CSE321 Introduction to Algorithm Design HW2

*Questions 1,3,5-a and 6 are programming questions and should be implemented to the python source file that is shared on the moodle. Zip the python source files and the photos of your solutions and upload them to the moodle.

1-)Let A[0.....n] be a sorted array whose elements are distinct integers. Design an algorithm to find an index such that $0 \leq i \leq n$ and A[i]=i provided that such an index exists. The algorithm returns only yes or no. The algoritm's running time should be O(logn).

2-)
a)Using mathematical induction prove that $3^n + 7^n - 2$ is divisible by 8. (for n>0 and n is integer)
b)Using mathematical induction prove that insertion sort is correct.

3-)Given an array A[1...n] whose elements are different real numbers ,
a)Write an iterative algoritm that finds the k smallest elements in the array in O(n) time.
b)Write the same algoritm in a) recursively.

4-)Solve the following recurrence relations. Do not use master's theorem.
a)T(n)=T(n/2)+(n*n) , T(1)=O(1)

b)In the following question you may assume that n is a power of 3. Your answer should be in the form n^a for some constant a. Remember that $a^{\log n} = n^{\log a}$.

T(n)=1 if n=1
T(n)=2T(n/3)+n if n>1

5-)Given an array A[1...n] whose elements are integers ,
a)Write a brute-force algorithm that finds interval in A whose sum is the greatest. Eg: A={-2,11,-4,13,-5,2} then A[2]+A[3]+A[4]=20 whose sum is the greatest among all the intervals in A.
b)What is the complexity of the algorithm in A?

6-)

Restricted Tower of Hanoi Consider the version of the Tower of Hanoi puzzle in which n disks have to be moved from peg A to peg C using peg B so that any move should either place a disk on peg B or move a disk from that peg. (Of course, the prohibition of placing a larger disk on top of a smaller one remains in place, too.) Design a recursive algorithm for this problem and find the number of moves made by it.

7-)

Consider the following algorithm to check whether a graph defined by its adjacency matrix is complete.

ALGORITHM *GraphComplete*$(A[0..n-1, 0..n-1])$

```
//Input: Adjacency matrix A[0..n − 1, 0..n − 1]) of an undirected graph G
//Output: 1 (true) if G is complete and 0 (false) otherwise
if n = 1 return 1   //one-vertex graph is complete by definition
else
    if not GraphComplete(A[0..n − 2, 0..n − 2]) return 0
    else for j ← 0 to n − 2 do
            if A[n − 1, j] = 0 return 0
        return 1
```

What is the algorithm's efficiency class in the worst case?

8-)

Celebrity problem A celebrity among a group of n people is a person who knows nobody but is known by everybody else. The task is to identify a celebrity by only asking questions to people of the form "Do you know him/her?" Design an efficient algorithm to identify a celebrity or determine that the group has no such person. How many questions does your algorithm need in the worst case?

Due Date : 9.11.2014

*Give the hard copy of the solutions of questions 2-a,2-b, 4-a,4-b,5-b,7 and 8 to M.Şekercioğlu (Room No: 108) and upload a photo of your those solutions to the moodle . Other questions are programming questions and u should implemet them in the python language and upload the source file to the moodle. (U can give the hard copy till 11.11.2014 5.00 pm)

*All questions are 12.5 points.