# 1 Maxwell's Equations

## 1.1 Electromotive Force

## 1.2 Electromagnetic Induction

## 1.3 Maxwell's Equations

Okk so now let us try to summarize everything we have learnt till now and what actually Maxwell had in front of him when he tried to fix the theory of electromagnetism. So let us write them down

$$\boldsymbol{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \tag{1}$$

$$\boldsymbol{\nabla} \cdot \vec{B} = 0 \tag{2}$$

$$\boldsymbol{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{3}$$

$$\boldsymbol{\nabla} \times \vec{B} = \mu_0 \vec{J} \tag{4}$$

Now let us try to understand the problem with the above equations. Take the Eq. 3 and take the divergence of it. We get $\boldsymbol{\nabla} \cdot \left(\boldsymbol{\nabla} \times \vec{E}\right) = -\frac{\mathrm{d}\boldsymbol{\nabla}\cdot\left(\vec{B}\right)}{\mathrm{d}t}$ Now we have both left and right hand sides of the equation to zero. Now for Eq. 4 we have $\boldsymbol{\nabla} \cdot \left(\boldsymbol{\nabla} \times \vec{B}\right) = \mu_0 \boldsymbol{\nabla} \cdot \vec{J}$ in which left hand side is zero but right hand side is not in general except we for steady current. So how do we fix it? We need to add a term to the equation which will make the right hand side zero. So by using equation of continuity we can write $\boldsymbol{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$ and hence we can write the corrected equation as and by using Eq. 1 we have $\boldsymbol{\nabla} \cdot \vec{J} = -\varepsilon_0 \frac{\partial \boldsymbol{\nabla}\cdot\left(\vec{E}\right)}{\partial t}$ and hence we have $\vec{J} = -\varepsilon_0 \frac{\partial \vec{E}}{\partial t}$ and hence the corrected equation is

$$\boldsymbol{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \tag{5}$$

and we have a quantity called **displacement current** that we define as

**Definition 1.1** (Displacement Current): We define the displacement current as

$$\vec{J}_d = \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \tag{6}$$

Now we can write all the maxwell's equations in differential form as

$$\boldsymbol{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \tag{7}$$

$$\boldsymbol{\nabla} \cdot \vec{B} = 0 \tag{8}$$

$$\boldsymbol{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{9}$$

$$\boldsymbol{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \tag{10}$$

We can also write the equations in integral form as

$$\oiint \vec{E} \cdot \mathrm{d}\vec{a} = \frac{Q}{\varepsilon_0} \tag{11}$$

$$\oiint \vec{B} \cdot \mathrm{d}\vec{a} = 0 \tag{12}$$

$$\oint \vec{E} \cdot \mathrm{d}\vec{l} = -\oiint \frac{\partial \vec{B} \cdot \mathrm{d}\vec{a}}{\partial t} \tag{13}$$

$$\oint \vec{B} \cdot \mathrm{d}\vec{l} = \mu_0 i + \mu_0 \varepsilon_0 \oiint \frac{\partial \vec{E} \cdot \mathrm{d}\vec{a}}{\partial t} \tag{14}$$

One can derive the above simply using the integral theorems discussed in earlier chapters. Now let us try to understand the physical significance of the Eq. 14. Suppose