Practical 12 : Simpson Rule

1

Figure 1:

$$I(f) \approx I_{2,\text{closed}}(f) = \frac{\Delta x}{3}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$
$$= \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right].$$

This formula may be recognized from calculus as Simpson's rule.

2

2.1

Approximate the value of the integrals using Simpsons Rule

Figure 2:

(a) $\int_1^2 \frac{1}{x}\, dx$

(b) $\int_0^1 e^{-x}\, dx$

(c) $\int_0^1 \frac{1}{1+x^2}\, dx$

(d) $\int_0^1 \tan^{-1} x\, dx.$

2.2

```
(%i1) kill(all);
(%o0) done
```

(%i1)
```
S(g, a, b):=block
(
   define(f(x), g),
   return( ((b−a)/6)·(f(a)+ (4·f((a+b)/2)) + f(b)))
);
```

(%o1) $S(g,a,b):=$

$$\mathrm{block}\left(\mathrm{define}(f(x),g),\mathrm{return}\left(\frac{b-a}{6}\left(f(a)+4\,f\left(\frac{a+b}{2}\right)+f(b)\right)\right)\right)$$

2.3

(%i2)
```
ratprint:false;
```

(%o2) *false*

(%i8)
```
f1(x):=1/x;
a:1.0;
b:2.0;
s:S(f1(x), a, b);
v:integrate(f1(x), x, a, b);
e:abs(v−s);
```

(%o3) $f1(x):=\frac{1}{x}$

(%o4) 1.0

(%o5) 2.0

(%o6) 0.6944444444444443

(%o7) 0.6931471805599453

(%o8) 0.001297263884499022

2.4

(%i14)
```
f1(x):=exp(−x);
a:0.0;
b:1.0;
s:S(f1(x), a, b);
v:integrate(f1(x), x, a, b);
e:abs(v−s);
```

(%o9) $f1(x):=\exp(-x)$

(%o10) 0.0

(%o11) 1.0

(%o12) 0.6323336800036626

(%o13) $\frac{17854706}{28245729}$

(%o14) $2.131211751048578\ 10^{-4}$

2.5

```
(%i20) f1(x):=1/(1+x^2);
       a:0.0;
       b:1.0;
       s:S(f1(x),   a,   b);
       v:integrate(f1(x),   x,   a,   b);
       e:abs(v-s);
```

(%o15) $f1(x):=\frac{1}{1+x^2}$

(%o16) 0.0

(%o17) 1.0

(%o18) 0.7833333333333333

(%o19) 0.7853981633974483

(%o20) 0.002064830064114953

2.6

```
(%i26) f1(x):=atan(x);
       a:0.0;
       b:1.0;
       s:S(f1(x),   a,   b);
       v:integrate(f1(x),   x,   a,   b);
       e:abs(v-s);
```

(%o21) $f1(x):=\operatorname{atan}(x)$

(%o22) 0.0

(%o23) 1.0

(%o24) 0.4399980999001121

(%o25) 0.4388245731174756

(%o26) 0.001173526782636491

3

3.1

Composite Simpsons Rule

Figure 3:

Composite Simpson's Rule

Since the basic Simpson's rule formula already divides the interval $[a, b]$ into two pieces, $[a, b]$ must be divided into an even number of subintervals to apply Simpson's rule in a composite manner. Therefore, let $n = 2m$, define

$$h = \frac{b-a}{n} = \frac{b-a}{2m},$$
$$x_i = a + ih \quad (0 \le i \le 2m),$$

Figure 4:

and apply the Simpson's rule formula m times, once over each $[x_{2j-2}, x_{2j}]$ for $j = 1, 2, 3, \ldots, m$. This produces

Figure 5:

$$\begin{aligned} I(f) &= \sum_{j=1}^{m} \int_{x_{2j-2}}^{x_{2j}} f(x)\, dx \\ &= \sum_{j=1}^{m} \frac{x_{2j} - x_{2j-2}}{6} \left[f(x_{2j-2}) + 4f(x_{2j-1}) + f(x_{2j})\right] - \sum_{j=1}^{m} \frac{(x_{2j} - x_{2j-2})^5}{2880} f^{(4)}(\xi_j) \\ &= \frac{h}{3}\left[f(x_0) + 4\sum_{j=1}^{m} f(x_{2j-1}) + 2\sum_{j=1}^{m-1} f(x_{2j}) + f(x_{2m})\right] - \frac{h^5}{90}\sum_{j=1}^{m} f^{(4)}(\xi_j). \end{aligned}$$

3.2 n must be even

```
(%i32) Sc(g1, a, b, n):=block
(
    m:n/2,
    define(f(x), g1),
    h:(b−a)/n,
    odd:0.0,
    even:0.0,
    for i:1 thru m−1 do even:even+f(a+(2·i)·h),
    for i:1 thru m do odd:odd+f(a+(2·i−1)·h),
    ans:(h/3)·(f(a)+4·odd+2·even+f(b)),
    return(float(ans))
);
```

(%o32) $\mathrm{Sc}(g1,a,b,n):=\mathrm{block}\left(m:\frac{n}{2},\mathrm{define}(f(x),g1),h:\frac{b-a}{n},\right.$ $odd:0.0,even:0.0,\mathrm{for}\ i\ \mathrm{thru}\ m-1\ \mathrm{do}\ even:even+f(a+2\,i\,h),$ $\mathrm{for}\ i\ \mathrm{thru}\ m\ \mathrm{do}\ odd:odd+f(a+(2\,i-1)\,h),ans:\frac{h}{3}$ $\left.(f(a)+4\,odd+2\,even+f(b)),\mathrm{return}(\mathrm{float}(ans))\right)$

3.3

Figure 6:

Reconsider the integral

$$I(f) = \int_0^\pi \sin x \, dx,$$

Its exact value is 2

```
(%i33) for k:2 next 2·k thru 150 do block(
        temp:Sc(sin(x),  0,  %pi,  k),
        print(k, "      ", temp)
        );
2         2.094395102393195
4         2.004559754984421
8         2.000269169948387
16        2.000016591047935
32        2.000001033369412
64        2.000000064530001
128       2.000000004032257
(%o33) done
```

check

Figure 7:

n	h	$S_h(f)$
2	$\frac{\pi}{2}$	2.09439510239
4	$\frac{\pi}{4}$	2.00455975498
8	$\frac{\pi}{8}$	2.00026916995
16	$\frac{\pi}{16}$	2.00001659105
32	$\frac{\pi}{32}$	2.00000103337
64	$\frac{\pi}{64}$	2.00000006453
128	$\frac{\pi}{128}$	2.00000000403

3.4

Figure 8:

Consider the definite integral

$$I(f) = \int_0^1 \sqrt{1+x^3}\, dx.$$

(%i34)
```
for k:2 next 2·k thru 150 do block(
    temp:Sc(sqrt(1+x^3),  0,   1,  k),
    print(k, "      ", temp)
    );
```

```
2        1.10947570824873
4        1.111363232263895
8        1.111442701555603
16        1.111447642677047
32        1.111447950064473
64        1.111447969253675
128        1.111447970452648
```

(%o34) *done*

4

Exercise

Do the above for the following using composite Simpsons method(n is even)

Figure 9:

6. $\int_1^2 \frac{1}{x}\, dx$
7. $\int_0^1 e^{-x}\, dx$
8. $\int_0^1 \tan^{-1} x\, dx$
9. $\int_1^2 \frac{\sin x}{x}\, dx$
10. $\int_0^1 \frac{1}{\sqrt{1+x^4}}\, dx$