Optimal control of a subway train with regard to the criteria of minimum energy consumption

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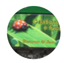
Ivan Meleshin
Moscow State University of Railway Engineering



TRINH LUONG MIEN
University of Transport and Communications

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ERRATA

Erratum to: "Optimal Control of a Subway Train with Regard to the Criteria of Minimum Energy Consumption" [Russian Electrical Engineering 82, 405 (2011)]

L. A. Baranov[a], *, I. S. Meleshin[a], and L. M. Trinh[b]

[a]*Russian University of Transport, Moscow, 127994 Russia*

[b]*University of Transport and Communications, Hanoi, Vietnam*

**e-mail: journal-elektrotechnika@mail.ru*

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Optimal Control of a Subway Train with Regard to the Criteria of Minimum Energy Consumption

L. A. Baranov, I. S. Meleshin, and L. M. Chin'

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Abstract—A mathematical formulation of the problem of determining energy optimal train control with regenerative breaking and its solution based on the maximum principle are presented. The results of analysis of the train model and train model errors influence on the optimal trajectory are presented. Examples of energy optimal trajectories on the lines of the Moscow and Hanoi Metro are shown.

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Subways consume a large amount of energy. In particular, the annual energy consumption of the Moscow Metro equals 1.8–2 bln kWh; almost 70% is spent for hauling operations. The increase of the subway length and traffic volume results in the increase of energy consumption up to 20 bln kWh per year [1]. This is why the development and implementation of energy saving technologies on subways are quite urgent.

Energy-saving technologies are connected with the decrease of energy consumption for hauling operations and, consequently, with the selection of energy optimal modes of train control. Many papers both in our country and abroad have been devoted to the task of energy optimal control of a train movement along the haul at a set period of time. Different mathematical methods of task solving are used (maximum principle, discrete variant of dynamic programming method, etc.) [2–5]. The implementation of a new generation of electric stocks, which use a traction drive based on asynchronous motors and provide regenerative, rheostat, and mechanical braking, made it necessary to solve the task of selection the energy optimal mode of train control in these conditions taking into consideration the limits of the phase coordinate. The full system of correlations for this task was studied by the authors in [6]. The problem of the *uncertainty* of the obtained solution still remains. In operation conditions the train weight is not defined exactly and the distribution of the regenerative energy between the consumers and rheostats (rheostat braking) depends on a significant number of factors, which were unknown before. This is why it is necessary to study the impact of partial lack of the prior information on the selection of the optimal control for different hauls. Hereinafter, hauls without steep declines and ascents will be called hauls with light profiles. The profile element, where the speed increases with the train inertial movement (slowing-down) is called a steep one (or harmful). If the train speed decreases at profile elements at maximum, this element is called a steep ascent. The hauls with steep declines and ascents are called the hauls with heavy profile.

FORMULATION OF THE PROBLEM OF OPTIMAL CONTROL [6]

In the differential equation of train movement in coordinates with speed v, the covered distance s is determined by the equation

$$\frac{dv}{ds} = \frac{\xi}{v}\left[\frac{F}{P+Q} - \omega_0(v) - \omega_g(s) - \frac{B}{P+Q}\right], \qquad (1)$$

where F is the traction effort; B is the summary braking effort, determined by the mechanical, rheostat, or regenerative braking; P is the weight of a locomotive or an electric train wagon; Q is the weight of a train or an electric train load; $\omega_0(v)$ is the specific net train resistance; $\omega_g(s)$ is a specific additional resistance to movement from slopes and curves; $\xi = \varepsilon/(1+\gamma)$, ε is a coefficient, including the dimensions of variables, preset in units that do not correspond to the International System of Units (SI); and γ is the inertial coefficient of the revolving masses.

The train is considered as an inextensible cord. The formula that determines the specific additional train resistance is the following:

$$\omega_g(s) = \sum_{i=i_1}^{i_2}\frac{g_i l_i}{l_c} + \sum_{j=j_1}^{j_2}\frac{630 l_j}{Rc_j l_c}, \qquad (2)$$

where i_1 and i_2 is an element profile number that includes the *front end* and the *rear end* of a train; g_i is the slope of the i-element; l_i is the length of the train within the limits of g_i; l_c is the train length; j_1 and j_2 are the numbers of the plan elements, including train parts; Rc_j is the track curve radius of a plan j-element; and l_j is the length of the train part on the Rc_j element.

We consider a task with the fixed ends: $s(0) = s_i$, $v(0) = v_i$, $s(T_{tm}) = s_f$; $v(T_{tm}) = v_f$, where s_i, s_f, v_i, and v_f are the ordinate and end coordinates of the haul and the initial and final speed; and T_{tm} is the time of train movement along the haul. Traction F and braking B efforts are limited; their maximum values are the functions of speed v, i.e.

$$0 \le F \le F_{max}(v); \quad 0 \le B \le B_{max}(v). \tag{3}$$

The braking effort B is determined by mechanical and electric (regenerative in general) braking $B = B_t + R$.

$$0 \le B_t \le B_{max}(v); \quad 0 \le R \le R_{max}(v). \tag{4}$$

The haul has the speed limits

$$0 \le v \le v_{max}(s). \tag{5}$$

Traction and braking efforts are the control means during train movement. The control limitations (maximum train efforts and braking) are functions of speed, which is also limited (so called combined limitations). Using the method from [2, 3], let us write the equation of train movement excluding the limitations;

$$\frac{dv}{ds} = \frac{1}{v}[u_f f_{max}(v) - u_b b_{tmax}(v) - u_r r_{max}(v) - \omega(v) - g(s)], \tag{6}$$

where $f_{max}(v) = \dfrac{\xi F_{max}(v)}{P+Q}$, $b_{tmax}(v) = \dfrac{\xi B_{tmax}(v)}{P+Q}$, $\omega(v) = \xi\omega_0(v)$, and $g(s) = \xi\omega_g(s)$.

The values u_f, u_b, and u_r are the controls; the control limitations are determined by the inequalities

$$0 \le u_f \le 1; \quad 0 \le u_b \le 1; \quad 0 \le u_r \le 1; \quad u_f u_r = 0. \tag{7}$$

The last equation is determined by the inability of the traction drive to function in the mode of traction and braking simultaneously. Since the right part of Eq. (6) depends on function s, the train, as a control object, becomes nonautonomous.

With T_{tm} set as a criterion of optimality, let us select the energy consumption for the traction:

$$A_e = \int_{s_i}^{s_f}\left[\frac{1}{\eta_t}F(s) - \eta_p k_b R(s)\right]ds, \tag{8}$$

where η_t is an efficiency factor in a traction mode, η_p is an efficiency factor in a regenerative mode, and k_b is a coefficient with the part of regenerative energy being dissipated on the train rheostats if there are no energy receivers.

Let us assume that η_t, η_p, and k_b are constant, so we write

$$A_e = \frac{P+Q}{\eta_t\xi}\int_{s_i}^{s_f}[u_f f_{max}(v) - \eta u_r r_{max}(v)](ds), \tag{9}$$

where $\eta = \eta_t\eta_p k_b$ is a returning coefficient of regenerative energy to the network.

The task of optimal control of the train movement along the haul is the following [2, 3]: for the train control object, determined by differential equation (6), including the regenerative braking, it is necessary to find controls u_f, u_b, and u_r with limitations (7) to minimize optimum criteria (9) at the time of the train movement along the haul T_{tm} and set limitations (5) for phase limitations (speed).

The time of the train movement along the haul at the selected independent variable s is determined by

$$T_{tm} = \int_{s_o}^{s_e}\frac{1}{v}ds. \tag{10}$$

Equation (10) is an isoperimetric condition for solving the optimization task. Taking into account (10), let us pass onto a general optimization criteria,

$$J^* = A_e + \lambda T_{tm}, \tag{11}$$

where λ is a Lagrange undetermined coefficient.

Substituting corresponding equations (9) and (10) to (11), we will obtain

$$J^* = \frac{P+Q}{\eta_t\xi}\int_{s_i}^{s_f}\left[u_f f_{max}(v) - \eta u_r r_{max}(v) + \frac{\lambda^*}{v}\right]ds, \tag{12}$$

where $\lambda^* = \dfrac{\lambda\eta_t\xi}{P+Q}$.

Let us study the physical meaning of the Lagrange undetermined coefficient. Let the optimal in the sense of criteria (11) dependence $v(s)$ be known, and let us expose it to variation. Since the first variation of the functional on the optimal trajectory equals 0,

$$dA_e + \lambda dT_{tm} = 0, \tag{13}$$

where dA_e and dT_{tm} are the operation variations of the traction effort and time of movement;

$$\lambda = -\frac{dA_e}{dT_{tm}}; \tag{14}$$

i.e., λ is a derivative of the dependency $A_e(T_{tm})$ when the traction effort is minimal for each value of T_{tm}. Since the dependence $A_e(T_{tm})$ decreases, $\lambda > 0$.

For direct solution of the optimal control task, it is more convenient to use a functional J in proportion to J^*:

$$J = \frac{P+Q}{\eta_t\xi}\int_{s_i}^{s_f}\left[u_f f_{max}(v) - \eta u_r r_{max}(v) + \frac{\lambda}{v}\right]ds. \tag{15}$$

RESULTS OF THE OPTIMAL CONTROL TASK

Using the maximum principle in the formulation offered by A.A. Milyutin and A.Ya. Dubovitsky for

A complex of control modes on the optimal trajectory

Description	Mode	Control
BM	A mode of full braking with maximum intensity	$u_f = 0,\ u_b = 1,\ u_r = 1$
ST	A mode of speed stabilization by regenerative and mechanical braking	$u_f = 0,\ u_r = 1,\ 0 \le u_b \le 1$
RB	(A mode of full regenerative braking	$u_f = 0,\ u_b = 0,\ u_r = 1$
SR	Speed stabilization by regenerative braking	$u_f = 0,\ u_b = 0,\ 0 \le u_r \le 1$
SD	Slowing-down—free running	$u_f = 0,\ u_b = 0,\ u_r = 0$
S	Speed stabilization in traction mode	$0 \le u_f \le 1,\ u_b = 0,\ u_r = 0$
TR	Full traction mode	$u_f = 1,\ u_b = 0,\ u_r = 0$

tasks with limitations imposed on phase coordinates [8], in [6] the authors obtained a complex of control modes at the optimal trajectory (table).

The account of the limitations per phase coordinate showed that, if the speed of movement is lower than the limitation of speed $v_{\max}(s)$ in the ST, SR, and S modes, the corresponding values v_{ST}, v_{SR}, v_{S}, which are determined by the following equations in compliance with that obtained in [3]:

$$\lambda = -\eta r'_{\max} v_{\mathrm{ST}}^2 = v_{\mathrm{SR}}^2 \eta \omega'(v_{\mathrm{SR}}) = v_{\mathrm{S}}^2 \omega'(v_{\mathrm{S}}). \quad (16)$$

If $v > v_{\max}(s)$, $v_{\max}(s)$ is selected.

From the persistence of values of the conjugated functions in the maximum principle [9], at $v < v_{\max}(s)$ the sequence of modes along the optimal trajectory is shown in Fig. 1.

Here is the p-function

$$p = -\frac{\psi_1}{\psi_0 v}, \quad (17)$$

where ψ_0 and ψ_1 are the conjugated functions.

The differential equation of the p-function is the following [6]:

$$\frac{dp}{ds} = \frac{1-p}{v} u_f f'_{\max}(v) + \frac{p-\eta}{v} u_r r'_{\max}(v) + \frac{p}{v} u_b b'_{\max}(v) + \frac{p}{v}\omega'(v) - \frac{\lambda}{v^3} - \frac{1}{\psi_0 v}\dot{\mu}(s). \quad (18)$$

Here $\dot{\mu}(s) = 0$ at $v < v_{\max}(s)$ and $\dot{\mu}(s) > 0$ at $v = v_{\max}(s)$. The derivatives $f_{\max}(v)$, $r_{\max}(v)$, and $b_{\max}(v)$ are taken based on v.

The mode of speed stabilization ST by regenerative and mechanical braking is implemented at $p = 0$ only on extra steep descents, when one of these brakes is not able to fulfill this task. In a subway the brakes of the electric stocks are located in such a way that any of the brakes accomplishes its task in a preset range of speeds. The mode of speed stabilization ST by the regenerative brake is implemented only on the steep descents at $p = \eta$. The speed S is stabilized in a traction mode at $p = 1$. There may be no ST, SR and S modes, because, at $p = 0$, $p = \eta$, and $p = 1$ the function is not discontinued during the shift, for example, from the FT to SD mode.

A modern electric stock (for example, the Moscow Metro) admits the use of electric braking (regenerative or rheostat) on all elements of the section from the maximum speed equal to 7 km/h. At $v < 7$ km/h, the electropneumatic (mechanical) braking is activated; formally, it is taken into account in the studied setting of the task in curves $R_{\max}(v)$ and $B_{\max}(v)$.

Equations (6) and (18), together with the obtained values of the p-function, are the full system of formulas for selection of the sequence of control modes and for creating the optimal trajectory $v(s)$.

IMPACT OF THE MAIN TRAIN RESISTANCE, TRAIN WEIGHT, AND REGENERATIVE ENERGY RESETTING RATIO TO THE NETWORK ON THE OPTIMAL TRAJECTORY

Based on a numerical study, it was obtained that inaccurate determination of the main resistance due to outside perturbations (the error is equal to 5% [11]) resulted in additional energy consumption, equal to 4%, and the maximum deviation of the switching coordinates, equal to 100 m. In subway conditions the analysis also showed that the calculation with additional resistance with regard to the simple model as a *material point* can result in unacceptable conditions, which can be avoided when the train is showed as an *inextensible cord*.

It is obvious that the value of a resetting ratio η is connected directly with the energy consumption for the hauling operations. The following questions are still open: how the optimal trajectory changes, how the mode switch coordinates shift when their sequence at

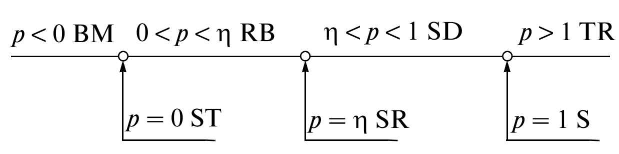

Fig. 1. A permissible sequence of control modes depending on the function p.

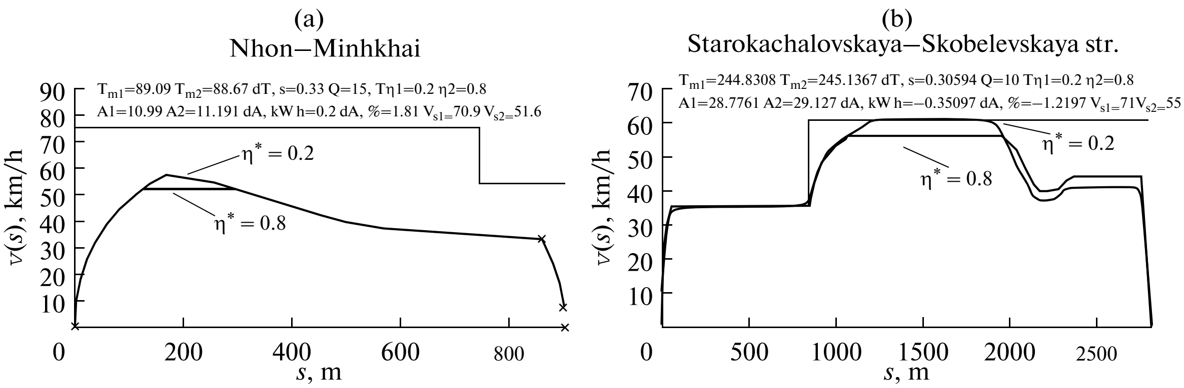

Fig. 2. A comparison of the trajectories at variation of η: (a) for the line Nhon—Minhkhai line of the first line during the construction of the Hanoi Metro; (b) for the Starokachalovskaya—Skobelevskaya str. line of the Butov Moscow Metro.

different values η is invariant, what hauls have the mode sequence changed, and how and where the energy variation is consumed by the train in traction mode along the optimal trajectory. Most probably, the answers to these questions depend on the preset time of train movement along the haul and the weight. The following qualitative and quantitative indices characterize the impact of η and loading weight Q of the train on the optimal trajectory:

—$\lambda(\eta, Q, T_m)$ is the dependence of the undetermined Lagrange coefficient on η, Q at different times of movement T_m;

—where what hauls and when the sequence of the control modes vary on the optimal trajectory depending on η, Q for preset T_{tm};

—the values of deviation of mode switch coordinates depending on η, Q for preset T_{tm} for the hauls, where the control mode sequence on the optimal trajectory remains unchanged;

—A_{et} (η, Q, T_m) is the dependence of energy, consumed by the train in traction mode during movement along the haul on η, Q and at different times of movement T_m;

—A_e (η, Q, T_m) is the dependence of the energy consumed by the train in traction mode during movement along the haul on η, Q and at different times of movement T_m.

The study of the planned train diagram for a subway shows that the train time is selected within the range $T_{m\,min} \le T_m \le T_{m\,min} + 30$ s with the spacing equal to 5 s. The train weight varies within the range from the train weight without passengers up to the fully loaded train in rush hour. The following variants are taken for modeling: the train without passengers (0 t load per wagon), average load (10 t per wagon), full load (15 t per wagon). The wagon weight P for the Moscow Metro is 44 t, while for the Hanoi Metro it is 32.5 t. In operating conditions information about train loading, correct to 2 t, can be obtained from automatic control units.

The value of the energy resetting ratio in the regenerative regime equals 0.2, 0.5, and 0.8. The dependency of the maximally permissible speed v_{max} on the trajectory has a significant impact on the view of the optimal trajectory.

Generally, η is a random value and depends on the line, amount of traffic, and traction power supply diagram. Since η was constant and preset in the calculations, it is necessary to study separately the impact of η on the image of the optimal trajectory and estimate the possible energy excess consumption due to inaccurate determination of this parameter. This study was carried out in the following way. The energy optimal modes were found for the preset conditions of movement and all preset values of η. With regard to all found modes, traditional traction calculation was executed with variation of the energy resetting ratio, which is different from the preset η. Possible energy output to the network and energy consumption were determined taking into consideration that the preset ratio in an energy optimal traction calculation differs from the real one.

The comparison of the energy optimal trajectories at the preset time taken and loading showed that, in more than 90% of cases, the trajectory does not depend on the parameter η. The obtained calculations also determined the dependences of λ on time taken, loadings and η. As can be seen from (16), λ is connected with the calculated speed of stabilization.

Due to η variation, the trajectory varies in two cases. In both cases the trajectories were compared at preset T_{tm}, Q, and η that possesses the value of η_1 and η_2. The first case of trajectory variation occurs when the trajectory obtained at $\eta = \eta_1$ lacks the stabilization mode and the traction mode shifts to the slowing-down mode; the new value of v_s at $\eta = \eta_2$ is lower than the speed of the shift from the traction to the slowing-down v_{tsd}; i.e., $v_s^{\eta 2} < v_{tsd}^{\eta 1}$.

Then the structure of the energy optimal trajectory varies: a stabilization mode is added (Fig. 2a). In real-

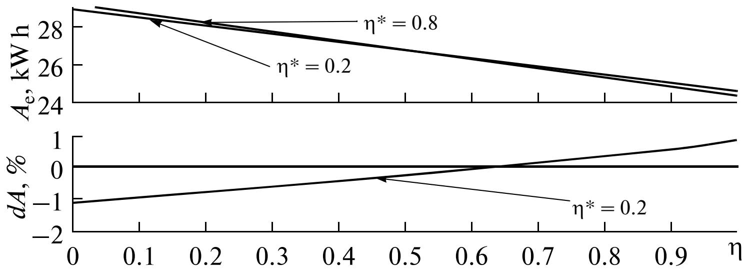

Fig. 3. A comparison of two control modes with regard to the energy consumption A_e depending on η.

ity, the speed of stabilization cannot exceed the speed limit. If for the preset T_{tm} and Q such a λ was found that the corresponding $v_s > v_{max}$, $v_s = v_{max}$ is taken for calculations. That is why there is a second case of the trajectory variation in which there is a stabilization mode and the value v_s at $\eta^* = \eta_2$ is below the speed limit v_{max} and below v_s at $\eta = \eta1$; i.e., $v_s^{\eta 2} < v_{max}$ and $v_s^{\eta 2} < v_s^{\eta 1}$. The example of such comparisons is shown in Fig. 2b. Thus, the dependence λ or v_s on η at different T_m and Q allows one to determine the conditions of variation of the energy optimal trajectory for all hauls.

Then it is necessary to evaluate possible energy excess consumption due to inaccurate determination of the energy resetting ratio to the network. Figure 3 shows a comparison of trajectories, given in Fig. 2b, with regard to the energy consumption ratio. The top diagram shows two dependences of the possible energy excess consumption of the train on the variation of the energy resetting ratio to the network in a traditional traction calculation with regard to two energy optimal modes of slowing-down, obtained at $\eta^* = 0.2$ and $\eta^* = 0.8$. The bottom diagram shows the difference of consumption as a percentage. Figure 3 shows that, if $\eta \leq 0.63$ is real, it is more profitable to use the modes obtained at $\eta^* = 0.2$. In comparison with the modes at $\eta^* = 0.8$, there is an advantage due to less consumption for traction. At $\eta > 0.63$, it is more profitable to use modes obtained at $\eta^* = 0.8$. In this case the excess consumption for traction is balanced out for a greater output of regenerative energy braking comparing to recuperative energy obtained when the train is maintained with regard to modes at $\eta^* = 0.2$.

The obtained data for all studied hauls show that the excess consumption does not exceed 5%. The error in coordinate determination of mode switching given an unvaried structure of the trajectory at preset T_{tm} and Q can be more than 100 m, but the difference in energy consumption does not exceed the obtained evaluations. Due to the impact of the train weight on specific net train resistance, which decreases with the increase of Q [11], different loads will result in various trajectories in the slowing-down mode. Thus, for the haul in Fig. 2b, the maximum difference in coordinates of mode switching is 40 m in comparison of the trajectories for $Q = 0$ and $Q = 15$ t. If the error of the automatic mode is 2 t, the error of coordinates of mode switching is insignificant. There was no variation of the structure of the optimal trajectory upon variation of weight on all studied hauls.

CONCLUSIONS

(1) The use of the energy resetting ratio (η) equal to its mathematical expectation for this line is acceptable for selection of train control modes with regard to the criteria of minimum energy consumption.

(2) The impact of the error in determination of the train loading on selection of the control modes is measured up to the impact of random disturbances; it can be compensated for during train control by a time device that uses the energy optimal advance traction calculation carried out periodically.

REFERENCES

1. Gaev, D.V., Ershov, A.V., Baranov, L.A., et al., Introduction of Energy Saving Technologies, *Mir Transporta*, 2010, no. 3.
2. Baranov, L.A., Astrakhan, V.I., Golovicher, Ya.M., et al., *Sistemy avtomaticheskogo i telemekhanicheskogo upravleniya elektropodvizhnym sostavom* (Systems for Automatic and Remote Operating Control of Electric Stock), Moscow: Transport, 1984.
3. Baranov, L.A., Golovicher, Ya.M., Erofeev, E.V., and Maksimov, V.M., *Mikroprotsessornye sistemy avtovedeniya elektropodvizhnogo sostava* (Microprocessor Systems for Electric Stock Automatic Drive), Moscow: Transport, 1990.
4. Muginshtein, L.A., Vinogradov, S.A., and Yabko, I.A., Energetically Optimal Traction Calculation of Trains Motion, *Zheleznodorozhn. Transport*, 2010, no. 2.
5. Ilyutovich, A.E., The Way to Choose Descent Variants for the Optimal Control Problem with Mixed Limita-

tions. Decomposition Approach, *Avtom. Telemekh.*, 1989, no. 9.

6. Baranov, L.A., Meleshin, I.S., and Chin', L.M., Energetically Optimal Control of Train Motion with Regenerative Braking by Considering the Phase Coordinate Limitation, *Nauka Tekhn. Transporta*, 2010, no. 4.
7. Pontryagin, L.S., Boltyanskii, V.G., Gamkrelidze, R.V., and Mishchenko, E.F., *Matematicheskaya teoriya optimal'nykh protsessov* (Mathematical Theory of Optimal Processes), Moscow: Nauka. Gl. red. fiz.-mat. lit, 1969.
8. Ioffe, A.D. and Tikhomirov, V.M., *Teoriya ekstremal'nykh zadach* (The Theory of Extreme Problems), Moscow: Nauka. Gl. red. fiz.-mat. lit, 1974.
9. Boltyanskii, V.G., *Optimal'noe upravlenie* (Optimal Control), Moscow: Nauka, 1972.
10. Grebenyuk, P.T., *Pravila tyagovykh raschetov dlya poezdnoi raboty* (Traction Calculations Rules for Train Operation), Moscow: Transport, 1985.
11. Radchenko, V.D., *Soprotivlenie dvizheniyu vagonov metropolitena* (Running Resistance of Metro Carriages), Moscow: Transzheldorizdat, 1956.