Practical 5

████████████

Creating lattices and determining whether or not, a given partially
ordered set is a lattice.

1

(%i1) kill(all);
(%o0) *done*

1.1

(%i1) findRelation(A):=block(
           [A2:cartesian_product_list(A, A), R:[]],
           for i:1 thru length(A2) do(
           t:A2[i],
           if(remainder(t[2], t[1])=0) then R:cons(t, R)
           ),
           R
       );

(%o1) findRelation$(A):=$ block$([A2:$ cartesian_product_list$(A,A),$
       $R:[]],$ for i thru length$(A2)$ do
       $(t:A2_i,$ if remainder$(t_2,t_1)=0$ then $R:$ cons(t,R))$,R)$

1.2

(%i2) findUpperBounds(A, R, P):=block(
           [s, t, C:[]],
           for k:1 thru length(A) do(
             s:0,
             t:A[k],
             for i:1 thru length(P) do(
             if(member([P[i], t], R)) then(s:s+1)
             ),
             if(s=length(P)) then(C:cons(t, C))
           ),
           return(C)
       );

(%o2) findUpperBounds$(A,R,P):=$ block$([s,t,C:[]],$ for k thru
       length(A) do $(s:0,t:A_k,$ for i thru length(P) do if
       member$([P_i,t],R)$ then $s:s+1$, if $s=$ length(P) then $C:$
       cons(t,C))$,$ return$(C))$

1.3

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(%i3)  findLowerBounds(A,  R,  P):=block(
           [s,  t,  C:[]],
           for k:1 thru length(A) do(
               s:0,
               t:A[k],
               for i:1 thru length(P) do(
               if(member([t,  P[i]],   R)) then(s:s+1)
               ),
               if(s=length(P)) then(C:cons(t,   C))
           ),
           return(C)
       );
```

(%o3) findLowerBounds$(A,R,P):=$block$([s,t,C:[]],$for k thru length(A) do $(s:0,t:A_k,$for i thru length(P) do if member$([t,P_i],R)$ then $s:s+1$,if $s=$length(P) then $C:$cons(t,C)),return$(C))$

1.4

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(%i4)  lub(A,  R,  P):=block(
           [U:findUpperBounds(A,  R,  P)],
           if(U=[]) then(return(U)) else(
               t:U[1],
               for i:2 thru length(U) do(
                   if(member([U[i],  t],  R)) then(t:U[i])
               ),
               return(t)
           )
       );
```

(%o4) lub$(A,R,P):=$block$([U:$findUpperBounds$(A,R,P)],$if $U=$ $[]$ then return(U) else $(t:U_1,$for i from 2 thru length(U) do if member$([U_i,t],R)$ then $t:U_i$,return$(t)))$

1.5

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(%i5)  glb(A, R, P):=block(
          [L:findLowerBounds(A, R, P)],
          if(L=[]) then(return(L)) else(
             t:L[1],
             for i:2 thru length(L) do(
                if(member([t, L[i]], R)) then(t:L[i])
             ),
             return(t)
          )
       );
```

(%o5) $glb(A,R,P):=block([L:findLowerBounds(A,R,P)],$ if $L=[$
    $]$ then return(L) else $(t:L_1,$ for i from 2 thru length(L) do if
    member$([t,L_i],R)$ then $t:L_i$, return$(t)))$

1.6

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(%i6)  checkLattice(A, R):=block(
          [s:0],
          for i:1 thru length(A) do(
             for j:1 thru length(A) do(
                if( (lub(A, R, [A[i], A[j]])#[]) and (glb(A, R, [A[i], A[j]])#[])) then(s:s+
             )
          ),
          if(s=(length(A))^2) then(return("lattice")) else(return("not a lattice"))
       );
```

(%o6)   checkLattice$(A,R):=block([s:0],$ for i thru length(A) do
    for j thru length(A) do if lub$(A,R,[A_i,A_j])\neq[] \wedge$
    glb$(A,R,[A_i,A_j])\neq[]$ then $s:s+1$, if $s=$ length$(A)^2$ then
    return$(lattice)$ else return$(not\ a\ lattice))$

1.7

Determine whether the posets ({1, 2, 3, 4, 5}, |) and ({1, 2, 4, 8, 16}, |) are lattices.

1.7.1

(%i9) A1:[1, 2, 3, 4, 5];
R1:findRelation(A1);
checkLattice(A1, R1);

(%o7) **[1,2,3,4,5]**

(%o8) **[[5,5],[4,4],[3,3],[2,4],[2,2],[1,5],[1,4],[1,3],[1,2],[1,1]]**

(%o9) *not a lattice*

1.7.2

(%i12) A2:[1, 2, 4, 8, 16];
R2:findRelation(A2);
checkLattice(A2, R2);

(%o10) **[1,2,4,8,16]**

(%o11) **[[16,16],[8,16],[8,8],[4,16],[4,8],[4,4],[2,16],[2,8],[2,4],[2,2],[1,16],[1,8],[1,4],[1,2],[1,1]]**

(%o12) *lattice*

1.7.3

44. Determine whether these posets are lattices.
a) ({1, 3, 6, 9, 12}, |)
b) ({1, 5, 25, 125}, |)

(%i15) A3:[1, 3, 6, 9, 12];
R3:findRelation(A3);
checkLattice(A3, R3);

(%o13) **[1,3,6,9,12]**

(%o14) **[[12,12],[9,9],[6,12],[6,6],[3,12],[3,9],[3,6],[3,3],[1,12],[1,9],[1,6],[1,3],[1,1]]**

(%o15) *not a lattice*

1.7.4

(%i18) A4:[1, 5, 25, 125];
R4:findRelation(A4);
checkLattice(A4, R4);

(%o16) **[1,5,25,125]**

(%o17) **[[125,125],[25,125],[25,25],[5,125],[5,25],[5,5],[1,125],[1,25],[1,5],[1,1]]**

(%o18) *lattice*