

CSE321 Introduction to Algorithm Design HW1

1-) Show the following. For each question state specific constants (eg. c_1, c_2, n_0).

a) $5n^2 - 7n + 9 = O(n^2 - 4n + 1)$

b) $2n^3 - 6n^2 + n - 3 = \Theta(n^3)$

c) $7n^2 + 5n(\log n)^3 = O(n^2)$

2-)

a) The following "sort" function sorts the given array "A". As a function of n ($n = \text{length}(A)$) express how many times the test in the if statement executed.

```
function sort(A):
    for i=0 to (length(A)-2):
        j=i+1
        while j<length(A):
            if A[j]<A[i]:
                swap A[i] with A[j]
            j+=1
```

b) Give the running time of the following code snippet as a function of N .

```
int sum = 0;
for (int n = N; n > 0; n /= 2)
    for(int i = 0; i < n; i++)
        sum++;
```

3-)

a) Given two asymptotically nonnegative functions $f(n)$ and $g(n)$. Using the definition of Θ -notation prove that $\max(f(n), g(n)) = \Theta(f(n) + g(n))$.

b) For $a, b \in \mathbb{R}$ and $b > 0$ prove that :

$$(n + a)^b = \Theta(n^b) .$$

4-)

For each of the following functions, indicate the class $\Theta(g(n))$ the function belongs to. (Use the simplest $g(n)$ possible in your answers.) Prove your assertions.

- | | |
|---|-----------------------------------|
| a. $(n^2 + 1)^{10}$ | b. $\sqrt{10n^2 + 7n + 3}$ |
| c. $2n \lg(n + 2)^2 + (n + 2)^2 \lg \frac{n}{2}$ | d. $2^{n+1} + 3^{n-1}$ |
| e. $\lfloor \log_2 n \rfloor$ | |

5-)

Door in a wall You are facing a wall that stretches infinitely in both directions. There is a door in the wall, but you know neither how far away nor in which direction. You can see the door only when you are right next to it. Design an algorithm that enables you to reach the door by walking at most $O(n)$ steps where n is the (unknown to you) number of steps between your initial position and the door. [Par95]

6-)

Find the order of growth of the following sums. Use the $\Theta(g(n))$ notation with the simplest function $g(n)$ possible.

- a. $\sum_{i=0}^{n-1} (i^2+1)^2$ b. $\sum_{i=2}^{n-1} \lg i^2$
c. $\sum_{i=1}^n (i+1)2^{i-1}$ d. $\sum_{i=0}^{n-1} \sum_{j=0}^{i-1} (i+j)$

Due Date : 17.10.2014

*Give the hard copy of the hw to M.Şekercioğlu (Room No: 108) and upload a photo of your hw to moodle.

*Question 1 is 15 points others are 17 points.