Practical 1 : Bisection Method

1

1.1

Pseudo code(from Cheney)

Figure 1:

```
procedure Bisection(f, a, b, nmax, ε)
integer n, nmax;    real a, b, c, fa, fb, fc, error
fa ← f(a)
fb ← f(b)
if sign(fa) = sign(fb) then
      output a, b, fa, fb
      output "function has same signs at a and b"
      return
end if
error ← b − a
for n = 0 to nmax do
      error ← error/2
      c ← a + error
      fc ← f(c)
      output n, c, fc, error
      if |error| < ε then
            output "convergence"
            return
      end if
      if sign(fa) ≠ sign(fc) then
            b ← c
            fb ← fc
      else
            a ← c
            fa ← fc
      end if
end for
end procedure Bisection
```

1.2

bisection method theorem

Figure 2:

BISECTION METHOD THEOREM

If the bisection algorithm is applied to a continuous function f on an interval $[a, b]$, where $f(a)f(b) < 0$, then, after n steps, an approximate root will have been computed with error at most $(b - a)/2^{n+1}$.

2

```
(%i1) kill(all);
(%o0) done
```

(%i1)
```
bisect(g,  a,  b,  kmax,  e):=block(
    [ya, yb, ym, err, iter, k],
   define(f(x), g),
    ya:f(a),
    yb:f(b),
    if(signum(ya)=signum(yb)) then ( print("fn has same sign at end points"))
    else
    (
        disp("iter                    m                         ym                    error"),
        err:(b−a),
        for k:1 thru kmax do
        (
            err:err/2,
            m:a+err,
            ym:f(m),
            iter:k,
            print(iter, "                 ", m, "               ", float(ym),"            ", err),
            if(abs(err)<e) then (return("bisection has converged") ),
            if(signum(ym) # signum(ya)) then ( b:m, yb:ym )
            else (a:m,  ya:ym)
        )

    )
)
;
```

(%o1) bisect $(g,a,b,kmax,e):=$ block $([ya,yb,ym,err,iter,k],$ define $(f(x),g),ya:f(a),yb:f(b),$ if signum $(ya)=$ signum (yb) then print $(fn\ has\ same\ sign\ at\ end\ points)$ else (disp $($
$iter \qquad m \qquad ym \qquad error$
$),err:b-a,$ for k thru $kmax$ do $(err:\frac{err}{2},m:a+err,ym:f(m),$
$iter:k,$ print
$(iter, \quad ,m, \quad ,$ float $(ym), \quad ,err),$ if $|err|$
$<e$ then return $(bisection\ has\ converged)$, if signum $(ym)\neq$
signum (ya) then $(b:m,yb:ym)$ else $(a:m,ya:ym))))$

2.1

2.1.1

```
(%i2) bisect(x^3+2·x^2−3·x−1, 1.0, 2.0, 20, 0.0005);
```

iter	*m*	*ym*	*error*
1	1.5	2.375	0.5
2	1.25	0.328125	0.25
3	1.125	−0.419921875	0.125
4	1.1875	−0.067626953125	0.0625
5	1.21875	0.124725341796875	0.03125
6	1.203125	0.02717971801757812	0.015625
7	1.1953125	−0.02056455612182617	0.0078125
8	1.19921875	0.003222167491912842	0.00390625
9	1.197265625	−0.008692525327205658	0.001953125
10	1.1982421875	−0.00274051446467638	$9.765625\ 10^{-4}$
11	1.19873046875	$2.394922776147723\ 10^{-4}$	$4.8828125\ 10^{-4}$

(%o2) *bisection has converged*

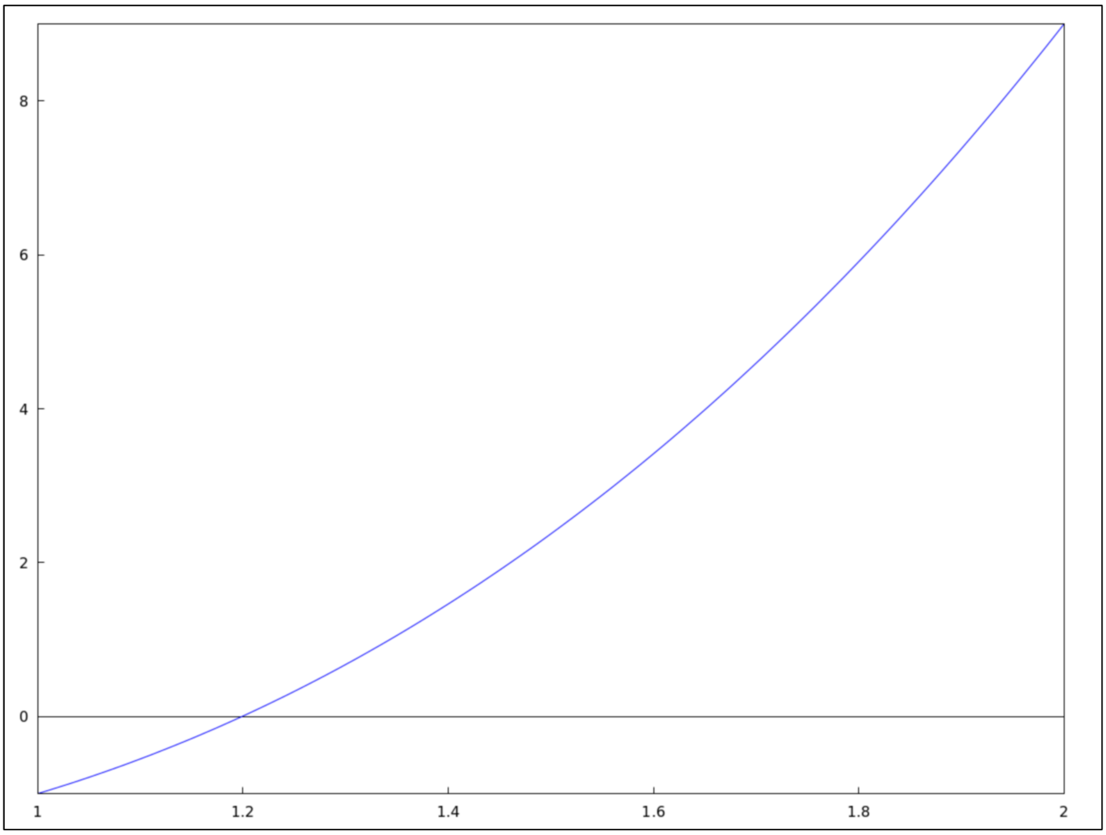
2.1.2

plot

(%i3)
```
wxdraw2d(
    xaxis = true, xaxis_type = solid,
    yaxis = true, yaxis_type = solid,

    explicit(x^3+2·x^2−3·x−1,   x,  1,  2)
);
```

(%t3)



(%o3)

2.2

2.2.1

(%i4) bisect(tan(%pi·x)−x−6, 0.40, 0.48, 20, 0.0005);

iter	*m*	*ym*	*error*
1	0.44	−1.197816418886826	0.03999999999999998
2	0.4599999999999999	1.455815088305811	0.01999999999999999
3	0.45	−0.136248485324959	0.009999999999999999
4	0.455	0.5713662290413843	0.004999999999999997
5	0.4525	0.1989448766616348	0.002499999999999998
6	0.45125	0.02705267432522884	0.001249999999999999
7	0.450625	−0.0556310035476173	$6.249999999999997\ 10^{-4}$
8	0.4509375	−0.01455237456134561	$3.124999999999998\ 10^{-4}$

(%o4) *bisection has converged*

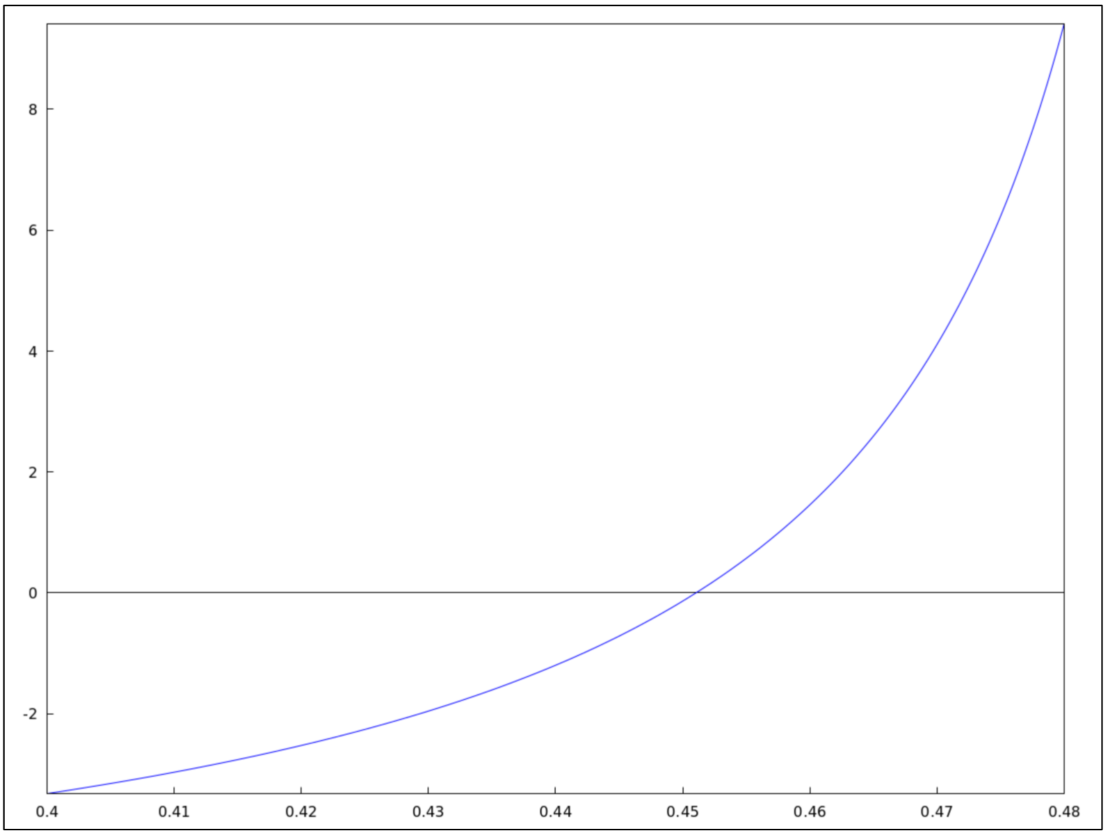
2.2.2

plot

```
(%i5) wxdraw2d(
        xaxis = true, xaxis_type = solid,
        yaxis = true, yaxis_type = solid,

        explicit(tan(%pi·x)−x−6,    x,   0.4,   0.48)
      );
```

(%t5)



(%o5)

2.3

2.3.1

(%i6) bisect(x^3−2·sin(x), 0.50, 2.00, 20, 0.0005);

iter	*m*	*ym*	*error*
1	1.25	0.0551557612888276	0.75
2	0.875	−0.8651651294720542	0.375
3	1.0625	−0.5476869797091422	0.1875
4	1.15625	−0.2847914007983883	0.09375
5	1.203125	−0.1247986155094702	0.046875
6	1.2265625	−0.03735980652509796	0.0234375
7	1.23828125	0.00825801590073083	0.01171875
8	1.232421875	−0.01471021624269308	0.005859375
9	1.2353515625	−0.003266014170569153	0.0029296875
10	1.23681640625	0.002486011901918328	0.00146484375
11	1.236083984375	$-3.924970675475148\ 10^{-4}$	$7.32421875\ 10^{-4}$
12	1.2364501953125	0.001046133270412585	$3.662109375\ 10^{-4}$

(%o6) *bisection has converged*

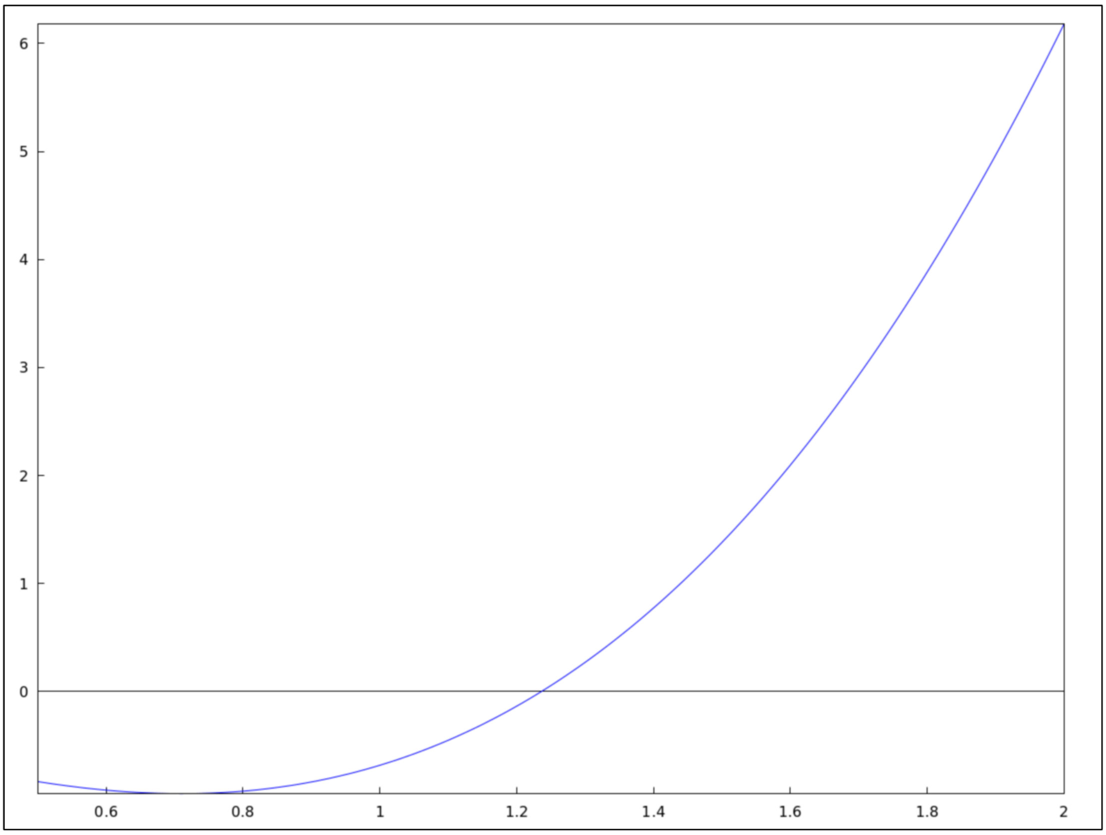
2.3.2

plot

(%i7)
```
wxdraw2d(
    xaxis = true, xaxis_type = solid,
    yaxis = true, yaxis_type = solid,

    explicit(x^3−2·sin(x),   x,  0.5,   2)
);
```

(%t7)



(%o7)

3

Exercise

Also plot the curves

3.1

Figure 3:

1. Verify that each of the following equations has a root on the interval $(0,1)$. Next, perform the bisection method to determine p_3, the third approximation to the location of the root, and to determine (a_4, b_4), the next enclosing interval.

(a) $\ln(1+x) - \cos x = 0$ **(b)** $x^5 + 2x - 1 = 0$

(c) $e^{-x} - x = 0$ **(d)** $\cos x - x = 0$

3.2

Figure 4:

In Exercises 2–5, verify that the given function has a zero on the indicated interval. Next, perform the first five (5) iterations of the bisection method and verify that each

Figure 5:

2. $f(x) = x^3 + x^2 - 3x - 3, \quad (1,2), \quad p = \sqrt{3}$

3. $f(x) = \sin x, \quad (3,4), \quad p = \pi$

4. $f(x) = 1 - \ln x, \quad (2,3), \quad p = e$

5. $f(x) = x^6 - 3, \quad (1,2), \quad p = \sqrt[6]{3}$

3.3

Figure 6:

16. For each of the functions given below, use the bisection method to approximate all real zeros. Use an absolute tolerance of 10^{-6} as a stopping criterion.

(a) $f(x) = e^x + x^2 - x - 4$

(b) $f(x) = x^3 - x^2 - 10x + 7$

(c) $f(x) = 1.05 - 1.04x + \ln x$

```
(%i8) bisect(x^2-2, 1.00, 2.00, 20, 0.0005);
```

iter	*m*	*ym*	*error*
1	1.5	0.25	0.5
2	1.25	−0.4375	0.25
3	1.375	−0.109375	0.125
4	1.4375	0.06640625	0.0625
5	1.40625	−0.0224609375	0.03125
6	1.421875	0.021728515625	0.015625
7	1.4140625	$-4.2724609375\ 10^{-4}$	0.0078125
8	1.41796875	0.0106353759765625	0.00390625
9	1.416015625	0.005100250244140625	0.001953125
10	1.4150390625	0.00233548400878906	$9.765625\ 10^{-4}$
11	1.41455078125	$9.539127349853516\ 10^{-4}$	$4.8828125\ 10^{-4}$

(%o8) *bisection has converged*