

Practical 6 : Gauss-Jacobi Method

1

Gauss_jacobi Method

Figure 1:

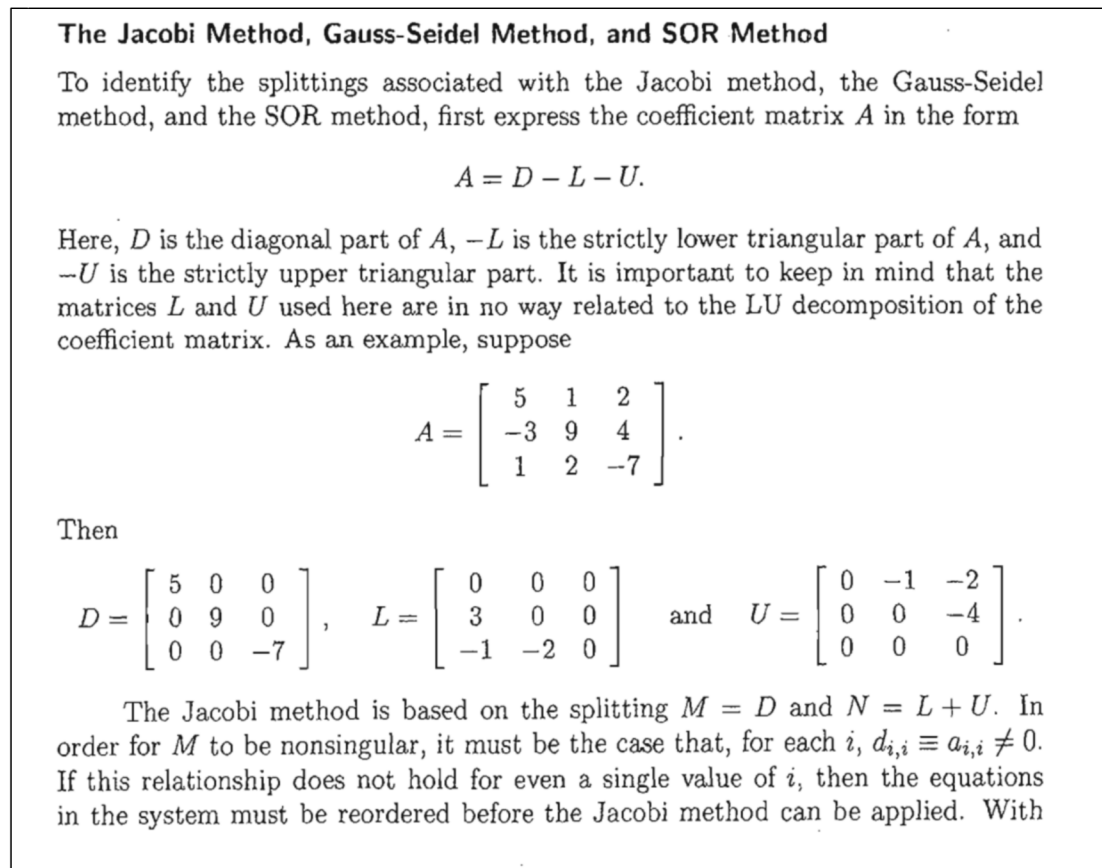
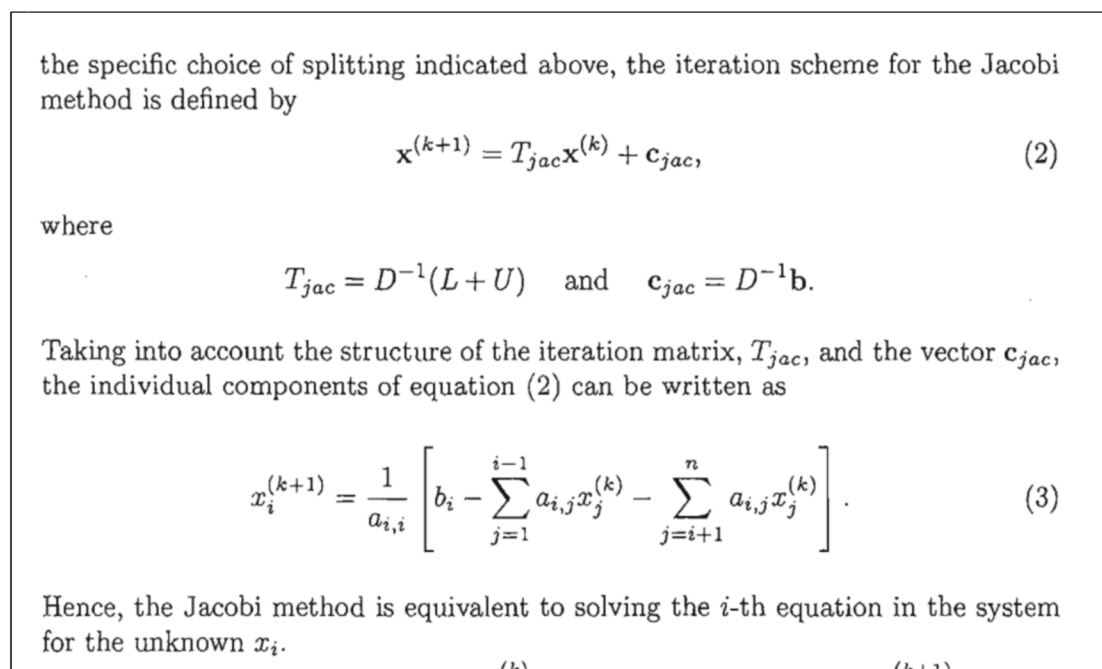


Figure 2:



2

2.1

Figure 3:

Consider the system of equations

$$\begin{array}{rrrrrrcl} 5x_1 & + & x_2 & + & 2x_3 & = & 10 \\ -3x_1 & + & 9x_2 & + & 4x_3 & = & -14 \\ x_1 & + & 2x_2 & - & 7x_3 & = & -33. \end{array}$$

The Jacobi method, when applied to this system, will produce the sequence of approximations $\{\mathbf{x}^{(k)}\}$ according to the rules

$$\begin{aligned} x_1^{(k+1)} &= \frac{1}{5} \left[10 - x_2^{(k)} - 2x_3^{(k)} \right] \\ x_2^{(k+1)} &= \frac{1}{9} \left[-14 + 3x_1^{(k)} - 4x_3^{(k)} \right] \\ x_3^{(k+1)} &= -\frac{1}{7} \left[-33 - x_1^{(k)} - 2x_2^{(k)} \right]. \end{aligned}$$

2.2

function for Gauss-Jacobi method

```
(%i1) kill(all);  
(%o0) done
```

```
(%i1) gauss_jacobi(A, b, iter):=block(
  n:matrix_size(A)[1],
  for i:1 thru n do
  (
    for j:1 thru n do
    (
      if(i > j) then (l[i, j]:A[i, j])
      else ( l[i, j]:0)
    )
  ),
  L:genmatrix(l, n, n),
  for i:1 thru n do
  (
    for j:1 thru n do
    (
      if(i < j) then (u[i, j]:A[i, j])
      else ( u[i, j]:0)
    )
  ),
  U:genmatrix(u, n, n),
  for i:1 thru n do
  (
    for j:1 thru n do
    (
      if(i = j) then (d[i, j]:A[i, j])
      else ( d[i, j]:0)
    )
  ),
  D:genmatrix(d, n, n),
  D1:invert(D),
  Tgj:D1.(-L-U),
  cgj:D1.b,
  for i:1 thru n do t[i, 1]:0.0,
  x0:genmatrix(t, n, 1),
  for i:1 thru iter do
  (
    x1:Tgj.x0+cgj,
    disp(i, "-----", x1),
    x0:x1
  )
);
```

```
(%o1) gauss_jacobi(A,b,iter):=block(n:(matrix_size(A))_1,for i
  thru n do for j thru n do if i>j then li,j:Ai,j else li,j:0,L:
  genmatrix(l,n,n),for i thru n do for j thru n do if i<j then
  ui,j:Ai,j else ui,j:0,U:genmatrix(u,n,n),for i thru n do for j
  thru n do if i=j then di,j:Ai,j else di,j:0,D:genmatrix(d,n,n),
  D1:invert(D),Tgj:D1 . (-L-U),cgj:D1 . b,for i thru n do ti,1
  :0.0,x0:genmatrix(t,n,1),for i thru iter do
  (x1=Tgj.x0+cgj,disp(i,"-----",x1),x0=x1)
```

```
(%i3) A1:matrix([5, 1, 2], [-3, 9, 4], [1, 2, -7]);
```

```
b1:matrix([10], [-14], [-33]);
```

```
(%o2) 
$$\begin{pmatrix} 5 & 1 & 2 \\ -3 & 9 & 4 \\ 1 & 2 & -7 \end{pmatrix}$$

```

```
(%o3) 
$$\begin{pmatrix} 10 \\ -14 \\ -33 \end{pmatrix}$$

```

```
(%i4) gauss_jacobi(A1, b1, 14);
```

```
1
```

$$\begin{pmatrix} 2.0 \\ -1.555555555555555 \\ 4.714285714285714 \end{pmatrix}$$

```
2
```

$$\begin{pmatrix} 0.4253968253968252 \\ -2.984126984126984 \\ 4.555555555555555 \end{pmatrix}$$

```
3
```

$$\begin{pmatrix} 0.7746031746031745 \\ -3.438447971781305 \\ 3.922448979591836 \end{pmatrix}$$

```
4
```

$$\begin{pmatrix} 1.118710002519526 \\ -3.040665154950869 \\ 3.842529604434366 \end{pmatrix}$$

```
5
```

$$\begin{pmatrix} 1.071121189216427 \\ -2.890443156686543 \\ 4.005339956088256 \end{pmatrix}$$

```
6
```

$$\begin{pmatrix} 0.9759526489020063 \\ -2.97866625074486 \\ 4.041462125120478 \end{pmatrix}$$

```
7
```

$$\begin{pmatrix} 0.9791484001007809 \\ -3.026443394863988 \\ 4.002660021058898 \end{pmatrix}$$

```
8
```

$$\begin{pmatrix} 1.004224670549238 \\ -3.008132764881472 \\ 3.989465944338972 \end{pmatrix}$$

Note :

`disp(i, "----", transpose(x1))`

can be used for printing as row vectors

2.3

check

Figure 4:

k	$\mathbf{x}^{(k)}$
0	$\begin{bmatrix} 0.000000 & 0.000000 & 0.000000 \end{bmatrix}^T$
1	$\begin{bmatrix} 2.000000 & -1.555556 & 4.714286 \end{bmatrix}^T$
2	$\begin{bmatrix} 0.425397 & -2.984127 & 4.555556 \end{bmatrix}^T$
3	$\begin{bmatrix} 0.774603 & -3.438448 & 3.922449 \end{bmatrix}^T$
4	$\begin{bmatrix} 1.118710 & -3.040665 & 3.842530 \end{bmatrix}^T$
5	$\begin{bmatrix} 1.071121 & -2.890443 & 4.005340 \end{bmatrix}^T$
6	$\begin{bmatrix} 0.975953 & -2.978666 & 4.041462 \end{bmatrix}^T$
7	$\begin{bmatrix} 0.979148 & -3.026443 & 4.002660 \end{bmatrix}^T$
8	$\begin{bmatrix} 1.004225 & -3.008133 & 3.989466 \end{bmatrix}^T$
9	$\begin{bmatrix} 1.005840 & -2.993910 & 3.998280 \end{bmatrix}^T$
10	$\begin{bmatrix} 0.999470 & -2.997289 & 4.002574 \end{bmatrix}^T$
11	$\begin{bmatrix} 0.998428 & -3.001321 & 4.000699 \end{bmatrix}^T$
12	$\begin{bmatrix} 0.999985 & -3.000835 & 3.999398 \end{bmatrix}^T$
13	$\begin{bmatrix} 1.000408 & -2.999738 & 3.999759 \end{bmatrix}^T$
14	$\begin{bmatrix} 1.000044 & -2.999757 & 4.000133 \end{bmatrix}^T$

```
(%i5) kill(all);
```

```
(%o0) done
```

```
(%i1) gauss_jacobi(A, b, init, iter):=block(
  n:matrix_size(A)[1],
  for i:1 thru n do
  (
    for j:1 thru n do
    (
      if(i > j) then (l[i, j]:A[i, j])
      else ( l[i, j]:0)
    )
  ),
  L:genmatrix(l, n, n),
  for i:1 thru n do
  (
    for j:1 thru n do
    (
      if(i < j) then (u[i, j]:A[i, j])
      else ( u[i, j]:0)
    )
  ),
  U:genmatrix(u, n, n),
  for i:1 thru n do
  (
    for j:1 thru n do
    (
      if(i = j) then (d[i, j]:A[i, j])
      else ( d[i, j]:0)
    )
  ),
  D:genmatrix(d, n, n),
  D1:invert(D),
  Tgj:D1.(-L-U),
  cgj:D1.b,

  x0:init,
  for i:1 thru iter do
  (
    x1:Tgj.x0+cgj,
    disp(i, "-----", transpose(x1)),
    x0:x1
  )
);
```

```
(%o1) gauss_jacobi(A,b,init,iter):=block(n:(matrix_size(A))_1,
for i thru n do for j thru n do if i>j then li,j:Ai,j else li,j:0,L
:genmatrix(l,n,n),for i thru n do for j thru n do if i<j then
ui,j:Ai,j else ui,j:0,U:genmatrix(u,n,n),for i thru n do for j
thru n do if i=j then di,j:Ai,j else di,j:0,D:genmatrix(d,n,n),
D1:=invert(D).Tgj:=D1.(-L-U).cgj:=D1.b.x0:=init.for i thru
```



```
(%i9) A1:matrix([5, 1, 2], [-3, 9, 4], [1, 2, -7]);
b1:matrix([10], [-14], [-33]);
start:matrix([0.0], [0.0], [0.0]);
gauss_jacobi(A1, b1, start, 14);
```

```
(%o6) 
$$\begin{pmatrix} 5 & 1 & 2 \\ -3 & 9 & 4 \\ 1 & 2 & -7 \end{pmatrix}$$

```

```
(%o7) 
$$\begin{pmatrix} 10 \\ -14 \\ -33 \end{pmatrix}$$

```

```
(%o8) 
$$\begin{pmatrix} 0.0 \\ 0.0 \\ 0.0 \end{pmatrix}$$

```

```
1
```

```
-----
```

```
(2.0 -1.5555555555555555 4.714285714285714)
```

```
2
```

```
-----
```

```
(0.4253968253968252 -2.984126984126984 4.555555555555555)
```

```
3
```

```
-----
```

```
(0.7746031746031745 -3.438447971781305 3.922448979591836)
```

```
4
```

```
-----
```

```
(1.118710002519526 -3.040665154950869 3.842529604434366)
```

```
5
```

```
-----
```

```
(1.071121189216427 -2.890443156686543 4.005339956088256)
```

```
6
```

```
-----
```

```
(0.9759526489020063 -2.97866625074486 4.041462125120478)
```

```
7
```

```
-----
```

```
(0.9791484001007809 -3.026443394863988 4.002660021058898)
```

```
8
```

```
-----
```

```
(1.004224670549238 -3.008132764881472 3.989465944338972)
```

```
9
```

```
-----
```

```
(1.005840175240705 -2.993909973967575 3.998279877255185)
```

```
10
```

```
-----
```

4

Exercise

Starting with initial vector $x_0 = 0$, perform 6 iterations of Gauss-Jacobi method

Figure 5:

$$\begin{array}{ll} \text{(a)} & \begin{bmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \\ 0 & 2 & 6 \end{bmatrix}, \quad \begin{bmatrix} -1 \\ 3 \\ 5 \end{bmatrix} \\ \text{(b)} & \begin{bmatrix} 3 & -1 & 1 \\ 2 & -6 & 3 \\ -9 & 7 & -20 \end{bmatrix}, \quad \begin{bmatrix} 4 \\ -13 \\ 7 \end{bmatrix} \\ \text{(c)} & \begin{bmatrix} 4 & 2 & -1 \\ 2 & 4 & 1 \\ -1 & 1 & 4 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \\ \text{(d)} & \begin{bmatrix} 4 & -1 & 0 & 0 \\ 2 & 4 & -1 & 0 \\ 0 & -2 & 4 & -1 \\ 0 & 0 & -2 & 4 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 2 \\ -3 \\ 1 \end{bmatrix} \end{array}$$