# Practical 3 : Secant Method

## 1

Secant Method

Figure 1:

$$x_{n+1} = x_n - \left( \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} \right) f(x_n)$$

Pseudo code for Secant Method

Figure 2:

**procedure** $Secant(f, a, b, nmax, \varepsilon)$
**integer** $n, nmax$; **real** $a, b, fa, fb, \varepsilon, d$
**external function** $f$
$fa \leftarrow f(a)$
$fb \leftarrow f(b)$

Figure 3:

```
if |fa| > |fb| then
    a ⟷ b
    fa ⟷ fb
end if
output 0, a, fa
output 1, b, fb
for n = 2 to nmax do
    if |fa| > |fb| then
        a ⟷ b
        fa ⟷ fb
    end if
    d ← (b − a)/(fb − fa)
    b ← a
    fb ← fa
    d ← d · fa
    if |d| < ε then
        output "convergence"
        return
    end if
    a ← a − d
    fa ← f(a)
    output n, a, fa
end for
end procedure Secant
```

## 2

(%i1) kill(all);

(%o0) *done*

(%i1)
```
secant(g,  a,  b,  e,  Nmax):=block
(
   [fa,  fb,  d,  t,  t1 ],
   define(f(x),   g),
   fa:f(a),
   fb:f(b),
   if(abs(fa) >abs(fb)) then (t:a, a:b, b:t, t1:fa, fa:fb, fb:t1),
   print("0       ", a, "       ", fa),
   print("1       ", b, "        ",  fb),
   for i:2 thru Nmax do
   (
      if(abs(fa) >abs(fb)) then (t:a, a:b, b:t, t1:fa, fa:fb, fb:t1),
      d:(b−a)/(fb−fa),
      b:a,
      fb:fa,
      d:d·fa,
      if(abs(d)<e)then(return("convergence")),
      a:a−d,
      fa:f(a),
      print(i, "     ", a, "     ",  fa)
   )
);
```

(%o1) secant$(g,a,b,e,Nmax):=$block$([fa,fb,d,t,t1],$ define$(f(x),g),fa:f(a),fb:f(b),$if $|fa|>|fb|$ then $(t:a,a:b,b:t,t1:fa,fa:fb,fb:t1)$ ,print$(0\quad ,a,\quad ,fa),$ print$(1\quad ,b,\quad ,fb),$for $i$ from 2 thru $Nmax$ do $($if $|fa|>|fb|$ then $(t:a,a:b,b:t,t1:fa,fa:fb,fb:t1)$ $,d:\frac{b-a}{fb-fa},b:a,fb:fa$ $,d:d\,fa,$if $|d|<e$ then return$(convergence)$ $,a:a-d,fa:f(a),$ print$(i,\quad ,a,\quad ,fa)))$

## 2.1

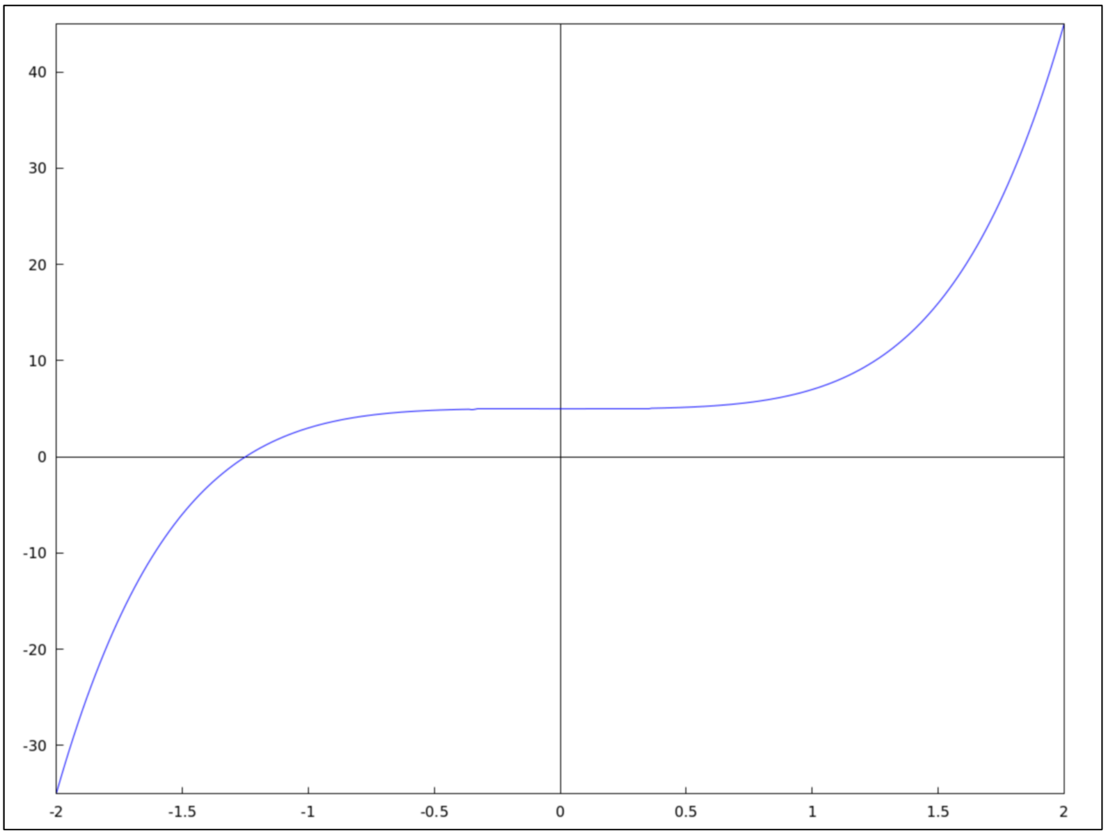
### 2.1.1

plot

(%i2)
```
wxdraw2d(
    xaxis = true, xaxis_type = solid,
    yaxis = true, yaxis_type = solid,

    explicit(x^5+x^3+5,    x,   −2,   2)
);
```

(%t2)




(%o2)

## 2.1.2

(%i3)
```
secant(x^5+x^3+5,   −1.0,  1.0,  0.00001,   10);
```

| | | |
|---|---|---|
| *0* | −1.0 | 3.0 |
| *1* | 1.0 | 7.0 |
| 2 | −2.5 | −108.28125 |
| 3 | −1.040438079191238 | 2.65449640625329 |
| 4 | −1.351122939874979 | −1.969233529771539 |
| 5 | −1.218803134584444 | 0.5000141393826882 |
| 6 | −1.245597438935572 | 0.06904655944420757 |
| 7 | −1.249890231744745 | −0.003028609776762891 |
| 8 | −1.249709847925851 | $1.721118287223433 \cdot 10^{-5}$ |

(%o3) *convergence*

## 2.2

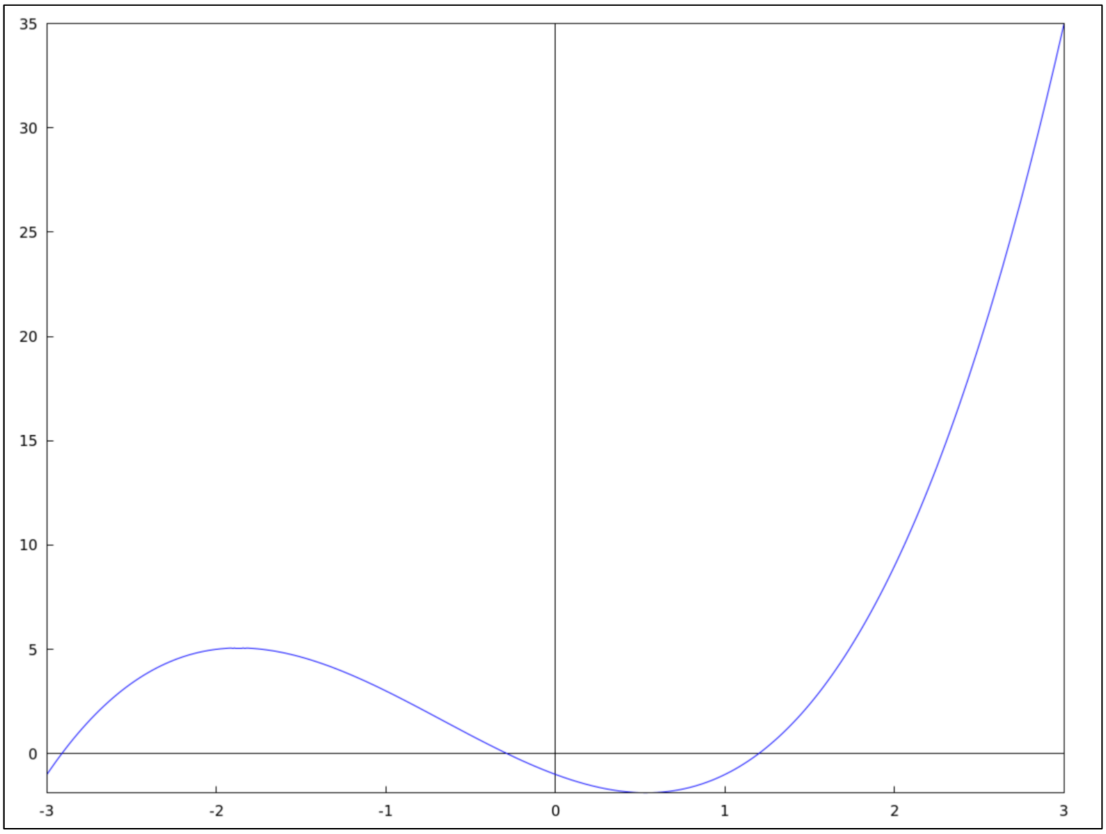
### 2.2.1

plot

(%i4)
```
wxdraw2d(
    xaxis = true, xaxis_type = solid,
    yaxis = true, yaxis_type = solid,

    explicit(x^3+2·x^2−3·x−1,   x,   −3,   3)
);
```

(%t4)




(%o4)

### 2.2.2

(%i5) secant(x^3+2·x^2−3·x−1, 2.0, 1.0, 0.00001, 10);

| | | |
|---|---|---|
| *0* | 1.0 | −1.0 |
| *1* | 2.0 | 9.0 |
| 2 | 1.1 | −0.5489999999999999 |
| 3 | 1.221729490022173 | 0.1436388950901026 |
| 4 | 1.19648532661955 | −0.01344066810262378 |
| 5 | 1.198645368415737 | $-2.800716317907792\ 10^{-4}$ |
| 6 | 1.198691336425769 | $5.672464513661879\ 10^{-7}$ |

(%o5) *convergence*

## 2.2.3

(%i6) secant(x^3+2·x^2−3·x−1, −2.0, −3.0, 0.00001, 10);

| | | |
|---|---|---|
| *0* | −3.0 | −1.0 |
| *1* | −2.0 | 5.0 |
| 2 | −2.833333333333333 | 0.8101851851851833 |
| 3 | −2.907928388746803 | 0.04629957161572662 |
| 4 | −2.912449640422374 | −0.002380064066290543 |
| 5 | −2.912228585591192 | $6.399876401275151\ 10^{-6}$ |

(%o6) *convergence*

# 3

Exercise(Bradie)

## 3.1

Figure 4:

1. Each of the following equations has a root on the interval $(0, 1)$. Perform the secant method to determine $p_4$, the fourth approximation to the location of the root.

   (a) $\ln(1+x) - \cos x = 0$ (b) $x^5 + 2x - 1 = 0$

   (c) $e^{-x} - x = 0$ (d) $\cos x - x = 0$

## 3.2

Figure 5:

In Exercises 5–8, an equation, an interval on which the equation has a root, and the exact value of the root are specified.

(a) Perform seven (7) iterations of the secant method.

Figure 6:

5. The equation $x^3 + x^2 - 3x - 3 = 0$ has a root on the interval $(1, 2)$, namely $x = \sqrt{3}$.
6. The equation $x^7 = 3$ has a root on the interval $(1, 2)$, namely $x = \sqrt[7]{3}$.
7. The equation $x^3 - 13 = 0$ has a root on the interval $(2, 3)$, namely $\sqrt[3]{13}$.
8. The equation $1/x - 37 = 0$ has a zero on the interval $(0.01, 0.1)$, namely $x = 1/37$.

# 3.3

Figure 7:

18. For each of the functions given below, use the secant method to approximate all real roots. Use an absolute tolerance of $10^{-6}$ as a stopping condition.
    (a) $f(x) = e^x + x^2 - x - 4$
    (b) $f(x) = x^3 - x^2 - 10x + 7$
    (c) $f(x) = 1.05 - 1.04x + \ln x$