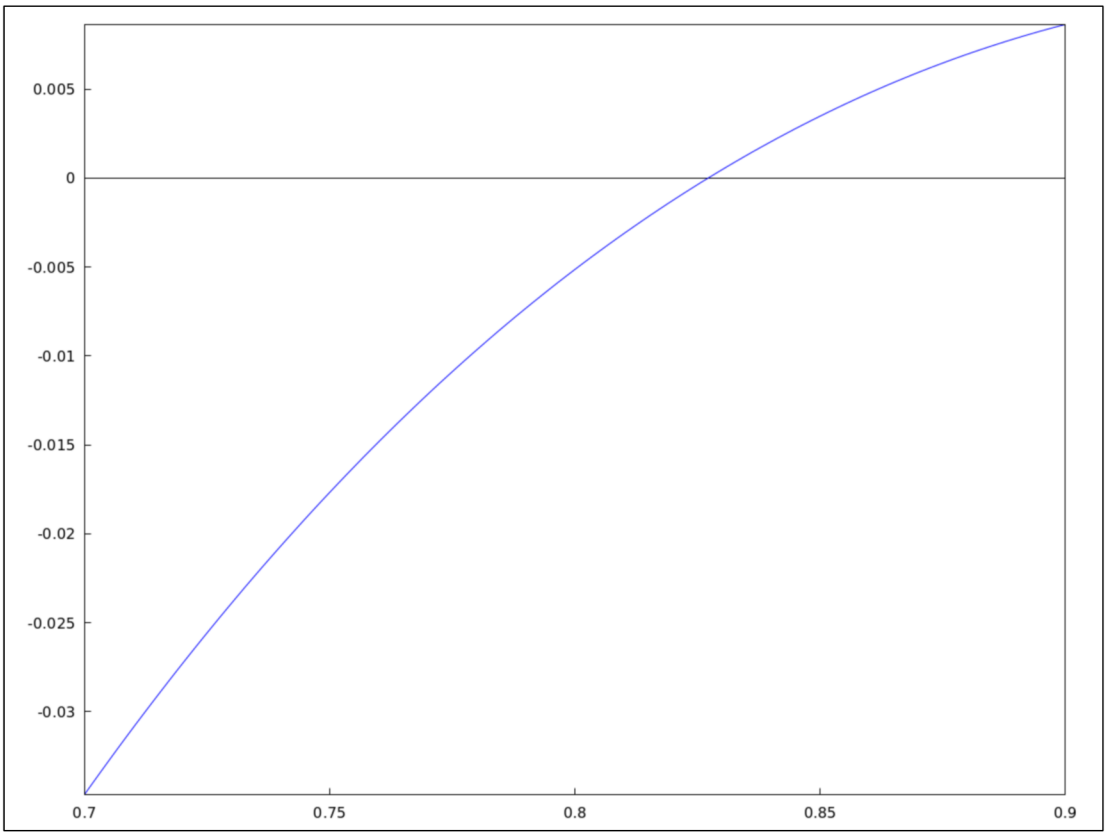
iter	*m*	*a*	*b*	*ym*	*error*
1	−1.0	−2.0	0.0	−1.0	1.0
2	−1.5	−2.0	−1.0	1.25	0.5
3	−1.25	−1.5	−1.0	0.1875	0.25
4	−1.125	−1.25	−1.0	−0.390625	0.125
5	−1.1875	−1.25	−1.125	−0.09765625	0.0625
6	−1.21875	−1.25	−1.1875	0.0458984375	0.03125
7	−1.203125	−1.21875	−1.1875	−0.025634765625	0.015625
8	−1.2109375	−1.21875	−1.203125	0.01019287109375	0.0078125
9	−1.20703125	−1.2109375	−1.203125	−0.0077056884765625	0.00390625
10	−1.208984375	−1.2109375	−1.20703125	0.001247406005859375	0.001953125
11	−1.2080078125	−1.208984375	−1.20703125	0.003228187561035156	$9.765625\ 10^{-4}$
12	−1.20849609375	−1.208984375	−1.2080078125	$9.90152359008789\ 10^{-4}$	$4.8828125\ 10^{-4}$
13	−1.208740234375	−1.208984375	−1.20849609375	$1.286864280700683\ 10^{-4}$	$2.44140625\ 10^{-4}$
14	−1.2086181640625	−1.208740234375	−1.20849609375	$4.307180643081665\ 10^{-4}$	$1.220703125\ 10^{-4}$
15	−1.20867919921875	−1.208740234375	−1.2086181640625	$-1.510120928287506\ 10^{-4}$	$6.103515625\ 10^{-5}$
16	−1.208709716796875	−1.208740234375	−1.20867919921875	$-1.116190105676651\ 10^{-5}$	$3.0517578125\ 10^{-5}$

2.3.3

```
(%i58) j3(x):=1.05−1.04·x+log(x);
```
(%o58) $j3(x):=1.05-1.04\,x+\log(x)$

plot

```
(%i62) wxdraw2d(
         xaxis = true, xaxis_type = solid,
         yaxis = true, yaxis_type = solid,

         explicit(j3(x),    x,   0.7,   0.9)
       );
```
(%t62)



(%o62)

```
(%i63) j3(0.7);
```
(%o63) -0.03467494393873238

```
(%i64) j3(0.9);
```
(%o64) 0.008639484342173709

plot

(%i65) bisect(j3(x), 0.7, 0.9, 30, 0.000001);

iter	*m*	*a*	*b*	*ym*	*error*
1	0.8	0.7	0.9	-0.005143551314209738	0.1
2	0.8500000000000001	0.8	0.9	0.003481070502225125	0.05000000000000002
3	0.8250000000000001	0.8	0.8500000000000001	$-3.718926474560458\ 10^{-4}$	0.02500000000000001
4	0.8375	0.8250000000000001	0.8500000000000001	0.001665984717084401	0.0125
5	0.83125	0.8250000000000001	0.8375	$6.753129879268904\ 10^{-4}$	0.006250000000000002
6	0.8281250000000001	0.8250000000000001	0.83125	$1.588301924499713\ 10^{-4}$	0.003125000000000001
7	0.8265625000000001	0.8250000000000001	0.8281250000000001	$-1.047444954180853\ 10^{-4}$	0.0015625
8	0.82734375	0.8265625000000001	0.8281250000000001	$2.7488687743632\ 10^{-5}$	$7.812500000000003\ 10^{-4}$
9	0.8269531250000001	0.8265625000000001	0.82734375	$-3.851633874302207\ 10^{-5}$	$3.906250000000001\ 10^{-4}$
10	0.8271484375000001	0.8269531250000001	0.82734375	$-5.485947398631286\ 10^{-6}$	$1.953125\ 10^{-4}$
11	0.8272460937500001	0.8271484375000001	0.82734375	$1.100833805220413\ 10^{-5}$	$9.765625000000003\ 10^{-5}$
12	0.8271972656250001	0.8271484375000001	0.8272460937500001	$-2.762937502287\ 51\ 10^{-6}$	