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title: "Assignment5"
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Load libraries required

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```{r}
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```
library(Benchmarking)
library(lpSolveAPI)
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```{r}
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```
DMU1<- read.lp("C:/Users/ramne/Desktop/QMM
Assignment/Assignment5/DMU1.lp")
DMU1
solve(DMU1)
get.objective(DMU1)
get.variables(DMU1)
```

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The lp acheives maximum efficiency 1 for DMU1.

Given inputs and outputs when we use the weights 5.17 and 1.12 for the outputs, 7.14 and 0.00 for the input for maximum efficiency.

```
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```
DMU2<- read.lp("C:/Users/ramne/Desktop/QMM
Assignment/Assignment5/DMU2.lp")
DMU2
solve(DMU2)
get.objective(DMU2)
get.variables(DMU2)
```

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The lp acheives maximum efficiency 1 for DMU2.

Given inputs and outputs when we use the weights 1.29 and 6.8 for the outputs, 0.00 and 4.7 for the input for maximum efficiency.

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```
DMU3<- read.lp("C:/Users/ramne/Desktop/QMM
Assignment/Assignment5/DMU3.lp")
DMU3
solve(DMU3)
get.objective(DMU3)
get.variables(DMU3)
```

```

The lp achieves maximum efficiency 1 for DMU3.

Given inputs and outputs when we use the weights 1.7 and 3.7 for the outputs, 2.3 and 0.00 for the input for maximum efficiency.

```{r}

```
DMU4<- read.lp("C:/Users/ramne/Desktop/QMM
Assignment/Assignment5/DMU4.lp")
DMU4
solve(DMU4)
get.objective(DMU4)
get.variables(DMU4)
```

```

The lp achieves efficiency 0.98 with DMU4.

Given inputs and outputs when we use the weights 1.9 and 0.0 for the outputs, 1.05 and 1.63 for the input for maximum efficiency. Even though we provide the greatest weight to deposits, DMU4 is not efficient.

```{r}

```
DMU5<- read.lp("C:/Users/ramne/Desktop/QMM
Assignment/Assignment5/DMU5.lp")
DMU5
solve(DMU5)
get.objective(DMU5)
get.variables(DMU5)
```

```

The lp achieves efficiency 0.96 for DMU5.

Given inputs and outputs when we use the weights 1.03 and 5.3 for the outputs, 1.11 and 2.99 for the input for maximum efficiency. Even though we provide the greatest weight to deposits, DMU5 is not efficient.

```{r}

```
DMU6<- read.lp("C:/Users/ramne/Desktop/QMM
Assignment/Assignment5/DMU6.lp")
DMU6
solve(DMU6)
get.objective(DMU6)
get.variables(DMU6)
```

```

The lp achieves efficiency 0.86 for DMU6.

Given inputs and outputs when we use the weights 1.46 and 7.56 for the outputs, 1.59 and 4.26 for the input for maximum efficiency.

Even though we provide the greatest weight to deposits, DMU6 is not efficient.

Let's define our inputs and outputs as vectors .There are 2 inputs (Staff hours, Supplies) and 2 outputs (Reimbursed Patient_Days, Privately Paid Patient_Day)

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```
x <- matrix(c(150, 400, 320, 520, 350, 320, 0.2, 0.7, 1.2, 2.0, 1.2, 0.7), ncol = 2)
y <-
matrix(c(14000,14000,42000,28000,19000,14000,3500,21000,10500,42000,25000,15000), ncol = 2)
colnames(x) <- c("Staff_Hours", "Supplies")
colnames(y) <- c("Reimbursed Patient_Days", "Privately Paid Patient_Days")
print(x)
print(y)
Matrix<- cbind(x,y)
row.names(Matrix) = c("Faci1", "Faci2", "Faci3", "Faci4", "Faci5", "Faci6")
Matrix
```

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1) Formulate and perform DEA analysis under all DEA assumptions of FDH, CRS, VRS, IRS, DRS, and FRH.

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```
#Free disposability hull
FDH <- dea(x,y, RTS = "fdh")
FDH
peers(FDH)
FDH_Weights <- lambda(FDH)
```

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The peer for each facility is same as the peer.

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```
#Constant returns to scale, convexity and free disposability
```

```
CRS <- dea(x,y, RTS = "crs")
CRS
#Identify Peers
peers(CRS)
#Identify lambda
CRS_Weights <- lambda(CRS)
```

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The results show DMU 1,2,3,4 are efficient and DMU 5 is 0.9775, DMU 6 0.867

The peer for 5 and 6 are 1,2,3

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```{r}
#Variable returns to scale, convexity and free disposability
VRS <- dea(x,y, RTS = "vrs")
VRS
peers(VRS)
VRS_Weights <- lambda(VRS)
```

```

All facilities are efficient except DMU5 which is 0.8963

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```{r}

#Increasing returns to scale, (up-scaling, but not down-scaling),
convexity and free disposability
IRS <- dea(x,y, RTS = "irs")
IRS
peers(IRS)
IRS_Weights <- lambda(IRS)
```

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Decreasing returns to scale, convexity, down-scaling and free disposability

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```{r}

DRS <- dea(x,y, RTS = "drs")
DRS
peers(DRS)
DRS_Weights <- lambda(DRS)
```

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```{r}

FRH <- dea(x,y, RTS="add")
FRH
peers(FRH)
FRH_Weights <- lambda(FRH)
```

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```{r}

as.data.frame(Matrix)
DataFrame<- data.frame(CRS = c(1.0000, 1.0000, 1.0000, 1.0000, 0.9775,
0.8675), FDH = c(1, 1, 1 ,1 ,1 ,1), VRS =c(1.0000, 1.0000, 1.0000, 1.0000
,1.0000, 0.8963), IRS = c(1.0000, 1.0000, 1.0000, 1.0000, 1.0000,
0.8963), DRS = c(1.0000, 1.0000, 1.0000 ,1.0000 ,0.9775, 0.8675), FRH =
c(1, 1, 1 ,1, 1, 1))
DataFrame
```

```

So, from the above results,

1.Facilities 1,2,3,4 are fully efficient for all the assumptions and
Facilities 5,6 are not efficient.

2. Facility 5 is fully efficient for FDH, VRS, IRS and FRH assumptions.
3. It is observed that 97.7% efficient for CRS and DRS assumptions.
4. Facility 6 is fully efficient for FDH and FRS assumptions.
5. For the Facility 6, CRS and DRS assumptions 86.7% efficient.
6. For the Facility 6, IRS and VRS assumptions 89.6% efficient.

Question 2 : GOAL PROGRAMMING

Maximize $Z = P - 6C - 3D$, where

P = total (discounted) profit over the life of the new products,

C = change (in either direction) in the current level of employment,

D = decrease (if any) in next year's earnings from the current year's level.

Profit P is expressed as:

$$P = 20x_1 + 15x_2 + 25x_3$$

Employment level is expressed as :

$$6x_1 + 4x_2 + 5x_3 = 50$$

Next year Earnings goal is expressed as:

$$8x_1 + 7x_2 + 5x_3 \geq 75$$

1) Model Formulation:

Let us consider

y_1 - Employment Level minus the target and

y_2 - Next Year Earnings minus the Target

y_1^+ - Penalty for employment level goal exceeding 50

y_1^- - Penalty for employment level goal decreasing below 50

y_2^+ - Exceed the next year earnings

y_2^- - Penalty for not reaching the next year earnings

$$y_1 = 6x_1 + 4x_2 + 5x_3 - 50$$

$$y_2 = 8x_1 + 7x_2 + 5x_3 - 75$$

For Employment level goal

$$y_1 = y_1^+ - y_1^- \text{ where } y_1^+, y_1^- \geq 0$$

$$y_1^+ - y_1^- = 6x_1 + 4x_2 + 5x_3 - 50$$

For Next year earnings goal

$$y_2 = y_2^+ - y_2^- \text{ where } y_2^+, y_2^- \geq 0$$

$$y_2^+ - y_2^- = 8x_1 + 7x_2 + 5x_3 - 75$$

Final Formulation is expressed as

$$\text{Max } P = 20x_1 + 15x_2 + 25x_3$$

$$6x_1 + 4x_2 + 5x_3 - (y_1^+ - y_1^-) = 50$$

$$8x_1 + 7x_2 + 5x_3 - (y_2^+ - y_2^-) = 75$$

$$x_j \geq 0, \text{ where } j=1,2,3$$

$$y_i \geq 0, \text{ where } i=1,2$$

$$y_i \geq 0, \text{ where } i=1,2$$

2) Managements objective function Objective Function

$$\text{Maximize } Z = P - 6C - 3D$$

Objective function in terms of $x_1, x_2, x_3, y_1^+, y_1^-, y_2^+$ and y_2^-

$$\text{Max } Z = 20x_1 + 15x_2 + 25x_3 - 6y_1^+ - 6y_1^- - 3y_2^-$$

S.T.:

$$6x_1 + 4x_2 + 5x_3 - y_{1+} + y_{1-} = 50$$

$$8x_1 + 7x_2 + 5x_3 - y_{2+} + y_{2-} = 75$$

$$x_j \geq 0 \text{ where } j=1,2,3$$

$$y_i \geq 0 \text{ where } i=1,2$$

$$y_i \geq 0 \text{ where } i=1,2$$

3) Formulate and solve the linear programming model

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```
DEA<- read.lp("C:/Users/ramne/Desktop/QMM
Assignment/Assignment5/Emax.lp")
DEA
solve(DEA)
get.objective(DEA)
get.variables(DEA)
get.constraints(DEA)
```

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From the above result, penalty for not satisfying the goals on the objective function is 225. The order shows the order in which the variables were written in the objective function. The results show that $x_1 = 0$, $x_2 = 0$, $x_3 = 15$, $y_{1+} = 25$, $y_{1-} = 0$, $y_{2+} = 0$, $y_{2-} = 0$, which indicates that the Next years Earnings (y_2) expectations are fully satisfied, but the Employment level goal is exceeded by 25 with the total profit of product 3, there is a negative result on its profit by 15.