Practical 3b

Finding the following for a given partially ordered set
i. Covering relations.
ii. Minimal and maximal elements.

1 Minimal and maximal elements

1.1

```
(%i1) kill(all);
(%o0) done

(%i2) A:[2, 3, 4, 6, 8];
(%o1) [2,3,4,6,8]

(%i3) findRelation(A):=block(
        [A2:cartesian_product_list(A, A), R:[]],
        for i:1 thru length(A2) do(
        t:A2[i],
        if(remainder(t[2], t[1])=0) then R:cons(t, R)
        ),
        R
      );
(%o2) findRelation(A):=block([A2:cartesian_product_list(A,A),
      R:[]],for i thru length(A2) do
      (t:A2_i,if remainder(t_2,t_1)=0 then R:cons(t,R) ),R)

(%i4) R:findRelation(A);
(%o3) [[8,8],[6,6],[4,8],[4,4],[3,6],[3,3],[2,8],[2,6],[
      2,4],[2,2]]

(%i5) a:A[1];
(%o4) 2

(%i6) s:0;
      for i:1 thru length(A) do(
        if(member([A[1], A[i]], R)) then(s:s+1)
      );
      if(s=0) then(print(A[1], "is maximal"));
(%o5) 0
(%o6) done
(%o7) false
```

```
for k:1 thru length(A) do(
   s:0,
   for i:1 thru length(A) do(
      if((k#i) and (member([A[k],  A[i]],  R))) then(s:s+1)
   ),
   if(s=0) then(print(A[k],  "is maximal"))
);
```

```
6  is maximal
8  is maximal
done
```

```
maximalElements(A,  R):=block(
   [s],
   for k:1 thru length(A) do(
      s:0,
      for i:1 thru length(A) do(
         if((k#i) and (member([A[k],  A[i]],  R))) then(s:s+1)
      ),
      if(s=0) then(print(A[k],  "is maximal"))
   )
);
```

$$\text{maximalElements}(A,R):=\text{block}([s],\text{for } k \text{ thru } \text{length}(A) \text{ do } (s:0,\text{for } i \text{ thru } \text{length}(A) \text{ do } \text{if } k\neq i \wedge \text{member}([A_k,A_i],R) \text{ then } s:s+1 ,\text{if } s=0 \text{ then } \text{print}(A_k,\text{is maximal})))$$

```
maximalElements(A,  R);
```

```
6  is maximal
8  is maximal
done
```

1.2

Rosen : EXAMPLE 14

Which elements of the poset ({2, 4, 5, 10, 12, 20, 25}, |) are maximal, and which are minimal?

```
A1:[2,  4,  5,  10,  12,  20,  25];
```

```
[2,4,5,10,12,20,25]
```

```
R1:findRelation(A1);
```

```
[[25,25],[20,20],[12,12],[10,20],[10,10],[5,25],[5,20],[5,10],[5,5],[4,20],[4,12],[4,4],[2,20],[2,12],[2,10],[2,4],[2,2]]
```

```
(%i13) maximalElements(A1, R1);
```
```
12 is maximal
20 is maximal
25 is maximal
(%o13) done
```

1.3

```
(%i14) for k:1 thru length(A) do(
    s:0,
    for i:1 thru length(A) do(
        if((k#i) and (member([A[i], A[k]], R))) then(s:s+1)
    ),
    if(s=0) then(print(A[k], "is minimal"))
);
```
```
2 is minimal
3 is minimal
(%o14) done
```

```
(%i15) minimalElements(A, R):=block(
    [s],
    for k:1 thru length(A) do(
        s:0,
        for i:1 thru length(A) do(
            if((k#i) and (member([A[i], A[k]], R))) then(s:s+1)
        ),
        if(s=0) then(print(A[k], "is minimal"))
    )
);
```
```
(%o15) minimalElements(A,R):=block([s],for k thru length(A)
do (s:0,for i thru length(A) do if k≠i ∧ member([A_i,A_k],R)
then s:s+1 ,if s=0 then print(A_k,is minimal) ))
```

```
(%i16) minimalElements(A, R);
```
```
2 is minimal
3 is minimal
(%o16) done
```

```
(%i17) minimalElements(A1, R1);
```
```
2 is minimal
5 is minimal
(%o17) done
```

1.4

Rosen 33.

Answer these questions for the poset
({3, 5, 9, 15, 24, 45}, |).
a) Find the maximal elements.
b) Find the minimal elements.

```
(%i18) A2:[3, 5, 9, 15, 24, 45];
(%o18) [3,5,9,15,24,45]

(%i19) R2:findRelation(A2);
(%o19) [[45,45],[24,24],[15,45],[15,15],[9,45],[9,9],[5,
45],[5,15],[5,5],[3,45],[3,24],[3,15],[3,9],[3,3]]

(%i20) maximalElements(A2, R2);
24 is maximal
45 is maximal
(%o20) done

(%i21) minimalElements(A2, R2);
3 is minimal
5 is minimal
(%o21) done
```