

Question 4

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4. Why does the same bijection also prove the following stronger statement for $n \geq 1$?

$p(n \mid \text{even number of odd parts}) =$

$p(n \mid \text{distinct parts, number of odd parts is even})$

as well as the same statement with both "even" changed to "odd." (Difficulty rating: 2)

For first case, we start out with even number of odd parts. When we combine two similar odd parts it becomes even, leaving the number of odd parts still even. Hence, we combine similar parts until all parts become distinct which proves the bijection.

For example,

$$5 + 3 + 3 + 3 + 1 + 1 \rightarrow 6 + 5 + 3 + 2$$

For the second case, we start out with odd number of odd parts. When we combine two similar odd parts it becomes even, leaving the number of odd parts still odd. Hence, we combine similar parts until all parts become distinct which proves the bijection.

For example,

$$5 + 3 + 3 + 3 + 1 \rightarrow 6 + 5 + 3 + 1$$