A simple method to determine train running resistance from full-scale measurements

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Abstract: This article proposes a simple method to determine train running resistance. The resistance is determined by calculating the change in kinetic and potential energy of a coasting train between successive measurement positions. The strength of this method is that the measuring equipment needed is kept at a minimum and it is not limited to a track having a constant grade, thus making this method suitable, in particular, for long freight trains running in mountain areas. An error analysis is performed for this method and the probable error sources are discussed.

Keywords: coasting energy method, running resistance, full-scale testing, coasting, non-constant grade, ambient wind, measurement error

1 INTRODUCTION

To calculate the journey time and capacity and to determine energy requirements, it is important to know the total forces acting on a train against its direction of travel, i.e. running resistance. Throughout this article, running resistance means the sum of mechanical rolling resistance and aerodynamic drag.

There are a number of full-scale methods available to determine the mechanical rolling resistance and aerodynamic drag. They can be divided into three main groups.

(a) tractive effort methods;
(b) coasting methods;
(c) dynamometer or drawbar methods.

In the first method, measurements are taken with the train under traction. The total running resistance can be estimated by measuring the power drawn by each motor. With this method, it is necessary to know the total inertia of the train. The measurements should be undertaken at constant speed, as any speed change would lead to acceleration that would have to be corrected for [**1**], and the energy efficiency of the tractive vehicle(s) must be known for various conditions. It is also possible to measure the force applied on the wheel rims by means of strain gauges mounted on the transmission.

In the second method, coasting, the normal procedure is first to bring the train to a certain speed on a track with constant gradient. Before the test train's point of reference arrives at a measuring section, the drive and braking are reduced to zero. From that moment, speed, time, and position along the measuring section are determined and recorded. A coasting train on a level track experiences a decrease in the speed and kinetic energy, making it possible to estimate the running resistance indirectly by calculating the deceleration or directly by measuring the deceleration [**2**]. Some methods use inertia pendulum accelerometers to measure deceleration, thus avoiding the effects of track profile up to a gradient of 5‰. The pendulum system is sensitive to track faults and inclination of the test vehicle.

The third method makes use of a dynamometer or drawbar. For measuring mechanical rolling resistance due to rotation of wheels, transmission, and motors, it is reported in reference [**1**] that a dynamometer can be used, which is placed between a winch and a cable connected to the train or vehicle. The winch must pull the train smoothly and the track must be straight and of constant profile. A method covered in reference [**3**] makes use of a drawbar for measuring the resistance. The drawbar itself may contribute to precision and bias errors, and the running resistance is determined only for the vehicles following the drawbar, so that running resistance cannot be obtained for a whole train.

Independent of which method is used, precise data of track altitude and grades, with accurate determination of speed and position of the train along the track, are essential.

Coasting methods include the resistance originating from losses in transmission and motors and they do not have the drawbacks of the tractive effort methods which have more error sources.

It is generally accepted that running resistance, F_R, can be expressed as a function of speed, v, described by the following formula

$$F_R = A + Bv + Cv^2 \tag{1}$$

where coefficients A and B mainly originate from mechanical rolling resistance [**4**, **5**] and coefficient C originates from aerodynamical drag.

A review and comparison of different formulae for calculating train resistances are made by Rochard and Schmid [**6**]. The authors also indicate that the preferred method for determining running resistance is the coasting method.

The main purpose of this article is to describe, analyse, and suggest an alternative coasting and evaluation method from what is reported in references [**1–3**] and [**7**] for determining train running resistance. The method for obtaining the total running resistance as presented here is similar to the method reported by Dayman [**7**]. However, there is a difference in the evaluation method for obtaining A, B, and C coefficients and how the effects of ambient wind are treated. In reference [**7**], coefficient C is obtained from wind tunnel tests and not from the full-scale masuremets.

The alternative coasting method together with the evaluation method presented in this article is named as the coasting energy method.

This article begins with a description of the governing equations. Thereafter, an error analysis is performed on this method and tested on a freight train application.

2 BASICS OF THE COASTING ENERGY METHOD

2.1 General

Train running resistance, F_R, is associated with energy dissipation. This dissipation equals the work done by F_R over a travelled distance.

If a coasting train's kinetic and potential energy is determined at successive measuring positions, $x_1, \ldots, x_a, \ldots, x_{a+k}, \ldots$, along the track separated by a measuring distance $\Delta x = x_{a+1} - x_a$ from each other, as shown in Fig. 1, the difference in the train's total energy between these measuring positions can be calculated. The energy must be determined with respect to some point of reference of the train.

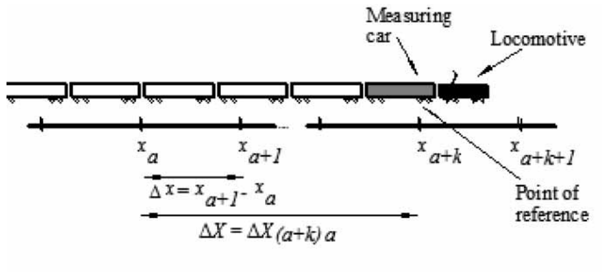

Fig. 1 Measuring positions along the track

The energy dissipation corresponding to the mean running resistance, $\bar{F}_{R_{(a+k)a}}$, can be calculated between two arbitrary measuring positions, x_a and x_{a+k}, separated by the evaluation distance of $\Delta X = \Delta X_{(a+k)a}$. This is advantageous for reducing output data scatter, which reduces as ΔX increases.

The mean running resistance is calculated from the energy balance equation

$$E_{\text{kin}_a} + E_{\text{pot}_a} = E_{\text{kin}_{a+k}} + E_{\text{pot}_{a+k}} + \bar{F}_{R_{(a+k)a}} \Delta X_{(a+k)a} \tag{2}$$

Provided that no energy is supplied from any external source and all vehicles of the train have the same speed. In order to estimate a train's potential energy at a measuring position, the train mass has to be distributed over the length of the train. The track altitude, h, at each vehicle's local point of reference in the train must be determined.

The total mean running resistance for a coasting train and the corresponding mean speed, $\bar{v}$, between two arbitrary measuring positions, x_a and x_{a+k}, is calculated by

$$\bar{F}_{R_{(a+k)a}} = \frac{1}{\Delta X_{(a+k)a}} \left(\sum_{i=1}^{n_v} m_i \left(\frac{1}{2}(1 + H_i) \times (v_a^2 - v_{a+k}^2) + g(h_{i_a} - h_{i_{a+k}}) \right) \right) \tag{3}$$

$$\bar{v}_{(a+k)a} = \frac{\Delta X_{(a+k)a}}{\Delta t_{(a+k)a}} \tag{4}$$

where i is the vehicle number, n_v the total number of vehicles in the train set, m_i the mass, and H_i a factor accounting for the rotary inertia of vehicle i. $\Delta t_{(a+k)a}$ is the travel time between measuring positions x_a and x_{a+k}.

By shifting $\Delta X_{(a+k)a}$ forward, $a = 1, \ldots, j$, along the measuring section by Δx, the mean running resistance with its corresponding mean speed can be estimated successively for each ΔX until the end of the measuring section is reached, or the train stops by itself.

In this way, j number of data pairs of $(\bar{v}, \bar{F}_R)$ is formed. The data pairs can then be used for fitting a polynomial having the form of equation (1) for each test run by means of the method of least squares. The error in the calculated mean running resistance is discussed in section 3.

2.2 Minimizing ambient wind influence

While measuring the wind direction and wind speed in the ambience of the test section, tests are run in both directions along the measuring section, thus forming the first pairs of fitted polynomials. Test runs in such a pair shall have about the same wind conditions with respect to the measuring section.

From each pair of fitted polynomials, a mean polynomial, thus mean running resistance, is determined for the two wind conditions.

Finally, all of the mean polynomials for the same train consist are averaged, thus forming a second mean polynomial. This results in wind averaged values of coefficients A, B, and C; thus the wind minimized running resistance is estimated. However, this is not exactly the same as running resistance for zero wind conditions, mainly due to the following reasons.

1. The magnitude of the increase in the aerodynamic drag of a train due to ambient wind, when running in one direction, is not necessarily the same as the decrease in the aerodynamic drag when running in the opposite direction, even though the wind conditions are exactly the same [**8**].
2. The average of the aerodynamic drag in side wind is not the same as at zero wind conditions, as the aerodynamic drag may increase considerably at large side wind angles.

3 PRECISION AND BIAS ERRORS IN ESTIMATED RUNNING RESISTANCE

A result based on experiments, such as field tests, is always associated with errors due to faulty data. These errors can be divided into two main groups [**9**].

1. Precision errors that are random and can have different signs and values for each successive measurement. They can be treated using statistical techniques.
2. Bias errors that are systematical and occur in the same way each time, often resulting in an offset from the true value.

3.1 Precision errors

Precision errors are due to the determination of:

(a) speed, v, from which also the evaluation distance can be calculated;
(b) each vehicle's altitude, h_i.

An estimation of the maximal error due to precision errors can be made by analytical means. By differentiating equation (3) with respect to v and h as shown in equation (5), it is possible to approximately estimate the maximal error $\Delta\bar{F}_{Rp}$ due to precision errors in v and h for the determined mean running resistance

$$\Delta\bar{F}_{Rp} \approx \left|\frac{\partial \bar{F}_R}{\partial v}\Delta v\right| + \left|\frac{\partial \bar{F}_R}{\partial h}\sum_{i=1}^{n_v}\Delta h_i\right| \tag{5}$$

The relative maximal error due to precision errors can be calculated by

$$\varepsilon_p = \frac{\Delta\bar{F}_{Rp}}{\bar{F}_R} \tag{6}$$

The precision error in speed, Δv, is estimated from:

(a) error in measuring precision of about ± 0.002 m/s (L. Andersson, 1995, personal communication);
(b) variation in wheel radii due to wheel conicity and variation in track gauge, if an ordinary passenger car with conical wheels is used as measuring car. This results in an additional error in speed of approximately $\pm 0.1-0.2$ per cent [**10**].
(c) longitudinal oscillatory motions of the measuring car.

Longitudinal oscillatory motions of the measuring car, with respect to the train's mean speed, superimpose an error in determined running resistance. By assuming for a short speed interval, a linear decrease in the train's mean speed, the magnitude of the speed variations can be determined approximately. An average speed variation of the measuring car of approximately ± 0.05 m/s with respect to the train's mean speed is observed in some test results reported in reference [**11**].

The estimated precision error, Δh_i, in the altitude of each vehicle with respect to track data is approximately $+10$ and -80 mm (P. Zachrisson, 1995, personal communication). However, for an evaluation distance of $\Delta X = 100$ m, an error of -80 mm with respect to track data is considered to be unusual (E. Andersson, 1995, personal communication). Therefore, it is assumed that for $\Delta X = 100$ m, the error in the altitude of each vehicle's centre of gravity is $+10$ and -40 mm.

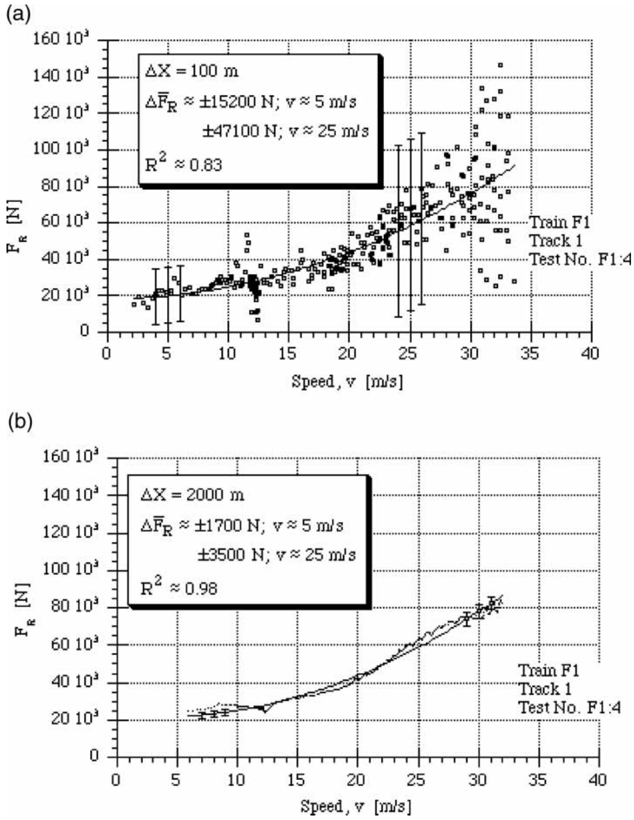

Fig. 2 Example of scatter and maximal error bars for estimated mean running resistance. Single test run with train F1 for (a) $\Delta X = 100$ m and (b) $\Delta X = 2000$ m [**11**, **12**]

The estimated values of maximal precision errors are given by

$$\Delta_{h_i} \approx \begin{cases} +10\,\text{mm}, & \text{for all } \Delta X \\ -40\,\text{mm}, & \Delta X = 100\,\text{m} \\ -80\,\text{mm}, & \Delta X \geqslant 1000\,\text{m} \end{cases}$$

$$\Delta v \approx \begin{cases} \pm 0.052\,\text{m/s for cylindrical measuring wheel} \\ \pm 0.05 + 0.002v\,\text{m/s for conical worn type measuring wheel.} \end{cases}$$

Some examples of scatter due to precision errors in test runs of a train are described in section 3.3 and shown in Fig. 2.

3.2 Bias errors

The bias errors in the evaluated running resistance are due to determination of:

(a) running resistance as a function of mean speed;
(b) train mass;
(c) rotary inertia.

The determination of running resistance as a function of mean speed for a given evaluation distance ΔX, gives a bias error. This is exemplified in the following. For simplicity, it is assumed that there are no gradients.

The work done by $F_{R_{(a+k)a}}$ as a function of speed along $\Delta X_{(a+k)a}$ must equal the decrease in total energy of the coasting train whose speed is in the range of $v_a, \ldots, v_{a+k}$. Hence, the calculated mean speed, $\bar{v}$, is given as

$$\bar{v}_{(a+k)a} = \frac{\Delta X_{(a+k)a}}{\Delta t_{(a+k)a}} \tag{7}$$

However, assuming the running resistance of the coasting train is given exactly by

$$F_R(v) = A + Bv + Cv^2 \tag{8}$$

then, for the calculated constant mean speed, the work done by $F_R(v = \bar{v})$ does not generally equal the dissipation according to the decrease in the energy. Instead by assuming a constant train speed, $v = \bar{v}_{corr}$, which satisfies equation (9) along $\Delta X_{(a+k)a}$, the work done by $F_R(v = \bar{v}_{corr})$ can be written as

$$(A + B\bar{v}_{corr} + C\bar{v}_{corr}^2)\Delta X_{(a+k)a} = \frac{m_T(1+H)}{2}(v_a^2 - v_{(a+k)^2}) \tag{9}$$

From equation (9), the constant speed, $\bar{v}_{corr}$, corresponding to the running resistance can be resolved as

$$\bar{v}_{corr} = \frac{B}{2C} \pm \sqrt{\left(\frac{B}{2C}\right)^2 - \frac{A}{C} + \frac{m_T(1+H)}{2C\Delta X_{(a+k)a}}(v_a^2 - v_{(a+k)^2})} \tag{10}$$

The error in determined running resistance, due to the difference between the determined mean speed and the corresponding speed, varies with train type, evaluation distance ΔX, and speed interval $v_a, \ldots, v_{a+k}$, which must be obtained from measurements.

The relative maximal bias error due to evaluation of running resistance as a function of mean speed can be calculated approximately by

$$\varepsilon = \frac{F_R(\bar{v})}{F_R(\bar{v}_{corr})} - 1 \tag{11}$$

Maximal error in determined running resistance due to the estimated train mass m_T and the term due to rotating masses $m_T H$ can be calculated in the same

Fig. 3 Hbis and Oms wagons forming test train F1

Table 1 Basic data of test train F1

Train	Configuration	Wagons	Axles	Total mass (tonnes)	Axel load (kN)	Train (m)
F1		36	72	1470	183	514

The locomotive is included in the train length and total mass but not in the mean axle load and number of axles.

way as the precision errors

$$\Delta\bar{F}_{\mathrm{Rb2}} \approx \left| \frac{\partial \bar{F}_{\mathrm{R}}}{\partial m_{\mathrm{T}}} \sum_{i=1}^{n_v} \Delta m_{\mathrm{T}i} \right| + \left| \frac{\partial \bar{F}_{\mathrm{R}}}{\partial m_{\mathrm{T}} H} \sum_{i=1}^{n_v} \Delta (m_{\mathrm{T}} H)_i \right| \tag{12}$$

The relative maximal error due to bias errors in mass and rotating inertia is calculated by

$$\varepsilon_{\mathrm{b2}} = \frac{\Delta\bar{F}_{\mathrm{Rb2}}}{\bar{F}_{\mathrm{R}}} \tag{13}$$

The error in determined train mass, m_{T}, is approximately ± 500 kg/vehicle. The estimated error in the term accounting for the rotary inertia $m_{\mathrm{T}}H$ is approximately ± 120 kg/wheelset [**11**], caused by different wheel radii due to wear.

The total relative maximal bias error is given as

$$\varepsilon_{\mathrm{b}} = \varepsilon_{\mathrm{b1}} + \varepsilon_{\mathrm{b2}} \tag{14}$$

3.3 Discussion and comparison of errors with a test run

In the following, the error analysis is applied and exemplified on a freight train consisting of an SJ Rc4 electric locomotive + measuring car + 36 wagons. The wagons are of the closed Hbis and open type Oms, alternated to form a mixed consist (Fig. 3). This mixed freight train is referred to as F1. The basic data of the test train are shown in Table 1. Running resistance coefficients together with their confidence intervals are shown in Table 2 [**11**].

Table 2 Running resistance coefficients together with their confidence intervals for train F1

Train	A (N)	B (Ns/m)	C (Ns^2/m^2)
F1	$15\,400 \pm 470$	279 ± 162	49.2 ± 2.1

The level of confidence is 95 per cent [**11**].

The relative confidence intervals for the A, B, and C coefficients are ± 3, ± 58, and ± 4 per cent, respectively. The relatively high confidence interval for coefficient B shows that its value is more uncertain and is difficult to measure with precision. Equation (1) is only an algebraic approximation of the running resistance. The parts of the running resistance that depend upon other powers of speed than those in equation (1) affect the values of the fitted A, B, and C coefficents.

Tests with this train configuration represent a relatively difficult case, due to scatter in output data caused mainly by ambient wind, in comparison with a smooth high-speed train.

The estimated relative maximal precision and bias errors, according to sections 3.1 and 3.2, are shown for comparison in Figs 4 and 5. The total relative maximal error is

$$\varepsilon_{\mathrm{T}} = \varepsilon_{\mathrm{p}} + \varepsilon_{\mathrm{b}} \tag{15}$$

An important effect of the evaluation distance ΔX is that precision errors, and thus the scatter around the fitted polynomial, decrease considerably for increasing values of ΔX, as shown in Fig. 4.

The total relative maximal error in determined running resistance decreases as the speed increases in the

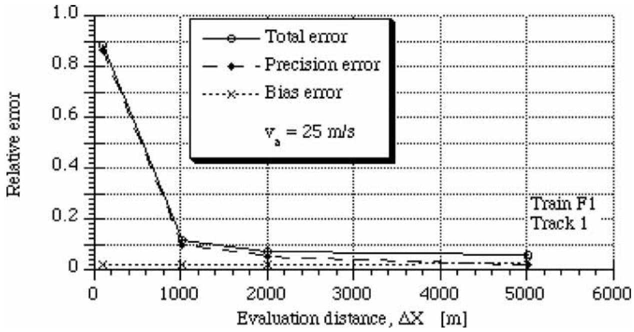

Fig. 4 Estimted relative maximal errors for train F1 for different ΔX at 25 m/s

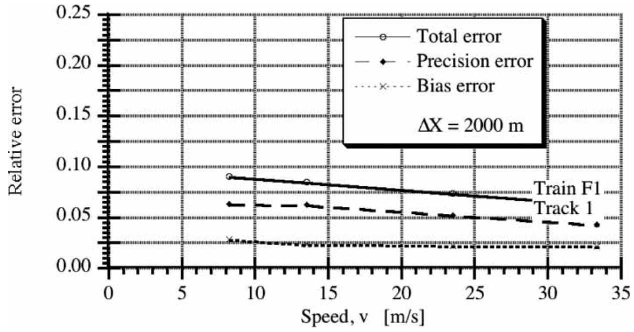

Fig. 5 Estimated relative maximal errors for train F1 at $\Delta X = 2000$ for different speed

evaluation (Fig. 5). This is because the running resistance increases with speed, thus making the relative precision error in equation (7) and relative bias error in equation (14) smaller.

These estimated maximal errors are shown in Fig. 2, as error bars around fitted polynomials from a single test run. The coefficient of determination, denoted by R^2, is a measure of how well the data are explained by the fitted polynomial. The higher the value of R^2, the better the fit.

A source of error difficult to control is the longitudinal relative motions of the vehicles in a train. This affects the estimation of the train speed, since the measured speed is in fact the speed of the measuring car; thus the determined running resistance is also affected. The big scatter in Fig. 2(a), in the speed interval of 30–34 m/s at the beginning of the test run, is most likely originating from the measuring car's longitudinal oscillatory motions as the locomotive's drive is cut off somewhat too late at the beginning of the test run.

The value of ΔX has another important influence upon the result of curve fitting with the method of least squares. As the value of ΔX increases, the number of data pairs, and the speed interval over which the running resistance polynomial is fitted, decreases.

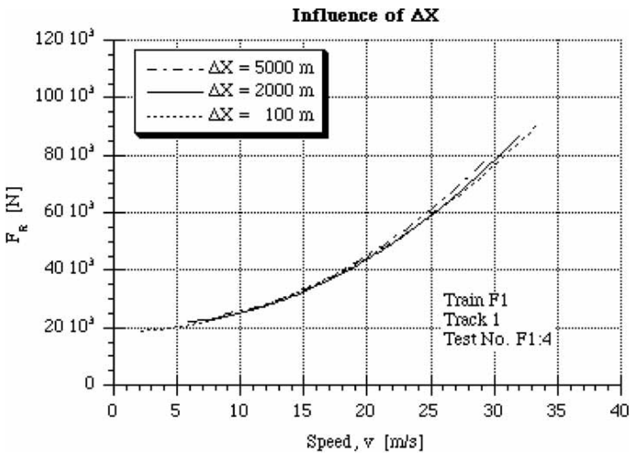

Fig. 6 Comparison between polynomials of estimated running resistance for the same test run with different ΔX [**11**, **12**]

This is exemplified by a test run in Fig. 6 evaluated with three different evaluation distances, $\Delta X = 5000$, 2000, and 100 m. A large value for ΔX may cause problems in attaining a correct curve fit for the running resistance, with respect to low and high speeds. The effect of the bias error due to the determination of running resistance as a function of mean speed is also clearly shown in Fig. 6.

From full-scale tests reported in references [**11**] and [**12**] with conventional loco-hauled passenger and various freight trains, it is determined by means of Student's- t-distribution that for a confidence level of 95 per cent the uncertainty in wind averaged running resistance is about ± 8 per cent, taken as an average for all speeds. However, from tests performed with passenger trains, a standard deviation of less than ± 3 per cent is achieved from individual test run pairs at 25 m/s. The full-scale tests were made by means of the coasting energy method.

4 CONCLUSIONS

The coasting energy method is a useful tool for estimating train running resistance. Running resistance of any train can be determined on any track regardless of gradients, provided the track altitude data are accurate. In comparison with other methods, it is not necessary to compensate in real time for acceleration due to gradients, thus making this method more simple to use.

Influence of ambient wind upon the determined running resistance can be minimized by averaging polynomials from test runs which are run in each direction along the same measuring section during similar wind conditions. The difference between the wind averaged coefficient C and the one evaluated for almost zero wind conditions is approximately less than ± 3 per cent for freight trains.

Theoretically, the total maximal relative error is approximately ± 8 per cent. From full-scale tests, a standard error of less than 3 per cent is achieved for the test run pairs. The method is sensitive, like all other methods, to precision errors in speed and determined track altitude. This can be coped with by choosing an appropriate evaluation distance, since scatter of the data from which a running resistance polynomial is fitted decreases with increasing evaluation distance. However, for obtaining enough output data for low speeds, a short evaluation distance is necessary.

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APPENDIX

Notation

A	resistance constant (N)
A_{f}	cross-section area of train (m^2)
B	resistance coefficient (N s/m)
C	resistance coefficient (Ns^2/m^2)
E	energy (Ws)
F	force (N)
$\bar{F}$	mean force (N)
g	gravitational acceleration (m/s^2)
h	absolute altitude of centre of gravity according to track profile (m)
H	relative factor accounting for rotary inertia
m	mass (kg)
n	number
R^2	coefficient of determination
t	time (s)
v	speed (m/s)
$\bar{v}$	mean speed (m/s)
x	measuring position (m)
$\Delta\bar{F}_{\mathrm{R}}$	maximal error in mean running resistance
Δh	precision error in altitude (m)
Δt	travel time between measuring positions (s)
Δv	precision error in speed (m/s)
ΔX	evaluation distance (m)
ε	maximal relative error

Subscripts

a	position index
b	bias
b1	bias due to speed

b2	bias due to mass and rotary inertia	kin	kinetic
corr	corresponding	p	precision
i	vehicle number	pot	potential
j	data pair number	R	running
k	increment	T	total

