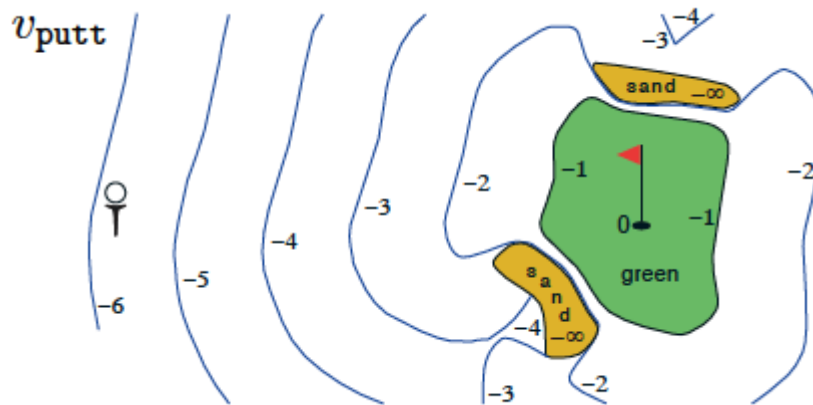


## Summary



State-value function for golf-playing agent (Sutton and Barto, 2017)

## Policies

- A **deterministic policy** is a mapping  $\pi: S \rightarrow A$ . For each state  $s \in S$ , it yields the action  $a \in A$  that the agent will choose while in state  $s$ .
- A **stochastic policy** is a mapping  $\pi: S \times A \rightarrow [0, 1]$ . For each state  $s \in S$  and action  $a \in A$ , it yields the probability  $\pi(a|s)$  that the agent chooses action  $a$  while in state  $s$ .

## State-Value Functions

- The **state-value function** for a policy  $\pi$  is denoted  $v_\pi$ . For each state  $s \in S$ , it yields the expected return if the agent starts in state  $s$  and then uses the policy to choose its actions for all time steps. That is,

$$v_\pi(s) \doteq E_\pi[G_t | S_t = s]$$

We refer to  $v_\pi(s)$  as the **value of state  $s$  under policy  $\pi$** .

- The notation  $E_\pi[\cdot]$  is borrowed from the suggested textbook, where  $E_\pi[\cdot]$  is defined as the expected value of a random variable, given that the agent follows policy  $\pi$ .

## Bellman Equations

- The **Bellman expectation equation for  $v_\pi$**  is:  $v_\pi(s) = E_\pi[R_{t+1} + \gamma v_\pi(S_{t+1}) | S_t = s]$ .

## Optimality

- A policy  $\pi'$  is defined to be better than or equal to a policy  $\pi$  if and only if  $v_{\pi'}(s) \geq v_\pi(s)$  for all  $s \in S$ .

- An **optimal policy**  $\pi^*$  satisfies  $\pi^* \geq \pi$  for all policies  $\pi$ . An optimal policy is guaranteed to exist but may not be unique.
- All optimal policies have the same state-value function  $v^*$ , called the **optimal state-value function**.

## Action-Value Functions

- The **action-value function** for a policy  $\pi$  is denoted  $q_\pi$ . For each state  $s \in \mathcal{S}$  and action  $a \in \mathcal{A}$ , it yields the expected return if the agent starts in state  $s$ , takes action  $a$ , and then follows the policy for all future time steps. That is,

$$q_\pi(s, a) \doteq \mathbb{E}_\pi[G_t | S_t = s, A_t = a]$$

We refer to  $q_\pi(s, a)$  as the **value of taking action  $a$  in state  $s$  under a policy  $\pi$**  (or alternatively as the **value of the state-action pair  $s, a$** ).

- All optimal policies have the same action-value function  $q^*$ , called the **optimal action-value function**.

## Optimal Policies

- Once the agent determines the optimal action-value function  $q^*$ , it can quickly obtain an optimal policy  $\pi^*$  by setting  $\pi^*(s) = \arg\max_{a \in \mathcal{A}(s)} q^*(s, a)$ .