

## ENGR 391 Numerical methods

### Midterm exam

- Read carefully all three questions
- Write all the steps you need to find the solution
- Please do not write in red (colour used for correction)

#### Exercise 1 – General understanding (3 marks)

- a) You are implementing on a computer a numerical algorithm to solve an equation like  $f(x) = 0$ . In order to check your algorithm you decide to test it with the function  $f(x) = x^3$  (where you know that the exact solution is  $x = 0$ ).

Is this a good choice? Justify your answer.

- b) Consider following functions. For each of them mention if it is possible to apply a bracketing method to locate their roots (if you cannot apply it give the reason):

$$f_1(x) = 10 - \exp(x)$$

$$f_2(x) = 5 + \frac{4}{x-3}$$

$$f_3(x) = \sin(x^2)$$

$$f_4(x) = (5x + 4)^2$$

- c) During your work you have to solve a numerical problem which is the solution of a non linear equation. You tried several algorithms you have learned in this lecture. However, all these algorithms give you quite different results.

I. What do you conclude about the problem you are solving?

- II. Your supervisor agrees to spend some money in buying a powerful commercial software to help you. You have the choice between two packages. The first one have an excellent root location algorithm which is smart enough to choose the most appropriate algorithm in a database in function of the problem you handle. The second one implements more or less standard algorithms but implements floating operations with 256 significant numbers. Which one of these two packages would you buy to solve your problem? Justify your answer in the light of answer I).

**Exercise 2 – System of equations (4 marks)**

Consider following equations

$$3x + 5y + 2z = 8$$

$$8y + 2z = -7$$

$$6x + 2y + 8z = 26$$

- a) Write the system in Matrix form
- b) Decompose the A matrix in LU
- c) Solve the system using your LU decomposition

Show the different steps of your calculations.

*Hint: Check your answers at each step!*

**Exercise 3 – Newton method for root location (4 marks)**

Find the positive solution ( $>0$ ) of the equation using Newton methods:

(1)  $2\sin x = x$

- a) Find an approximate solution (to the nearest integer value) of this equation graphically.
- b) Write equation (1) in the form  $f(x) = 0$  and write down the Newton algorithm.
- c) Choose as initial guess the approximate solution you found in a) and then apply the Newton algorithm until you get 4 significant digits.

Present your calculations in a table like :

| $i$ | $x_i$ | $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$ | Error estimation |
|-----|-------|--|------------------|
|     |       |  |                  |

*Hint: Be careful when you evaluate trigonometric functions! You have to use radians and not degrees!*