Assignment - 03

3D - LiDAR DIRECT GEOREFERENCING Using 3D Conformal Coordinate Transform

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LiDAR data Registration described by

$$\vec{G} = \vec{r_L} + \vec{S}$$

$\vec{G}$ = Vector from Earth Center to ground point

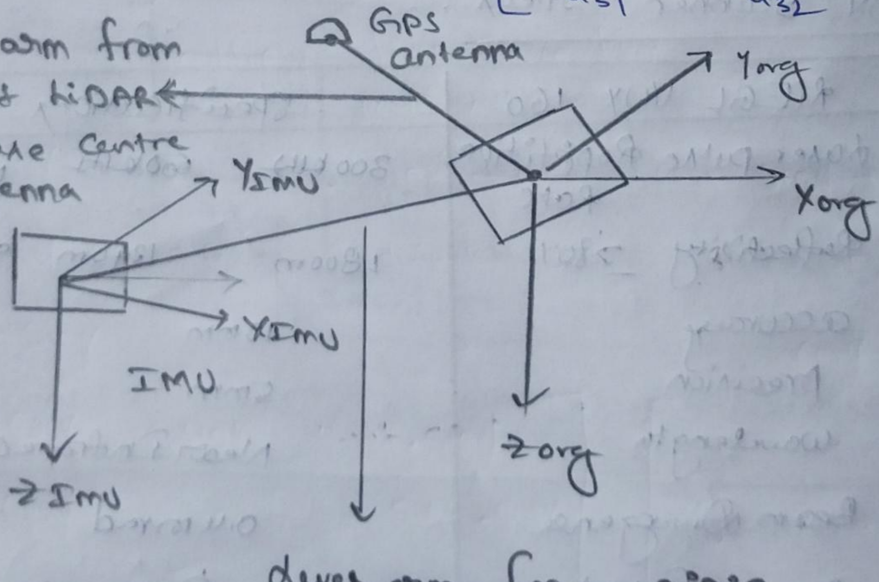
$\vec{r_L}$ = Vector from Earth Center to LiDAR point of origin

$\vec{S}$ = slant range vector

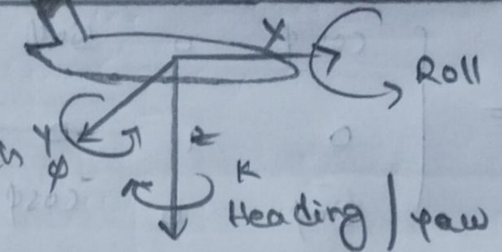
$$\boxed{G_{WGS84} = \vec{r}_{L\,WGS84} + (\_)_{H}^{WGS84} \cdot (\_)_{IMU}^{H} \cdot (\_)_{LiDAR}^{IMU} \cdot \vec{S_L}}$$

$$(\_)_{IMU}^{H} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

lever arm from origin of LiDAR to phase centre of antenna

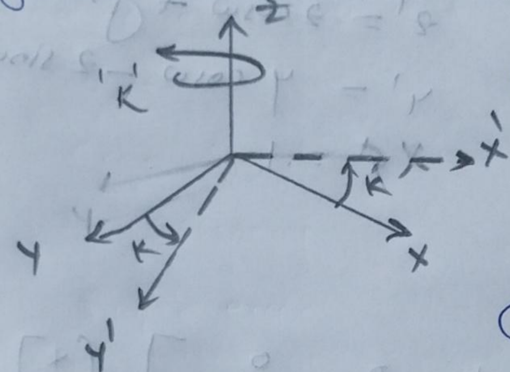


lever arm from origin of LiDAR to IMU



$\begin{bmatrix} C.C.W = +ve \\ C.W = -ve \end{bmatrix}$ Rotations.

Right Handed coordinate system



① applying the rotations in z, y, x axes with rotations (K, ϕ, ω)

② Let's rotate about z-axis with angle 'K' (Kappa)

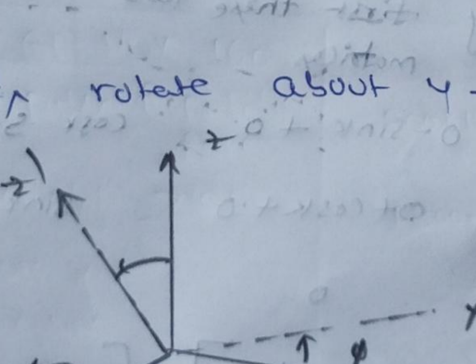
$y' = y \cos K + x \sin K$

$x' = x \cos K - y \sin K$

$z' = 1$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos K & -\sin K & 0 \\ \sin K & \cos K & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

③ Let's rotate about y-axis with angle ϕ



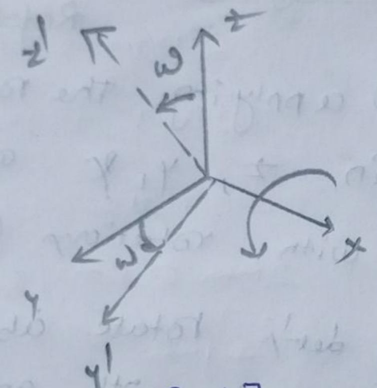
$x' = x \cos\phi + z \sin\phi$

$z' = z \cos\phi - x \sin\phi$

$y' = 1$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

(7) derx rotate in 'x' direction



$z' = z\cos\omega + y\sin\omega$

$y' = y\cos\omega - z\sin\omega$

$x' = 1$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & -\sin\omega \\ 0 & \sin\omega & \cos\omega \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{bmatrix} \cos k & -\sin k & 0 \\ \sin k & \cos k & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & -\sin\omega \\ 0 & \sin\omega & \cos\omega \end{bmatrix}$$

multiply first there two matrices

$$\begin{bmatrix} \cos k \cos\phi + 0 + 0 & 0 - \sin k + 0 & \cos k \sin\phi + 0 + 0 \\ \sin k \cos\phi + 0 + 0 & 0 + \cos k + 0 & \sin k \cdot \sin\phi + 0 + 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix}$$

$$\begin{bmatrix} \cos k \cdot \cos\phi & -\sin k & \cos k . \sin\phi \\ \sin k \cdot \cos\phi & \cos k & \sin k \cdot \cos\phi \\ -\sin\phi & 0 & \cos\phi \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & -\sin\omega \\ 0 & \sin\omega & \cos\omega \end{bmatrix}$$

$$\begin{bmatrix} \cos\kappa \cos\phi & \sin\phi \cdot \sin\omega \cos\kappa - \sin\kappa \cos\omega & \cos\kappa \sin\phi \cos\omega + \sin\kappa \cdot \sin\omega \\ \sin\kappa \cos\phi & \cos\kappa \cdot \cos\omega + \sin\kappa \cdot \sin\phi \cdot \sin\omega & \sin\kappa \cdot \sin\phi \cdot \cos\omega - \cos\kappa \sin\omega \\ -\sin\phi & \cos\phi + \sin\phi\omega & \cos\phi \cos\omega \end{bmatrix}$$

$$\therefore (-)^{H}_{IMU} \text{ matrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

The Same above parameters to be applied & to transform coordinates from LiDAR center of origin to IMU.

$$(-)^{IMU}_{LiDAR} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

apply rotations $\delta\kappa$, $\delta\phi$, $\delta\omega$ in x, y, z axes

$$\therefore \text{final} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} =$$

$$\begin{bmatrix} \cos(\delta\kappa)\cos(\delta\phi) & \sin(\delta\phi)\cos(\delta\omega)\cdot\cos(\delta\kappa) - \sin(\delta\kappa)\cdot\cos(\delta\omega) & \cos(\delta\kappa)\cdot\sin(\delta\phi)\cos(\delta\omega) + \sin(\delta\kappa)\cdot\sin(\delta\omega) \\ \sin(\delta\kappa)\cdot\cos(\delta\phi) & \cos(\delta\kappa)\cdot\cos(\delta\omega) + \sin(\delta\kappa)\cdot\sin(\delta\phi)\cdot\sin(\delta\omega) & \sin(\delta\kappa)\cdot\sin(\delta\phi)\cos(\delta\omega) - \cos(\delta\kappa)\sin(\delta\omega) \\ -\sin(\delta\phi) & \cos(\delta\phi) + \sin(\delta\omega) & \cos(\delta\phi)\cdot\cos(\delta\omega) \end{bmatrix}$$