Examples : numerical.mac

1

```
load("/home/jvr/Downloads/numericalPackage/v2/numerical.mac");
(%o1) /home/jvr/Downloads/numericalPackage/v2/numerical.mac
ratprint:false;
(%o2) false
```

2

The examples are from the books by :

1. Ward Cheney and David Kincaid :: Numerical Mathematics and Computing
2. Brian Bradie :: A Friendly Introduction to Numerical Analysis

3 Bisection Method

3.1

```
➔ bisect(t^3−2·sin(t),   t,   0.50,   2.00,   20,   0.0005);
```

iter	*m*	*ym*	*error*
1	1.25	0.0551557612888276	0.75
2	0.875	−0.8651651294720542	0.375
3	1.0625	−0.5476869797091422	0.1875
4	1.15625	−0.2847914007983883	0.09375
5	1.203125	−0.1247986155094702	0.046875
6	1.2265625	−0.03735980652509796	0.0234375
7	1.23828125	0.00825801590073083	0.01171875
8	1.232421875	−0.01471021624269308	0.005859375
9	1.2353515625	−0.003266014170569153	0.0029296875
10	1.23681640625	0.002486011901918328	0.00146484375
11	1.236083984375	$-3.924970675475148\ 10^{-4}$	$7.32421875\ 10^{-4}$
12	1.2364501953125	0.001046133270412585	$3.662109375\ 10^{-4}$

(%o3) *bisection has converged*

3.2

```
bisect(sin(x), x, 2, 4, 20, 0.0005);
```

iter	*m*	*ym*	*error*
1	3	0.1411200080598672	1
2	3.5	−0.3507832276896198	0.5
3	3.25	−0.1081951345301083	0.25
4	3.125	0.0165918922293479	0.125
5	3.1875	−0.04589122327277969	0.0625
6	3.15625	−0.01465682159049232	0.03125
7	3.140625	$9.676534387822795\ 10^{-4}$	0.015625
8	3.1484375	−0.006844792961296519	0.0078125
9	3.14453125	−0.002938592180907726	0.00390625
10	3.142578125	$-9.854712506993688\ 10^{-4}$	0.001953125
11	3.1416015625	$-8.908910206643689\ 10^{-6}$	$9.765625\ 10^{-4}$
12	3.14111328125	$4.793723214334506\ 10^{-4}$	$4.8828125\ 10^{-4}$

(%o4) *bisection has converged*

3.3

```
→ bisect(x^2−2, x, 1, 2, 20, 0.0005);
```

iter *m* *ym* *error*
1 1.5 0.25 0.5
2 1.25 −0.4375 0.25
3 1.125 −0.734375
0.125
4 1.0625 −0.87109375
0.0625
5 1.03125 −0.9365234375
0.03125
6 1.015625 −0.968505859375
0.015625
7 1.0078125 −0.98431396484375
0.0078125
8 1.00390625 −
0.9921722412109375 0.00390625
9 1.001953125 −
0.9960899353027344 0.001953125
10 1.0009765625 −
0.9980459213256836 $9.765625\ 10^{-4}$
11 1.00048828125 −
0.9990231990814209 $4.8828125\ 10^{-4}$
(%o5) *bisection has converged*

3.4

This example was suggested by Professor Barton Willis

```
→ bisect(x^3−2·x^2+x−%pi, x, 2, 3, 10, 0.0001);
```

iter *m* *ym* *error*
1 2.5 2.483407346410207
0.5
2 2.25 0.374032346410207
0.25
3 2.125 −0.4521395285897931
0.125
4 2.1875 −0.05687585671479311
0.0625
5 2.21875 0.1540311257070818
0.03125
6 2.203125
0.04745229880278501 0.015625
7 2.1953125 −
0.004991682367869288 0.0078125
8 2.19921875
0.02116015355055722 0.00390625
9 2.197265625
0.008066719276360601 0.001953125
10 2.1962890625
0.001533142169467538 $9.765625\ 10^{-4}$

(%o6) *done*

4 Newton's Method

4.1

```
→ newton(x^3−2·x^2+x−3, x, 3, 0.0001, 0.00001, 8);
```

1 2.4375 2.036865234375
2 2.213032716315109 0.256363850614177
3 2.175554938721488 0.006463361488812325

4 2.174560100666446 $4.479068049789703\ 10^{-6}$
5 2.174559410293312 $2.156497203031904\ 10^{-12}$

(%o7) *convergence*

4.2

```
→ newton(sin(x), x, 3.0, 0.0001, 0.00001, 8);
```

1 3.142546543074278 $-9.538893398264409\ 10^{-4}$
2 3.141592653300477 $2.893162490762184\ 10^{-10}$
3 3.141592653589793 $1.224646799147353\ 10^{-16}$

(%o8) *convergence*

4.3

```
→ newton(x^2−2, x, 1, 0.0001, 0.00001, 8);
```

1 1.5 0.25
2 1.416666666666666 0.006944444444444642
3 1.414215686274509 $6.007304882871267\ 10^{-6}$
4 1.414213562374689 $4.510614104447086\ 10^{-12}$

(%o9) *convergence*

```
→ newton(x^3−2·x^2+x−%pi, x, 3, 0.0001, 0.00001, 8);
```

1 2.446349540849362 1.975991988443487
2 2.230829413793257 0.237982391674409
3 2.196863381645069 0.005374496729482203
4 2.196060159694346 $2.961172185678151\ 10^{-6}$
5 2.196059716657127 $9.00612917575927\ 10^{-13}$

(%o10) *convergence*

5 Secant Method

5.1

```
secant(x^3+2·x^2−3·x−1, x, −2, −3, 0.00001, 10);
```

0	−3	−1
1	−2	5
2	−2.833333333333333	0.8101851851851833
3	−2.907928388746803	0.04629957161572662
4	−2.912449640422374	−0.002380064066290543
5	−2.912228585591192	$6.399876401275151\ 10^{-6}$

(%o11) *convergence*

5.2

```
secant(sin(x), x, 1, 4, 0.00001, 10);
```

0	4	−0.7568024953079282
1	1	0.8414709848078965
2	2.579462454848934	0.5329898131328342
3	3.166481028136553	−0.02488580518710502
4	3.140295209333355	0.001297443892426365
5	3.141592780561861	$-1.269720678483891\ 10^{-7}$

(%o12) *convergence*

5.3

```
secant(x^2−2, x, 1, 2, 0.00001, 10);
```

0	1	−1
1	2	2
2	1.333333333333333	−0.2222222222222223
3	1.428571428571428	0.04081632653061229
4	1.413793103448275	−0.001189060642093009
5	1.41421143847487	$-6.007286838860537\ 10^{-6}$

(%o13) *convergence*

6 Regula-Falsi Method

6.1

```
→ regula(x^3+2·x^2−3·x−1, x, 1, 2, 20, 0.0005);
```

iter *m* *ym*
1 1.1 −0.5489999999999999
2 1.221729490022173
0.1436388950901026
3 1.193880682944684 −
0.02924075169187978
4 1.199720665327255
0.006290908737020207
5 1.198472095487697 −
0.001337706025862806
6 1.198737948438891
$2.851619710524389\ 10^{-4}$

(%o14) *regula falsi method has converged*

6.2

```
→ regula(sin(x), x, 2, 4, 20, 0.0005);
```

iter *m* *ym*
1 3.091528082734958
0.05004365932452196
2 3.147874957380742 −
0.006282262466726139
3 3.141590357955694
$2.295634098490862\ 10^{-6}$

(%o15) *regula falsi method has converged*

6.3

```
→ regula(x^2−2, x, 1, 2, 20, 0.0005);
```

iter *m* *ym*
1 1.333333333333333 −
0.2222222222222218
2 1.428571428571428
0.04081632653061229
3 1.411764705882353 −
0.006920415224913157
4 1.414634146341463
0.001189767995240842
5 1.414141414141414 −
$2.040608101214758\ 10^{-4}$

(%o16) *regula falsi method has converged*

7 LU Factorization

7.1

```
A:matrix([1, 2, 3], [3, 5, 7], [4, 6, 9]);
luFactor(A);
luFactor(A)[1];
```

(%o17) $\begin{pmatrix} 1 & 2 & 3 \\ 3 & 5 & 7 \\ 4 & 6 & 9 \end{pmatrix}$

(%o18) $[L=\begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 4 & 2 & 1 \end{pmatrix}, U=\begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & 0 & 1 \end{pmatrix}]$

(%o19) $L=\begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 4 & 2 & 1 \end{pmatrix}$

8 Solving by LU Factorization

8.1

```
A:matrix([1, 2, 3], [3, 5, 7], [4, 6, 9]);
b:matrix([1], [2], [3]);
solve_by_lu(A, b);
```

(%o20) $\begin{pmatrix} 1 & 2 & 3 \\ 3 & 5 & 7 \\ 4 & 6 & 9 \end{pmatrix}$

(%o21) $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

(%o22) $[z=\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, x=\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}]$

8.2

```
A:matrix([2, 7, 5], [6, 20, 10], [4, 3, 0]);
b1:matrix([14], [36], [7]);
b2:matrix([−4], [−16], [−7]);
b3:matrix([−3], [−12], [6]);
```

(%o23) $\begin{pmatrix} 2 & 7 & 5 \\ 6 & 20 & 10 \\ 4 & 3 & 0 \end{pmatrix}$

(%o24) $\begin{pmatrix} 14 \\ 36 \\ 7 \end{pmatrix}$

(%o25) $\begin{pmatrix} -4 \\ -16 \\ -7 \end{pmatrix}$

(%o26) $\begin{pmatrix} -3 \\ -12 \\ 6 \end{pmatrix}$

```
luFactor(A);
```

(%o27) $[L=\begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 11 & 1 \end{pmatrix}, U=\begin{pmatrix} 2 & 7 & 5 \\ 0 & -1 & -5 \\ 0 & 0 & 45 \end{pmatrix}]$

```
solve_by_lu(A, b1);
```

(%o28) $[z=\begin{pmatrix} 14 \\ -6 \\ 45 \end{pmatrix}, x=\begin{pmatrix} 1.0 \\ 1.0 \\ 1.0 \end{pmatrix}]$

```
solve_by_lu(A, b2);
```

(%o29) $[z=\begin{pmatrix} -4 \\ -4 \\ 45 \end{pmatrix}, x=\begin{pmatrix} -1.0 \\ -1.0 \\ 1.0 \end{pmatrix}]$

```
solve_by_lu(A, b3);
```

(%o30) $[z=\begin{pmatrix} -3 \\ -3 \\ 45 \end{pmatrix}, x=\begin{pmatrix} 3.0 \\ -2.0 \\ 1.0 \end{pmatrix}]$

9 Gauss-Jacobi Method

9.1

```
A1:matrix([5,   1,   2], [−3,   9,   4], [1,   2,   −7]);
b1:matrix([10],   [−14],   [−33]);
start:matrix([0],   [0],   [0]);
gauss_jacobi(A1,   b1,   start,   14);
```

(%o31) $\begin{pmatrix} 5 & 1 & 2 \\ -3 & 9 & 4 \\ 1 & 2 & -7 \end{pmatrix}$

(%o32) $\begin{pmatrix} 10 \\ -14 \\ -33 \end{pmatrix}$

(%o33) $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

1
————
$\begin{pmatrix} 2.0 & -1.555555555555555 & 4.714285714285714 \end{pmatrix}$
2
————
$\begin{pmatrix} 0.4253968253968252 & -2.984126984126984 & 4.555555555555555 \end{pmatrix}$
3
————
$\begin{pmatrix} 0.7746031746031745 & -3.438447971781304 & 3.922448979591836 \end{pmatrix}$
4
————
$\begin{pmatrix} 1.118710002519526 & -3.040665154950868 & 3.842529604434367 \end{pmatrix}$
5
————
$\begin{pmatrix} 1.071121189216427 & -2.890443156686543 & 4.005339956088256 \end{pmatrix}$
6
————
$\begin{pmatrix} 0.9759526489020063 & -2.97866625074486 & 4.041462125120478 \end{pmatrix}$
7
————
$\begin{pmatrix} 0.9791484001007809 & -3.026443394863987 & 4.002660021058898 \end{pmatrix}$
8
————
$\begin{pmatrix} 1.004224670549238 & -3.008132764881471 & 3.989465944338972 \end{pmatrix}$
9
————
$\begin{pmatrix} 1.00584017524070 5 & -2.993909973967574 & 3.998279877255185 \end{pmatrix}$
10
————
$\begin{pmatrix} 0.9994700438914408 & -2.997288775922069 & 4.002574318186508 \end{pmatrix}$
11

9.2

```
A1:matrix([2, −1, 0], [−1, 4, 2], [0, 2, 6]);
b1:matrix([−1], [3], [5]);
start:matrix([0], [0], [0]);
gauss_jacobi(A1, b1, start, 14);
```

(%o35) $\begin{pmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \\ 0 & 2 & 6 \end{pmatrix}$

(%o36) $\begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix}$

(%o37) $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

1

$\begin{pmatrix} -0.5 & 0.75 & 0.8333333333333333 \end{pmatrix}$
2

$\begin{pmatrix} -0.125 & 0.2083333333333333 & 0.5833333333333333 \end{pmatrix}$
3

$\begin{pmatrix} -0.3958333333333333 & 0.4270833333333333 & 0.7638888888888888 \end{pmatrix}$
4

$\begin{pmatrix} -0.2864583333333333 & 0.2690972222222222 & 0.6909722222222221 \end{pmatrix}$
5

$\begin{pmatrix} -0.3654513888888888 & 0.3328993055555556 & 0.7436342592592592 \end{pmatrix}$
6

$\begin{pmatrix} -0.3335503472222222 & 0.2868200231481482 & 0.722366898148148 \end{pmatrix}$
7

$\begin{pmatrix} -0.3565899884259259 & 0.3054289641203704 & 0.7377266589506172 \end{pmatrix}$
8

$\begin{pmatrix} -0.3472855179398147 & 0.2919891734182099 & 0.7315236786265431 \end{pmatrix}$
9

$\begin{pmatrix} -0.354005413290895 & 0.297416781201747 & 0.736003608860596 \end{pmatrix}$
10

$\begin{pmatrix} -0.3512916093991126 & 0.293496842246978 & 0.734194406266075 \end{pmatrix}$
11

10 SOR Method

10.1

```
A1:matrix([5,  1,  2], [-3,  9,  4], [1,  2,  -7]);
b1:matrix([10],  [-14],  [-33]);
start:matrix([0],  [0],  [0]);
sor(A1,  b1,  0.9,  start,  10);
```

(%o39) $\begin{pmatrix} 5 & 1 & 2 \\ -3 & 9 & 4 \\ 1 & 2 & -7 \end{pmatrix}$

(%o40) $\begin{pmatrix} 10 \\ -14 \\ -33 \end{pmatrix}$

(%o41) $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

1
————
$\begin{pmatrix} 1.799999999999999 & -0.8599999999999999 & 4.253142857142857 \end{pmatrix}$
2
————
$\begin{pmatrix} 0.6036685714285714 & -3.006156571428571 & 3.972774269387755 \end{pmatrix}$
3
————
$\begin{pmatrix} 0.9712763030204081 & -2.998342473991836 & 3.994010601157784 \end{pmatrix}$
4
————
$\begin{pmatrix} 0.9989854592037691 & -2.997742850101166 & 3.999851029130248 \end{pmatrix}$
5
————
$\begin{pmatrix} 0.9995458884516973 & -2.999850930126706 & 3.999965049395661 \end{pmatrix}$
6
————
$\begin{pmatrix} 0.9999403384855388 & -2.999989011225273 & 3.99999165985835 \end{pmatrix}$
7
————
$\begin{pmatrix} 0.9999950583200967 & -2.999997047569838 & 3.999999289823317 \end{pmatrix}$
8
————
$\begin{pmatrix} 0.9999992300581864 & -2.999999651668854 & 3.999999919560678 \end{pmatrix}$
9
————
$\begin{pmatrix} 0.9999998892643681 & -2.999999966211847 & 3.999999986407011 \end{pmatrix}$
10
————
$\begin{pmatrix} 0.999999987738045 & -2.999999994862575 & 3.999999998385215 \end{pmatrix}$

(%o42) *done*

```
A1:matrix([2, -1, 0], [-1, 4, 2], [0, 2, 6]);
b1:matrix([-1], [3], [5]);
start:matrix([0], [0], [0]);
sor(A1, b1, 0.9, start, 14);
```

(%o43) $\begin{pmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \\ 0 & 2 & 6 \end{pmatrix}$

(%o44) $\begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix}$

(%o45) $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

1

$\begin{pmatrix} -0.45 & 0.5737500000000001 & 0.577875 \end{pmatrix}$
2

$\begin{pmatrix} -0.2368124999999999 & 0.4190484375000001 & 0.68207296875 \end{pmatrix}$
3

$\begin{pmatrix} -0.2851094531249999 & 0.3458223808593751 & 0.7144605826171875 \end{pmatrix}$
4

$\begin{pmatrix} -0.3228908739257812 & 0.3154245292749024 & 0.726818699479248 \end{pmatrix}$
5

$\begin{pmatrix} -0.3403480492188721 & 0.3028957270875825 & 0.7318131518216502 \end{pmatrix}$
6

$\begin{pmatrix} -0.3477317277324751 & 0.2977340156492088 & 0.7338611104874024 \end{pmatrix}$
7

$\begin{pmatrix} -0.3507928657311035 & 0.2956075070560916 & 0.7347038589319128 \end{pmatrix}$
8

$\begin{pmatrix} -0.3520559083978691 & 0.2947314347967279 & 0.735050955454173 \end{pmatrix}$
9

$\begin{pmatrix} -0.3525764451812593 & 0.2943705133595117 & 0.7351939415375639 \end{pmatrix}$
10

$\begin{pmatrix} -0.3527909135063456 & 0.2942218221051197 & 0.7352528475222205 \end{pmatrix}$
11

10.2

```
A1:matrix([2,   −1,   0], [−1,   4,   2], [0,   2,   6]);
b1:matrix([−1],  [3],  [5]);
start:matrix([0],  [0],  [0]);
sor(A1,  b1,  1.1,   start,  14);
```

(%o47) $\begin{pmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \\ 0 & 2 & 6 \end{pmatrix}$

(%o48) $\begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix}$

(%o49) $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

1
−−−−
$\begin{pmatrix} -0.55 & 0.6737500000000001 & 0.6696249999999999 \end{pmatrix}$
2
−−−−
$\begin{pmatrix} -0.1244374999999999 & 0.3551109375000001 & 0.7194968229166666 \end{pmatrix}$
3
−−−−
$\begin{pmatrix} -0.342245234375 & 0.2996482141927084 & 0.7348459725043401 \end{pmatrix}$
4
−−−−
$\begin{pmatrix} -0.3509689587565104 & 0.2943534300453017 & 0.7352524783996219 \end{pmatrix}$
5
−−−−
$\begin{pmatrix} -0.353008717599433 & 0.2940983965358337 & 0.7353053400968987 \end{pmatrix}$
6
−−−−
$\begin{pmatrix} -0.3529450101453482 & 0.2941123455031516 & 0.7352949393058211 \end{pmatrix}$
7
−−−−
$\begin{pmatrix} -0.3529437089587318 & 0.294117028867832 & 0.735294262151212 7 \end{pmatrix}$
8
−−−−
$\begin{pmatrix} -0.3529412632268192 & 0.2941176055426745 & 0.7352941184192313 \end{pmatrix}$
9
−−−−
$\begin{pmatrix} -0.3529411906288471 & 0.2941176468922224 & 0.7352941176309286 \end{pmatrix}$
10
−−−−
$\begin{pmatrix} -0.352941175146393 & 0.294117647448509 & 0.7352941175057871 \end{pmatrix}$
11

11 Gauss-Seidel Method

11.1

```
A1:matrix([5, 1, 2], [−3, 9, 4], [1, 2, −7]);
b1:matrix([10], [−14], [−33]);
start:matrix([0], [0], [0]);
gauss_seidel(A1, b1, start, 10);
```

(%o51) $\begin{pmatrix} 5 & 1 & 2 \\ -3 & 9 & 4 \\ 1 & 2 & -7 \end{pmatrix}$

(%o52) $\begin{pmatrix} 10 \\ -14 \\ -33 \end{pmatrix}$

(%o53) $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

1
−−−−
$\begin{pmatrix} 2.0 & -0.8888888888888887 & 4.746031746031746 \end{pmatrix}$

2
−−−−
$\begin{pmatrix} 0.2793650793650792 & -3.571781305114638 & 3.7336860670194 \end{pmatrix}$

3
−−−−
$\begin{pmatrix} 1.220881834215167 & -2.808010973936899 & 4.086408555191624 \end{pmatrix}$

4
−−−−
$\begin{pmatrix} 0.9270387727107303 & -3.062724211403811 & 3.971655764271872 \end{pmatrix}$

5
−−−−
$\begin{pmatrix} 1.023882536572013 & -2.979441716374605 & 4.0092855862604 \end{pmatrix}$

6
−−−−
$\begin{pmatrix} 0.992174108770761 & -3.006735557636591 & 3.996957570499654 \end{pmatrix}$

7
−−−−
$\begin{pmatrix} 1.002564083327456 & -2.997793114668471 & 4.000996836284359 \end{pmatrix}$

8
−−−−
$\begin{pmatrix} 0.9991598884199506 & -3.000723075541953 & 3.999673391048006 \end{pmatrix}$

9
−−−−
$\begin{pmatrix} 1.00027525868918 & -2.99976308756938 4 & 4.000107011935774 \end{pmatrix}$

10
−−−−
$\begin{pmatrix} 0.9999098127395674 & -3.00007762328048 8 & 3.99996493802551 3 \end{pmatrix}$

(%o54) *done*

11.2

```
→ A1:matrix([2, −1, 0], [−1, 4, 2], [0, 2, 6]);
  b1:matrix([−1], [3], [5]);
  start:matrix([0], [0], [0]);
  gauss_seidel(A1, b1, start, 14);
```

(%o55) $\begin{pmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \\ 0 & 2 & 6 \end{pmatrix}$

(%o56) $\begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix}$

(%o57) $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

1
\----
$\begin{pmatrix} -0.5 & 0.625 & 0.6249999999999999 \end{pmatrix}$
2
\----
$\begin{pmatrix} -0.1875 & 0.390625 & 0.7031249999999999 \end{pmatrix}$
3
\----
$\begin{pmatrix} -0.3046875 & 0.322265625 & 0.7259114583333332 \end{pmatrix}$
4
\----
$\begin{pmatrix} -0.3388671875 & 0.302327473958334 & 0.7325575086805554 \end{pmatrix}$
5
\----
$\begin{pmatrix} -0.3488362630208332 & 0.296512179904514 & 0.7344959400318285 \end{pmatrix}$
6
\----
$\begin{pmatrix} -0.3517439100477429 & 0.29481605247215 & 0.7350613158426166 \end{pmatrix}$
7
\----
$\begin{pmatrix} -0.352591973763925 & 0.2943213486377105 & 0.7352262171207631 \end{pmatrix}$
8
\----
$\begin{pmatrix} -0.3528393256811447 & 0.2941770600193322 & 0.7352743133268891 \end{pmatrix}$
9
\----
$\begin{pmatrix} -0.3529114699903338 & 0.2941349758389719 & 0.7352883413870093 \end{pmatrix}$
10
\----
$\begin{pmatrix} -0.35293251208051 4 & 0.2941227012863668 & 0.7352924329045443 \end{pmatrix}$
11

12 Lagrange Interpolation

12.1

```
xval:[0, 1, −1, 2, −2]$
yval:[−5, −3, −15, 39, −9]$
LP(xval, yval, x);
```

(%o61) $1.625\,(x-1)\,x\,(x+1)\,(x+2)+0.5\,(x-2)\,x\,(x+1)\,(x+2)+1.25\,(1-x)\,(x-2)\,(x+1)\,(x+2)+2.5\,(x-2)\,(x-1)\,x\,(x+2)-0.375\,(x-2)\,(x-1)\,x\,(x+1)$

```
ratsimp(%);
```

(%o62) $3\,x^4+2\,x^3-7\,x^2+4\,x-5$

```
LP(xval, yval, 3);
```

(%o63) 241.0

12.2

```
xval:[−3, 1, 2, 5]$
yval:[−23, −11, −23, 1]$
LP(xval, yval, x);
```

(%o66) $0.01041666666666666\,(x-2)\,(x-1)\,(x+3)+1.533333333333333\,(x-5)\,(x-1)\,(x+3)-0.6875\,(x-5)\,(x-2)\,(x+3)+0.14375\,(x-5)\,(x-2)\,(x-1)$

```
ratsimp(%);
```

(%o67) $x^3-3\,x^2-10\,x+1$

12.3

```
xval:[−1, 0, 1, 2]$
yval:[5, 1, 1, 11]$
LP(xval, yval, x);
```

(%o70) $1.833333333333333\,(x-1)\,x\,(x+1)-0.5\,(x-2)\,x\,(x+1)+0.5\,(x-2)\,(x-1)\,(x+1)-0.8333333333333333\,(x-2)\,(x-1)\,x$

```
ratsimp(%);
```

(%o71) $x^3+2\,x^2-3\,x+1$

13 Newton Interpolation

13.1

13.2

```
dd_table([1, 3/2, 0, 2], [3, 13/4, 3, 5/3]);
```

(%o72)

$$\begin{pmatrix} 1 & 3 & 0.5 & 0.3333333333333333 & -2.0 \\ 1.5 & 3.25 & 0.1666666666666666 & -1.666666666666666 & A_{2,5} \\ 0 & 3 & -0.6666666666666666 & A_{3,4} & A_{3,5} \\ 2 & 1.666666666666666 & A_{4,3} & A_{4,4} & A_{4,5} \end{pmatrix}$$

13.3

```
dd_table([−1, 0, 1, 2, −2, 3], [5, 1, 1, 11, 5, 35]);
```

(%o73)

$$\begin{pmatrix} -1 & 5 & -4 & 2 & 1 & -0.08333333333333326 & -1.7347234 \\ 0 & 1 & 0 & 5 & 1.083333333333333 & -0.08333333333333333 & \\ 1 & 1 & 10 & 2.833333333333333 & 0.8333333333333333 & A_{3,6} & \\ 2 & 11 & 1.5 & 4.5 & A_{4,5} & A_{4,6} & \\ -2 & 5 & 6 & A_{5,4} & A_{5,5} & A_{5,6} & \\ 3 & 35 & A_{6,3} & A_{6,4} & A_{6,5} & A_{6,6} & \end{pmatrix}$$

13.4

13.5

```
xval:[−1, 0, 1, 2]$
yval:[5, 1, 1, 11]$
NP(xval, yval, x);
```

(%o76) $(x-1)\,x\,(x+1)+2\,x\,(x+1)-4\,(x+1)+5$

```
ratsimp(%);
```

(%o77) $x^3+2\,x^2-3\,x+1$

13.6

```
xval:[−1, 0, 1, 2, −2, 3]$
yval:[5, 1, 1, 11, 5, 35]$
NP(xval, yval, x);
```

(%o80) $-1.734723475976807\ 10^{-17}(x-2)(x-1)x(x+1)(x+2)-0.08333333333333326(x-2)(x-1)x(x+1)+(x-1)x(x+1)+2x(x+1)-4(x+1)+5$

```
NP(xval, yval, 1.5);
```

(%o81) 4.453125

14

```
ratprint:false;
```

(%o82) ***false***

14.1 Trapezoidal Rule

Basic Trapezoidal Rule

```
T(1/s, s, 1.0, 2.0);
```

(%o83) 0.75

Comparison

```
compareT(g, x, a, b):=block(

    [t, v, e, numer:true],
    local(f),

    define(f(x), g),
    t:T(f(x), x, a, b),
    v:integrate(f(x), x, a, b),
    e:abs(v−t),
    print(t, "  ", v, "     ", e)
);
```

(%o84) compareT(g,x,a,b):=block$\left([t,v,e,numer:\mathbf{true}],\right.$local$(f)$,define$(\mathrm{f}(x),g),t:$T

14.1.1

```
→ compareT(1/x, x, 1, 2);
0.75 0.6931471805599453 0.05685281944005471
(%o85) 0.05685281944005471
```

14.1.2

```
→ compareT(exp(−x), x, 0, 1);
0.6839397205857212 0.6321205588285578
0.05181916175716339
(%o86) 0.05181916175716339
```

14.2

Composite Trapzoidal Rule

14.2.1

```
→ for k:2 next 2·k thru 150 do block(
          temp:Tc(sin(x), x, 0, %pi, k),
          print(k, "      ", temp)
          );
2      1.570796326794896
4      1.896118897937039
8      1.97423160194555
16      1.993570343772339
32      1.998393360970144
64      1.999598388640037
128      1.999899600184203
(%o87) done
```

14.2.2

```
→ for k:2 next 2·k thru 150 do block(
    temp:Tc(sqrt(1+x^3), x, 0.0, 1.0, k),
    print(k, "      ", temp)
    );
2      1.133883476483184
4      1.116993293318717
8      1.112830349496382
16      1.111793319381881
32      1.111534292393827
64      1.111469550038714
128      1.111453365349166
(%o88) done
```

15 Simpson Rule

Basic Simpson Rule

15.1

```
S(1/u, u, 1.0, 2.0);
(%o89) 0.6944444444444443
```

15.2

```
compareS(g, x, a, b):=block(
   [t, v, e, numer:true],
   local(f),

   define(f(x), g),
   t:S(f(x), x, a, b),
   v:integrate(f(x), x, a, b),
   e:abs(v−t),
   print(t, "  ", v, "    ", e)
);
```

(%o90) compareS (g,x,a,b):=block $\left([t,v,e,numer:\textbf{true}],\text{local}(f),\text{define}(f(x),g),t:S\right.$

15.2.1

```
compareS(1/x, x, 1, 2);
0.6944444444444443    0.6931471805599453
0.001297263884499022
(%o91) 0.001297263884499022
```

15.2.2

```
compareS(exp(−x), x, 0, 1);
0.6323336800036626    0.6321205588285578
```

$2.131211751048578\ 10^{-4}$

(%o92) $2.131211751048578\ 10^{-4}$

15.3

Composite Simpson rule

15.3.1

```
→   for k:2 next 2·k thru 150 do block(
          temp:Sc(sin(t),  t,   0,  %pi,   k),
          print(k, "      ", temp)
          );
```

```
2        2.094395102393195
4        2.00455975498442
8        2.00026916994838
16        2.000016591047935
32        2.000001033369412
64        2.000000064530001
128        2.000000004032257
```

(%o93) *done*

15.3.2

```
→   for k:2 next 2·k thru 150 do block(
      temp:Sc(sqrt(1+x^3),  x,   0,   1,  k),
      print(k, "      ", temp)
      );
```

```
2        1.109475708248729
4        1.111363232263895
8        1.111442701555604
16        1.111447642677047
32        1.111447950064475
64        1.111447969253676
128        1.111447970452649
```

(%o94) *done*

16 Euler Method

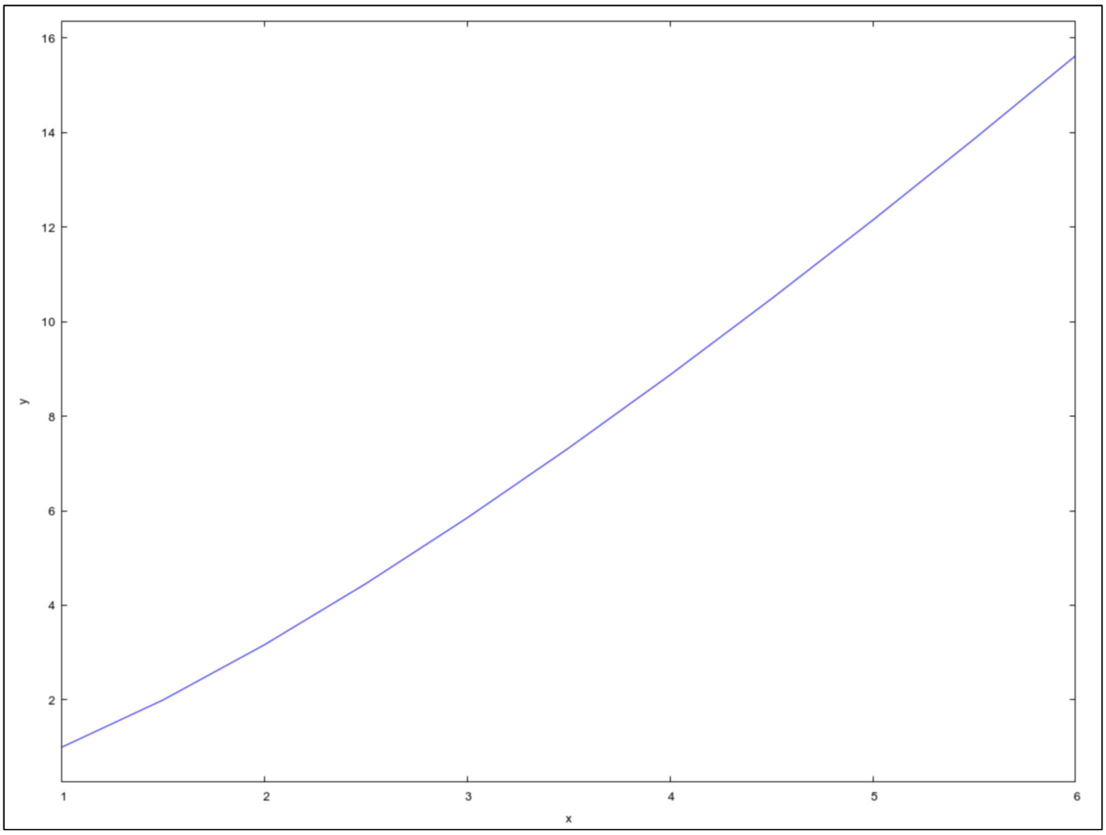
16.1

```
→   ans:eulerm(1+(x/t),   t,   x,   1,   1,   6,   10);
```

(%o95) [[1,1],[1.5,2.0],[2.0,3.166666666666666],[2.5,4.458333333333333],[3.0,5.85],[3.5,7.324999999999999],[4.0,8.871428571428571],[4.5,10.48035714285714],[5.0,12.14484126984126],[5.5,13.85932539682539],[6.0,15.61926406926406]]

plot of the solution

```
→      wxplot2d([discrete,  ans]);
(%t96)
```



(%o96)

print the values

```
→      for k:1 thru 11 do print(ans[k][1],"------------", ans[k][2] );
       1 ------------ 1
       1.5 ------------ 2.0
       2.0 ------------ 3.166666666666666
       2.5 ------------ 4.458333333333333
       3.0 ------------ 5.85
       3.5 ------------ 7.324999999999999
       4.0 ------------ 8.87142857142857
       4.5 ------------ 10.48035714285714
       5.0 ------------ 12.14484126984126
       5.5 ------------ 13.85932539682539
       6.0 ------------ 15.61926406926406
(%o97) done
```

16.2

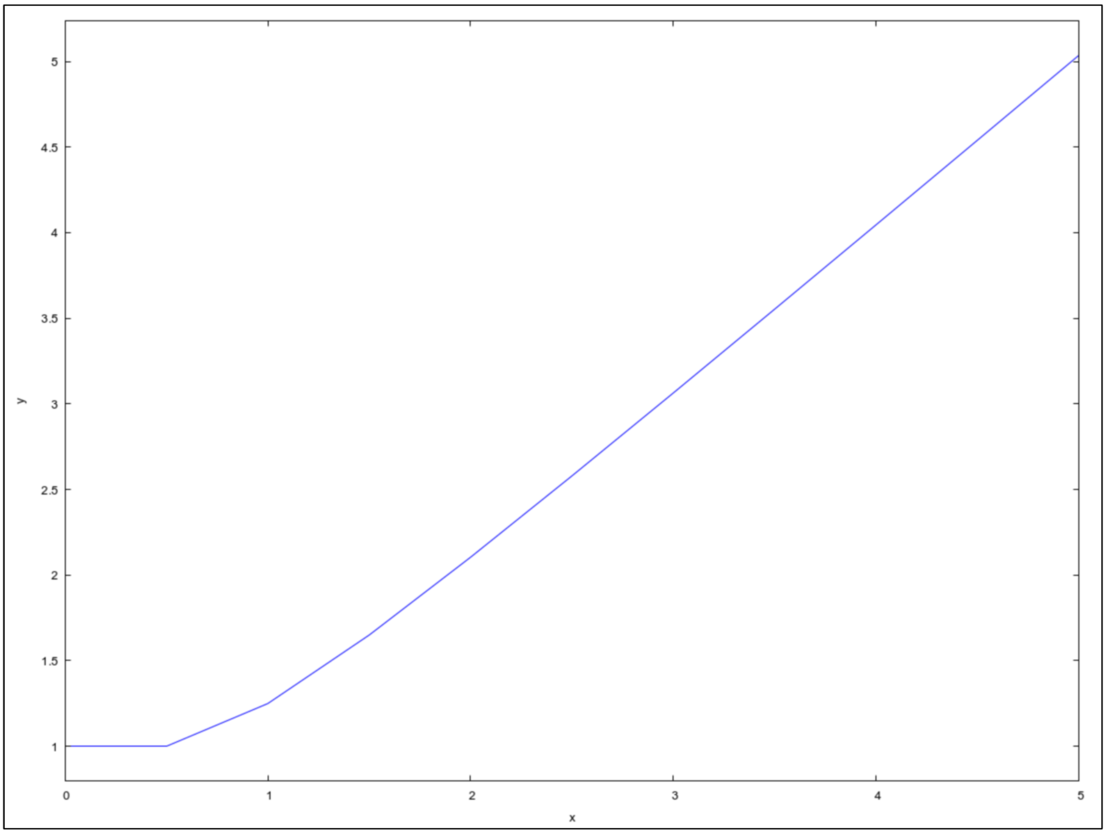
```
→ ans:eulerm((t/x), t, x, 0, 1, 5, 10);
```

(%o98) [[0,1],[0.5,1],[1.0,1.25],[1.5,1.65],[2.0,
2.104545454545454],[2.5,2.579707441586491],[3.0,
3.064258511157103],[3.5,3.553773346324708],[4.0,
4.046207677233808],[4.5,4.54049767901822],[5.0,
5.036038071076412]]

plot of the solution

```
→ wxplot2d([discrete, ans]);
```

(%t99)



(%o99)

print the values

```
→ for k:1 thru 11 do print(ans[k][1],"------------", ans[k][2] );
0 ------------ 1
0.5 ------------ 1
1.0 ------------ 1.25
1.5 ------------ 1.65
2.0 ------------ 2.104545454545454
2.5 ------------ 2.579707441586491
3.0 ------------ 3.064258511157103
3.5 ------------ 3.553773346324708
4.0 ------------ 4.046207677233808
4.5 ------------ 4.54049767901822
5.0 ------------ 5.036038071076412
(%o100) done
```

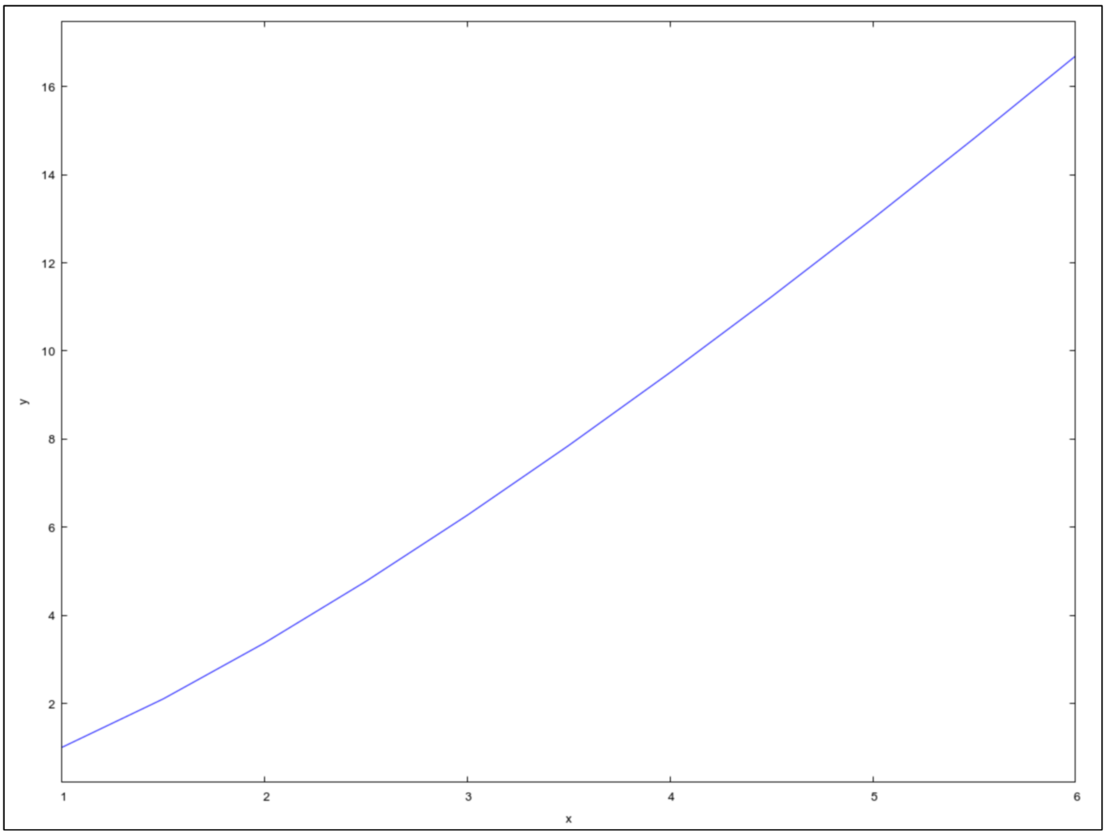
17 Second-Order Runge-Kutta Methods

17.1 Modified-Euler Method

17.1.1

```
→ ans:eulermod(1+(x/t), t, x, 1.0, 1.0, 6, 10);
(%o101) [[1.0,1.0],[1.5,2.1],[2.0,3.371428571428571],[2.5,
4.76984126984127],[3.0,6.26926406926407],[3.5,
7.852602952602953],[4.0,9.507736707736708],[4.5,
11.22561556090967],[5.0,12.99922196826221],[5.5,
14.82295368889796],[6.0,16.69223406377801]]
```

```
→ wxplot2d([discrete, ans]);
(%t102)
```



(%o102)

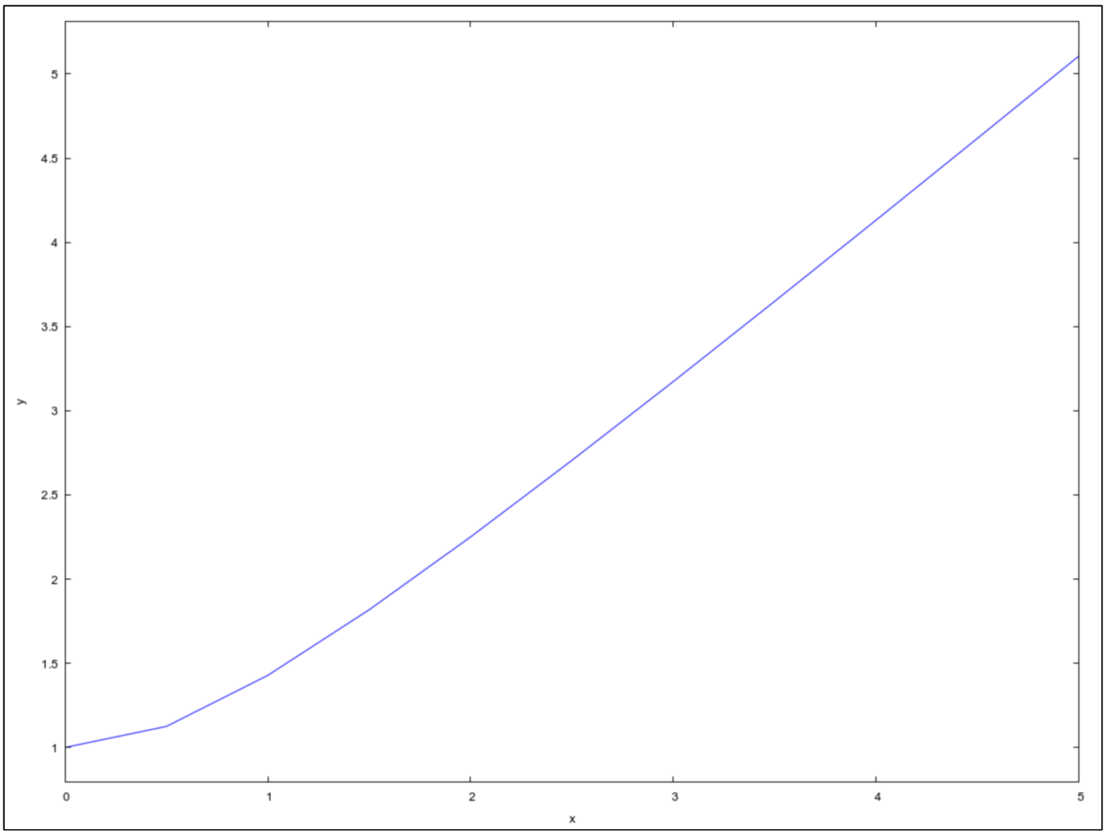
```
→ for k:1 thru 11 do print(ans[k][1],"————————————", ans[k][2] );
1.0 ———————————— 1.0
1.5 ———————————— 2.1
2.0 ———————————— 3.371428571428571
2.5 ———————————— 4.76984126984127
3.0 ———————————— 6.26926406926407
3.5 ———————————— 7.852602952602953
4.0 ———————————— 9.507736707736708
4.5 ———————————— 11.22561556090967
5.0 ———————————— 12.99922196826221
5.5 ———————————— 14.82295368889796
6.0 ———————————— 16.69223406377801
(%o103) done
```

17.1.2

```
→ ans:eulermod((t/x), t, x, 0, 1, 5, 10);
(%o104) [[0,1],[0.5,1.125],[1.0,1.428370786516854],[1.5,
1.818168594488258],[2.0,2.250391136810608],[2.5,
2.705382439640636],[3.0,3.173642355339167],[3.5,
3.650186992527909],[4.0,4.132204433515342],[4.5,
4.618006883981802],[5.0,5.106527324792223]]
```

```
wxplot2d([discrete,   ans]);
```

(%t105)



(%o105)

```
for k:1 thru 11 do print(ans[k][1],"————————————", ans[k][2] );
```

0 ———————————— 1
0.5 ———————————— 1.125
1.0 ———————————— 1.428370786516854
1.5 ———————————— 1.818168594488258
2.0 ———————————— 2.250391136810608
2.5 ———————————— 2.705382439640636
3.0 ———————————— 3.173642355339167
3.5 ———————————— 3.650186992527909
4.0 ———————————— 4.132204433515342
4.5 ———————————— 4.618006883981802
5.0 ———————————— 5.106527324792223

(%o106) *done*

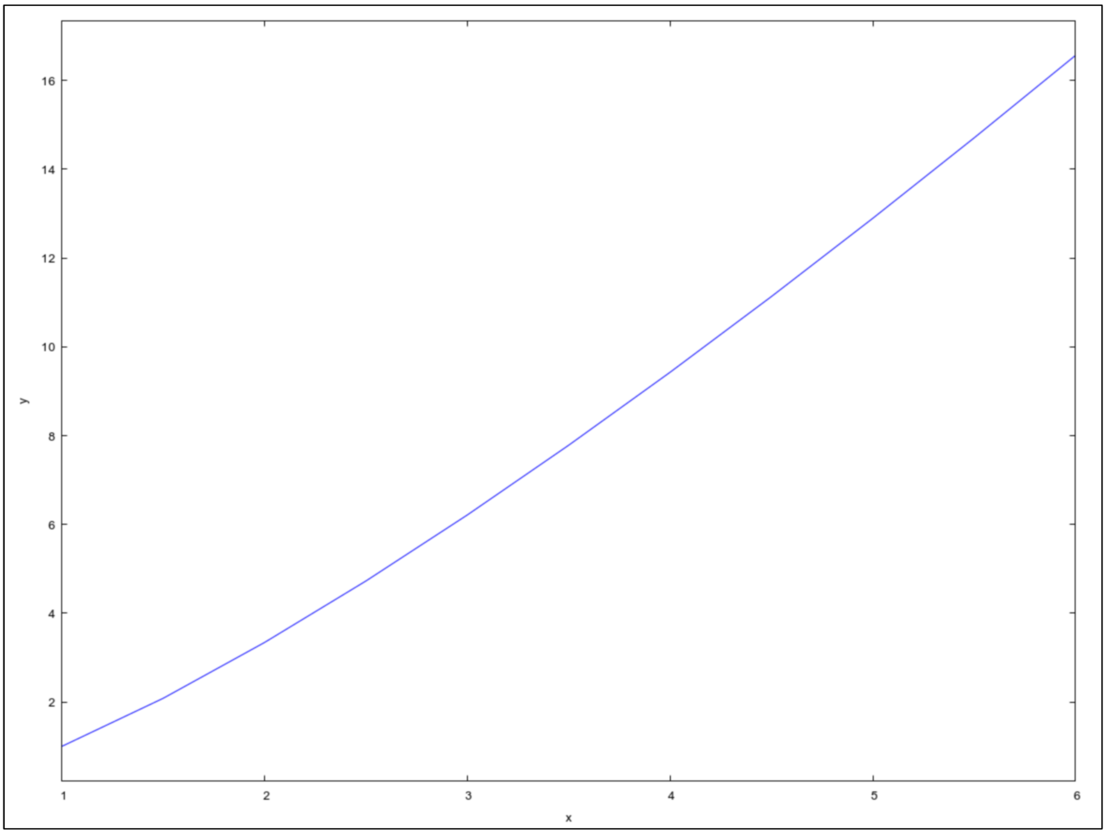
17.2 Heun's Method

17.2.1

```
ans:heun(1+(x/t),   t,   x,   1.0,    1.0,   6,   10);
wxplot2d([discrete,  ans]);
for k:1 thru 11 do print(ans[k][1],"------------", ans[k][2] );
```

(%o107) [[1.0,1.0],[1.5,2.083333333333333],[2.0,3.340277777777777],[2.5,4.725347222222221],[3.0,6.212083333333332],[3.5,7.78314484126984],[4.0,9.426272675736959],[4.5,11.13233453798185],[5.0,12.89426059775761],[5.5,14.70641393026065],[6.0,16.56419398452677]]

(%t108)



(%o108)

```
1.0 ------------ 1.0
1.5 ------------ 2.083333333333333
2.0 ------------ 3.340277777777777
2.5 ------------ 4.725347222222221
3.0 ------------ 6.212083333333332
3.5 ------------ 7.78314484126984
4.0 ------------ 9.426272675736959
4.5 ------------ 11.13233453798185
5.0 ------------ 12.89426059775761
5.5 ------------ 14.70641393026065
6.0 ------------ 16.56419398452677
```

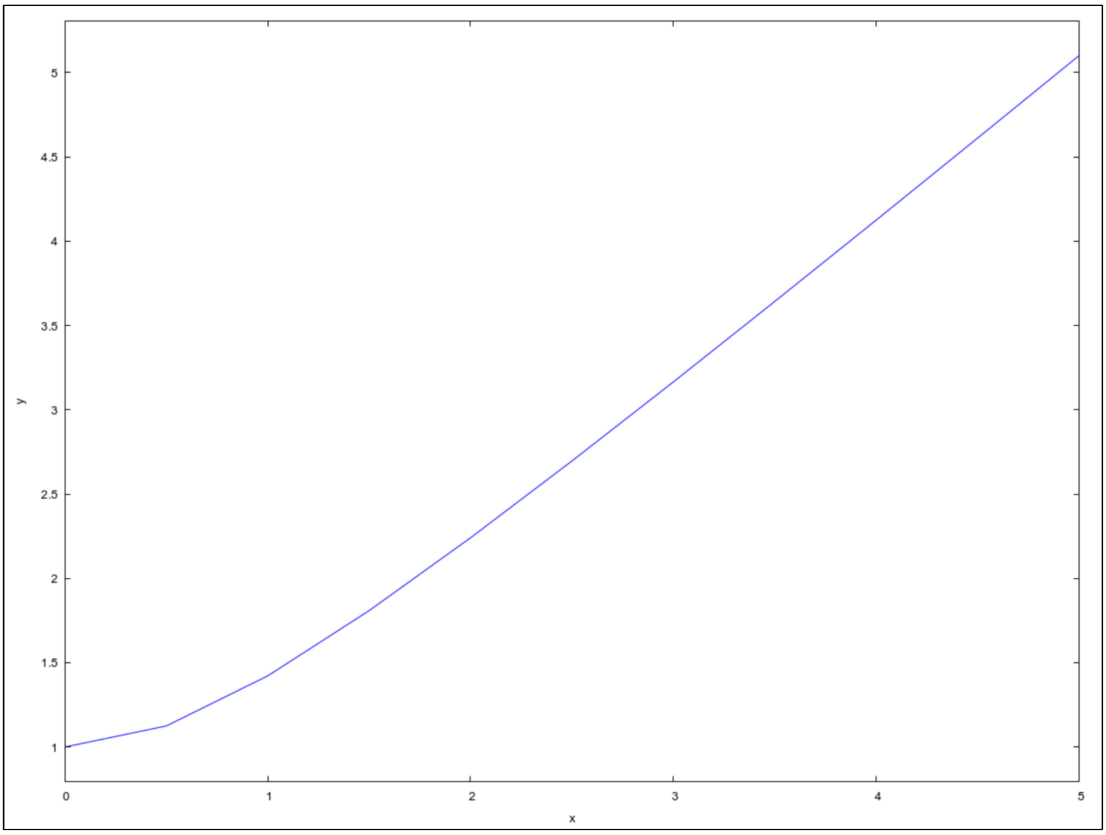
(%o109) *done*

17.2.2

```
ans:heun((t/x),   t,    x,    0,   1,    5,    10);
wxplot2d([discrete,   ans]);
for k:1 thru 11 do print(ans[k][1],"------------", ans[k][2] );
```

(%o110) [[0,1],[0.5,1.125],[1.0,1.421678121420389],[1.5,
1.808987822487826],[2.0,2.241148239963114],[2.5,
2.696819448943987],[3.0,3.165891040921445],[3.5,
3.643196174669014],[4.0,4.125879763718132],[4.5,
4.612253947754017],[5.0,5.101263359939439]]

(%t111)



(%o111)

0 ------------ 1
0.5 ------------ 1.125
1.0 ------------ 1.421678121420389
1.5 ------------ 1.808987822487826
2.0 ------------ 2.241148239963114
2.5 ------------ 2.696819448943987
3.0 ------------ 3.165891040921445
3.5 ------------ 3.643196174669014
4.0 ------------ 4.125879763718132
4.5 ------------ 4.612253947754017
5.0 ------------ 5.101263359939439

(%o112) *done*

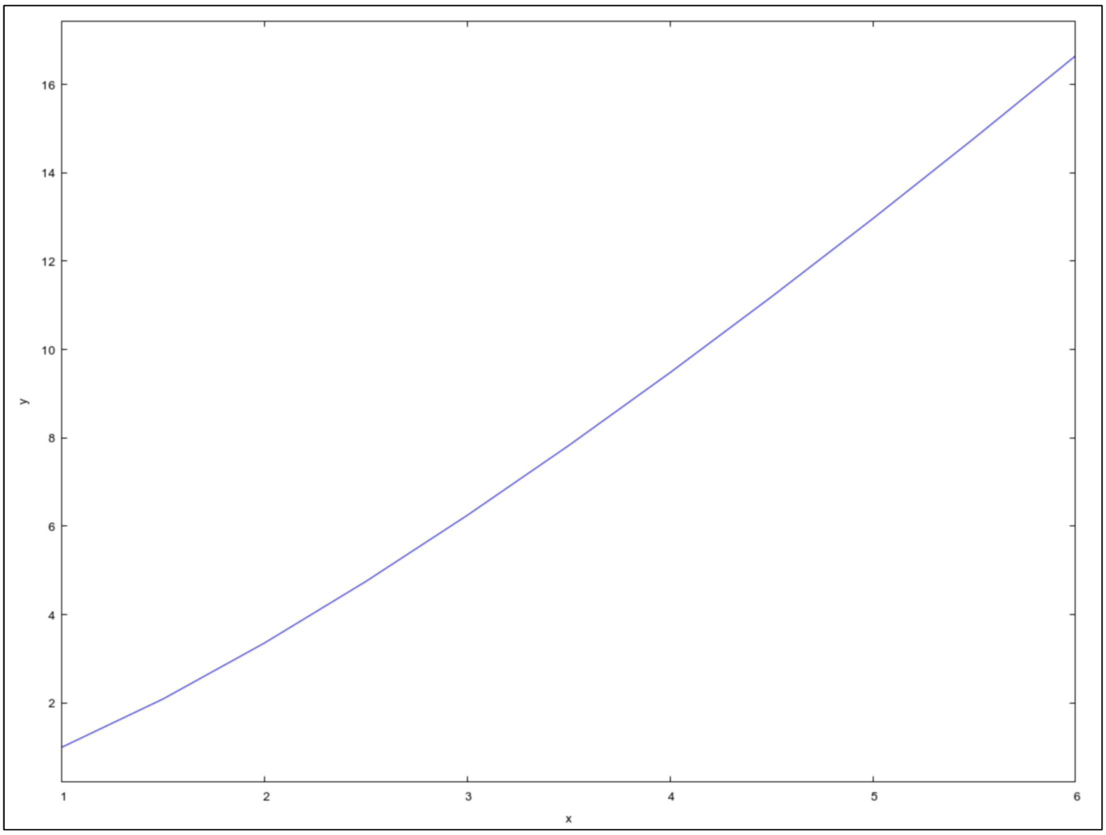
17.3 Optimal RK2 Method

17.3.1

```
ans:rk2(1+(x/t),   t,   x,   1.0,    1.0,   6,   10);
wxplot2d([discrete,  ans]);
for k:1 thru 11 do print(ans[k][1],"------------", ans[k][2] );
```

(%o113) [[1.0,1.0],[1.5,2.09375],[2.0,3.359848484848484],[2.5,4.753382034632034],[3.0,6.248176088617265],[3.5,7.827038770053475],[4.0,9.477795861427574],[4.5,11.19136649795217],[5.0,12.96071373335682],[5.5,14.7802226066925],[6.0,16.64530777872948]]

(%t114)



(%o114)

```
1.0 ------------ 1.0
1.5 ------------ 2.09375
2.0 ------------ 3.359848484848484
2.5 ------------ 4.753382034632034
3.0 ------------ 6.248176088617265
3.5 ------------ 7.827038770053475
4.0 ------------ 9.477795861427574
4.5 ------------ 11.19136649795217
5.0 ------------ 12.96071373335682
5.5 ------------ 14.7802226066925
6.0 ------------ 16.64530777872948
```

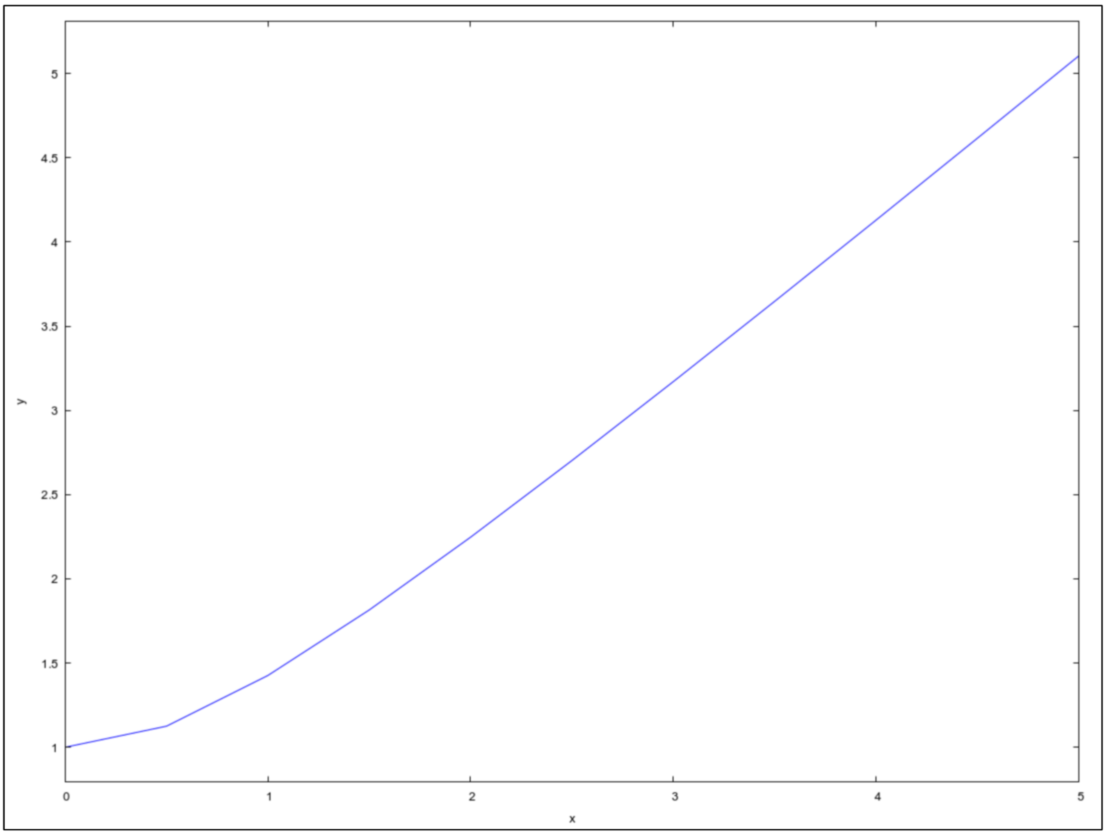
(%o115) *done*

17.3.2

```
ans:rk2((t/x),   t,    x,    0,   1,    5,    10);
wxplot2d([discrete,   ans]);
for k:1 thru 11 do print(ans[k][1],"————————————", ans[k][2] );
```

(%o116) [[0,1],[0.5,1.125],[1.0,1.426010101010101],[1.5,
1.814915177090553],[2.0,2.24710861815986],[2.5,
2.702338151234642],[3.0,3.170885177773465],[3.5,
3.64769967070791],[4.0,4.129953831171324],[4.5,
4.615959602475442],[5.0,5.104654003183263]]

(%t117)



(%o117)

0 ———————————— 1
0.5 ———————————— 1.125
1.0 ———————————— 1.426010101010101
1.5 ———————————— 1.814915177090553
2.0 ———————————— 2.24710861815986
2.5 ———————————— 2.702338151234642
3.0 ———————————— 3.170885177773465
3.5 ———————————— 3.64769967070791
4.0 ———————————— 4.129953831171324
4.5 ———————————— 4.615959602475442
5.0 ———————————— 5.104654003183263

(%o118) *done*

18 A Check

```
A1:matrix([5,  1,  2], [−3,  9,  4], [1,  2,  −7]);
b1:matrix([10],  [−14],  [−33]);
start:matrix([0],  [0],  [0]);
gauss_jacobi(A1,  b1,  start,  14);
```

(%o119) $\begin{pmatrix} 5 & 1 & 2 \\ -3 & 9 & 4 \\ 1 & 2 & -7 \end{pmatrix}$

(%o120) $\begin{pmatrix} 10 \\ -14 \\ -33 \end{pmatrix}$

(%o121) $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

1

$\begin{pmatrix} 2.0 & -1.555555555555555 & 4.714285714285714 \end{pmatrix}$
2

$\begin{pmatrix} 0.4253968253968252 & -2.984126984126984 & 4.555555555555555 \end{pmatrix}$
3

$\begin{pmatrix} 0.7746031746031745 & -3.438447971781304 & 3.922448979591836 \end{pmatrix}$
4

$\begin{pmatrix} 1.118710002519526 & -3.040665154950868 & 3.842529604434367 \end{pmatrix}$
5

$\begin{pmatrix} 1.071121189216427 & -2.890443156686543 & 4.005339956088256 \end{pmatrix}$
6

$\begin{pmatrix} 0.9759526489020063 & -2.97866625074486 & 4.041462125120478 \end{pmatrix}$
7

$\begin{pmatrix} 0.9791484001007809 & -3.026443394863987 & 4.002660021058898 \end{pmatrix}$
8

$\begin{pmatrix} 1.004224670549238 & -3.008132764881471 & 3.989465944338972 \end{pmatrix}$
9

$\begin{pmatrix} 1.005840175240705 & -2.993909973967574 & 3.998279877255185 \end{pmatrix}$
10

$\begin{pmatrix} 0.9994700438914408 & -2.997288775922069 & 4.002574318186508 \end{pmatrix}$
11