Question 3

Ram Balaji S

May 2020

3. For odd n, there can be no partitions into even parts, nor into parts with an even number of each size. Why? For even $n \geq 2$, find an alternative bijective proof of the above identity by finding bijections for each of the two equalities $p(n|even \quad parts) = p(n/2) = p(n|even \quad number \quad of \quad each \quad part)$ **(Difficulty rating: 2)**

For odd n, there should be odd number of odd parts which proves there can't be partitions of odd n into even parts nor into parts with an even number of each size.

$$p(n|even \quad parts) = p(n/2) = p(n|even \quad number \quad of \quad each \quad part)$$

For, $p(n|even \quad parts) = p(n/2)$, divide all parts by two. The bijection will be to multiply all parts by two.

For, $p(n|even \quad number \quad of \quad each \quad part) = p(n/2)$, eliminate one among the pair of each part and for bijection, just clone every parts.

Hence,
$p(n|even \quad parts) = p(n/2) = p(n|even \quad number \quad of \quad each \quad part)$