PROPOSED ERRATA (4) FOR *EXCURSIONS IN MULTIPLICATIVE NUMBER THEORY*

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These are additional proposed errata (4th set) for Olivier Ramaré, *Excursions in Multiplicative Number Theory*. Most are typographical errors.

- (p. 56, line 3) "there exists forcibly": "forcibly" is the wrong word, it probably should be "necessarily"
- (p. 87, line 3 from bottom) "the problematic we address": "problematic" is the wrong word, it should be probably be "problem" or perhaps "challenge"
- (p. 147, last line of Theorem 13.4 statement) "where C is as in Theorem 13.3." should be "where C is as in Theorem 13.3, taking $g(d) = f(d)/d$."
- (p. 148, lines 6–7) Q should be D throughout
- (p. 235, Exercise 23-1) "Let $D(t) = \sum_{n\geq 1} a_n n^{it}$ and $D^*(t) = \sum_{n\geq 1} a_n^* n^{it}$ be two Dirichlet series, both absolutely convergent for $\Re s \geq 0$." I was confused by this phrasing. First, $D(t)$ and $D^*(t)$ are not Dirichlet series because they do not have the correct form (although they can be considered as Dirichlet series restricted to the line $\sigma = 0$). Also, s is not mentioned in their definition, so $\Re s \geq 0$ is not clear; this probably means that they are absolutely convergent for all real t. We don't consider any $\sigma > 0$. Could we instead say "Let $D(t) = \sum_{n\geq 1} a_n n^{it}$ and $D^*(t) = \sum_{n\geq 1} a_n^* n^{it}$ be two series, both absolutely convergent for all real t."?
- (p. 240, Exercise 23-9) $\mathscr{C}_1$ should be $\sqrt{2}\mathscr{C}_1$
- (p. 243, line 6) "due to the K. Iseki" should be"due to K. Iseki"
- (p. 251, line 7 from bottom) "to every integers" should be "to every integer"
- (p. 251, last displayed formula) $\zeta(2-a)$ should be $C\zeta(2-a)$
- (p. 252, line 4) Exer. 3-4 should be Exer. 3-6
- (p. 252, Exercise 25-2 part 1: "any nonnegative σ" should be "any positive σ" ($\sigma = 0$ causes a division by zero)
- (p. 255, line 13)
$$\sum_{\substack{d\leq D\\ d|P(z)}} \mu(d) \text{ should be } \sum_{\substack{d\leq D\\ d|Q}} \mu(d)$$
- (p. 258, line 1) "The sieve problematic" should be "The sieve method"
- (p. 258, reference [5]) This reference is to a book and should be formatted as a book
- (p. 260, line 3) $(\rho - \rho^{-1})\rho^{-2k}$ should be $(\rho + \rho^{-1})\rho^{-2k}$ (wrong sign)

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- (p. 260, line 8 from bottom) $N_k \geq (1+\sqrt{2})^k/2$ should be $N_k \leq (1+\sqrt{2})^k/2$
- (p. 260, line 2 from bottom) "coprime positive integer" should be "coprime positive integers"
- (p. 261, line 5) "the points $h\alpha$ are distant from one another by at least $1/(2q)$" should be "the points $h\alpha$ modulo 1 are distant from one another by at least $1/(2q)$"
- (p. 262, line 9) "loose a logarithmic factor" should be "lose a logarithmic factor"
- (p. 264, line 7 from bottom) "forcibly larger than or equal to z": The word "forcibly" is wrong here; I don't know what word is intended.
- (pp. 264–265, proof of Theorem 26.2) The symbol Q is not defined but is used in two places. I think it means the same as $P(z)$.
- (p. 265, proof of Theorem 26.3) The function g to use in the application of Theorem 26.2 is not stated; it should be $g(n) = \mathrm{e}(\rho m)$.
- (p. 266, line 13) $Q \leq X/n$ should be $Q \leq x/n$ (lower case x)
- (p. 266, last displayed equation) There seems to be a missing step in this proof: to pick the value of z. At this point we are almost at the end of the proof, but all we know is $z \geq 2$ and $\log z \leq \sqrt{\log x}/2$. I think we should pick $\log z = \sqrt{\log x}/2$, which we can do at the beginning of the proof. This gives the indicated bound in this last line. The bound is not true for all z in the above range; for example, $z = 2$ gives a bound of $O(x \log x)$, which is worse than the trivial bound $O(x)$.
- (p. 268, line 6) "if only finitely primes p where such that $\{\rho p\} \notin [a, b]$" should be "if there are only finitely primes p such that $\{\rho p\} \notin [a, b]$"
- (p. 273, lines 1–2) "the sequence $(\cos 2\pi\rho n)$ where n ranges the irregular integers dense in $[-1, 1]$" should be "the sequence $(\cos 2\pi\rho n)_n$ where n ranges over the irregular integers is dense in $[-1, 1]$"
- (p. 273, reference [3]) Distjointness should be Disjointness
- (p. 274, reference [19] This reference is to a book and should be formatted as a book
- (p. 275, line 11) E. Cohen should be H. Cohen
- (p. 276, line 4) $x \geq 2 \cdot 10^6$ should be $x \leq 2 \cdot 10^6$
- (p. 277, line 6) $\sqrt{x} \geq 3500$ is not correct; from $x \geq 2 \cdot 10^6$ we only get $\sqrt{x} \geq 1414$.
- (p. 278, line 4) $\frac{2}{25} + \frac{27 \log x}{x^{1/3}} \leq 4$ should be $\frac{2 \cdot 3}{25} + \frac{27 \log x}{x^{1/3}} \leq 4$ (add factor for $D(|h_6|, 0)$).
- (p. 279, line 5) "the *large sieve*. and stems from" should be "the *large sieve*, and stems from" (replace period with comma)
- (p. 281, Exercise 28-3) This result is incorrect as stated. As a counterexample, consider f defined by $f(1, 2) = 1$ and $f(i, j) = 0$ otherwise. Then the left-hand side is

  $$\left|\xi_1 \overline{\xi_2}\right| = |\xi_1| \, |\xi_2|$$

  and the right-hand side is

  $$|\xi_1|^2 .$$

The exercise then asserts that $|\xi_1|\,|\xi_2| \le |\xi_1|^2$, which implies $|\xi_2| \le |\xi_1|$. One way to repair this would be to add the hypothesis that $f(i,j) = f(j,i)$ for all i and j. In the application of this item to Exercise 29-2, this additional symmetry hypothesis is satisfied.

- (p. 285, p. 291) The ordering of parameters of $S(q;b)$ is inconsistent between Chapters 28 and 29; the $S(q;b)$ of Chapter 28 is the $S(b;q)$ of Chapter 29.
- (p. 285, line 10) $W^*(d) = q\sum_{\delta|q}$ should be $W^*(q) = \sum_{\delta|q}$ (change argument of function, omit factor q)
- (p. 285, line 13) $V^*(d) = q^2\sum_{\delta|q}$ should be $V^*(q) = q\sum_{\delta|q}$ (change argument of function, omit one factor q)
- (p. 285, last displayed equation)
$$\frac{N\log^2 Q}{Q\sqrt{p}} \text{ should be } \frac{N\log Q}{Q\sqrt{p}} \text{ (omit one factor of } \log Q)$$
- (p. 285, line 6 from bottom) “almost every pairs” should be “almost every pair”
- (p. 292, Exercise 29-2, part 2) This is not true as stated. For example, for $t = T$ we have $\hat{K}(t) = 0$.
- (p. 293, Exercise 29-3) The statement of part 2 is not clear. I think it means, “Enumerate the triples $(n, n+2, n+4)$ where $n \le 3^{100}$ and each element of the ordered triple is a prime power.”
- (p. 295, reference [4]) $2^k\ p$ should be $2^k + p$
- (p. 295, reference [8]) $p\ 2^k$ should be $p + 2^k$
- (p. 295, references [9] and [12]) These references are to books and should be formatted as books
- (p. 295, reference [14]) “14 O. Ramaré” should be “O. Ramaré” (omit repeated 14)
- (p. 318, Hint for 23-3) Lemma 19.2 should be Lemma 19.1 (there is no Lemma 19.2)
- (p. 318, Hint for 23-8) Lemma 12.2.1 should be Lemma 12.2.
- (p. 320, line 13 from bottom) `for(y = 1, x,` should be `for(y = begx, endx,`
- (p. 324, line 2) $\hat{K}(t)$ should be $\hat{K}(t)\pi^2 T$
- (p. 325, line 1) ofenly should be often (“oftenly” is archaic)

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