

# Practical 1

Expressing relations as ordered pairs and creating relations.

## 1

```
→ kill(all);
(%o0) done

→ A:setify(makelist(i, i, 1, 3));
   B:setify(makelist(i, i, 1, 3));
(%o1) {1,2,3}
(%o2) {1,2,3}

→ R:cartesian_product(A, B);
(%o3) {[1,1],[1,2],[1,3],[2,1],[2,2],[2,3],[3,1],[3,2],
      [3,3]}
```

## 2

Figure 1: ex 4 : Rosen

Let  $A$  be the set  $\{1, 2, 3, 4\}$ . Which ordered pairs are in the relation  $R = \{(a, b) \mid a \text{ divides } b\}$ ?

```
→ kill(all);
(%o0) done

→ A:makelist(k, k, 1, 4);
(%o1) [1,2,3,4]

→ length(A);
(%o2) 4

→ remainder(4, 2);
(%o3) 0
```

```

→ remainder(5, 2);
(%o4) 1

→ A2:cartesian_product_list(A, A);
(%o5) [[1,1],[1,2],[1,3],[1,4],[2,1],[2,2],[2,3],[2,4],[
      3,1],[3,2],[3,3],[3,4],[4,1],[4,2],[4,3],[4,4]]

→ R:[];

(%o6) []

→ for i:1 thru length(A2) do(
    t:A2[i],
    if(remainder(t[1], t[2])=0) then R:cons(t, R)
  );
(%o7) done

→ R;
(%o8) [[4,4],[4,2],[4,1],[3,3],[3,1],[2,2],[2,1],[1,1]]

```

### 3

Figure 2:

Let  $A$  be the set  $\{1, 2, 3, 4\}$ . Which ordered pairs are in the relation  $R = \{(a, b) \mid a \text{ divides } b\}$ ?

```

→ kill(all);
(%o0) done

→ findRelation(A):=block(
  [A2:cartesian_product_list(A, A), R:[]],
  for i:1 thru length(A2) do(
    t:A2[i],
    if(remainder(t[2], t[1])=0) then R:cons(t, R)
  ),
  R
);
(%o1) findRelation(A):=block([A2:cartesian_product_list(A,A),
  R:[]],for i thru length(A2) do
  (t:A2[i],if remainder(t[2],t[1])=0 then R:cons(t,R) ),R)

→ findRelation([1, 2]);
(%o2) [[2,2],[1,2],[1,1]]

```

```
→ findRelation([1, 2, 3]);
(%o3) [[3,3],[2,2],[1,3],[1,2],[1,1]]
```

```
→ findRelation([1, 2, 3, 4]);
(%o4) [[4,4],[3,3],[2,4],[2,2],[1,4],[1,3],[1,2],[1,1]]
```

## 4

Let A be the set {1, 2, 3, 4}.

Which ordered pairs are in the relation  $R = \{(a, b) \mid b \text{ divides } a\}$ ?

```
→ kill(all);
(%o0) done

→ findRelation(A):=block(
  [A2:cartesian_product_list(A, A), R:[]],
  for i:1 thru length(A2) do(
    t:A2[i],
    if(remainder(t[1], t[2])=0) then R:cons(t, R)
  ),
  R
);
(%o1) findRelation(A):=block([A2:cartesian_product_list(A,A),
  R:[]],for i thru length(A2) do
  (t:A2[i],if remainder(t[1],t[2])=0 then R:cons(t,R) ),R)

→ findRelation([1, 2]);
(%o2) [[2,2],[2,1],[1,1]]

→ findRelation([1, 2, 3]);
(%o3) [[3,3],[3,1],[2,2],[2,1],[1,1]]

→ findRelation([1, 2, 3, 4]);
(%o4) [[4,4],[4,2],[4,1],[3,3],[3,1],[2,2],[2,1],[1,1]]
```

## 5

Figure 3:ex 5 : Rosen

Consider these relations on the set of integers:

$$R_1 = \{(a, b) \mid a \leq b\},$$

$$R_2 = \{(a, b) \mid a > b\},$$

$$R_3 = \{(a, b) \mid a = b \text{ or } a = -b\},$$

$$R_4 = \{(a, b) \mid a = b\},$$

$$R_5 = \{(a, b) \mid a = b + 1\},$$

$$R_6 = \{(a, b) \mid a + b \leq 3\}.$$

## 5.1

```
→ kill(all);
(%o0) done

→ findRelation(A):=block(
  [A2:cartesian_product_list(A, A), R:[]],
  for i:1 thru length(A2) do(
    t:A2[i],
    if(is(t[1] <= t[2]) = true) then R:cons(t, R)
  ),
  R
);
(%o1) findRelation(A):=block([A2:cartesian_product_list(A,A),
  R:[]],for i thru length(A2) do
  (t:A2[i],if is(t[1]≤t[2])=true then R:cons(t,R) ),R)

→ findRelation([1, 2]);
(%o2) [[2,2],[1,2],[1,1]]

→ findRelation([1, 2, 3]);
(%o3) [[3,3],[2,3],[2,2],[1,3],[1,2],[1,1]]

→ findRelation([1, 2, 3, 4]);
(%o4) [[4,4],[3,4],[3,3],[2,4],[2,3],[2,2],[1,4],[1,3],[1,2],[1,1]]
```

## 5.2

```
→ kill(all);
(%o0) done
```

```

→ findRelation(A):=block(
    [A2:cartesian_product_list(A, A), R:],
    for i:1 thru length(A2) do(
        t:A2[i],
        if(is(t[1] > t[2]) = true) then R:cons(t, R)
    ),
    R
);
(%o1) findRelation(A):=block([A2:cartesian_product_list(A,A),
    R:],for i thru length(A2) do
    (t:A2[i],if is(t1>t2)=true then R:cons(t,R) ),R)

```

```

→ findRelation([1, 2]);
(%o2) [[2,1]]

```

```

→ findRelation([1, 2, 3]);
(%o3) [[3,2],[3,1],[2,1]]

```

```

→ findRelation([1, 2, 3, 4]);
(%o4) [[4,3],[4,2],[4,1],[3,2],[3,1],[2,1]]

```

### 5.3

```

→ kill(all);
(%o0) done

```

```

→ findRelation(A):=block(
    [A2:cartesian_product_list(A, A), R:],
    for i:1 thru length(A2) do(
        t:A2[i],
        if(is((t[1] = t[2]) or (t[1] = -t[2])) = true) then R:cons(t, R)
    ),
    R
);
(%o1) findRelation(A):=block([A2:cartesian_product_list(A,A),
    R:],for i thru length(A2) do
    (t:A2[i],if is(t1=t2 ∨ t1=-t2)=true then R:cons(t,R) ),R)

→ findRelation([1, 2]);
(%o2) [[2,2],[1,1]]

→ findRelation([1, 2, 3]);
(%o3) [[3,3],[2,2],[1,1]]

```

```
→ findRelation([1, 2, 3, 4]);
(%o4) [[4,4],[3,3],[2,2],[1,1]]
```

## 5.4

```
→ kill(all);
(%o0) done
```

```
→ findRelation(A):=block(
  [A2:cartesian_product_list(A, A), R:[]],
  for i:1 thru length(A2) do(
    t:A2[i],
    if(is(t[1] = t[2]) = true) then R:cons(t, R)
  ),
  R
);
(%o1) findRelation(A):=block([A2:cartesian_product_list(A,A),
R:[]],for i thru length(A2) do
(t:A2[i],if is(t1=t2)=true then R:cons(t,R) ),R)
```

```
→ findRelation([1, 2]);
(%o2) [[2,2],[1,1]]
```

```
→ findRelation([1, 2, 3]);
(%o3) [[3,3],[2,2],[1,1]]
```

```
→ findRelation([1, 2, 3, 4]);
(%o4) [[4,4],[3,3],[2,2],[1,1]]
```

## 5.5

```
→ kill(all);
(%o0) done
```

```

→ findRelation(A):=block(
    [A2:cartesian_product_list(A, A), R:],
    for i:1 thru length(A2) do(
        t:A2[i],
        if(is(t[1] = t[2] + 1) = true) then R:cons(t, R)
    ),
    R
);

(%o1) findRelation(A):=block([A2:cartesian_product_list(A,A),
    R:],for i thru length(A2) do
    (t:A2[i],if is(t1=t2+1)=true then R:cons(t,R) ),R)

→ findRelation([1, 2]);
(%o2) [[2,1]]

→ findRelation([1, 2, 3]);
(%o3) [[3,2],[2,1]]

→ findRelation([1, 2, 3, 4]);
(%o4) [[4,3],[3,2],[2,1]]

```

## 5.6

```

→ kill(all);
(%o0) done

→ findRelation(A):=block(
    [A2:cartesian_product_list(A, A), R:],
    for i:1 thru length(A2) do(
        t:A2[i],
        if(is(t[1] + t[2] <= 3) = true) then R:cons(t, R)
    ),
    R
);

(%o1) findRelation(A):=block([A2:cartesian_product_list(A,A),
    R:],for i thru length(A2) do
    (t:A2[i],if is(t1+t2≤3)=true then R:cons(t,R) ),R)

→ findRelation([1, 2]);
(%o2) [[2,1],[1,2],[1,1]]

→ findRelation([1, 2, 3]);
(%o3) [[2,1],[1,2],[1,1]]

```

```
→ findRelation([1, 2, 3, 4]);  
(%o4) [[2,1],[1,2],[1,1]]
```