1 Formulation of Quantum Mechanics

Now in here is the starting of my notes and in this note I will particularly try to motivate us towards quantum mechanics. Interesting thing about QM is that I don't what examples to cite and what not? Actually working of QM is at very small scale whether it is in terms of size or temperature. One way can be I'll just of start with the postulates of QM like my professor KP Yogendran and built up the subject in more mathematical taste with axioms and went on to develop different theorems or we can try to built up some understanding of things from ground. So for now we'll not talk about anything fancy but we'll start with experiments and problems which lead us to Quantum Mechanics first problem that I think I should discuss is of interference.

1.1 Interference

Now as anyone would ask what is interference? Interference is the phenomenon of superposition of two wave forms either reinforcing or cancelling each other. Now few of the examples of the interference in the daily life is the different color bands we observe on the surface of a bubble or bands due to oil film etc. Now for the further discussion of the matter I would like to take a up a very famous experiment in the optics that is Young's Double slit experiment.

Consider two slits illuminated by a single source and then these two slits will further act as two independent sources with temporal coherence.Now let us try to understand how things are happening? Now let us mark the upper slit as S_1 and lower slit as S_2. Let the electric field due to upper slit is given as $\boldsymbol{E_1}(\boldsymbol{t})$ and for lower slit we have $\boldsymbol{E_2}(\boldsymbol{t})$.

$$\boldsymbol{E_1}(\boldsymbol{t}) = \boldsymbol{E_{01}} \exp(i(\boldsymbol{k_1} \cdot \boldsymbol{x} - \omega_1 t + \varphi_1(t))) \tag{1}$$

$$\boldsymbol{E_2}(\boldsymbol{t}) = \boldsymbol{E_{02}} \exp(i(\boldsymbol{k_2} \cdot \boldsymbol{x} - \omega_2 t + \varphi_2(t))) \tag{2}$$

Now adding these two waves we'll have

$$\boldsymbol{E}(\boldsymbol{x}, \boldsymbol{t}) = \boldsymbol{E_1}(\boldsymbol{x}, \boldsymbol{t}) + \boldsymbol{E_2}(\boldsymbol{x}, \boldsymbol{t}) \tag{3}$$

Now definitely our experiment is about intensity which is will be given as $I = |\boldsymbol{E}(\boldsymbol{x}, \boldsymbol{t})|^2$ so let us try to find this out

$$|\boldsymbol{E_2} + \boldsymbol{E_1}|^2 = |\boldsymbol{E_1}|^2 + |\boldsymbol{E_2}|^2 + \boldsymbol{E_1^*} \cdot \boldsymbol{E_2} + \boldsymbol{E_2^*} \cdot \boldsymbol{E_1} \tag{4}$$

Now we I may define this inner product (I am not sure about the mathematical correctness of what I am going say?) of $\boldsymbol{A}^* \cdot \boldsymbol{B} = \boldsymbol{A}^* \boldsymbol{B} \cos(\chi)$, where χ is the angle between two vectors in real space, Now clearly these two electric field vectors point in the same direction as they are propagating in a same plane thus $\cos(\chi) = 1$. Now take a look back and try to understand it more. Ok so we have an understanding that definitely we are not having interference because of the angle between two electric field vectors but then what exactly is there?

Let us try and put things back in Eq. 4, with condition that $\omega_1 = \omega_2$ and $\varphi_1(t) = \varphi_2(t)$ we'll have

$$|\boldsymbol{E_2} + \boldsymbol{E_1}|^2 = |\boldsymbol{E_1}|^2 + |\boldsymbol{E_2}|^2 + E_{01} E_{02} \big(e^{\iota(\boldsymbol{k_1} - \boldsymbol{k_2}) \cdot \boldsymbol{r}}\big) + E_{01} E_{02} \big(e^{-\iota(\boldsymbol{k_1} - \boldsymbol{k_2}) \cdot \boldsymbol{r}}\big) \tag{5}$$

which further reduce to

$$|\boldsymbol{E_2} + \boldsymbol{E_1}|^2 = |\boldsymbol{E_1}|^2 + |\boldsymbol{E_2}|^2 + 2 E_{01} E_{02} \cos((\boldsymbol{k_2} - \boldsymbol{k_1}) \cdot \boldsymbol{r}) \tag{6}$$

this quantity $(\boldsymbol{k_2} - \boldsymbol{k_1}) \cdot \boldsymbol{r} \equiv \delta$ this δ is the phase difference responsible for the maxima and minima in the pattern. I will not extend the analysis of this topic further as it is of no use to us any further but let us start asking us some questions.

Exercise 1.1: What this analysis is all about?

Clearly, this whole analysis is about waves and thing that shows this same phenomenon should be a wave right? so it means if sound shows this behavior it should be a wave, wave ripples should be wave?

Exercise 1.2: Think of some experiment to get interference for
1. Sound waves
2. Water ripples

Now these examples are quite normal and nothing exciting about them we knew for centuries that they are waves but if I use electrons and they also give me the same pattern will I call it a wave or particle but I know electrons are particle right?

Exercise 1.3: Think of some experiment to show that electrons are particles.

Anyways Davidson Germer experiment show that the electrons get diffracted through a crystal.To most astonishing fact about this experiment done repeatedly with objects of different size largest that I know is Iodine molecule. So clearly now as one may conclude from this experiment that matter is wave but wait didn't you proved earlier that matter is like particle so is it both particle and wave or like neither. To be honest I don't know it is something that we perfer to call as matter waves. In rest of our course we'll spend our time in just trying develop a formalism and understand this new area of physics by applying these ideas to different problems.

1.2 Nature of Quantum Mechanics and it's Formulation

In this section I'll talk about the different formulations of quantum mechanics few of them are in terms of wavefunctions, density matrix and path-integral formulation each one of them have it's own advantage.

1.2.1 Formulation of Quantum Mechanics in Wavefunction

Any theory have few postulates over which it works and now I'll present these postulates to you
1. The state of a Quantum system is a vector in a vector space, Hilbert Space $(\mathcal{H})$ or as a mathematician will call it $L_2(a, b)$.
2. Multiparticle states can be assumed to be in tensor product of single particle vector space.
3. The dynamics of the system is governed by Schrondinger's Equation.

$$\mathcal{H}|\psi\rangle = E|\psi\rangle \tag{7}$$

4. An experiment is given as an Hermitian operator on a vector space.
5. Probability of finding system in one of the state is given by Born Rule

$$P(x,t) = |\psi(x,t)|^2 \tag{8}$$

6. Eigenvalues are possible results of an experiment.
7. Measurement collapse the quantum state into one of the eigenstates of the operator.
8. For identical particles, the tensor product states are to be either symmetrized (Bosons) or anti-symmetrized (Fermions).

1.2.2 Formulation of Quantum Mechanics in Density Matrix

Postulates of quantum mechanics in density matrix formalism

1. To each dynamical variable (physical concept) there corresponds a linear operator(mathematical object),and the possible values of the dynamical variable are the eigenvalues of the operator.
2. To each dynamical variable there is a Hermitian operator whose eigenvalues are the possible values of the dynamical variable.
3. To each state there corresponds a unique state operator. The average value of a dynamical variable R, represented by the operator R, in the virtual ensemble ofevents that may result from a preparation procedure for the state,represented by the operator ρ,is

$$\langle R \rangle = \frac{\mathrm{Tr}(\rho R)}{\mathrm{Tr}(\rho)} \tag{9}$$

4. To each state there corresponds a unique state operator, which must be Hermitian,non-negative,and of unit trace.

1.2.3 Formulation of Quantum Mechanics in Path-Integral Formulation