1 Electromagnetic Waves

1.1 Introduction

We have maxwell's equations as

$$\boldsymbol{\nabla}\cdot\vec{E}=\frac{\rho}{\varepsilon_0} \tag{1}$$

$$\boldsymbol{\nabla}\cdot\vec{B}=0 \tag{2}$$

$$\boldsymbol{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t} \tag{3}$$

$$\boldsymbol{\nabla}\times\vec{B}=\mu_0 J+\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t} \tag{4}$$

Now we this set of equations couple both $\vec{E}(\vec{r},t)$ and $\vec{B}(\vec{r},t)$ and now we can decouple them and get two decoupled sets of equations and we can simply have wave equations in different situations.

1.2 Electromagnetic Waves in Free Space

Now let us try to decouple the above equations and get the wave equations. We can decouple the above equations and get the wave equations assuming there is no charge in the media and nor we have any current flowing. So we have the Maxwell's equations as Eq. 1 to Eq. 4 for vaccum them we have

$$\boldsymbol{\nabla}\cdot\vec{E}=0 \tag{5}$$

$$\boldsymbol{\nabla}\cdot\vec{B}=0 \tag{6}$$

$$\boldsymbol{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t} \tag{7}$$

$$\boldsymbol{\nabla}\times\vec{B}=\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t} \tag{8}$$

Now by taking curl for Eq. 3 and by using the identity $\boldsymbol{\nabla}\times\left(\boldsymbol{\nabla}\times\vec{A}\right)=\boldsymbol{\nabla}\left(\boldsymbol{\nabla}\cdot\vec{A}\right)-\nabla^2\left(\vec{A}\right)$ so we have the resulting equation as

$$\boldsymbol{\nabla}\left(\boldsymbol{\nabla}\cdot\vec{E}\right)-\nabla^2\left(\vec{E}\right)=-\frac{\partial\boldsymbol{\nabla}\times\vec{B}}{\partial t} \tag{9}$$

Now by using Eq. 5 and Eq. 8 we have the wave equation as

$$\nabla^2\vec{E}=\mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2} \tag{10}$$

and similarly one can find one equation for magentic field as by taking curl of Eq. 8 and using Eq. 2 have

$$\nabla^2\vec{B}=\mu_0\varepsilon_0\frac{\partial^2\vec{B}}{\partial t^2} \tag{11}$$

Now these two are typical PDEs for wave equations so we can compare it with the standard wave equation PDE $\nabla^2 f = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$ and hence we can have the velocity of the wave as $v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$ and this is interestingly is the speed of light.

1.3 Electromagnetic Waves in a Medium

1.4 Electromagnetic Waves in Conducting Medium