Aerodynamic drag on trains in tunnels Part 1: synthesis and definitions

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Aerodynamic drag on trains in tunnels includes friction drag and pressure drag, which are respectively the algebraic sums of the longitudinal components of all shear and normal forces on the train surfaces. The first of these is broadly similar to its counterpart in the open. The second is shown to include two effects that are usually negligible in the open.

It is shown that the overall drag force must be regarded as the sum of individual components, each of which behaves differently from the others. The components can be represented by non-dimensional coefficients whose numerical values are nearly constant for a wide range of train journeys. In contrast, the overall *drag coefficient is shown to vary significantly, even during any particular journey.*

The principal causes of aerodynamic drag in tunnels are also the principal causes of pressure waves that give rise to potential aural discomfort for passengers. It is argued that a common method of analysis is appropriate for the prediction of both of these effects.

Ill-defined train areas are shown to be a potentially serious source of confusion in the estimation and interpretation of drag coefficients. The relevant train area is shown to be its aerodynamic area, *the definition of which is explained.*

Key words: railway tunnels, high-speed trains, aerodynamic drag, pressure transients, loss coefficients, drag coefficients, train resistance

NOTATION

A	area (m^2)
C_D	drag coefficient
f	skin friction coefficient $\equiv \tau_w/(\frac{1}{2}\rho V^2)$
F	force (N)
k	stagnation pressure loss coefficient
l	perimeter length (m)
p	static pressure (Pa)
p_0	stagnation pressure (Pa)
V	velocity (m/s)
β	area blockage ratio $\equiv A/A_t$
Δx	arbitrary length (m)
λ	skin friction coefficient $= 4f$
ρ	mass density (kg/m^3)
τ	shear stress (Pa)

Superscripts

*	relative to train

Subscripts

ann	annulus
D	drag
F	friction
N	train nose
ref	reference value
t	tunnel
T	train tail
w	wall
z	train (zug)
1–4	specific locations

1 INTRODUCTION

Aerodynamic drag on trains in tunnels exceeds that in the open air, often by a substantial margin. Despite this, current practice has not yet reached a stage where operators can predict aerodynamic drag in tunnels with confidence. Suppose, for instance, that the drag is known for a particular train in a particular tunnel. Even with this background, there exists no reliable method of predicting the drag on the same train in other tunnels of different diameter. This is the first of two papers designed to reduce the uncertainty by providing a rational basis for the description of drag.

The uncertainties arise in part from misconceptions about the origin of some components of drag. Some key parameters (for example the train cross-sectional area) are commonly determined in an unnecessarily arbitrary manner and, as a consequence, the apparent values of other parameters are distorted; that is derived values of these parameters are influenced by errors in the nominal area. This effect is especially important in the case of train nose loss coefficients. It also influences the assumed values of skin friction coefficients.

In *long* tunnels, the main significance of aerodynamic drag is its influence on train performance. It limits the power available for acceleration at lower speeds and it is the principal determining factor governing the maximum sustainable speed. Historically, it has been assumed, either implicitly or explicitly, that flows are quasi-steady for large parts of the train journey and that use can be made of this characteristic to deduce resistance coefficients for complete trains. This approach dominated early work [for example see reference (**1**)] because there was no practicable means of using more complex methodologies—even though authors clearly recognized limitations of global approaches. They did not have high-speed digital computers for theoretical analysis and they did not have high-speed data acquisition systems for detailed experimental measurement.

In *short* tunnels, the main significance of aerodynamic drag is different. The emphasis changes from train performance to an even more sensitive issue, namely passenger comfort. The origins of aerodynamic drag are

The MS was received on 16 November 1995 and was accepted for publication on 23 May 1996.

identical to the origins of pressure fluctuations, which are a potential source of aural discomfort. The relative importance of the various components of drag is not the same in the two cases, but the components themselves are the same. It is therefore appropriate to consider the two phenomena simultaneously.

1.1 Prediction methods

Until about thirty years ago, *analytical* methods were used to simulate air flows induced by trains in tunnels. For these purposes, it was convenient to aggregate the various contributions to train drag in a single composite drag coefficient. The advantage of this approach was simplicity; the disadvantage was the difficulty in choosing appropriate values for the coefficient, especially when it was necessary to allow for accelerating flows. As pointed out by Sabjen (**2**), for instance, the overall drag coefficient is a property of the whole train/tunnel system, not the train alone, even when the flows are approximately steady (see Section 6).

Today, it is more convenient to simulate the air flows numerically, using one of the computer programs developed in various countries expressly for this purpose [for example see references (**3**) to (**6**)]. With this approach, it is not merely unnecessary to use overall drag coefficients; it is almost impossible to do so. The programs simulate flows locally throughout the tunnel system—at the nose and tail of each train, at numerous positions along the tunnel, etc. Individual contributions to drag are necessarily analysed independently of one another.

Suppose that all individual contributions have been calculated at a typical instant in a simulation. In this case, their sum may be evaluated to give the overall drag. This is useful for output purposes and, sometimes, for train performance calculations within the program itself. Nevertheless, the result is not used in the calculation of subsequent air flows. Indeed, most programs do not even use *local* drag coefficients; it is usually more convenient to use friction coefficients and pressure loss coefficients.

The remainder of this paper deals with each of the major contributions to aerodynamic drag (and/or pressure fluctuations), showing their roles in the overall picture and highlighting inconsistencies in existing methodology. Subsequent papers will deal with prediction methods and measurements; the first step is to understand the overall relationships.

2 PRINCIPAL COMPONENTS OF DRAG

2.1 Trains in open air

For trains travelling over open ground, the most obvious contributions to aerodynamic drag are skin friction along the train and local flow disturbances at the nose and tail. Effects due to cross-winds are neglected herein because they have no counterpart in tunnels. Effects due to wayside structures and passing trains are neglected because they are of short duration.

The skin friction contribution is often the most important. The smallest value that is theoretically achievable would result if the train surfaces were perfectly smooth and there were no protuberances. Even in this case, there would nevertheless be a boundary layer alongside the train because of the no-slip condition at its surface. Designers of real trains seek to approximate this ideal case as closely as reasonable, but surfaces are not perfectly smooth and protuberances exist—bogies, pantographs, etc. For present purposes, it is acceptable to regard the overall effect as equivalent to a uniformly rough surface despite obvious inconsistencies in this approximation both longitudinally and circumferentially. With this idealization, skin friction is imagined to exist uniformly around the train and to vary gradually along the train—due to the growth of the boundary layer. There is assumed to be no variation in pressure, either radially or longitudinally.

The other main contribution to the aerodynamic drag is often called form drag, but is called *pressure drag* herein for reasons that will become clear in Section 2.2. In the simplest case, this drag is the difference between the longitudinal components of the pressure forces on the nose and tail of the train. In the absence of viscosity, the forces on an identical nose and tail would be equal and so there would be no pressure drag. In practice, a large wake region exists behind the tail and the pressures on the trailing surface are smaller than they would be in an ideal (inviscid) flow. Likewise, there is a boundary layer and there may be flow separations—just downstream of the nose, for example. If so, the pressures on the nose will exceed those for the equivalent inviscid case.

2.1.1 Definitions

In general, the overall aerodynamic drag may be regarded as the sum of skin friction drag and pressure drag, where:

(a) *skin friction drag* is regarded as the sum of the longitudinal components of all shear forces on the whole of the (idealized) train surface and
(b) *pressure drag* is regarded as the sum of the longitudinal components of all pressure forces on the whole of the (idealized) train surface.

2.2 Trains in tunnels

When the identical train passes through a tunnel, a number of complications arise in comparison with the open air case. These include:

(a) the magnitudes of the particular effects discussed above are altered,
(b) significant pressure waves are generated during train entry and train exit and
(c) longitudinal pressure gradients develop alongside the train.

These are discussed briefly in turn.

2.2.1 Modification of open-air effects

1. *Skin friction drag.* The velocity distribution alongside the train is constrained by a second no-slip condition, namely at the tunnel wall. As a consequence, the velocity gradient at the train surface differs from that in the open and so the shear force is different.
2. *Pressure drag.* The lateral extent of any region of separated flow just downstream of the nose is con-

strained by the tunnel wall, which therefore influences the contribution of the nose to pressure drag. The wake region behind the tail may also change.

These changes can be quite large in tunnels of small cross-section, but they are less strong in tunnels of larger cross-section.

2.2.2 Pressure waves

When a train enters a tunnel, it displaces air (Fig. 1). Some of this flows back alongside the train and out of the portal; the remainder passes down the tunnel behind a pressure wavefront. At high speeds and/or in tunnels of small cross-section, the pressure rise ahead of the train can be quite large (2 kPa or more). This pressure acts on the nose of the train, causing additional aerodynamic drag, which is typically much larger than drag due to local flow separations. The underlying phenomenon is inertial, not viscous; it would exist even in an ideal inviscid fluid. In a tunnel, therefore, pressure drag includes *inertial drag* as well as drag attributable to flow separations.

Further pressure waves are generated when the nose leaves the tunnel, when the tail enters and leaves the tunnel, and when the nose and tail pass alongside airshafts and cross-passages, etc. More gradual wavefronts are generated by skin friction during train entry and exit.

2.2.3 Friction pressure gradients

In the open air, the boundary layer thickness varies along the train, but the pressure is constant. In a tunnel, the opposite is true; the boundary layer is constrained by the tunnel wall so its thickness is constant (except near its upstream end) and the pressure varies along the train (Fig. 2). For the usual direction of flow relative to the train, the pressure tends to be greater alongside the nose than alongside the tail.

For the idealized (constant cross-section) train, variations in the pressure acting on the sides of the train have no direct influence on the drag—because there are no components in the direction of movement. However, the resulting difference in pressure on the nose and tail surfaces can have a substantial effect; that is skin friction on the lateral surfaces gives rise to pressure drag on the leading and trailing surfaces. In this paper, this contribution to the drag is called the *friction pressure drag.* It exists because of friction, but its effect is an additional contribution to pressure drag.

2.2.4 Terminology

The definitions of skin friction drag and pressure drag given in Section 2.1.1 are equally valid for travel in the open air or in tunnels. In the case of skin friction drag, the origins are essentially the same in both cases. In the case of pressure drag, however, there are important differences. In tunnels, it includes (a) inertia drag and (b) friction pressure drag as well as local pressure drag. Only the last of these is important at steady speeds in the open air.

3 DEFINITION OF EMPIRICAL PARAMETERS

It is helpful to use dimensionless coefficients to describe fluid phenomena that cannot be represented exactly by theoretical means. In the case of air flows in railway tunnels, the most important examples are *skin friction*

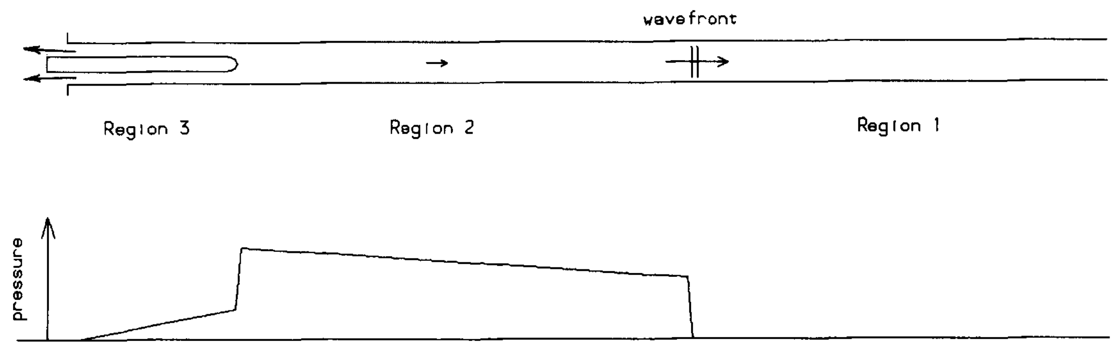


Fig. 1 Pressure distribution during train entry

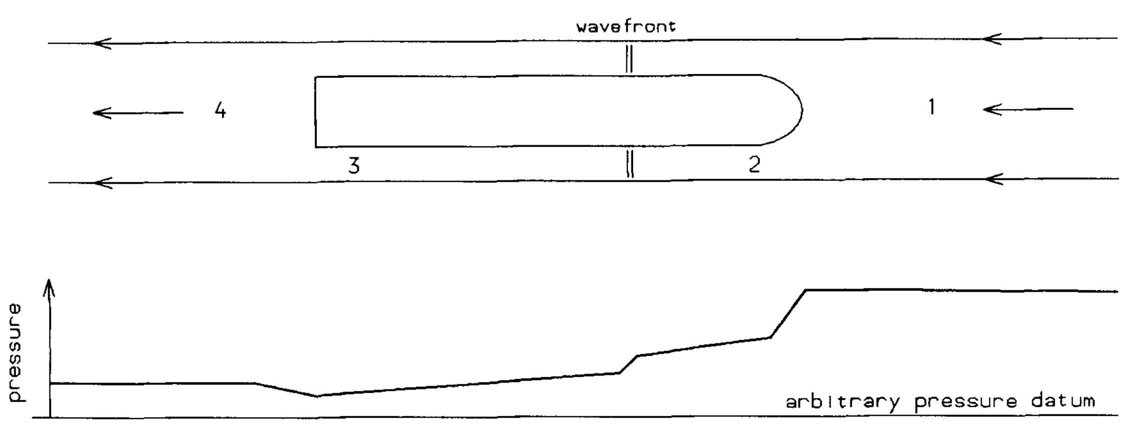


Fig. 2 Near-field flow relative to a train

coefficients, *stagnation pressure loss coefficients* and *drag coefficients*. These describe different aspects of identical phenomena and they can often be related theoretically. Typically, stress coefficients (friction and pressure) are preferred for analytical purposes and drag coefficients are used for practical assessments.

With one notable exception (pipe friction), stress coefficients in fluid flow are created by comparing a stress with the product $\frac{1}{2}\rho V^2$ in which ρ is a representative fluid density and V is a representative fluid velocity. In the case of railway tunnels, the choice of these representative parameters is usually straightforward. In the present paper, they are usually chosen at an appropriate flow section alongside the train, the velocity being the mean axial velocity in the cross-section. Train coefficients are based on velocities relative to the train. Tunnel coefficients are based on velocities relative to the tunnel. Force coefficients are usually defined using $\frac{1}{2}\rho A V^2$, where A denotes a representative area. For present purposes, use is made of cross-sectional areas and surface areas of the tunnel and the train.

Although the meanings of these areas seem intuitively obvious, there is no universally agreed method of defining them. Indeed, many authors implicitly assume that they may be defined arbitrarily. Unfortunately, however, this is physically unacceptable in calculations of air flows.

3.1 Geometrical parameters

3.1.1 Tunnels

Usually, the *cross-sectional area* of a tunnel can be deduced with sufficient accuracy from geometrical considerations alone. It may be necessary to make appropriate allowances for obstructions such as cables, signals and other equipment, but the resulting uncertainties typically amount to only a small percentage of the final value.

The *average* cross-sectional area of a tunnel could usefully be defined as the volume of contained air divided by the tunnel length. In this context, air included in the volume must be free to flow along the tunnel, not trapped in containers. This is because the mass of air that can be accelerated by trains is of interest. Although trapped air is excluded from the calculation of average area, it may nevertheless influence pressures and velocities. Provided that it is not in a sealed container, it will contribute to compressibility—and hence to wave speeds—and it may have a damping effect.

The *average cross-sectional perimeter* can be defined in a similar manner, but there is greater uncertainty. In the simplest case of a smooth, constant-area tunnel, it is the surface area of the designated air volume divided by the tunnel length. More generally, where surface irregularities are to be regarded as roughness elements, only the area in the tangential plane of the mean surface is included.

Uncertainties in the definition/interpretation of the cross-sectional perimeter will rarely be of practical importance, because the perimeter always appears in calculations as a product with the skin friction coefficient. The numerical value of the latter is usually a relatively crude estimate, with a typical accuracy of ± 20 per cent (or worse).

3.1.2 Trains

The train *cross-sectional area* is not easy to define and is even less easy to measure. Since the aerodynamic influence of the train is being dealt with, the appropriate definition is its *aerodynamic area*. The average area of a train or an individual coach/wagon can be defined as its enclosed volume divided by its length. In this context, any air included in the volume must be trapped air that is carried along with the train. Air that is free to move relative to the train is *not* included. This is because it is the displacement effect of trains that is being investigated. Trapped air contributes to the blockage effect as well as to compressibility. Air inside open-topped wagons and air between coaches should normally be regarded as trapped.

The average cross-sectional perimeter of a train may be defined in a similar manner to that of a tunnel. In the simplest case, it is the surface area of the enclosed volume divided by its length. More generally, however, complications arise from complex surfaces such as wheels, bogies and boxed equipment beneath the train body.

3.1.3 Blockage ratio

The ratio of the train and tunnel areas is used in many equations describing train-induced air flows in tunnels. It is known as the *blockage ratio*, β_z, and is defined by

$$\beta_z \equiv \frac{A_z}{A_t} \tag{1}$$

where A_z and A_t denote the areas of the train (zug) and tunnel respectively. In the past, greater use was made of the inverse of this ratio than of β_z itself, but this practice has almost disappeared in recent years. With this notation, the area of the annulus around a train in a tunnel can be expressed as $(1 - \beta_z)A_t$.

3.1.4 Alternative definitions

Historically, many different definitions have been used for the cross-sectional area of a train. These have ranged from a low estimate—the area of the passenger compartment—to a high estimate—the product of the maximum width and the maximum height above rail level. Arbitrary definitions such as these have the great advantage of being easy to evaluate, but the great disadvantage of having no obvious relationship to the true aerodynamic area. If they are used to deduce empirical coefficients for drag and pressure, etc., the resulting values will not be directly meaningful in other tunnels.

Even if attention is restricted to one particular train in one particular tunnel, it is *not* valid to use an arbitrary definition of the train area in air flow calculations. This is because the volume of air displaced by the train has to be represented correctly. Difficulties inherent in determining this volume do not justify resorting to alternative (false) definitions.

3.2 Stress coefficients

Attention now turns to the definitions of stagnation pressure loss coefficients, firstly those describing flows in very short lengths of tunnel (one or two diameters, say) and then distributed losses due to skin friction.

3.2.1 Local losses

At tunnel portals, the most suitable representative velocity is the mean axial velocity just inside the portal. The stagnation pressure loss is $k \times \frac{1}{2}\rho V^2$, where k is the local stagnation pressure loss coefficient. Typically, the loss coefficients for flows into and out of portals are about 0.6 and 1.0 respectively; that is the losses of stagnation pressure during inflow and outflow are typically $0.6 \times \frac{1}{2}\rho V^2$ and $1.0 \times \frac{1}{2}\rho V^2$ respectively. Assuming these values, the corresponding changes in *static* pressure are $1.6 \times \frac{1}{2}\rho V^2$ and zero respectively.

Similar coefficients can be defined at other discontinuities such as tunnel junctions or changes in cross-section. In many such cases, no general consensus exists about the most appropriate choice of representative velocities. Different choices are made in different publications [for example see references (**7**) and (**8**)]. It is important to inspect definitions carefully before making use of published data.

Loss coefficients at the nose and tail of a train are defined in a similar manner to those at tunnel portals. For this purpose, reference axes are chosen relative to the train, thereby achieving quasi-steady flow conditions at any particular instant throughout most of the journey.

Consider the conditions at a nose (see Fig. 3). There are two possible choices of a reference velocity, namely V_1^* or V_2^*, in which an asterisk denotes axes relative to the train. Either velocity is suitable in principle, but it is usual to choose the latter. This is consistent with the choice of reference velocity for inflows through a tunnel portal. Assuming this choice, a nose loss coefficient k_N implies a stagnation pressure loss of

$$(\Delta p_0)_{\text{nose}} = k_N \times \tfrac{1}{2}\rho V_2^{*2} \tag{2}$$

The stagnation pressure loss behind the tail (Fig. 4) can be treated similarly. The use of the upstream velocity V_3^* (in the annulus) in preference to the downstream velocity V_4^* is consistent with the choice of reference velocity for outflows through a tunnel portal. In this case, a tail loss coefficient k_T implies a stagnation pressure loss of

$$(\Delta p_0)_{\text{tail}} = k_T \times \tfrac{1}{2}\rho V_3^{*2} \tag{3}$$

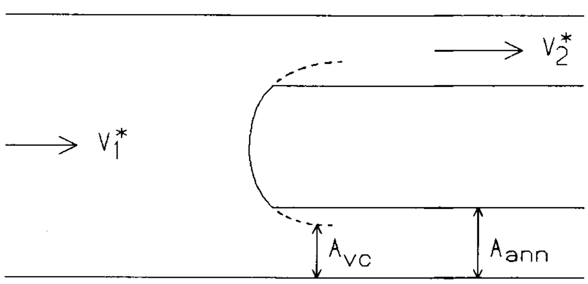
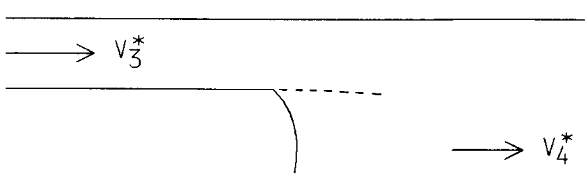


Fig. 3 Flow relative to a train nose



3.2.2 Skin friction

The method used to define skin friction coefficients is similar to that used to define pressure loss coefficients, except that the stress to be made dimensionless is a shear stress, not a pressure difference. Typically, a friction coefficient f implies a local wall shear stress of $\tau_w = f \times \frac{1}{2}\rho V^2$, where V is the local mean velocity relative to the surface on which the shear stress acts. At any particular cross-section i in the annulus around a train, the shear stresses on the tunnel and train surfaces are

$$\tau_{wt,i} = f_t \times \tfrac{1}{2}\rho V_i^2 \tag{4}$$

and

$$\tau_{wz,i} = f_z \times \tfrac{1}{2}\rho V_i^{*2} \tag{5}$$

respectively.

A different friction coefficient is sometimes used. Denoting this variant by λ, the shear stress τ_w on the wall of the tunnel, say, is equal to $\lambda\, \rho V^2/8$. The origins of this definition lie in the special case of steady, turbulent flows in pipes of circular cross-section. No further use is made of it herein.

Although the dominant direction of flow in the annulus is parallel to the tunnel axis, the magnitude varies strongly between the train and tunnel surfaces. In two-dimensional calculations [for example see references (**9**) and (**10**)], the flow may be regarded as two boundary layers, one adjacent to the tunnel wall and one adjacent to the train. In this case, it is appropriate to use mean velocities in the individual boundary layers in the evaluation of equations (4) and (5). This is not possible in one-dimensional analyses and so the average velocity over the whole section is used for both surfaces. By implication, the numerical values of the friction coefficients will be different for the two types of analysis.

Whichever definition is used, the outcome is a wall shear stress that can be used to determine the shear force resisting a steady flow along the tunnel (or train) surface. There may also be an unsteady contribution to skin friction [see reference (**11**), for example], but this is neglected herein.

3.3 Force coefficients

The preceding parameters are sufficient for most calculation purposes, but it is also useful to define *drag* coefficients. These provide an alternative way of expressing the results of calculations or measurements.

3.3.1 Local pressure drag

Local pressure drag coefficients can be defined at all absolute quantity. It is more common to use the excess of the actual force over a theoretically achievable minimum.

As an example, consider the train nose shown in Fig. 3. If the numerical value of the nose loss coefficient is known, the pressure difference $(p_1 - p_2)$ can be determined using the Bernoulli equation. It is a minimum when the nose coefficient is zero.

Now compare two cases, namely a 'real' train with a finite nose loss coefficient and an 'ideal' train for which the nose loss coefficient is zero. For the purposes of comparison, the velocities at sections 1 and 2 and the pressures at section 2 are chosen to be the same in the real and ideal cases. The pressures at flow section 1 are *not* the same, however.

The real pressure p_1 exceeds the ideal pressure $p_{1,\,\text{ideal}}$. Since the conditions at section 2 are stipulated to be identical for the two trains, the whole of the additional force, namely $(p_1 - p_{1,\,\text{ideal}})A_1$, must be resisted by the train nose. This is the force used in the definition of the nose drag coefficient.

The definition of the nose drag coefficient requires the choice of a reference area and a reference velocity. The author prefers to use the train area and the annulus velocity V_2^*, but it is more common to use the train area and the open tunnel velocity V_1^*. In this case, the definition is

$$C_{\text{DN}} \equiv \frac{p_1 - p_{1,\,\text{ideal}}}{\frac{1}{2}\beta_z \rho V_1^{*2}} \tag{6}$$

A similar method is used to define the tail drag coefficient. If the train area and the open tunnel velocity V_4^* are used as reference values (see Fig. 4), the definition is

$$C_{\text{DT}} \equiv \frac{p_{4,\,\text{ideal}} - p_4}{\frac{1}{2}\beta_z \rho V_4^{*2}} \tag{7}$$

The choice of reference values for these coefficients is arbitrary and there is no universally accepted convention to fall back on. For example, use is sometimes made of the tunnel area instead of the train area. In this case, the blockage ratio β_z disappears from the above definitions. It is essential to inspect definitions of drag coefficients before making use of published numerical values.

3.3.2 Skin friction drag

It is not usual to express friction forces in terms of drag coefficients, but this is sometimes done as a step towards the derivation of an overall drag coefficient. Consider, for instance, the skin friction force F_{wz} on a suitable length of the train. If the reference area and velocity are chosen as the lateral surface area and the local annulus velocity relative to the train, then the skin friction drag coefficient C_{DF} will be identical to the skin friction coefficient f_z defined by equation (5) (see also Section 4.3). In practice, however, the chosen reference area is more likely to be the cross-sectional area of the train. In this case, the definition may be expressed as

$$C_{\text{DF}} \equiv \frac{F_{\text{wz}}}{\frac{1}{2}A_z \rho V_{\text{ref}}^{*2}} \tag{8}$$

The reference velocity V_{ref}^* has been left undefined in this expression. Logically, the most appropriate choice is the local velocity in the annulus relative to the train. Practically, however, the relative velocity in the open tunnel ahead of the train is often chosen. When the flow may be regarded as quasi-steady, no ambiguity results with either of these choices. When pressure waves exist, however, no simple relationship exists between the two velocities and so no meaningful values can be deduced from equation (8) if the open tunnel velocity is used.

3.3.3 Overall aerodynamic drag

The drag coefficient for a complete train may be defined as the total aerodynamic drag force F_z on the train divided by an appropriate reference value of $\frac{1}{2}\rho A V^2$. The choice of the reference values will depend upon the purpose for which the coefficient is being defined. Usually the choices will be the same as those discussed above for skin friction drag. Typically, use might be made of the train area and the air speed in front of the nose (location 1 in Fig. 2). In this case, the overall drag coefficient will be defined by

$$C_{\text{DZ}} \equiv \frac{F_z}{\frac{1}{2}A_z \rho V_1^{*2}} \tag{9}$$

In common with the skin friction drag coefficient, the overall drag coefficient is *not* a constant—even for a particular train in a particular tunnel. It varies rapidly during train entry and train exit and when the train passes other discontinuities in the tunnel—ventilation shafts, cross passages, etc. Also, like the (local) skin friction drag, it varies whenever pressure waves exist because no direct relationships exist between the chosen reference velocity and the actual velocities local to the various sources of drag. In short, its ability to convey accurate information is severely limited.

4 USE OF EMPIRICAL PARAMETERS

The use of the various train coefficients is now illustrated and sources of potential uncertainty are clarified.

4.1 Train nose

Figure 3 depicts flow past a train nose. For generality, the boundary layer is assumed to separate from the train surface, creating a vena contracta with a smaller effective cross-section than the annulus surrounding the train. Assuming quasi-steady conditions in the immediate neighbourhood of the nose and neglecting compressibility effects for simplicity, the continuity equation may be written as

$$\rho A_1 V_1^* = \rho A_2 V_2^* \tag{10}$$

and the extended Bernoulli equation is

$$(p_1 + \tfrac{1}{2}\rho V_1^{*2}) - (p_2 + \tfrac{1}{2}\rho V_2^{*2}) = k_{\text{N}} \times \tfrac{1}{2}\rho V_2^{*2} \tag{11}$$

in which k_{N} is the nose loss coefficient defined in Section 3.2.1.

These equations can be combined to give a relationship between the pressure difference $(p_1 - p_2)$ and the rate of flow. On eliminating the velocity V_1^*, for example, the following equation is obtained:

$$p_1 - p_2 = \tfrac{1}{2}\rho V_2^{*2}(k_{\text{N}} + 2\beta_z - \beta_z^2) \tag{12}$$

in which $\beta_z \equiv A_z/A_t$ is the train tunnel blockage ratio.

4.1.1 Contributions of k_N and β_z to $(p_1 - p_2)$

Equation (12) shows that the magnitude of the pressure difference $(p_1 - p_2)$ at the train nose increases with increasing k_N and with increasing β_z. This important result is intuitively obvious, but it has an unfortunate consequence, namely that the contributions of the two parameters cannot easily be disentangled.

Suppose that the blockage ratio β_z is known for a particular train journey and that the pressure difference $(p_1 - p_2)$ and the velocity V_2^* are measured. In this case, the value of the nose loss coefficient k_N can be deduced from equation (12). In principle, this is a reliable method of deducing k_N; it is closely related to a popular method in common use.

Now suppose that the blockage ratio used in the above process is incorrect. In this case, the above procedure will lead to an incorrect value of k_N (too small if the assumed β_z is too large, and vice versa). For the purpose of representing the pressure difference at the nose, the two errors will be mutually self-compensating, but they will be errors nonetheless. The percentage error in β_z will lead to bigger proportional errors in the deduced nose drag coefficient and will confuse attempts to quantify the dependence of k_N on β_z.

The inevitable conclusion is that every effort should be made to use the 'correct' value of β_z. This reinforces the argument in Section 3.1.4 that it is not acceptable to define cross-sectional areas in an arbitrary manner.

4.1.2 Relationship between k_N and C_{DN}

The relationship between the nose loss coefficient and the nose drag coefficient can be deduced from a comparison of equations (6) and (11). The *additional* pressure change arising because of k_N is seen from equation (11) to be $k_N \times \frac{1}{2}\rho V_2^{*2}$. On substituting this into equation (6), the following equation is obtained:

$$C_{DN} = \frac{k_N}{\beta_z(1 - \beta_z)^2} \tag{13}$$

The existence of the term $(1 - \beta_z)^2$ in this expression is a consequence of using different reference velocities in the definitions of the two coefficients. If V_2^* were used instead of V_1^* in the definition of the drag coefficient [equation (6)], the relationship would be $C_{DN} = k_N/\beta_z$.

A simple, but important, conclusion can be drawn from equation (13), namely that it is not possible for *both* k_N and C_{DN} to be independent of the blockage ratio (unless both are equal to zero). If one is assumed to be independent of β_z, then equation (13) shows that the other will depend on β_z. In practice, neither is completely independent of β_z, but the loss coefficient is less dependent than the drag coefficient; that is the loss coefficient is more closely representative of the train itself whereas the drag coefficient is representative of the train in a particular size of tunnel.

4.1.3 Predicted values of k_N

An approximate theoretical method of estimating the dependence of k_N on β_z is given in an accompanying paper (Part 2 which follows). It is shown therein that k_N is small (less than 0.1, say) for all trains with reasonably streamlined noses. The nose illustrated in Fig. 3 is not well streamlined.

4.2 Train tail

The flow past the tail of a train can be analysed similarly. Briefly, using the notation in Fig. 4, the continuity and extended Bernoulli equations are

$$\rho A_3 V_3^* = \rho A_4 V_4^* \tag{14}$$

$$(p_3 + \tfrac{1}{2}\rho V_3^{*2}) - (p_4 + \tfrac{1}{2}\rho V_4^{*2}) = k_T \times \tfrac{1}{2}\rho V_3^{*2} \tag{15}$$

and so the pressure difference may be expressed as

$$p_3 - p_4 = \tfrac{1}{2}\rho V_3^{*2}(k_T - 2B_z + \beta_z^2) \tag{16}$$

4.2.1 Contributions of k_T and β_z to $(p_3 - p_4)$

By inspection, the contributions of k_T and β_z to the pressure difference $(p_3 - p_4)$ are interlinked in a similar manner to their counterparts at the nose. There is one important difference, however, namely that $(p_3 - p_4)$ is usually negative and so moderate increases in k_T tend to *reduce* the pressure difference. The drag increases, but the pressure difference decreases—offering a potential advantage from the perspective of passengers experiencing pressure waves.

4.2.2 Relationship between k_T and C_{DT}

The relationship between the loss coefficient k_T used in equation (3) (reference velocity V_3^*) and the drag coefficient C_{DT} defined by equation (7) (reference velocity V_4^*) is

$$C_{DT} = \frac{k_T}{\beta_z(1 - \beta_z)^2} \tag{17}$$

4.2.3 Predicted values of k_T

An approximate method of estimating the dependence of k_T on β_z is given in an accompanying paper (Part 2). It is shown that k_T is usually less than or equal to β_z^2.

4.3 Skin friction

When the train and tunnel skin friction coefficients are known for any particular journey, their use in numerical or analytical methods of prediction is straightforward. Strictly, they are not constants, but vary with the Reynolds number. In practice, however, the Reynolds numbers in practical train/tunnel cases are sufficiently large for such variations to be small in comparison with uncertainties in the estimated values of the friction coefficients themselves.

Although drag coefficients are rarely used explicitly in the description of skin friction, their relationship with conventional skin friction coefficients is included for completeness. The skin friction force on a length Δx of train is

$$F_{wz} = \tau_{wz} l_z \Delta x \tag{18}$$

where τ_{wz} is the local wall shear stress on the train surface and l_z is the length of the train cross-sectional perimeter. On substituting equation (18) into the definition of the skin friction drag coefficient [equation (8)] and replacing τ_{wz} by $f_z \times \frac{1}{2}\rho V_{ann}^{*2}$, the following equation is obtained:

$$C_{DF} = \frac{l_z \,\Delta x}{A_z} \frac{V_{ann}^{*2}}{V_{ref}^{*2}} f_z \qquad (19)$$

If the reference velocity is chosen in the open tunnel just ahead of the train and the flow is steady and incompressible, the ratio of the two velocities (both chosen relative to the train) is $(1 - \beta_z)$ and so the relationship between C_{DF} and f_z becomes

$$\text{Special case 1:} \qquad C_{DF} = \frac{l_z \,\Delta x}{A_z(1 - \beta_z)^2} f_z \qquad (20)$$

Alternatively, if the reference velocity is defined to be the same as V_{ann} and the reference area is defined to be equal to $l_z \,\Delta x$, then the relationship reduces to

$$\text{Special case 2:} \qquad C_{DF} = f_z \qquad (21)$$

which is directly equivalent to the relationship used for flows in pipes (or open tunnels).

4.4 Comparison of magnitudes of local pressure drag and friction drag

It is useful to compare typical magnitudes of the local pressure drag at the nose and tail of a train with the drag caused by skin friction. The local pressure drag at the nose and tail is

$$\text{Drag}_{local} = (k_N + k_T) \times \tfrac{1}{2}\rho V_{ann}^{*2} A_t \qquad (22)$$

and the total drag due to skin friction is

$$\text{Drag}_{friction} = f_z \times \tfrac{1}{2}\rho V_{ann}^{*2} l_z L_z\left(1 + \frac{A_t}{A_{ann}}\right) \qquad (23)$$

which is the sum of the shear force on the train surfaces F_{wz} and the consequential friction pressure drag $F_{wz}(A_t/A_{ann})$.

To make a meaningful comparison, it is necessary to consider typical numerical values for the various parameters. Suppose, for example, that $f_z = 0.0035$, $k_N = 0.05$, $k_T = 0.02$, $l_z = 12$ m, $L_z = 300$ m, $A_t = 70$ m^2 and $A_{ann} = 60$ m^2. With these values, the total drag due to friction is about 5.5 times as large as the local pressure drag.

Although these values are only indicative, it is reasonable to conclude that neither type of drag should be neglected universally in comparison with the other. With long, well-streamlined trains, friction drag will dominate, whereas the opposite could be true for very short, poorly streamlined trains.

4.5 Other losses

In this section, attention has been focused on the nose, tail and skin friction coefficients of the train because these usually have the greatest influence on train drag. When required, similar analyses can be developed at other train discontinuities—inter-coach boundaries, steps in the areas between locomotives and coaches, etc. Alternatively, the effects of such discontinuities might be allowed for pragmatically by increasing the assumed skin friction coefficient.

No account has been taken of tunnel resistance coefficients in this section. Tunnel resistance must be considered, however, before the magnitudes of the various drag forces can be deduced from equations (22) and (23), etc. Otherwise, the value of the velocity V_{ann}^* will be unknown.

5 ACCELERATIONS AND PRESSURE WAVES

So far, attention has focused on flows that may be regarded as steady in some sense. In a full analysis, however, it is also necessary to allow for strong influences of accelerations. Ideally, account should be taken of pressure waves generated during train entry and exit. Consider the conditions depicted in Fig. 1 during the entry of a train to a tunnel. There are three regions of flow bounded by the entrance portal, the train nose, the nose entry wavefront and the exit portal:

1. *Region 1* is the undisturbed flow ahead of the wavefront. The conditions within it have not yet been influenced by the train entry. In the simplest case, the air will be stationary and the pressure will be constant.
2. *Region 2* is between the nose and the nose entry wavefront. Usually, the direction of air flow is the same as that of the train, but the velocity is much smaller. A small part of the pressure gradient in this region is due to local skin friction. Most of the pressure gradient is due to waves generated continuously during train entry because of frictional resistance to flow along the annulus. The greater the length of train in the tunnel, the greater this resistance.
3. *Region 3* is alongside the train. The mean direction of flow is usually from the nose towards the portal (although this can reverse if the train is sufficiently long). The pressure gradient is due to (a) skin friction on the train and tunnel surfaces and (b) axial accelerations. The latter are commonly (but unjustifiably) neglected when interpreting full-scale measurements.

5.1 Pressure drag

Figure 1 provides a simple illustration of the influence of inertia. Although the wavefront is far ahead of the train, it has a strong influence on the pressure just in front of the train. This is most easily seen by imagining an equivalent case with no skin friction. In such a case, the pressure in region 3 alongside the train would be zero (assuming the usual portal exit loss) and that in region 2 in front of the train would equal the magnitude of the nose entry wavefront. The pressure on the nose would remain constant until some other disturbance arrived (for example the tail entry wavefront or a reflection of the nose entry wavefront).

In Section 2, the pressure drag was subdivided into local pressure drag, friction pressure drag and inertia drag. An inspection of Fig. 1 shows, however, that these classifications are somewhat arbitrary. It is not possible to state unambiguously what proportion of the pressure in front of the nose is due to inertia and what is due to friction. If the assessment is undertaken in region 2, the pressure gradient will be attributed dominantly to inertia; if it is undertaken in region 3, the gradient will be attributed primarily to skin friction. Neither of these assessments is more correct than the other. In reality, the result is a balance between inertia and friction forces in the two regions considered together.

5.2 Quasi-steady conditions

Figure 1 can also be used to justify the use of quasi-steady loss coefficients. Consider the conditions at the nose, for instance. The pressure increases continuously as the train enters, but the difference in pressure across the nose changes much more slowly. The velocity relative to the train similarly changes only slowly; that is the local conditions are quasi-steady.

Similar arguments can be used at all other geometrical discontinuities and they can also apply, as a first approximation, in regions of distributed flow—in region 3, for instance. The pressure gradient in the annulus changes only slowly, as does the velocity of flow along it. Although unsteady skin friction forces exist in the annulus, they are usually much smaller than the quasi-steady contribution on the train surfaces.

As the train progresses through the tunnel, more and more pressure wavefronts exist and so the average lengths of the regions between them reduce. In compensation, however, their magnitudes tend to reduce as their number increases. Even at later stages of the train passage, therefore, the use of quasi-steady approximations is plausible for most of the coefficients.

In sufficiently long tunnels, the influence of discrete pressure waves eventually becomes very small. For practical purposes, it can be acceptable to imagine that conditions of constant, uniform velocity exist in front of, alongside and behind the train. Pressure gradients in these regions are almost constant. Such conditions rarely develop in real tunnels, but they are a useful asymptotic approximation for design purposes because they enable qualitative information to be deduced about the complete train/tunnel system [for example see reference (**2**)]. They are also the basis of the older methods of drag measurement in tunnels [for example see reference (**12**)].

6 OVERALL AERODYNAMIC DRAG

The overall drag on a train is obtained by summing the various contributions due to friction and pressure drag. In typical numerical analyses, the train is considered in several sections, in each of which the local flow conditions are assessed independently of the others. Calculations are repeated at successive instants throughout the train journey. With this methodology, the overall drag cannot be calculated as an isolated entity in its own right. Instead, it has to be deduced as the sum of the individual components. This reflects the physical reality.

Despite the sudden fluctuations that characterize the pressure histories, the overall aerodynamic drag on a single train in a simple tunnel rarely changes by more than about 20 per cent, except during entry and exit. For this reason—and also because of the long-term asymptotic condition in long tunnels—it is possible that use will continue to be made of overall drag coefficients. Nevertheless, they should not be regarded as properties of the train alone; they also depend upon its speed and upon the tunnel.

6.1 Variation of overall drag with blockage ratio

The overall drag does not depend in a simple way on the blockage ratio because each component behaves differently. The mathematical nature of the different behaviours can be seen by examining equations presented herein. The physical reasons for the differences are even more straightforward. At a given speed, inertial drag is influenced primarily by blockage, local drag is influenced primarily by shape and friction drag is influenced primarily by viscosity and surface roughness. The scaling laws for the three types of drag necessarily reflect these different origins.

More detailed information about the dependence of the various train coefficients on the blockage ratio is given in a subsequent paper in this Journal (Part 2).

7 CONCLUSIONS

1. The overall aerodynamic drag on trains in tunnels has been shown to be composed of individual contributions that can be expressed in a rational manner. Dimensionless coefficients that behave in a predictable manner have been defined for each component that depends on empirical information.
2. Although an overall aerodynamic drag coefficient can be defined and can be used for approximate calculations, it is a property of the complete train/tunnel system, not of the train alone. Moreover, it is not a constant, even for a particular train in a particular tunnel.
3. In tunnels, pressure drag includes contributions due to inertia and friction as well as to local flow separations analogous to those causing form drag in the open air.
4. For aerodynamic calculations, the relevant train area is its aerodynamic area, defined in Section 3.1.2. In contrast with historical practice, it is not acceptable to define train areas arbitrarily for this purpose. If an alternative definition is used for any reason, then its relationship with the aerodynamic area should be quantified.
5. Drag due to flow separations at the noses and tails of trains in tunnels is usually smaller than that due to skin friction—except for very short trains.
6. The principal factors governing aerodynamic drag on trains in tunnels are the same as those governing pressure fluctuations. It is sensible and convenient to use the same methods of analysis for both purposes.

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