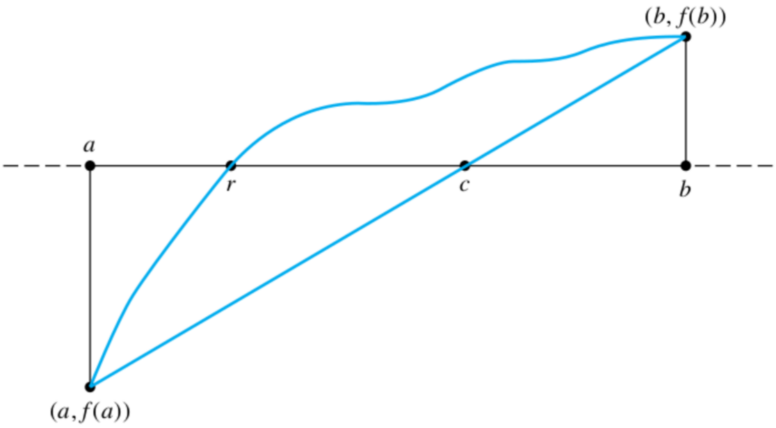
Practical 4 : Regula Falsi Method

1

Regula Falsi method

Figure 1:



In Figure 3.2, the secant line over the interval $[a, b]$ is the chord between $(a, f(a))$ and $(b, f(b))$. The two right triangles in the figure are *similar*, which means that

$$\frac{b-c}{f(b)} = \frac{c-a}{-f(a)}$$

It is easy to show that

$$c = b - f(b)\left[\frac{a-b}{f(a)-f(b)}\right] = a - f(a)\left[\frac{b-a}{f(b)-f(a)}\right] = \frac{af(b)-bf(a)}{f(b)-f(a)}$$

We then compute $f(c)$ and proceed to the next step with the interval $[a, c]$ if $f(a)f(c) < 0$ or to the interval $[c, b]$ if $f(c)f(b) < 0$.

2

```
→ kill(all);
(%o0) done
```

```
→ regula(g,  a,  b, kmax,  e):=block
  (  [ya, yb, ym,  m, k, iter],
     define(f(x),  g),
     ya:f(a),
     yb:f(b),
     if(signum(ya)=signum(yb))then( print("fn has same sign at end points"))
     else
     (
        disp("iter                    m                              ym  "),
        for k:1 thru kmax do
        (
           m:b−(yb·(b−a)/(yb−ya)),
           ym:f(m),
           iter:k,
           print(iter, "                 ", m, "              ", float(ym) ),
           if(abs(float(ym))<e)then( return("regula falsi method has converged") ),
           if(signum(ym) # signum(ya)) then ( b:m, yb:ym )
           else (a:m,  ya:ym)
        )

     )
  )
  ;
```

(%o1) regula$(g,a,b,kmax,e):=$block$([ya,yb,ym,m,k,iter],$ define$(f(x),g),ya:$f$(a),yb:$f$(b),$if signum$(ya)=$signum(yb) then print$(fn\ has\ same\ sign\ at\ end\ points)$ else $($ disp$(iter\qquad m\qquad ym\ ),$for k thru $kmax$ do $(m:b-\frac{yb\,(b-a)}{yb-ya},ym:f(m),iter:k,$ print$(iter,\qquad ,m,\qquad ,$float$(ym)),$if $|$float$(ym)|<$ e then return$(regula\ falsi\ method\ has\ converged)$,if signum$(ym)\neq$signum(ya) then $(b:m,yb:ym)$ else $(a:m,ya:ym))))$

2.1

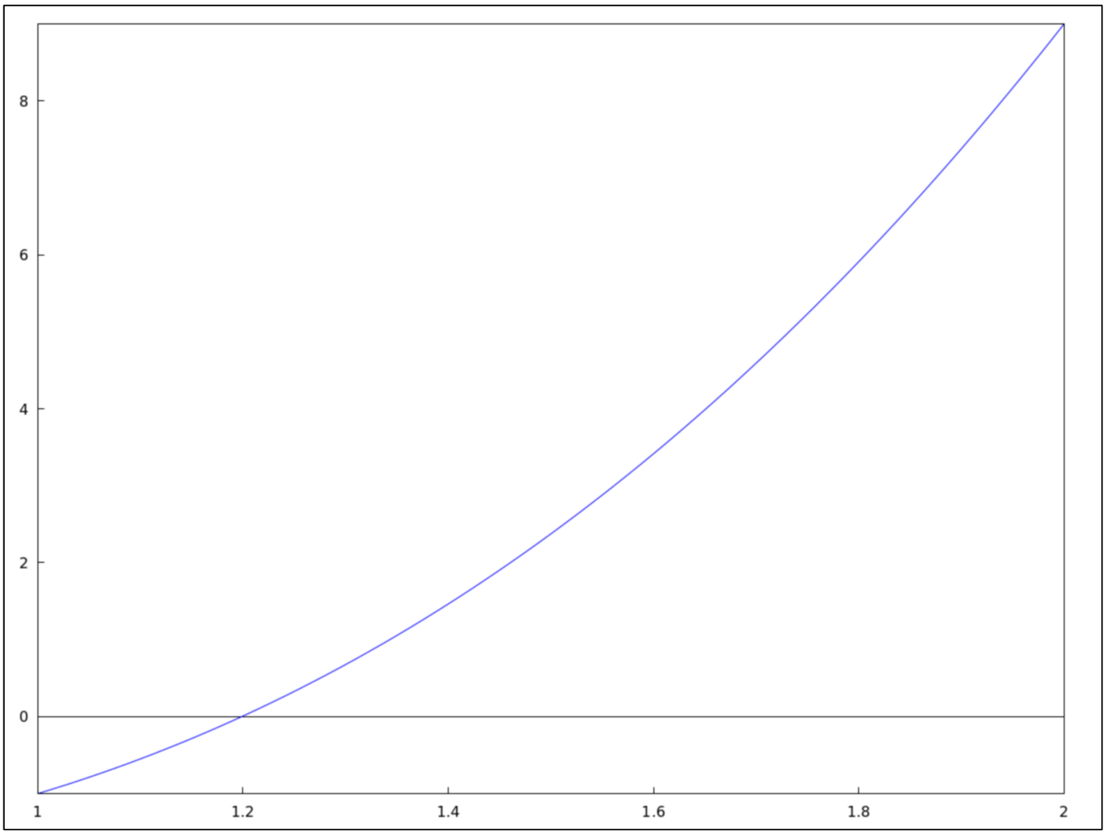
2.1.1

plot

```
→  wxdraw2d(
      xaxis = true, xaxis_type = solid,
      yaxis = true, yaxis_type = solid,

      explicit(x^3+2·x^2−3·x−1,   x,  1,  2)
   );
```

(%t2)



(%o2)

2.1.2

```
regula(x^3+2·x^2−3·x−1,   1.0,  2.0,  20,   0.0005);
```

iter	*m*	*ym*
1	1.1	−0.5489999999999999
2	1.151743638077285	−0.2744007202116687
3	1.176840909982786	−0.1307425288092163
4	1.188627673293828	−0.06087586326028837
5	1.194078911293239	−0.02804093844229971
6	1.196582088205247	−0.01285224023453723
7	1.197727754386817	−0.005877241523802867
8	1.198251317792008	−0.002684816279181046
9	1.198490418455614	−0.001225881007669516
10	1.198599576406594	$-5.596124951825487\ 10^{-4}$
11	1.19864940371845	$-2.554366889526704\ 10^{-4}$

(%o2) *regula falsi method has converged*

2.2

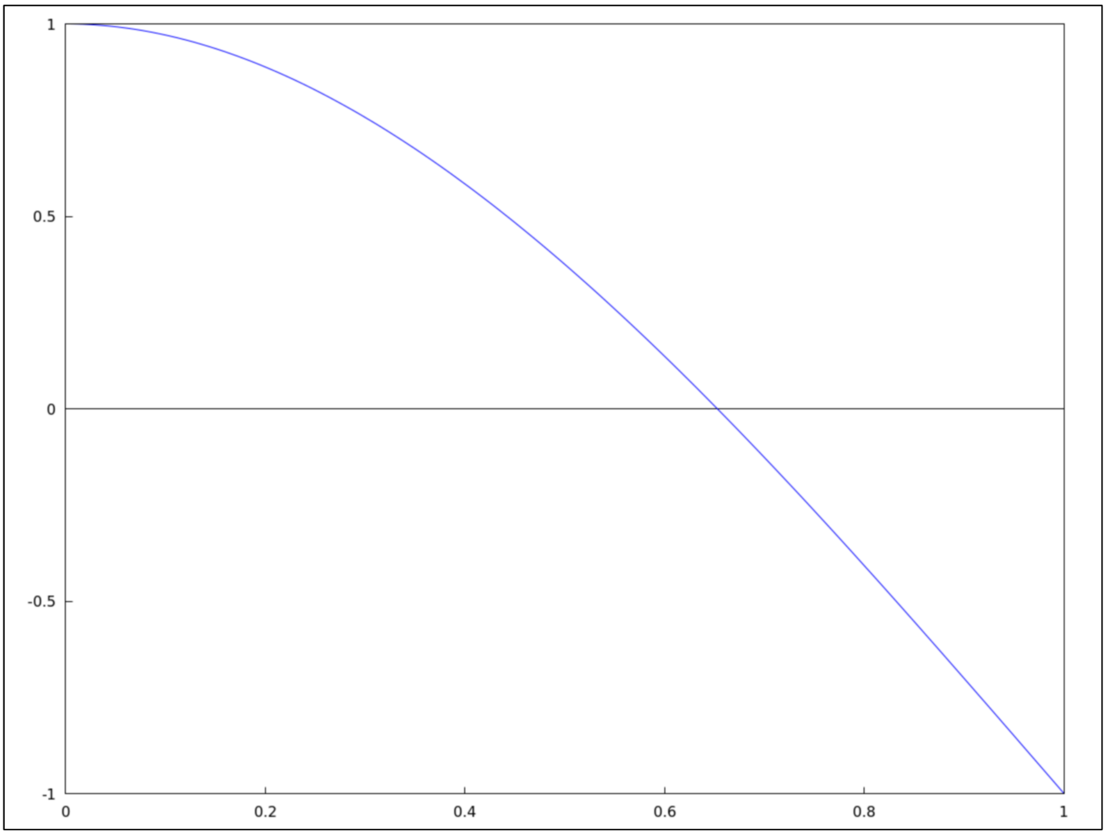
2.2.1

plot

```
wxdraw2d(
    xaxis = true, xaxis_type = solid,
    yaxis = true, yaxis_type = solid,

    explicit(x^3−3·x^2+1,   x,  0,  1)
);
```

(%t3)



(%o3)

2.2.2

```
regula(x^3−3·x^2+1,  0.0,  1.0,  20,  0.00001);
```

iter *m* *ym*
1 0.5 0.375
2 0.6363636363636364
0.04282494365138994
3 0.6512968299711816
0.003709331425458761
4 0.652585505473389
$3.116550445095289\ 10^{-4}$
5 0.65269374521968
$2.611617924025422\ 10^{-5}$
6 0.6527028152952065
$2.188008957682008\ 10^{-6}$

(%o3) *regula falsi method has converged*

3

Exercises

3.1

Figure 2:

1. Each of the following equations has a root on the interval $(0,1)$. Perform the method of false position to determine p_3, the third approximation to the location of the root, and to determine (a_4, b_4), the next enclosing interval.
   (a) $\ln(1+x) - \cos x = 0$
   (b) $x^5 + 2x - 1 = 0$
   (c) $e^{-x} - x = 0$
   (d) $\cos x - x = 0$

3.2

Figure 3:

In Exercises 4–7, an equation, an interval on which the equation has a root, and the exact value of the root are specified.

(1) Perform the first five (5) iterations of the method of false position.

Figure 4:

4. The equation $x^3 + x^2 - 3x - 3 = 0$ has a root on the interval $(1,2)$, namely $x = \sqrt{3}$.
5. The equation $x^7 = 3$ has a root on the interval $(1,2)$, namely $x = \sqrt[7]{3}$.
6. The equation $x^3 - 13 = 0$ has a root on the interval $(2,3)$, namely $\sqrt[3]{13}$.
7. The equation $1/x - 37 = 0$ has a zero on the interval $(0.01, 0.1)$, namely $x = 1/37$.

3.3

Figure 5:

11. For each of the functions given below, use the method of false position to approximate all real roots. Use an absolute tolerance of 10^{-6} as a stopping condition.
    (a) $f(x) = e^x + x^2 - x - 4$
    (b) $f(x) = x^3 - x^2 - 10x + 7$
    (c) $f(x) = 1.05 - 1.04x + \ln x$