Practical 3

[redacted]

Finding the following for a given partially ordered set
i. Covering relations.
ii. Minimal and maximal elements.

1 Covering relations

1.1

```
kill(all);
```

(%o0) *done*

```
findCoveringRelation(A, R):=block(
    [C:[], nR:[], s, e, f],
    for k:1 thru length(R) do(
        if(R[k][1]#R[k][2]) then(nR:cons([R[k][1], R[k][2]], nR))
    ),
    for j:1 thru length(nR) do(
        e:nR[j][1],
        f:nR[j][2],
        s:0,
        for i:1 thru length(A) do(
            if(member([e, A[i]], nR) and member([A[i], f], nR)) then(s:s+1)
        ),
        if(s=0) then(C:cons([e, f], C))
    ),
    return(C)
);
```

(%o1) findCoveringRelation(A,R):=block$([C:[],nR:[],s,e,f],$ for k thru length(R) do if $(R_k)_1 \neq (R_k)_2$ then $nR:$ cons$([(R_k)_1,(R_k)_2],nR)$,for j thru length(nR) do $(e:(nR_j)_1,f$ $:(nR_j)_2,s:0,$for i thru length(A) do if member$([e,A_i],nR) \wedge$ member$([A_i,f],nR)$ then $s:s+1$,if $s=0$ then $C:$ cons$([e,f],C)$),return$(C))$

1.2

```
A:[2, 3, 4, 6, 8];
```

(%o2) $[2,3,4,6,8]$

```
→ R:[[2,4],[2,6],[2,8],[3,6],[4,8]];
(%o3) [[2,4],[2,6],[2,8],[3,6],[4,8]]

→ R1:[[2,2],[2,4],[2,6],[2,8],[3,3],[3,6],[4,4],[4,8],[6,6],[8,8]];
(%o4) [[2,2],[2,4],[2,6],[2,8],[3,3],[3,6],[4,4],[4,8],[
6,6],[8,8]]

→ C1:findCoveringRelation(A, R);
(%o5) [[2,4],[2,6],[3,6],[4,8]]

→ D1:findCoveringRelation(A, R1);
(%o6) [[2,4],[2,6],[3,6],[4,8]]
```

1.3

Rosen : {(a, b) | a divides b} on {1, 2, 3, 4, 6, 8, 12}

```
→ kill(all);
(%o0) done

→ A:[1, 2, 3, 4, 6, 8, 12];
(%o1) [1,2,3,4,6,8,12]

→ findRelation(A):=block(
    [A2:cartesian_product_list(A, A), R:[]],
    for i:1 thru length(A2) do(
    t:A2[i],
    if(remainder(t[2], t[1])=0) then R:cons(t, R)
    ),
    R
  );
```

(%o2) findRelation(A):=block([$A2$:cartesian_product_list(A,A), R:[]],for i thru length$(A2)$ do (t:$A2_i$, if remainder$(t_2,t_1)=0$ then R:cons(t,R)),R)

```
→ R:findRelation(A);
(%o3) [[12,12],[8,8],[6,12],[6,6],[4,12],[4,8],[4,4],[3,
12],[3,6],[3,3],[2,12],[2,8],[2,6],[2,4],[2,2],[1,12],[
1,8],[1,6],[1,4],[1,3],[1,2],[1,1]]
```

```
→ findCoveringRelation(A, R):=block(
    [C:[], nR:[], s, e, f],
    for k:1 thru length(R) do(
      if(R[k][1]#R[k][2]) then(nR:cons([R[k][1], R[k][2]], nR))
    ),
    for j:1 thru length(nR) do(
      e:nR[j][1],
      f:nR[j][2],
      s:0,
      for i:1 thru length(A) do(
        if(member([e, A[i]], nR) and member([A[i], f], nR)) then(s:s+1)
      ),
      if(s=0) then(C:cons([e, f], C))
    ),
    return(C)
  );
```

(%o4) findCoveringRelation(A,R):=block$([C:[],nR:[],s,e,f],$ for k thru length(R) do if $(R_k)_1 \neq (R_k)_2$ then $nR:$ cons$([(R_k)_1,(R_k)_2],nR)$,for j thru length(nR) do $(e:(nR_j)_1,f$ $:(nR_j)_2,s:0,$for i thru length(A) do if member$([e,A_i],nR) \wedge$ member$([A_i,f],nR)$ then $s:s+1$,if $s=0$ then $C:$ cons$([e,f],C)$),return(C))

```
→ C1:findCoveringRelation(A, R);
```

(%o5) [[6,12],[4,12],[4,8],[3,6],[2,6],[2,4],[1,3],[1,2]]

```
→ C2:setify(C1);
```

(%o6) {[1,2],[1,3],[2,4],[2,6],[3,6],[4,8],[4,12],[6,12]}

1.4

Rosen p663 : 28.
What is the covering relation of the partial ordering
{(a, b) | a divides b} on {1, 2, 3, 4, 6, 12}?

```
→ A1:[1, 2, 3, 4, 6, 8, 12];
```

(%o7) [1,2,3,4,6,8,12]

```
→ R1:findRelation(A1);
```

(%o8) [[12,12],[8,8],[6,12],[6,6],[4,12],[4,8],[4,4],[3,12],[3,6],[3,3],[2,12],[2,8],[2,6],[2,4],[2,2],[1,12],[1,8],[1,6],[1,4],[1,3],[1,2],[1,1]]

```
→ D1:findCoveringRelation(A1, R1);
(%o9) [[6,12],[4,12],[4,8],[3,6],[2,6],[2,4],[1,3],[1,2]
]

→ D2:setify(D1);
(%o10) {[1,2],[1,3],[2,4],[2,6],[3,6],[4,8],[4,12],[6,12
]}
```

1.5

({2, 4, 5, 10, 12, 20, 25}, |)

```
→ A2:[2, 4, 5, 10, 12, 20, 25];
(%o11) [2,4,5,10,12,20,25]

→ R2:findRelation(A2);
(%o12) [[25,25],[20,20],[12,12],[10,20],[10,10],[5,25],[
5,20],[5,10],[5,5],[4,20],[4,12],[4,4],[2,20],[2,12],[2
,10],[2,4],[2,2]]

→ D2:findCoveringRelation(A2, R2);
(%o13) [[10,20],[5,25],[5,10],[4,20],[4,12],[2,10],[2,4]
]

→ D3:setify(D2);
(%o14) {[2,4],[2,10],[4,12],[4,20],[5,10],[5,25],[10,20]
}
```