SOME EULER PRODUCTS WITH HIGH ACCURACY

S. ETTAHRI, O. RAMARÉ, AND L. SUREL

ABSTRACT. (File `LoeschianConstant-SomeEulerProducts-01.tex`)

We proposed in [1] an algorithm to compute some Euler products with high precision. Here are some examples for $s = 2$ and small q's. We decided to produce 100 decimal digits each time. Each computation took at most five seconds and we selected uniformly $P = 100$.

Modulo 3.

$$\prod_{p\equiv 1[3]} (1-p^{-2})^{-1} = 1.03401\,48754\,14341\,88053\,90306\,44413\,04762\,85789\,65428\,48909\\ 98864\,16825\,03842\,12222\,45871\,09635\,80496\,21707\,98262\,05962\cdots$$

$$\prod_{p\equiv 2[3]} (1-p^{-2})^{-1} = 1.41406\,43908\,92147\,63756\,55018\,19079\,82937\,99076\,95069\,39316\\ 21750\,39924\,96242\,39281\,06992\,08849\,94537\,54858\,50247\,51141\cdots$$

Modulo 4.

$$\prod_{p\equiv 1[4]} (1-p^{-2})^{-1} = 1.05618\,21217\,26816\,14173\,79307\,65316\,21989\,05875\,80425\,46070\\ 80120\,04306\,19830\,27928\,16062\,22693\,04895\,12958\,37291\,59718\cdots$$

$$\prod_{p\equiv 3[4]} (1-p^{-2})^{-1} = 1.16807\,55854\,10514\,28866\,96967\,37064\,04040\,13646\,79021\,45554\\ 79928\,40563\,68111\,38106\,59377\,71094\,66904\,07472\,79588\,48702\cdots$$

Modulo 5.

$$\prod_{p\equiv 1[5]} (1-p^{-2})^{-1} = 1.01091\,51606\,01019\,52260\,49565\,84289\,51492\,09845\,38627\,58173\\ 85237\,32024\,20089\,25161\,37424\,56726\,37093\,96197\,69455\,89218\cdots$$

$$\prod_{p\equiv 2,3[5]} (1-p^{-2})^{-1} = 1.55437\,60727\,20889\,22081\,75902\,82565\,55177\,56056\,30147\,34257\\ 40072\,50077\,94457\,39239\,00871\,38641\,44091\,80733\,87878\,70683\cdots$$

$$\prod_{p\equiv 4[5]} (1-p^{-2})^{-1} = 1.00496\,03239\,22297\,55899\,37496\,24810\,25218\,47955\,10294\,18802\\ 28801\,99528\,37852\,15071\,27700\,70076\,98854\,32491\,36118\,00619\cdots$$

Modulo 7.

$$\prod_{p\equiv 1[7]} (1-p^{-2})^{-1} = 1.00222\,95338\,19740\,42627\,18641\,59138\,22019\,24486\,37565\,40128\\ 87922\,82973\,79678\,21741\,90308\,08041\,42707\,36575\,28295\,76151\cdots$$

$$\prod_{p\equiv 2,4[7]} (1-p^{-2})^{-1} = 1.34984\,62543\,65273\,20787\,74772\,44978\,62277\,76508\,69021\,24860\\ 12031\,69999\,35719\,21654\,93824\,75777\,02051\,36300\,53459\,76601\cdots$$

$$\prod_{p\equiv 3,5[7]} (1-p^{-2})^{-1} = 1.18274\,26007\,67364\,09208\,00286\,83933\,15918\,51718\,05360\,46335\\ 82633\,06344\,66854\,90324\,90537\,21799\,81486\,90001\,86365\,91391\cdots$$

$$\prod_{p\equiv 6[7]} (1-p^{-2})^{-1} = 1.00705\,20326\,03074\,04805\,67193\,52428\,88870\,69289\,36714\,73687\\ 58335\,65893\,11634\,74829\,60947\,12069\,41243\,26265\,99553\,53536\cdots$$

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Modulo 8.

$$\prod_{p\equiv 1[8]}(1-p^{-2})^{-1} = 1.00483\,50650\,34191\,18711\,83598\,31169\,10411\,95979\,07317\,54340\\ 88789\,55156\,06711\,74639\,62051\,31056\,35207\,32105\,88068\,58783\cdots$$

$$\prod_{p\equiv 3[8]}(1-p^{-2})^{-1} = 1.13941\,87771\,08211\,51502\,70589\,30773\,34020\,88725\,59961\,09629\\ 48302\,25821\,27411\,02101\,65577\,60742\,91446\,59374\,91512\,33349\cdots$$

$$\prod_{p\equiv 5[8]}(1-p^{-2})^{-1} = 1.05109\,99849\,42183\,30793\,68775\,56006\,33505\,68012\,01018\,45817\\ 85080\,59912\,94207\,39729\,30485\,58783\,38889\,50479\,59255\,34495\cdots$$

$$\prod_{p\equiv 7[8]}(1-p^{-2})^{-1} = 1.02515\,03739\,25759\,17991\,61954\,35560\,94158\,79433\,11002\,76024\\ 41530\,69566\,94982\,17644\,97960\,41007\,90076\,26943\,14236\,43529\cdots$$

Modulo 9.

$$\prod_{p\equiv 1[9]}(1-p^{-2})^{-1} = 1.00403\,38350\,51288\,79798\,24781\,19924\,74748\,94825\,22895\,79877\\ 28822\,86701\,42359\,63409\,37977\,93839\,33608\,94316\,94860\,37141\cdots$$

$$\prod_{p\equiv 2,5[9]}(1-p^{-2})^{-1} = 1.40783\,70719\,96538\,05093\,52684\,03433\,79823\,18382\,56159\,80878\\ 18858\,21039\,93308\,74959\,08486\,21687\,68292\,75777\,90984\,34896\cdots$$

$$\prod_{p\equiv 4,7[9]}(1-p^{-2})^{-1} = 1.02986\,05876\,77826\,18491\,88642\,35135\,21663\,16312\,01666\,87293\\ 15881\,63094\,56123\,55333\,65628\,89969\,28513\,96515\,60005\,36245\cdots$$

$$\prod_{p\equiv 8[9]}(1-p^{-2})^{-1} = 1.00442\,33235\,64550\,15978\,66082\,58390\,58205\,39661\,19672\,30788\\ 17744\,79626\,23017\,18753\,96410\,76663\,34579\,95134\,16501\,66760\cdots$$

Modulo 11.

$$\prod_{p\equiv 1[11]}(1-p^{-2})^{-1} = 1.00232\,82408\,97736\,52733\,78057\,92469\,42582\,04345\,78064\,14879\\ 23124\,99895\,44150\,38255\,72926\,07516\,98484\,87460\,03110\,08712\cdots$$

$$\prod_{p\equiv 2,6,7,8[11]}(1-p^{-2})^{-1} = 1.38240\,11448\,05788\,71773\,39824\,35954\,70441\,91351\,16435\,84157\\ 13863\,06101\,70250\,01900\,59181\,34321\,25138\,72741\,06748\,64687\cdots$$

$$\prod_{p\equiv 3,4,5,9[11]}(1-p^{-2})^{-1} = 1.17640\,19224\,41514\,71776\,56838\,81699\,54785\,03151\,42210\,45715\\ 72819\,38133\,44304\,81040\,93008\,74341\,67383\,61950\,21979\,26318\cdots$$

$$\prod_{p\equiv 10[11]}(1-p^{-2})^{-1} = 1.00079\,37707\,14740\,00680\,22327\,79981\,38075\,30993\,79972\,81556\\ 86828\,01966\,59824\,89326\,65924\,56171\,20791\,11742\,28212\,98769\cdots$$

Modulo 12.

$$\prod_{p\equiv 1[12]}(1-p^{-2})^{-1} = 1.00761\,32452\,14144\,96616\,93493\,12247\,73229\,37895\,47142\,90433\\ 17666\,43368\,44819\,49208\,97861\,01855\,78530\,60579\,11129\,80649\cdots$$

$$\prod_{p\equiv 5[12]}(1-p^{-2})^{-1} = 1.04820\,19036\,00769\,93683\,49374\,34895\,79267\,34804\,13674\,49481\\ 52581\,07376\,14495\,24161\,71571\,43788\,23594\,04990\,88566\,94968\cdots$$

$$\prod_{p\equiv 7[12]}(1-p^{-2})^{-1} = 1.02620\,21468\,31233\,70070\,72018\,66966\,36157\,23611\,09321\,31334\\ 95148\,10400\,66496\,54603\,29393\,86454\,19299\,91782\,63867\,91609\cdots$$

$$\prod_{p\equiv 11[12]}(1-p^{-2})^{-1} = 1.01177\,86368\,50332\,58370\,51194\,10267\,33127\,80584\,01230\,89520\\ 87028\,35959\,40756\,15016\,41704\,56300\,54442\,19591\,32980\,62727\cdots$$

Modulo 13.

$$\prod_{p\equiv 1[13]}(1-p^{-2})^{-1} = 1.00065\,68661\,98289\,66605\,74722\,84730\,77197\,91777\,00717\,07399\,33554\,44837\,12988\,36602\,52536\,84343\,79642\,73590\,88077\,31673\cdots$$

$$\prod_{p\equiv 2,6,7,11[5]}(1-p^{-2})^{-1} = 1.38005\,21671\,19142\,93623\,73358\,95833\,59312\,88490\,63922\,76216\,00813\,27801\,96170\,83570\,07037\,00666\,02382\,19997\,07055\,85939\cdots$$

$$\prod_{p\equiv 3,9[13]}(1-p^{-2})^{-1} = 1.12706\,12738\,77030\,37596\,05291\,90459\,70008\,03562\,53668\,12081\,48604\,51380\,13290\,89754\,69987\,12664\,24897\,64722\,52303\,29593\cdots$$

$$\prod_{p\equiv 4,10[13]}(1-p^{-2})^{-1} = 1.00628\,51383\,85264\,35654\,79220\,78630\,88874\,03212\,24553\,50607\,59162\,40959\,77321\,01204\,89381\,53735\,74182\,12805\,59112\,51752\cdots$$

$$\prod_{p\equiv 5,8[13]}(1-p^{-2})^{-1} = 1.04384\,79529\,58163\,48325\,64453\,12135\,62867\,13038\,05109\,49630\,56435\,71738\,46465\,77456\,29690\,71263\,29350\,03766\,17988\,29979\cdots$$

$$\prod_{p\equiv 12[13]}(1-p^{-2})^{-1} = 1.00019\,47228\,43353\,09720\,12251\,29852\,70839\,19867\,65951\,93000\,49665\,62593\,02690\,92410\,34974\,82067\,06364\,88262\,34074\,53639\cdots$$

Modulo 15.

$$\prod_{p\equiv 1[15]}(1-p^{-2})^{-1} = 1.00148\,97422\,73492\,93695\,62022\,82152\,29804\,06202\,71822\,24183\,85046\,92061\,06460\,33370\,47461\,16170\,34094\,66709\,13158\,03303\cdots$$

$$\prod_{p\equiv 2,8[15]}(1-p^{-2})^{-1} = 1.34246\,04551\,54995\,30799\,30100\,63345\,72665\,24298\,78723\,72380\,96524\,03928\,73058\,62457\,83670\,07480\,09151\,10334\,06933\,31380\cdots$$

$$\prod_{p\equiv 4[15]}(1-p^{-2})^{-1} = 1.00317\,84700\,07976\,58539\,76886\,54009\,35749\,55893\,69169\,67588\,37351\,26980\,45622\,46578\,84368\,96080\,28447\,94669\,19055\,69351\cdots$$

$$\prod_{p\equiv 7,13[15]}(1-p^{-2})^{-1} = 1.02920\,54524\,88970\,30487\,46169\,68199\,34620\,53972\,85734\,20801\,87576\,81344\,73863\,39397\,51683\,30560\,76995\,20714\,09590\,99521\cdots$$

$$\prod_{p\equiv 11[15]}(1-p^{-2})^{-1} = 1.00941\,13977\,70415\,34074\,11140\,07967\,71715\,31828\,38502\,83487\,41065\,68439\,10926\,98429\,51008\,47969\,06005\,15885\,02338\,55701\cdots$$

$$\prod_{p\equiv 14[15]}(1-p^{-2})^{-1} = 1.00177\,62082\,89544\,73626\,10915\,43079\,96283\,15610\,57061\,98467\,19519\,14691\,39870\,02036\,75682\,26376\,90944\,75824\,69831\,96091\cdots$$

Modulo 16.

$$\prod_{p\equiv 1[16]}(1-p^{-2})^{-1} = 1.00378\,12963\,11174\,37714\,94711\,72280\,61816\,45658\,26785\,28441\,57268\,63521\,48911\,54134\,99502\,87194\,19254\,71100\,10645\,46873\cdots$$

$$\prod_{p\equiv 3,11[16]}(1-p^{-2})^{-1} = 1.13941\,87771\,08211\,51502\,70589\,30773\,34020\,88725\,59961\,09629\,48302\,25821\,27411\,02101\,65577\,60742\,91446\,59374\,91512\,33349\cdots$$

$$\prod_{p\equiv 5,13[16]}(1-p^{-2})^{-1} = 1.05109\,99849\,42183\,30793\,68775\,56006\,33505\,68012\,01018\,45817\,85080\,59912\,94207\,39729\,30485\,58783\,38889\,50479\,59255\,34495\cdots$$

$$\prod_{p\equiv 7[16]}(1-p^{-2})^{-1} = 1.02325\,48781\,97407\,08067\,95776\,68614\,06977\,00372\,89157\,54600\,19844\,97929\,83355\,91253\,99909\,55714\,70317\,40567\,85934\,05044\cdots$$

$$\prod_{p\equiv 9[16]}(1-p^{-2})^{-1} = 1.00104\,97991\,21471\,31637\,83963\,95210\,10070\,68052\,00181\,57035\,98663\,81304\,47589\,89310\,55217\,86340\,51978\,44383\,63621\,58656\cdots$$

$$\prod_{p\equiv 15[16]}(1-p^{-2})^{-1} = 1.00185\,24179\,73996\,13159\,93578\,02219\,51678\,26622\,68517\,41444\,99996\,30754\,09303\,19958\,16127\,21985\,97936\,04820\,77136\,34947\cdots$$

References

[1] S. Ettahri, O. Ramaré, and L. Surel. “Fast multi-precision computation of some Euler products”. In: *Submitted* (2019). arxiv.org/pdf/1908.06808.pdf, 23p (cit. on p. 1).

Aix Marseille Univ, CNRS, Centrale Marseille, I2M, Marseille, France
Email address: salma.ettahri@etu.univ-amu.fr

CNRS / Aix Marseille Univ. / Centrale Marseille, I2M, Marseille, France
Email address: olivier.ramare@univ-amu.fr

Aix Marseille Univ, CNRS, Centrale Marseille, I2M, Marseille, France
Email address: leon.surel@etu.univ-amu.fr