

# Examples : numerical.mac

1

```
(%i1) load("/home/jvr/Downloads/numericalPackage/v2/numerical.mac");
(%o1) /home/jvr/Downloads/numericalPackage/v2/numerical.mac

(%i2) ratprint:false;
(%o2) false
```

## 2 Bisection Method

### 2.1

```
(%i3) bisect(t^3-2·sin(t), t, 0.50, 2.00, 20, 0.0005);
      iter          m                  ym          error
      1            1.25        0.0551557612888276
                  0.75
      2            0.875       -0.8651651294720542
                  0.375
      3            1.0625      -0.5476869797091422
                  0.1875
      4            1.15625     -0.2847914007983883
                  0.09375
      5            1.203125    -
      0.1247986155094702  0.046875
      6            1.2265625   -
      0.03735980652509796 0.0234375
      7            1.23828125
      0.00825801590073083 0.01171875
      8            1.232421875  -
      0.01471021624269308 0.005859375
      9            1.2353515625 -
      0.003266014170569153 0.0029296875
      10           1.23681640625
      0.002486011901918328 0.00146484375
      11           1.236083984375 -
      3.924970675475148 10^-4      7.32421875 10^-4
      12           1.2364501953125
      0.001046133270412585 3.662109375 10^-4
(%o3) bisection has converged
```

## 2.2

```
(%i4) bisect(sin(x), x, 2, 4, 20, 0.0005);
          iter      m           ym           error
          1          3   0.1411200080598672
          2          1   -0.3507832276896198
          3          0.5  -0.1081951345301083
          4          0.25 0.0165918922293479
          5          0.125
          6          0.0625 -0.04589122327277969
          7          3.1875
          8          0.03125
          9          3.140625 9.676534387822795
          10         3.15625
          11         0.0078125
          12         3.14453125
          13         0.00390625
          14         3.142578125
          15         0.001953125
          16         3.1416015625
          17         9.765625 10^-4
          18         3.14111328125
          19         4.8828125 10^-4
          20         4.793723214334506 10^-4
(%o4) bisection has converged
```

## 2.3

```
(%i5) bisect(x^2-2, x, 1, 2, 20, 0.0005);
```

iter	m	ym	error
1	1.5	0.25	0.5
2	1.25	-0.4375	0.25
3	1.125	-0.734375	
0.125			
4	1.0625	-0.87109375	
0.0625			
5	1.03125	-0.9365234375	
0.03125			
6	1.015625	-0.968505859375	
0.015625			
7	1.0078125	-0.98431396484375	
0.0078125			
8	1.00390625	-	
0.9921722412109375	0.00390625		
9	1.001953125	-	
0.9960899353027344	0.001953125		
10	1.0009765625	-	
0.9980459213256836	9.765625 10 <sup>-4</sup>		
11	1.00048828125	-	
0.9990231990814209	4.8828125 10 <sup>-4</sup>		

```
(%o5) bisection has converged
```

```
(%i6) bisect(x^3-2*x^2+x-%pi, x, 2, 3, 10, 0.0001);
      iter          m          ym          error
      1            2.5        2.483407346410207
      2            0.5        0.3740323464102069
      3            0.25       -0.4521395285897931
      4            0.125      -0.05687585671479311
      5            0.0625
      6            0.03125    0.1540311257070818
      7            0.015625
      8            0.0078125
      9            0.00390625
      10           0.001953125
      10           0.001533142169467538   9.765625 10-4
(%o6) done
```

### 3 Newton's Method

#### 3.1

```
(%i7) newton(x^3-2*x^2+x-3, x, 3, 0.0001, 0.00001, 8);
      1            2.4375        2.036865234375
      2            2.213032716315109     0.2563633850614177
      3            2.175554938721488     0.006463361488812325
      4            2.174560100666446     4.479068049789703
      10-6
      5            2.174559410293312     2.156497203031904
      10-12
(%o7) convergence
```

#### 3.2

```
(%i8) newton(sin(x), x, 3.0, 0.0001, 0.00001, 8);
      1      3.142546543074278      -9.538893398264409
      10-4
      2      3.141592653300477      2.893162490762184
      10-10
      3      3.141592653589793      1.224646799147353
      10-16
(%o8) convergence
```

### 3.3

```
(%i9) newton(x^2-2, x, 1, 0.0001, 0.00001, 8);
      1      1.5      0.25
      2      1.416666666666666      0.00694444444444642
      3      1.414215686274509      6.007304882871267
      10-6
      4      1.414213562374689      4.510614104447086
      10-12
(%o9) convergence
```

```
(%i10) newton(x^3-2*x^2+x-%pi, x, 3, 0.0001, 0.00001, 8);
      1      2.446349540849362      1.975991988443487
      2      2.230829413793257      0.237982391674409
      3      2.196863381645069      0.005374496729482203
      4      2.196060159694346      2.961172185678151
      10-6
      5      2.196059716657127      9.00612917575927
      10-13
(%o10) convergence
```

## 4 Secant Method

### 4.1

```
(%i11) secant(x^3+2*x^2-3*x-1, x, -2, -3, 0.00001, 10);
0      -3      -1
1      -2      5
2      -2.833333333333333      0.8101851851851833
3      -2.907928388746803      0.04629957161572662
4      -2.912449640422374      -0.002380064066290543
5      -2.912228585591192      6.399876401275151 10-6
```

(%o11) *convergence*

## 4.2

```
(%i12) secant(sin(x), x, 1, 4, 0.00001, 10);
0      4      -0.7568024953079282
1      1      0.8414709848078965
2      2.579462454848934      0.5329898131328342
3      3.166481028136553      -0.02488580518710502
4      3.140295209333355      0.001297443892426365
5      3.141592780561861      -1.269720678483891 10-7
```

(%o12) *convergence*

## 4.3

```
(%i13) secant(x^2-2, x, 1, 2, 0.00001, 10);
0      1      -1
1      2      2
2      1.333333333333333      -0.222222222222223
3      1.428571428571428      0.04081632653061229
4      1.413793103448275      -0.001189060642093009
5      1.41421143847487      -6.007286838860537 10-6
```

(%o13) *convergence*

# 5 Regula-Falsi Method

## 5.1

```
(%i14) regula(x^3+2*x^2-3*x-1, x, 1, 2, 20, 0.0005);
iter          m                      ym
1             1.1                   -0.5489999999999999
2             1.221729490022173
0.1436388950901026
3             1.193880682944684
0.02924075169187978
4             1.199720665327255
0.006290908737020207
5             1.198472095487697
0.001337706025862806
6             1.198737948438891
2.851619710524389 10-4
(%o14) regula falsi method has converged
```

## 5.2

```
(%i15) regula(sin(x), x, 2, 4, 20, 0.0005);
iter          m                      ym
1             3.091528082734958
0.05004365932452196
2             3.147874957380742
0.006282262466726139
3             3.141590357955694
2.295634098490862 10-6
(%o15) regula falsi method has converged
```

## 5.3

```
(%i16) regula(x^2-2, x, 1, 2, 20, 0.0005);
iter          m                      ym
1             1.333333333333333
0.2222222222222218
2             1.428571428571428
0.04081632653061229
3             1.411764705882353
0.006920415224913157
4             1.414634146341463
0.001189767995240842
5             1.414141414141414
2.040608101214758 10-4
(%o16) regula falsi method has converged
```

## 6 LU Factorization

### 6.1

```
(%i19) A:matrix([1, 2, 3], [3, 5, 7], [4, 6, 9]);
IuFactor(A);
IuFactor(A)[1];
(%o17) ⎛ 1 2 3 ⎞
      ⎜ 3 5 7 ⎟
      ⎜ 4 6 9 ⎟
(%o18) ⎡L = ⎛ 1 0 0 ⎤, U = ⎛ 1 2 3 ⎤⎤
      ⎣            ⎜ 3 1 0 ⎟   ⎜ 0 -1 -2 ⎟⎦
      ⎣            ⎜ 4 2 1 ⎟   ⎜ 0 0 1 ⎟⎦
(%o19) L = ⎛ 1 0 0 ⎞
      ⎜ 3 1 0 ⎟
      ⎜ 4 2 1 ⎟
```

## 7 Solving by LU Factorization

### 7.1

```
(%i22) A:matrix([1, 2, 3], [3, 5, 7], [4, 6, 9]);
b:matrix([1], [2], [3]);
solve_by_lu(A, b);
(%o20) ⎛ 1 2 3 ⎞
      ⎜ 3 5 7 ⎟
      ⎜ 4 6 9 ⎟
(%o21) ⎛ 1 ⎞
      ⎜ 2 ⎟
      ⎜ 3 ⎟
(%o22) ⎡z = ⎛ 1 ⎤, x = ⎛ 0 ⎤⎤
      ⎣           ⎜ -1 ⎟   ⎜ -1 ⎟⎦
      ⎣           ⎜ 1 ⎟   ⎜ 1 ⎟⎦
```

### 7.2

(%i26)  $A: \text{matrix}([2, 7, 5], [6, 20, 10], [4, 3, 0]);$   
 $b1: \text{matrix}([14], [36], [7]);$   
 $b2: \text{matrix}([-4], [-16], [-7]);$   
 $b3: \text{matrix}([-3], [-12], [6]);$

(%o23) 
$$\begin{pmatrix} 2 & 7 & 5 \\ 6 & 20 & 10 \\ 4 & 3 & 0 \end{pmatrix}$$

(%o24) 
$$\begin{pmatrix} 14 \\ 36 \\ 7 \end{pmatrix}$$

(%o25) 
$$\begin{pmatrix} -4 \\ -16 \\ -7 \end{pmatrix}$$

(%o26) 
$$\begin{pmatrix} -3 \\ -12 \\ 6 \end{pmatrix}$$

(%i27)  $\text{luFactor}(A);$

(%o27)  $L = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 11 & 1 \end{pmatrix}, U = \begin{pmatrix} 2 & 7 & 5 \\ 0 & -1 & -5 \\ 0 & 0 & 45 \end{pmatrix} I$

(%i28)  $\text{solve\_by\_lu}(A, b1);$

(%o28)  $Z = \begin{pmatrix} 14 \\ -6 \\ 45 \end{pmatrix}, X = \begin{pmatrix} 1.0 \\ 1.0 \\ 1.0 \end{pmatrix} I$

(%i29)  $\text{solve\_by\_lu}(A, b2);$

(%o29)  $Z = \begin{pmatrix} -4 \\ -4 \\ 45 \end{pmatrix}, X = \begin{pmatrix} -1.0 \\ -1.0 \\ 1.0 \end{pmatrix} I$

(%i30)  $\text{solve\_by\_lu}(A, b3);$

(%o30)  $Z = \begin{pmatrix} -3 \\ -3 \\ 45 \end{pmatrix}, X = \begin{pmatrix} 3.0 \\ -2.0 \\ 1.0 \end{pmatrix} I$

## 8 **Gauss-Jacobi Method**

### 8.1

```
(%i34) A1:matrix([5, 1, 2], [-3, 9, 4], [1, 2, -7]);  
b1:matrix([10], [-14], [-33]);  
start:matrix([0], [0], [0]);  
gauss_jacobi(A1, b1, start, 14);  
  
(%o31) 
$$\begin{pmatrix} 5 & 1 & 2 \\ -3 & 9 & 4 \\ 1 & 2 & -7 \end{pmatrix}$$
  
(%o32) 
$$\begin{pmatrix} 10 \\ -14 \\ -33 \end{pmatrix}$$
  
(%o33) 
$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$
  
-----  
(%o34) 
$$\begin{pmatrix} 2.0 & -1.555555555555555 & 4.714285714285714 \end{pmatrix}$$
  
2  
-----  
(%o35) 
$$\begin{pmatrix} 0.4253968253968252 & -2.984126984126984 & 4.555555555555555 \end{pmatrix}$$
  
3  
-----  
(%o36) 
$$\begin{pmatrix} 0.7746031746031745 & -3.438447971781304 & 3.922448979591836 \end{pmatrix}$$
  
4  
-----  
(%o37) 
$$\begin{pmatrix} 1.118710002519526 & -3.040665154950868 & 3.842529604434367 \end{pmatrix}$$
  
5  
-----  
(%o38) 
$$\begin{pmatrix} 1.071121189216427 & -2.890443156686543 & 4.005339956088256 \end{pmatrix}$$
  
6  
-----  
(%o39) 
$$\begin{pmatrix} 0.9759526489020063 & -2.97866625074486 & 4.041462125120478 \end{pmatrix}$$
  
7  
-----  
(%o40) 
$$\begin{pmatrix} 0.9791484001007809 & -3.026443394863987 & 4.002660021058898 \end{pmatrix}$$
  
8  
-----  
(%o41) 
$$\begin{pmatrix} 1.004224670549238 & -3.008132764881471 & 3.989465944338972 \end{pmatrix}$$
  
9  
-----  
(%o42) 
$$\begin{pmatrix} 1.005840175240705 & -2.993909973967574 & 3.998279877255185 \end{pmatrix}$$
  
10  
-----  
'
```

## 8.2

```
(%i38) A1:matrix([2, -1, 0], [-1, 4, 2], [0, 2, 6]);  
b1:matrix([-1], [3], [5]);  
start:matrix([0], [0], [0]);  
gauss_jacobi(A1, b1, start, 14);  
  
(%o35) 
$$\begin{pmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \\ 0 & 2 & 6 \end{pmatrix}$$
  
  
(%o36) 
$$\begin{pmatrix} -1 \\ 3 \\ 5 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$
  
  
-----  
(%o37) 
$$\begin{pmatrix} -0.5 & 0.75 & 0.8333333333333333 \end{pmatrix}$$
  
2  
  
-----  
(%o38) 
$$\begin{pmatrix} -0.125 & 0.2083333333333333 & 0.5833333333333333 \end{pmatrix}$$
  
3  
  
-----  
(%o39) 
$$\begin{pmatrix} -0.3958333333333333 & 0.4270833333333333 & 0.7638888888888888 \end{pmatrix}$$
  
4  
  
-----  
(%o40) 
$$\begin{pmatrix} -0.2864583333333333 & 0.2690972222222222 & 0.6909722222222221 \end{pmatrix}$$
  
5  
  
-----  
(%o41) 
$$\begin{pmatrix} -0.3654513888888888 & 0.3328993055555556 & 0.7436342592592592 \end{pmatrix}$$
  
6  
  
-----  
(%o42) 
$$\begin{pmatrix} -0.3335503472222222 & 0.2868200231481482 & 0.722366898148148 \end{pmatrix}$$
  
7  
  
-----  
(%o43) 
$$\begin{pmatrix} -0.3565899884259259 & 0.3054289641203704 & 0.7377266589506172 \end{pmatrix}$$
  
8  
  
-----  
(%o44) 
$$\begin{pmatrix} -0.3472855179398147 & 0.2919891734182099 & 0.7315236786265431 \end{pmatrix}$$
  
9  
  
-----  
(%o45) 
$$\begin{pmatrix} -0.354005413290895 & 0.2974167812017747 & 0.7360036088605966 \end{pmatrix}$$
  
10  
  
-----  
'
```

## 9 ***SOR Method***

### 9.1

```
(%i42) A1:=matrix([5, 1, 2], [-3, 9, 4], [1, 2, -7]);  
b1:=matrix([10], [-14], [-33]);  
start:=matrix([0], [0], [0]);  
sor(A1, b1, 0.9, start, 10);  
  
(%o39) 
$$\begin{pmatrix} 5 & 1 & 2 \\ -3 & 9 & 4 \\ 1 & 2 & -7 \end{pmatrix}$$
  
  
(%o40) 
$$\begin{pmatrix} 10 \\ -14 \\ -33 \end{pmatrix}$$
  
  
(%o41) 
$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$
  
-----  

$$\begin{pmatrix} 1.799999999999999 & -0.859999999999999 & 4.253142857142857 \end{pmatrix}$$
  
2  
-----  

$$\begin{pmatrix} 0.6036685714285714 & -3.006156571428571 & 3.972774269387755 \end{pmatrix}$$
  
3  
-----  

$$\begin{pmatrix} 0.9712763030204081 & -2.998342473991836 & 3.994010601157784 \end{pmatrix}$$
  
4  
-----  

$$\begin{pmatrix} 0.9989854592037691 & -2.997742850101166 & 3.999851029130248 \end{pmatrix}$$
  
5  
-----  

$$\begin{pmatrix} 0.9995458884516973 & -2.999850930126706 & 3.999965049395661 \end{pmatrix}$$
  
6  
-----  

$$\begin{pmatrix} 0.9999403384855388 & -2.999989011225273 & 3.99999165985835 \end{pmatrix}$$
  
7  
-----  

$$\begin{pmatrix} 0.9999950583200967 & -2.999997047569838 & 3.999999289823317 \end{pmatrix}$$
  
8  
-----  

$$\begin{pmatrix} 0.9999992300581864 & -2.999999651668854 & 3.999999919560678 \end{pmatrix}$$
  
9  
-----  

$$\begin{pmatrix} 0.9999998892643681 & -2.999999966211847 & 3.999999986407011 \end{pmatrix}$$
  
10  
-----  
'
```

```
(%i46) A1:matrix([2, -1, 0], [-1, 4, 2], [0, 2, 6]);  
b1:matrix([-1], [3], [5]);  
start:matrix([0], [0], [0]);  
sor(A1, b1, 0.9, start, 14);  
(%o43) 
$$\begin{pmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \\ 0 & 2 & 6 \end{pmatrix}$$
  
(%o44) 
$$\begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix}$$
  
(%o45) 
$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$
  
-----  
(%o46) 
$$\begin{pmatrix} -0.45 & 0.5737500000000001 & 0.577875 \end{pmatrix}$$
  
2  
-----  
(%o47) 
$$\begin{pmatrix} -0.2368124999999999 & 0.4190484375000001 & 0.68207296875 \end{pmatrix}$$
  
3  
-----  
(%o48) 
$$\begin{pmatrix} -0.2851094531249999 & 0.3458223808593751 & 0.7144605826171875 \end{pmatrix}$$
  
4  
-----  
(%o49) 
$$\begin{pmatrix} -0.3228908739257812 & 0.3154245292749024 & 0.726818699479248 \end{pmatrix}$$
  
5  
-----  
(%o50) 
$$\begin{pmatrix} -0.3403480492188721 & 0.3028957270875825 & 0.7318131518216502 \end{pmatrix}$$
  
6  
-----  
(%o51) 
$$\begin{pmatrix} -0.3477317277324751 & 0.2977340156492088 & 0.7338611104874024 \end{pmatrix}$$
  
7  
-----  
(%o52) 
$$\begin{pmatrix} -0.3507928657311035 & 0.2956075070560916 & 0.7347038589319128 \end{pmatrix}$$
  
8  
-----  
(%o53) 
$$\begin{pmatrix} -0.3520559083978691 & 0.2947314347967279 & 0.735050955454173 \end{pmatrix}$$
  
9  
-----  
(%o54) 
$$\begin{pmatrix} -0.3525764451812593 & 0.2943705133595117 & 0.7351939415375639 \end{pmatrix}$$
  
10  
-----  
'
```

## 9.2

```
(%i50) A1:matrix([2, -1, 0], [-1, 4, 2], [0, 2, 6]);  
b1:matrix([-1], [3], [5]);  
start:matrix([0], [0], [0]);  
sor(A1, b1, 1.1, start, 14);  
  
(%o47) 
$$\begin{pmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \\ 0 & 2 & 6 \end{pmatrix}$$
  
  
(%o48) 
$$\begin{pmatrix} -1 \\ 3 \\ 5 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$
  
  
-----  
(%o49) 
$$\begin{pmatrix} -0.55 & 0.6737500000000001 & 0.6696249999999999 \end{pmatrix}$$
  
2  
-----  
(%o50) 
$$\begin{pmatrix} -0.1244374999999999 & 0.3551109375000001 & 0.7194968229166666 \end{pmatrix}$$
  
3  
-----  
(%o51) 
$$\begin{pmatrix} -0.342245234375 & 0.2996482141927084 & 0.7348459725043401 \end{pmatrix}$$
  
4  
-----  
(%o52) 
$$\begin{pmatrix} -0.3509689587565104 & 0.2943534300453017 & 0.7352524783996219 \end{pmatrix}$$
  
5  
-----  
(%o53) 
$$\begin{pmatrix} -0.353008717599433 & 0.2940983965358337 & 0.7353053400968987 \end{pmatrix}$$
  
6  
-----  
(%o54) 
$$\begin{pmatrix} -0.3529450101453482 & 0.2941123455031516 & 0.7352949393058211 \end{pmatrix}$$
  
7  
-----  
(%o55) 
$$\begin{pmatrix} -0.3529437089587318 & 0.294117028867832 & 0.7352942621512127 \end{pmatrix}$$
  
8  
-----  
(%o56) 
$$\begin{pmatrix} -0.3529412632268192 & 0.2941176055426745 & 0.7352941184192313 \end{pmatrix}$$
  
9  
-----  
(%o57) 
$$\begin{pmatrix} -0.3529411906288471 & 0.2941176468922224 & 0.7352941176309286 \end{pmatrix}$$
  
10  
-----  
'
```

## 10 Gauss-Seidel Method

### 10.1

```
(%i54) A1:matrix([5, 1, 2], [-3, 9, 4], [1, 2, -7]);  
b1:matrix([10], [-14], [-33]);  
start:matrix([0], [0], [0]);  
gauss_seidel(A1, b1, start, 10);  
  
(%o51) 
$$\begin{pmatrix} 5 & 1 & 2 \\ -3 & 9 & 4 \\ 1 & 2 & -7 \end{pmatrix}$$
  
(%o52) 
$$\begin{pmatrix} 10 \\ -14 \\ -33 \end{pmatrix}$$
  
(%o53) 
$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$
  
-----  
(%o54) 
$$\begin{pmatrix} 2.0 & -0.8888888888888887 & 4.746031746031746 \end{pmatrix}$$
  
2  
-----  
(%o55) 
$$\begin{pmatrix} 0.2793650793650792 & -3.571781305114638 & 3.7336860670194 \end{pmatrix}$$
  
3  
-----  
(%o56) 
$$\begin{pmatrix} 1.220881834215167 & -2.808010973936899 & 4.086408555191624 \end{pmatrix}$$
  
4  
-----  
(%o57) 
$$\begin{pmatrix} 0.9270387727107303 & -3.062724211403811 & 3.971655764271872 \end{pmatrix}$$
  
5  
-----  
(%o58) 
$$\begin{pmatrix} 1.023882536572013 & -2.979441716374605 & 4.0092855862604 \end{pmatrix}$$
  
6  
-----  
(%o59) 
$$\begin{pmatrix} 0.992174108770761 & -3.006735557636591 & 3.996957570499654 \end{pmatrix}$$
  
7  
-----  
(%o60) 
$$\begin{pmatrix} 1.002564083327456 & -2.997793114668471 & 4.000996836284359 \end{pmatrix}$$
  
8  
-----  
(%o61) 
$$\begin{pmatrix} 0.9991598884199506 & -3.000723075541953 & 3.999673391048006 \end{pmatrix}$$
  
9  
-----  
(%o62) 
$$\begin{pmatrix} 1.000275258689188 & -2.999763087569384 & 4.000107011935774 \end{pmatrix}$$
  
10  
-----  
'
```

## 10.2

```
(%i58) A1:matrix([2, -1, 0], [-1, 4, 2], [0, 2, 6]);  
b1:matrix([-1], [3], [5]);  
start:matrix([0], [0], [0]);  
gauss_seidel(A1, b1, start, 14);  
(%o55) 
$$\begin{pmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \\ 0 & 2 & 6 \end{pmatrix}$$
  
(%o56) 
$$\begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix}$$
  
(%o57) 
$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$
  
-----  
(%o58) 
$$\begin{pmatrix} -0.5 & 0.625 & 0.6249999999999999 \end{pmatrix}$$
  
2  
-----  
(%o59) 
$$\begin{pmatrix} -0.1875 & 0.390625 & 0.7031249999999999 \end{pmatrix}$$
  
3  
-----  
(%o60) 
$$\begin{pmatrix} -0.3046875 & 0.322265625 & 0.7259114583333332 \end{pmatrix}$$
  
4  
-----  
(%o61) 
$$\begin{pmatrix} -0.3388671875 & 0.3023274739583334 & 0.7325575086805554 \end{pmatrix}$$
  
5  
-----  
(%o62) 
$$\begin{pmatrix} -0.3488362630208332 & 0.296512179904514 & 0.7344959400318285 \end{pmatrix}$$
  
6  
-----  
(%o63) 
$$\begin{pmatrix} -0.3517439100477429 & 0.29481605247215 & 0.7350613158426166 \end{pmatrix}$$
  
7  
-----  
(%o64) 
$$\begin{pmatrix} -0.352591973763925 & 0.2943213486377105 & 0.7352262171207631 \end{pmatrix}$$
  
8  
-----  
(%o65) 
$$\begin{pmatrix} -0.3528393256811447 & 0.2941770600193322 & 0.7352743133268891 \end{pmatrix}$$
  
9  
-----  
(%o66) 
$$\begin{pmatrix} -0.3529114699903338 & 0.2941349758389719 & 0.7352883413870093 \end{pmatrix}$$
  
10  
-----  
,
```

## 11 Lagrange Interpolation

### 11.1

```
(%i61) xval:[0, 1, -1, 2, -2]$
      yval: [-5, -3, -15, 39, -9]$
      LP(xval, yval, x);
(%o61) 1.625 (x-1) x (x+1) (x+2)+0.5 (x-2) x (x+1) (x+2)
      +1.25 (1-x) (x-2) (x+1) (x+2)+2.5 (x-2) (x-1) x (x+2)-
      0.375 (x-2) (x-1) x (x+1)

(%i62) ratsimp(%);
(%o62) 3 x4+2 x3-7 x2+4 x-5

(%i63) LP(xval, yval, 3);
(%o63) 241.0
```

### 11.2

```
(%i66) xval:[-3, 1, 2, 5]$
      yval: [-23, -11, -23, 1]$
      LP(xval, yval, x);
(%o66) 0.0104166666666666 (x-2) (x-1) (x+3) +
      1.53333333333333 (x-5) (x-1) (x+3)-0.6875 (x-5) (x-2)
      (x+3)+0.14375 (x-5) (x-2) (x-1)

(%i67) ratsimp(%);
(%o67) x3-3 x2-10 x+1
```

### 11.3

```
(%i70) xval:[-1, 0, 1, 2]$
      yval: [5, 1, 1, 11]$
      LP(xval, yval, x);
(%o70) 1.83333333333333 (x-1) x (x+1)-0.5 (x-2) x (x+1)
      +0.5 (x-2) (x-1) (x+1)-0.83333333333333 (x-2) (x-1)
      x

(%i71) ratsimp(%);
(%o71) x3+2 x2-3 x+1
```

## 12 Newton Interpolation

### 12.1

### 12.2

```
(%i72) dd_table([1, 3/2, 0, 2], [3, 13/4, 3, 5/3]);
```

```
(%o72)
```

1	3	0.5	0.333333333333333	-2.0	
1.5	3.25	0.166666666666666	-1.66666666666666	$A_{2,5}$	
0	3	-0.666666666666666		$A_{3,4}$	$A_{3,5}$
2	1.666666666666666	$A_{4,3}$		$A_{4,4}$	$A_{4,5}$

### 12.3

```
(%i73) dd_table([-1, 0, 1, 2, -2, 3], [5, 1, 1, 11, 5, 35]);
```

```
(%o73)
```

-1	5	-4	2	1	-0.083333333333326	-1.7347234
0	1	0	5	1.08333333333333	-0.083333333333333	
1	1	10	2.83333333333333	0.833333333333333		$A_{3,6}$
2	11	1.5	4.5	$A_{4,5}$		$A_{4,6}$
-2	5	6	$A_{5,4}$	$A_{5,5}$		$A_{5,6}$
3	35	$A_{6,3}$	$A_{6,4}$	$A_{6,5}$		$A_{6,6}$

### 12.4

### 12.5

```
(%i76) xval: [-1, 0, 1, 2]$  
yval: [5, 1, 1, 11]$  
NP(xval, yval, x);
```

```
(%o76) (x-1)x(x+1)+2x(x+1)-4(x+1)+5
```

```
(%i77) ratsimp(%);
```

```
(%o77)  $x^3 + 2x^2 - 3x + 1$ 
```

### 12.6

```
(%i80) xval: [-1, 0, 1, 2, -2, 3]$  

      yval: [5, 1, 1, 11, 5, 35]$  

      NP(xval, yval, x);  

(%o80) 
$$\frac{-1.734723475976807 \cdot 10^{-17} (x-2)(x-1)x(x+1)}{(x+2)-0.0833333333333326 (x-2)(x-1)x(x+1)+(x-1)x(x+1)+2x(x+1)-4(x+1)+5}$$
  

(%i81) NP(xval, yval, 1.5);  

(%o81) 4.453125
```

## 13

```
(%i82) ratprint:false;  

(%o82) false
```

### 13.1 Trapezoidal Rule

Basic Trapezoidal Rule

```
(%i83) T(1/s, s, 1.0, 2.0);  

(%o83) 0.75
```

Comparison

```
(%i84) compareT(g, x, a, b):=block(  

    [t, v, e, numer:true],  

    local(f),  

    define(f(x), g),  

    t:T(f(x), x, a, b),  

    v:integrate(f(x), x, a, b),  

    e:abs(v-t),  

    print(t, " ", v, " ", e)  

);
```

```
(%o84) compareT(g,x,a,b):=block(
$$[t,v,e,numer:\text{true}],\text{local}(f),\text{define}(f(x),g),t:T(f(x),x,a,b),v:\int f(x)dx\Big|_a^b,e:|v-t|$$
)
```

#### 13.1.1

```
(%i85) compareT(1/x, x, 1, 2);
      0.75    0.6931471805599453    0.05685281944005471
(%o85) 0.05685281944005471
```

## 13.1.2

```
(%i86) compareT(exp(-x), x, 0, 1);
      0.6839397205857212    0.6321205588285578
      0.05181916175716339
(%o86) 0.05181916175716339
```

## 13.2

Composite Trapzoidal Rule

### 13.2.1

```
(%i87) for k:2 next 2·k thru 150 do block(
      temp:Tc(sin(x), x, 0, %pi, k),
      print(k, " ", temp)
);
      2    1.570796326794896
      4    1.896118897937039
      8    1.97423160194555
     16    1.993570343772339
     32    1.998393360970144
     64    1.999598388640037
    128    1.999899600184203
(%o87) done
```

### 13.2.2

```
(%i88) for k:2 next 2·k thru 150 do block(
    temp:Tc(sqrt(1+x^3), x, 0.0, 1.0, k),
    print(k, " ", temp)
);
2      1.133883476483184
4      1.116993293318717
8      1.112830349496382
16     1.111793319381881
32     1.111534292393827
64     1.111469550038714
128    1.111453365349166
(%o88) done
```

## 14 Simpson Rule

Basic Simpson Rule

### 14.1

```
(%i89) S(1/u, u, 1.0, 2.0);
(%o89) 0.6944444444444443
```

### 14.2

```
(%i90) compareS(g, x, a, b):=block(
    [t, v, e, numer:true],
    local(f),
    define(f(x), g),
    t:S(f(x), x, a, b),
    v:integrate(f(x), x, a, b),
    e:abs(v-t),
    print(t, " ", v, " ", e)
);
```

```
(%o90) compareS(g,x,a,b):=block
      [t,v,e,numer:true],local(f),define(f(x),g),t:S(f(x),x,a,b),
      v:integrate(f(x),x,a,b),
      e:abs(v-t),
      print(t," ",v," ",e)
```

#### 14.2.1

```
(%i91) compareS(1/x, x, 1, 2);  
0.6944444444444443 0.6931471805599453  
0.001297263884499022  
(%o91) 0.001297263884499022
```

## 14.2.2

```
(%i92) compareS(exp(-x), x, 0, 1);  
0.6323336800036626 0.6321205588285578  
2.131211751048578 10-4  
(%o92) 2.131211751048578 10-4
```

## 14.3

Composite Simpson rule

### 14.3.1

```
(%i93) for k:2 next 2·k thru 150 do block(  
    temp:Sc(sin(t), t, 0, %pi, k),  
    print(k, " ", temp)  
)  
2 2.094395102393195  
4 2.00455975498442  
8 2.000269169948387  
16 2.000016591047935  
32 2.000001033369412  
64 2.000000064530001  
128 2.00000004032257  
(%o93) done
```

### 14.3.2

```
(%i94) for k:2 next 2·k thru 150 do block(  
    temp:Sc(sqrt(1+x^3), x, 0, 1, k),  
    print(k, " ", temp)  
);  
2      1.109475708248729  
4      1.111363232263895  
8      1.111442701555604  
16     1.111447642677047  
32     1.111447950064475  
64     1.111447969253676  
128    1.111447970452649  
(%o94) done
```

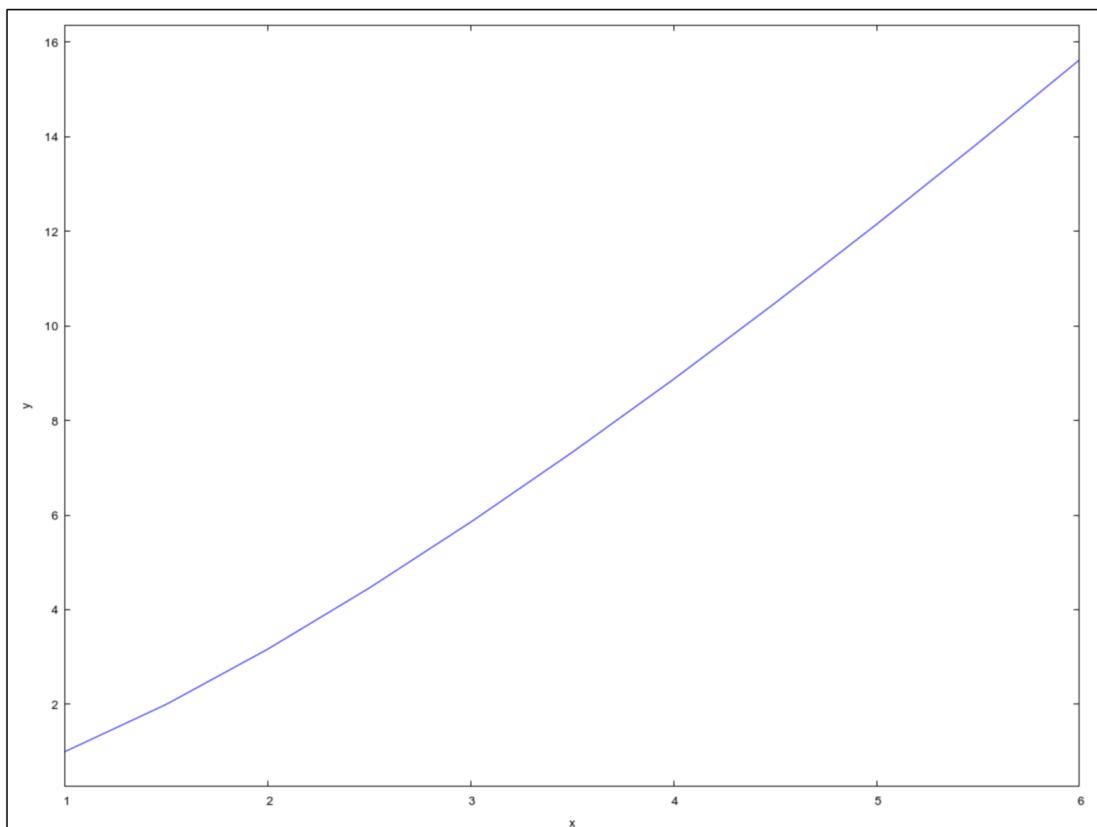
## 15 Euler Method

### 15.1

```
(%i95) ans:eulerm(1+(x/t), t, x, 1, 1, 6, 10);  
(%o95) [[1,1],[1.5,2.0],[2.0,3.166666666666666],[2.5,  
4.458333333333333],[3.0,5.85],[3.5,7.324999999999999],[4.0,  
8.87142857142857],[4.5,10.48035714285714],[5.0,  
12.14484126984126],[5.5,13.85932539682539],[6.0,  
15.61926406926406]]
```

plot of the solution

```
(%i96) wxplot2d([discrete, ans]);  
(%t96)
```



```
(%o96)
```

print the values

```
(%i97) for k:1 thru 11 do print(ans[k][1], "-----", ans[k][2]);  
1 ----- 1  
1.5 ----- 2.0  
2.0 ----- 3.166666666666666  
2.5 ----- 4.458333333333333  
3.0 ----- 5.85  
3.5 ----- 7.324999999999999  
4.0 ----- 8.87142857142857  
4.5 ----- 10.48035714285714  
5.0 ----- 12.14484126984126  
5.5 ----- 13.85932539682539  
6.0 ----- 15.61926406926406
```

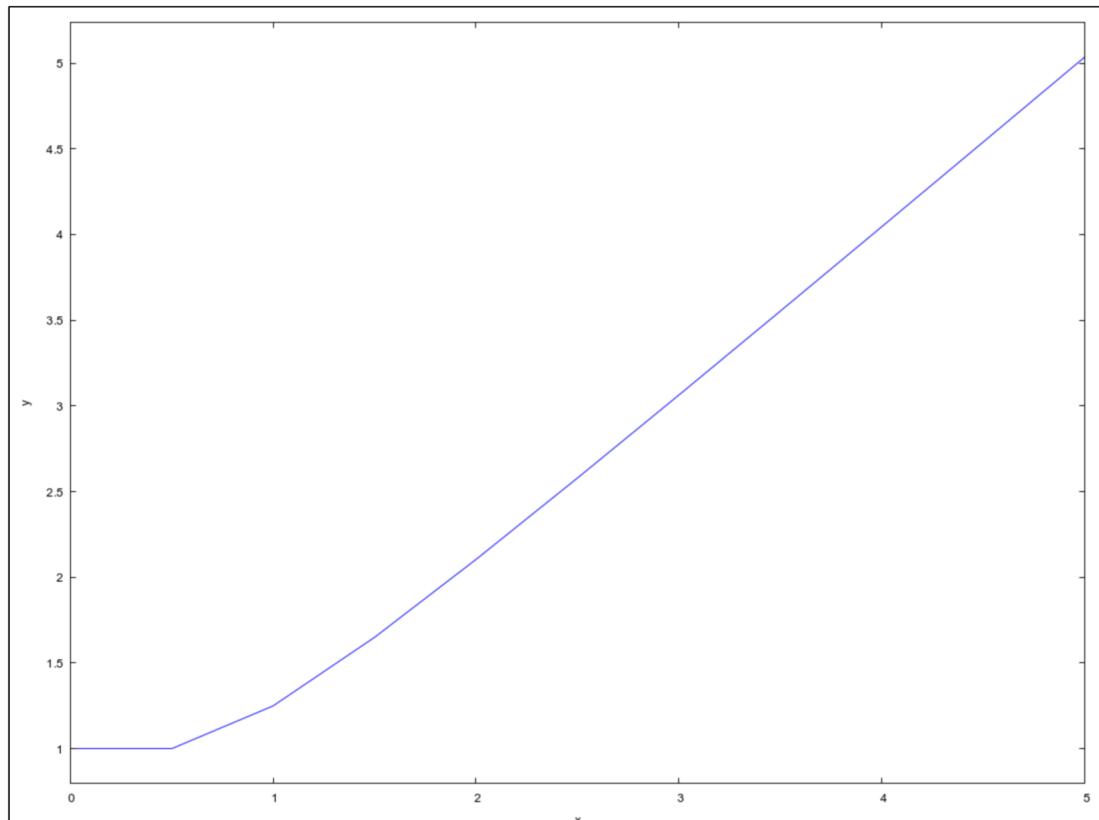
```
(%o97) done
```

## 15.2

```
(%i98) ans:eulerm((t/x), t, x, 0, 1, 5, 10);  
(%o98) [[0,1],[0.5,1],[1.0,1.25],[1.5,1.65],[2.0,  
2.104545454545454],[2.5,2.579707441586491],[3.0,  
3.064258511157103],[3.5,3.553773346324708],[4.0,  
4.046207677233808],[4.5,4.54049767901822],[5.0,  
5.036038071076412]]
```

plot of the solution

```
(%i99) wxplot2d([discrete, ans]);  
(%t99)
```



```
(%o99)
```

print the values

```
(%i100) for k:1 thru 11 do print(ans[k][1],"-----", ans[k][2] );  
0 ----- 1  
0.5 ----- 1  
1.0 ----- 1.25  
1.5 ----- 1.65  
2.0 ----- 2.104545454545454  
2.5 ----- 2.579707441586491  
3.0 ----- 3.064258511157103  
3.5 ----- 3.553773346324708  
4.0 ----- 4.046207677233808  
4.5 ----- 4.54049767901822  
5.0 ----- 5.036038071076412  
(%o100) done
```

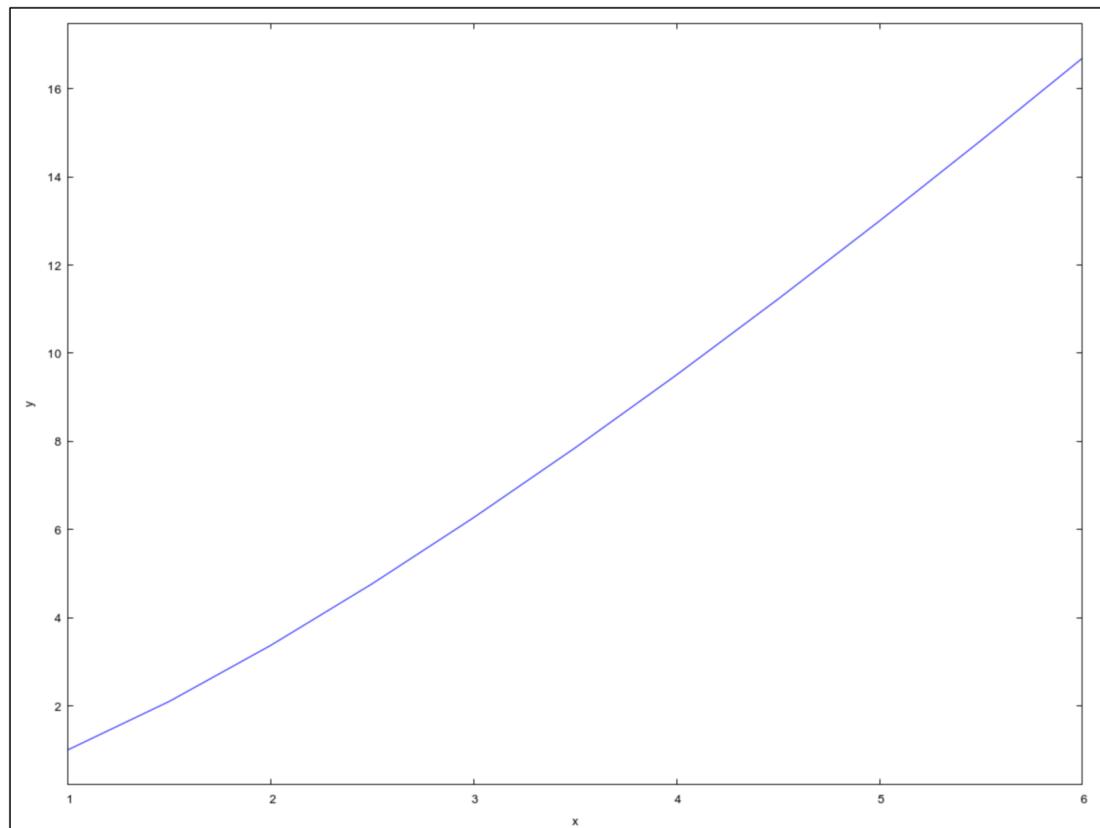
## 16 Second-Order Runge-Kutta Methods

### 16.1 Modified-Euler Method

#### 16.1.1

```
(%i101) ans:eulermod(1+(x/t), t, x, 1.0, 1.0, 6, 10);  
(%o101) [[1.0, 1.0], [1.5, 2.1], [2.0, 3.371428571428571], [2.5,  
4.76984126984127], [3.0, 6.26926406926407], [3.5,  
7.852602952602953], [4.0, 9.507736707736708], [4.5,  
11.22561556090967], [5.0, 12.99922196826221], [5.5,  
14.82295368889796], [6.0, 16.69223406377801]]
```

```
(%i102) wxplot2d([discrete, ans]);  
(%t102)
```



```
(%o102)
```

```
(%i103) for k:1 thru 11 do print(ans[k][1],"-----", ans[k][2]);  
1.0 ----- 1.0  
1.5 ----- 2.1  
2.0 ----- 3.371428571428571  
2.5 ----- 4.76984126984127  
3.0 ----- 6.26926406926407  
3.5 ----- 7.852602952602953  
4.0 ----- 9.507736707736708  
4.5 ----- 11.22561556090967  
5.0 ----- 12.99922196826221  
5.5 ----- 14.82295368889796  
6.0 ----- 16.69223406377801
```

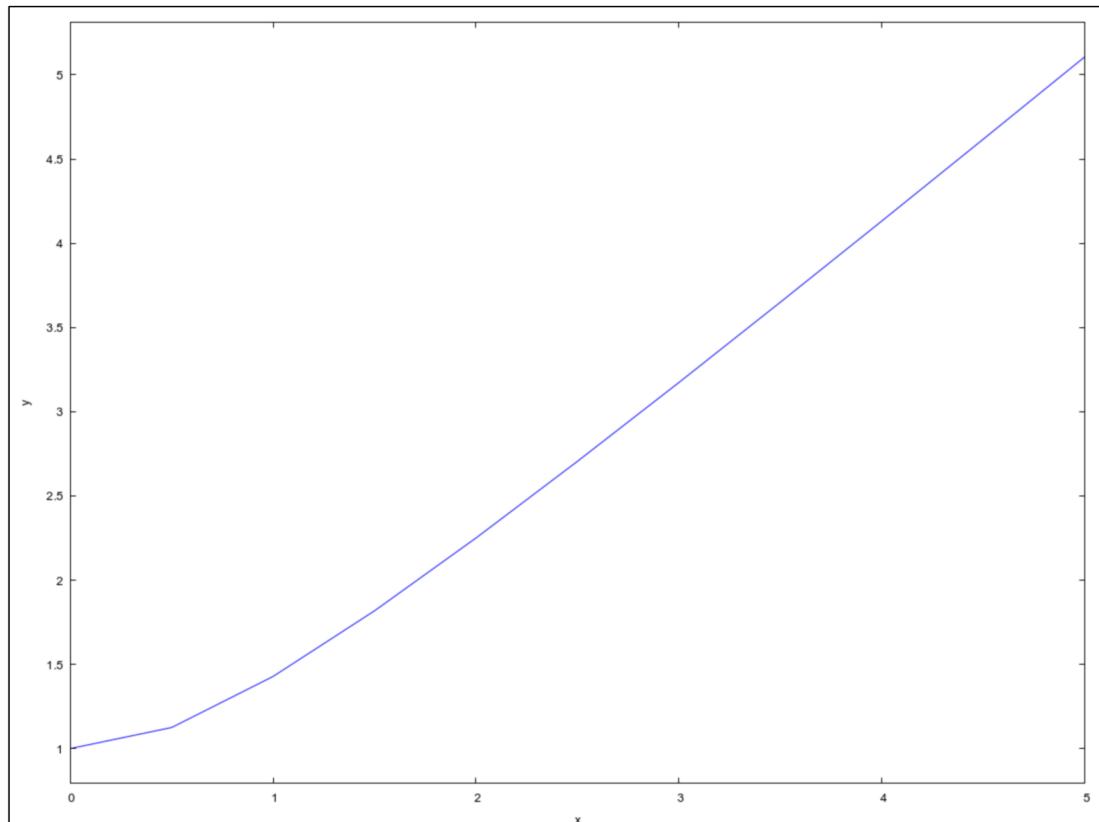
```
(%o103) done
```

## 16.1.2

```
(%i104) ans:eulermod((t/x), t, x, 0, 1, 5, 10);  
(%o104) [[0,1],[0.5,1.125],[1.0,1.428370786516854],[1.5,  
1.818168594488258],[2.0,2.250391136810608],[2.5,  
2.705382439640636],[3.0,3.173642355339167],[3.5,  
3.650186992527909],[4.0,4.132204433515342],[4.5,  
4.618006883981802],[5.0,5.106527324792223]]
```

```
(%i105) wxplot2d([discrete, ans]);
```

```
(%t105)
```



```
(%o105)
```

```
(%i106) for k:1 thru 11 do print(ans[k][1],"-----", ans[k][2]);  
0 ----- 1  
0.5 ----- 1.125  
1.0 ----- 1.428370786516854  
1.5 ----- 1.818168594488258  
2.0 ----- 2.250391136810608  
2.5 ----- 2.705382439640636  
3.0 ----- 3.173642355339167  
3.5 ----- 3.650186992527909  
4.0 ----- 4.132204433515342  
4.5 ----- 4.618006883981802  
5.0 ----- 5.106527324792223
```

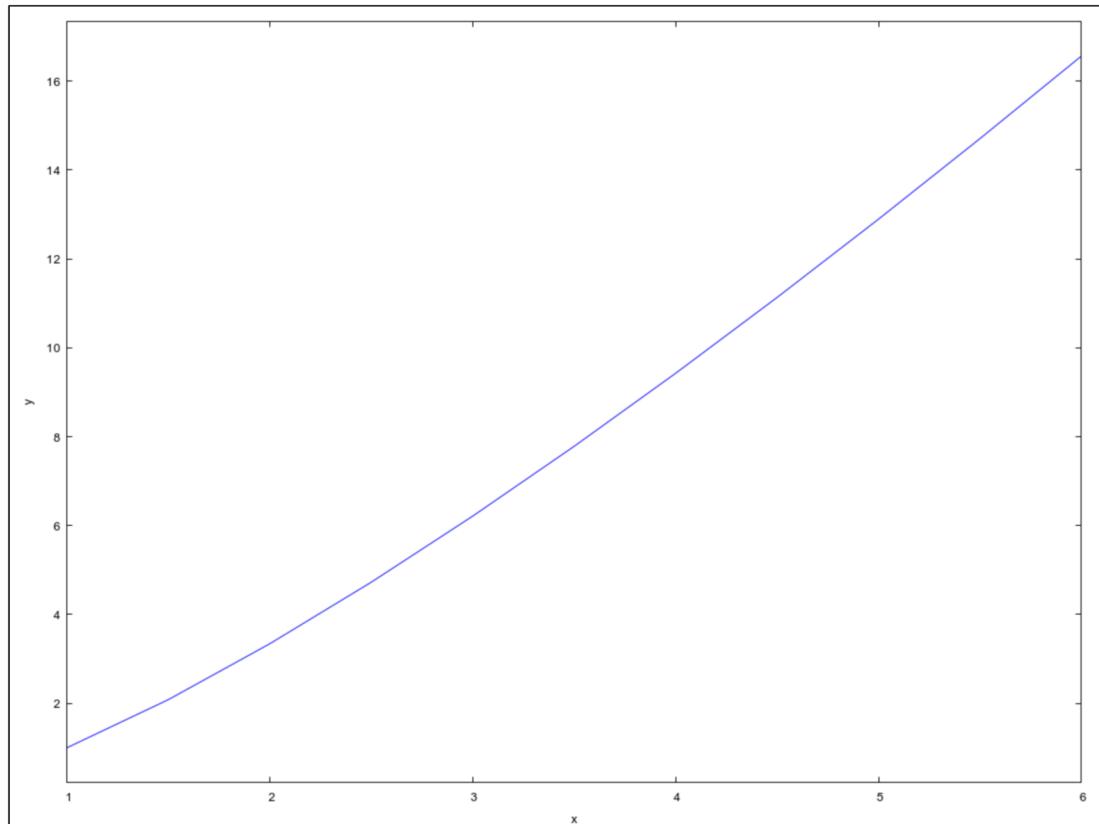
```
(%o106) done
```

## 16.2 Heun's Method

### 16.2.1

```
(%i109) ans:heun(1+(x/t), t, x, 1.0, 1.0, 6, 10);  
wxplot2d([discrete, ans]);  
for k:1 thru 11 do print(ans[k][1],"-----", ans[k][2] );  
(%o107) [[1.0,1.0],[1.5,2.08333333333333],[2.0,  
3.34027777777777],[2.5,4.72534722222221],[3.0,  
6.21208333333332],[3.5,7.78314484126984],[4.0,  
9.426272675736959],[4.5,11.13233453798185],[5.0,  
12.89426059775761],[5.5,14.70641393026065],[6.0,  
16.56419398452677]]
```

(%t108)



(%o108)

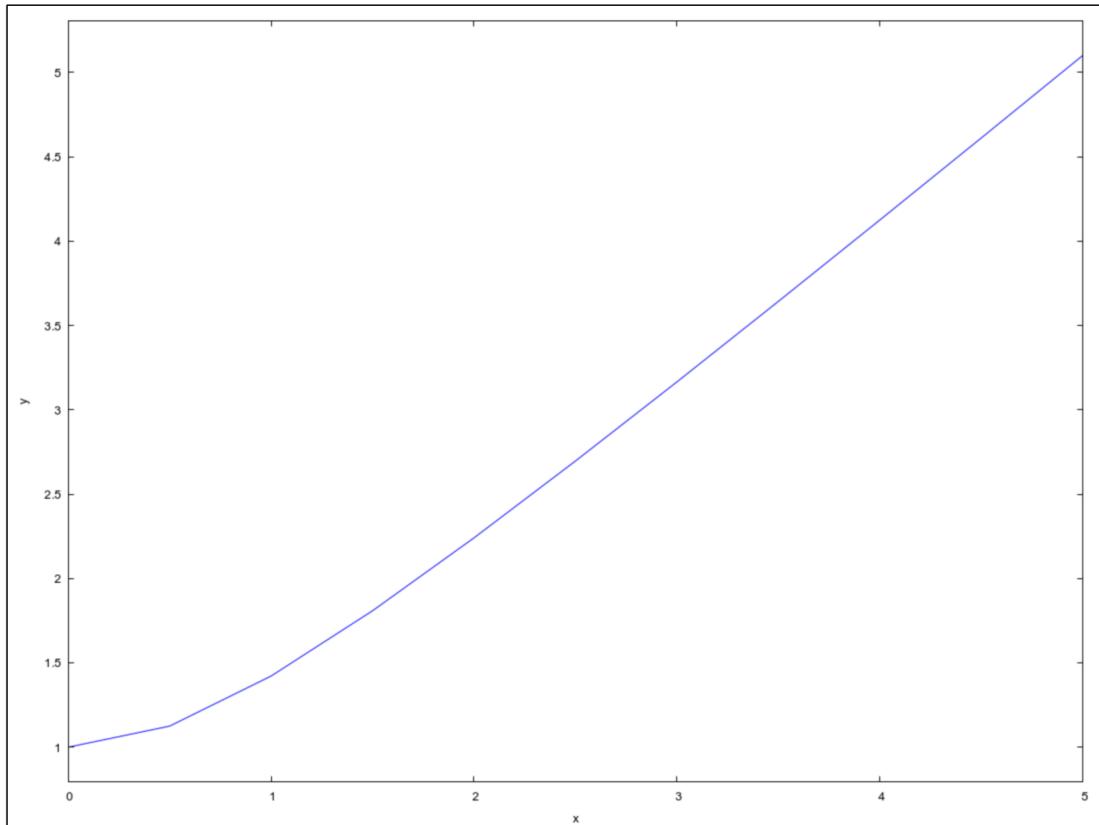
```
1.0 ----- 1.0  
1.5 ----- 2.08333333333333  
2.0 ----- 3.34027777777777  
2.5 ----- 4.72534722222221  
3.0 ----- 6.21208333333332  
3.5 ----- 7.78314484126984  
4.0 ----- 9.426272675736959  
4.5 ----- 11.13233453798185  
5.0 ----- 12.89426059775761  
5.5 ----- 14.70641393026065  
6.0 ----- 16.56419398452677
```

(%o109) done

## 16.2.2

```
(%i112) ans:heun((t/x), t, x, 0, 1, 5, 10);
wxplot2d([discrete, ans]);
for k:1 thru 11 do print(ans[k][1],"-----", ans[k][2] );
(%o110) [[0,1],[0.5,1.125],[1.0,1.421678121420389],[1.5,
1.808987822487826],[2.0,2.241148239963114],[2.5,
2.696819448943987],[3.0,3.165891040921445],[3.5,
3.643196174669014],[4.0,4.125879763718132],[4.5,
4.612253947754017],[5.0,5.101263359939439]]
```

```
(%t111)
```



```
(%o111)
```

0	-----	1
0.5	-----	1.125
1.0	-----	1.421678121420389
1.5	-----	1.808987822487826
2.0	-----	2.241148239963114
2.5	-----	2.696819448943987
3.0	-----	3.165891040921445
3.5	-----	3.643196174669014
4.0	-----	4.125879763718132
4.5	-----	4.612253947754017
5.0	-----	5.101263359939439

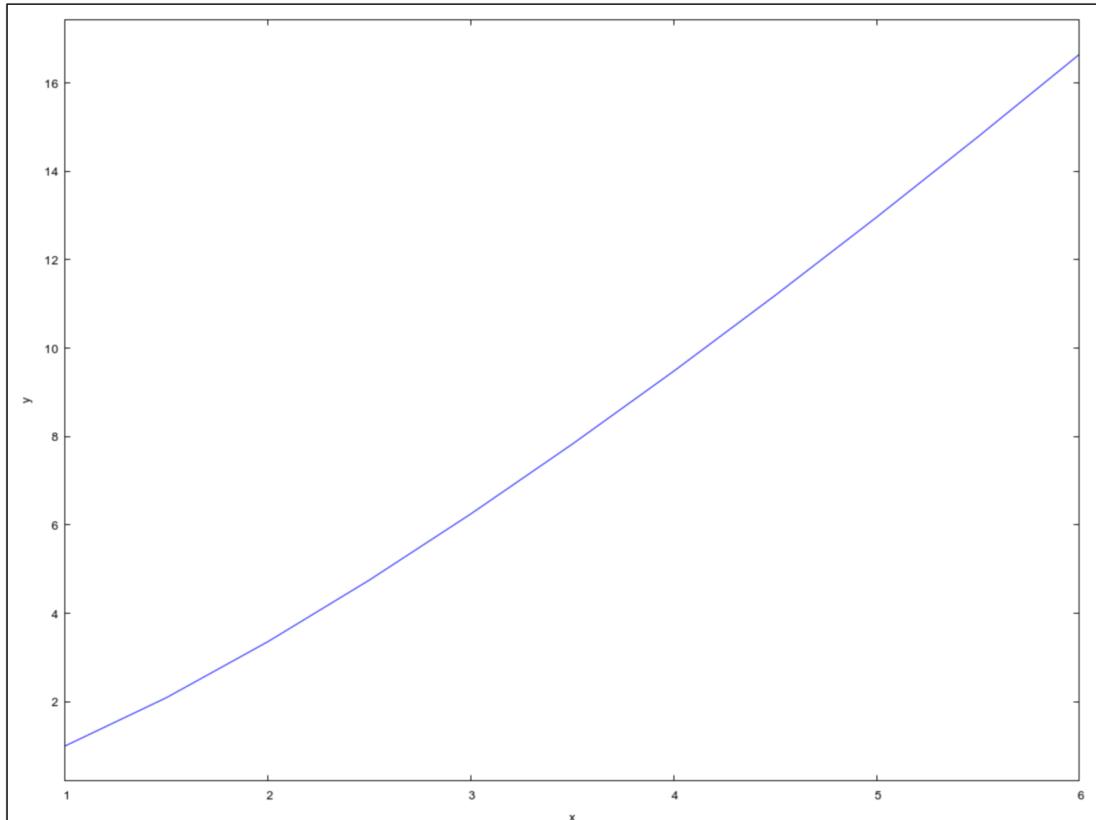
```
(%o112) done
```

## 16.3 Optimal RK2 Method

### 16.3.1

```
(%i115) ans:=rk2(1+(x/t), t, x, 1.0, 1.0, 6, 10);  
wxplot2d([discrete, ans]);  
for k:1 thru 11 do print(ans[k][1],"-----", ans[k][2] );  
(%o113) [[1.0,1.0],[1.5,2.09375],[2.0,3.3598484848484],[  
2.5,4.753382034632034],[3.0,6.248176088617265],[3.5,  
7.827038770053475],[4.0,9.477795861427574],[4.5,  
11.19136649795217],[5.0,12.96071373335682],[5.5,  
14.7802226066925],[6.0,16.64530777872948]]
```

```
(%t114)
```



```
(%o114)
```

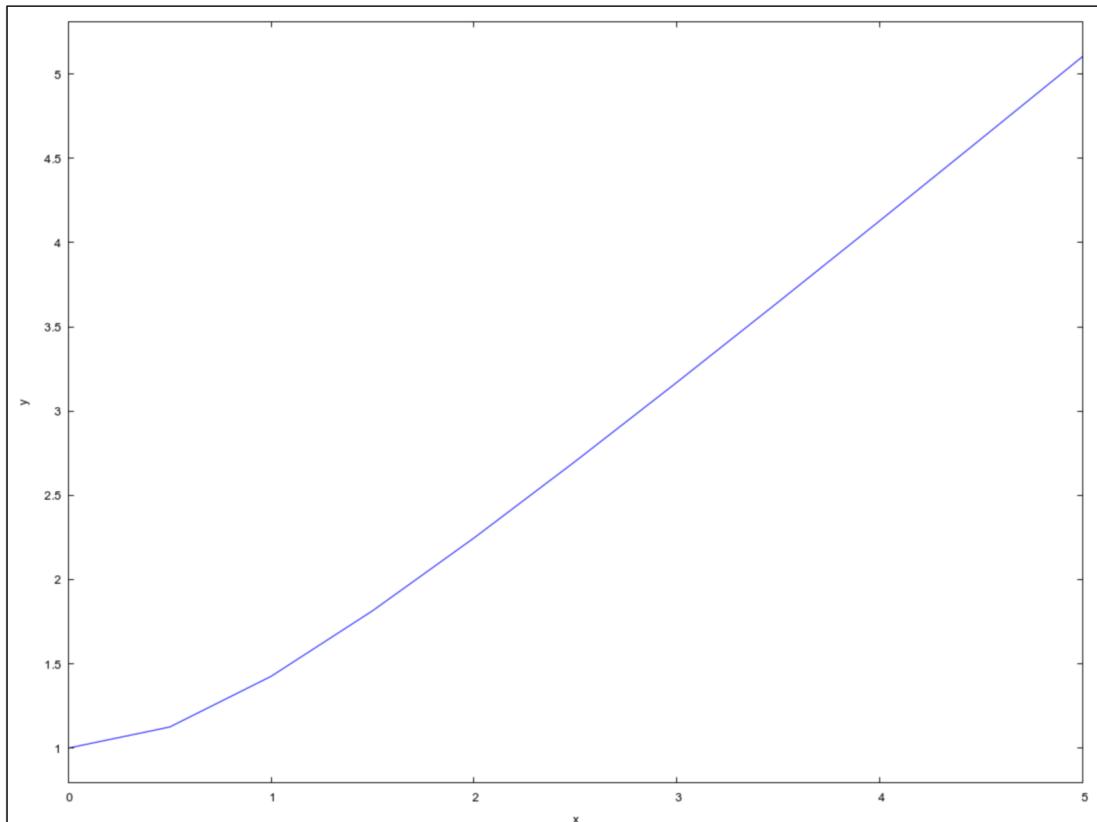
1.0	-----	1.0
1.5	-----	2.09375
2.0	-----	3.3598484848484
2.5	-----	4.753382034632034
3.0	-----	6.248176088617265
3.5	-----	7.827038770053475
4.0	-----	9.477795861427574
4.5	-----	11.19136649795217
5.0	-----	12.96071373335682
5.5	-----	14.7802226066925
6.0	-----	16.64530777872948

```
(%o115) done
```

### 16.3.2

```
(%i118) ans:=rk2((t/x), t, x, 0, 1, 5, 10);
wxplot2d([discrete, ans]);
for k:1 thru 11 do print(ans[k][1],"-----", ans[k][2] );
(%o116) [[0,1],[0.5,1.125],[1.0,1.426010101010101],[1.5,
1.814915177090553],[2.0,2.24710861815986],[2.5,
2.702338151234642],[3.0,3.170885177773465],[3.5,
3.64769967070791],[4.0,4.129953831171324],[4.5,
4.615959602475442],[5.0,5.104654003183263]]
```

```
(%t117)
```



```
(%o117)
```

```
0 ----- 1
0.5 ----- 1.125
1.0 ----- 1.426010101010101
1.5 ----- 1.814915177090553
2.0 ----- 2.24710861815986
2.5 ----- 2.702338151234642
3.0 ----- 3.170885177773465
3.5 ----- 3.64769967070791
4.0 ----- 4.129953831171324
4.5 ----- 4.615959602475442
5.0 ----- 5.104654003183263
```

```
(%o118) done
```

## 17 A Check

```
(%i122) A1:matrix([5, 1, 2], [-3, 9, 4], [1, 2, -7]);
```

```
b1:matrix([10], [-14], [-33]);
```

```
start:matrix([0], [0], [0]);
```

```
gauss_jacobi(A1, b1, start, 14);
```

$$\begin{pmatrix} 5 & 1 & 2 \\ -3 & 9 & 4 \\ 1 & 2 & -7 \end{pmatrix}$$

$$\begin{pmatrix} 10 \\ -14 \\ -33 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

-----

$$\begin{pmatrix} 2.0 & -1.555555555555555 & 4.714285714285714 \end{pmatrix}$$

2

-----

$$\begin{pmatrix} 0.4253968253968252 & -2.984126984126984 & 4.555555555555555 \end{pmatrix}$$

3

-----

$$\begin{pmatrix} 0.7746031746031745 & -3.438447971781304 & 3.922448979591836 \end{pmatrix}$$

4

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$$\begin{pmatrix} 1.118710002519526 & -3.040665154950868 & 3.842529604434367 \end{pmatrix}$$

5

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$$\begin{pmatrix} 1.071121189216427 & -2.890443156686543 & 4.005339956088256 \end{pmatrix}$$

6

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$$\begin{pmatrix} 0.9759526489020063 & -2.97866625074486 & 4.041462125120478 \end{pmatrix}$$

7

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$$\begin{pmatrix} 0.9791484001007809 & -3.026443394863987 & 4.002660021058898 \end{pmatrix}$$

8

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$$\begin{pmatrix} 1.004224670549238 & -3.008132764881471 & 3.989465944338972 \end{pmatrix}$$

9

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$$\begin{pmatrix} 1.005840175240705 & -2.993909973967574 & 3.998279877255185 \end{pmatrix}$$

10

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