ECE 421 Project 1

Authors:
Pedro Benitez-Villegas
Ram Bala

ECE 421 - Project 1
Main x(n) Plots:

Part b.)

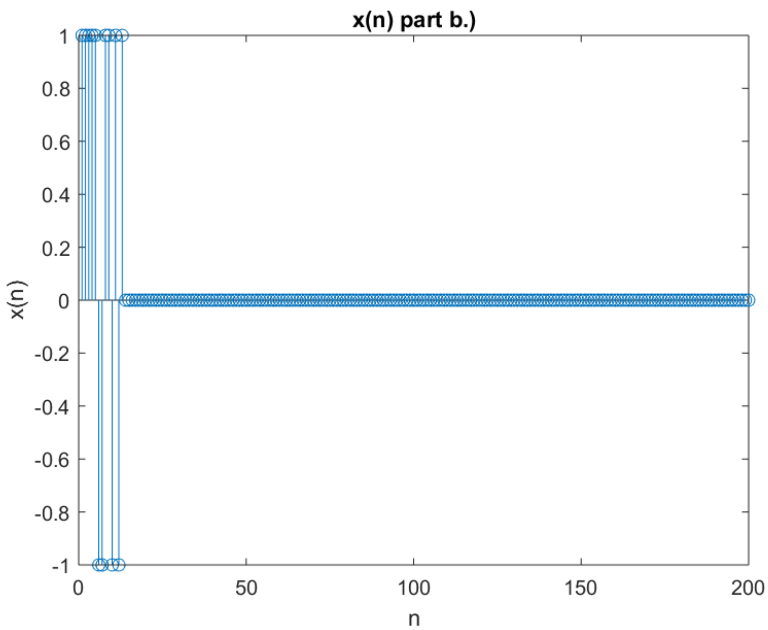

Figure 1: Plot of x(n) as described by part b

Part e.)

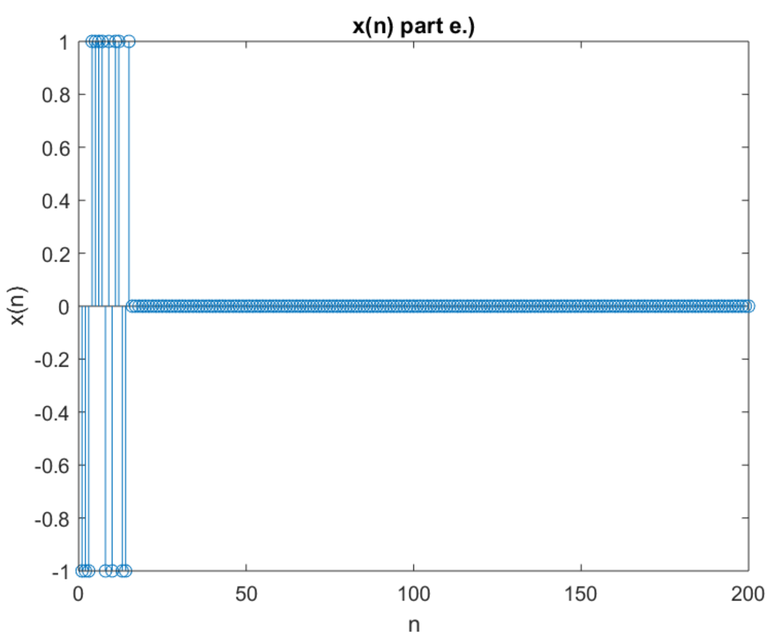

Figure 2: Plot of x(n) as described by part e

Part f.)

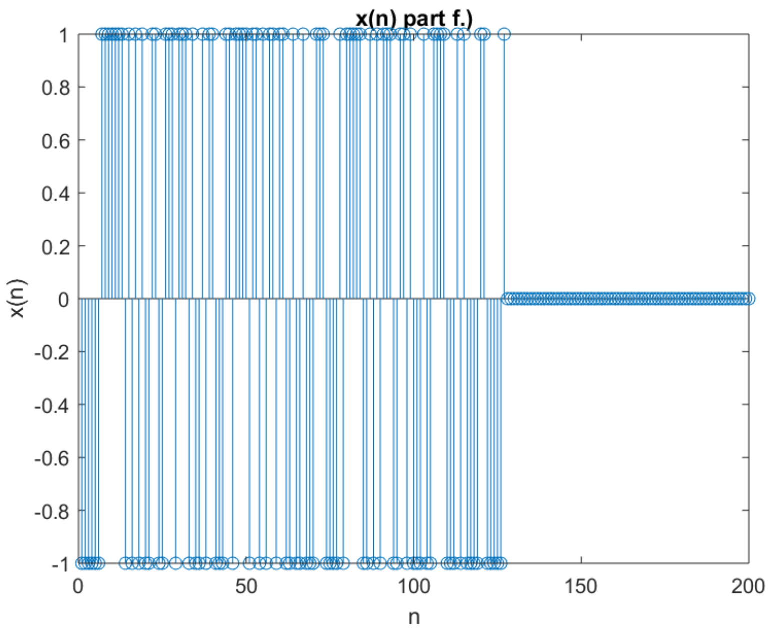

Figure 3: Plot of x(n) as described by part f

Code for x(n) plots:

```
% x(n) sequence given by part b.)
% zeros(1,187) adds extra zeros to the array to reach 200
% 13 given values + 187 zeros = 200
xn = [1,1,1,1,1,-1,-1,1,1,-1,1,-1,1, zeros(1,187)];
% plotting x(n) sequence for part b.)
figure;
stem(xn);
title('x(n) part b.)');
xlabel('n'); ylabel('x(n)');


%x(n) sequence given by part e.)
%padded the same way as before
%15 given values + 185 zeros = 200
xn2 = [-1,-1,-1,1,1,1,1,-1,1,-1,1,1,-1,-1,1, zeros(1,185)];

%plotting x(n) sequence for part e.)
figure
stem(xn2);
title('x(n) part e.)');
xlabel('n'); ylabel('x(n)');
```

```
%plotting x(n) sequence for part f.)
prex = LFSR([1,0,0,0,0,0,0],[1,7]); %using Feedback Shift Register
Function to create all 1s
prex = strrep(prex, 0, -1); %replace all zeros with -1 to build
completed sequence
xn3 = [prex, zeros(1,73)]; %replace the rest of the sequence with
zeros

%plotting x(n) sequence for part e.)
figure
stem(xn3);
title('x(n) part f.)');
xlabel('n'); ylabel('x(n)');
```

Generated y(n) Plots:

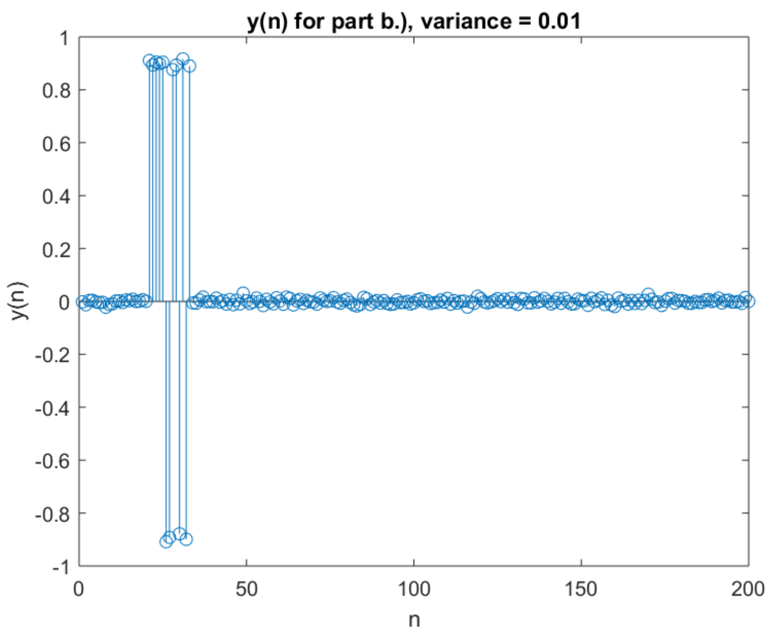

Figure 7: Plot of y(n) for sequence b with variance = 0.01

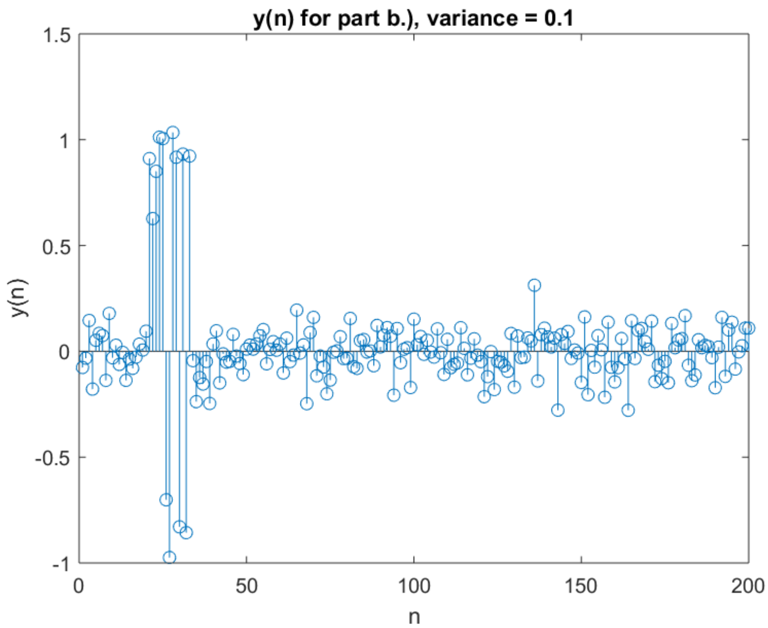

Figure 8: Plot of y(n) for sequence b with variance = 0.1

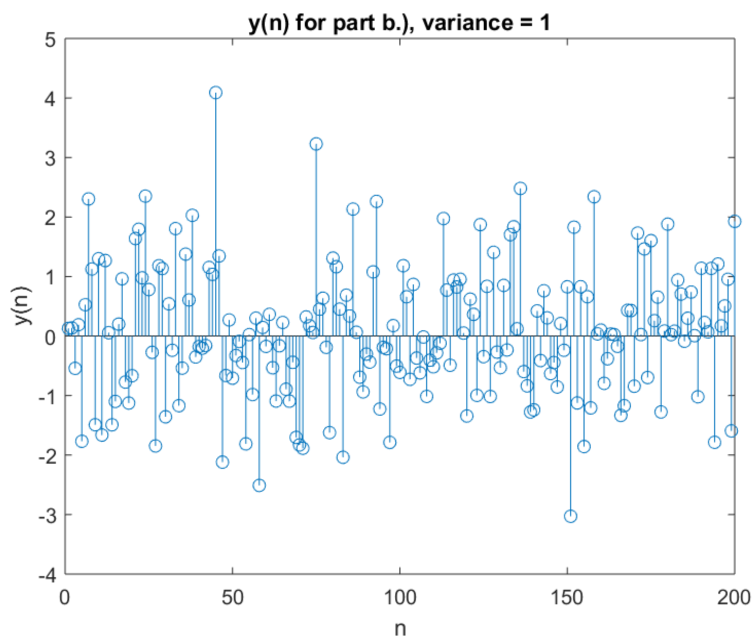

Figure 9: Plot of y(n) for sequence b with variance = 1

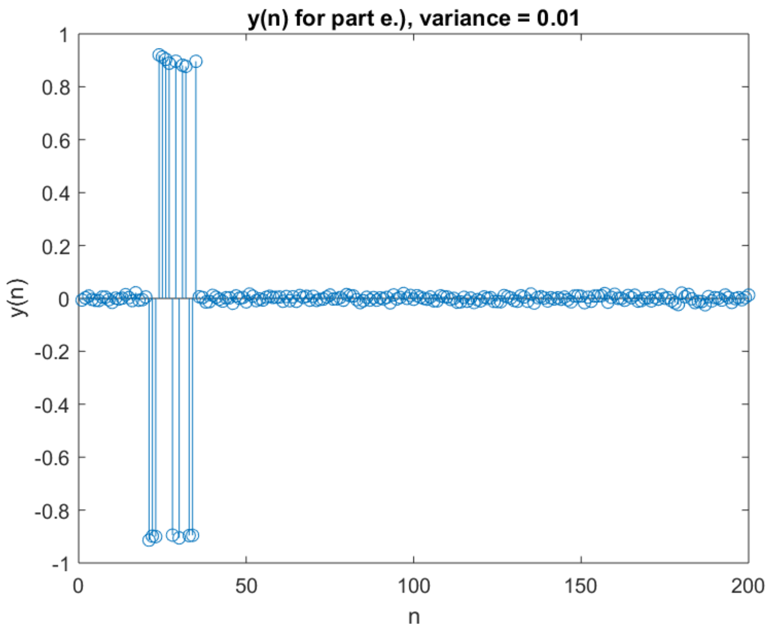

Figure 10: Plot of y(n) for sequence e with variance = 0.01

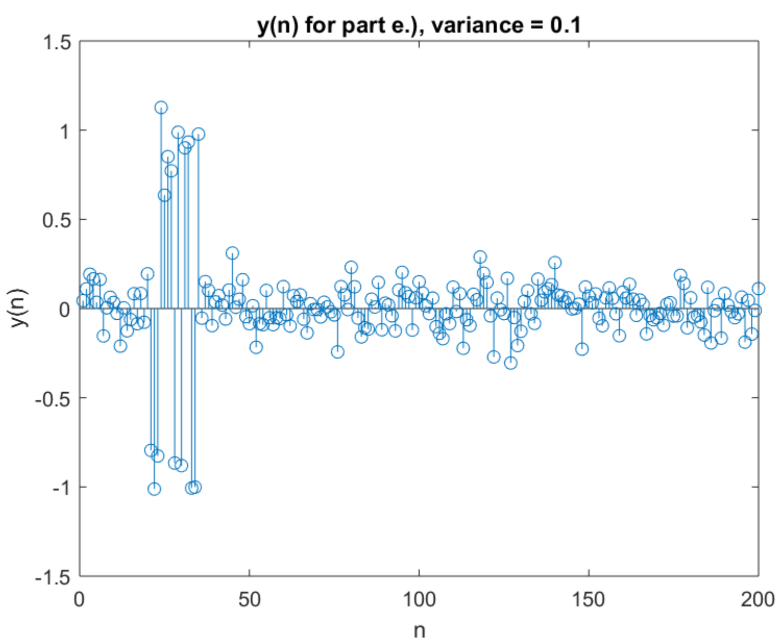

Figure 11: Plot of y(n) for sequence e with variance = 0.1

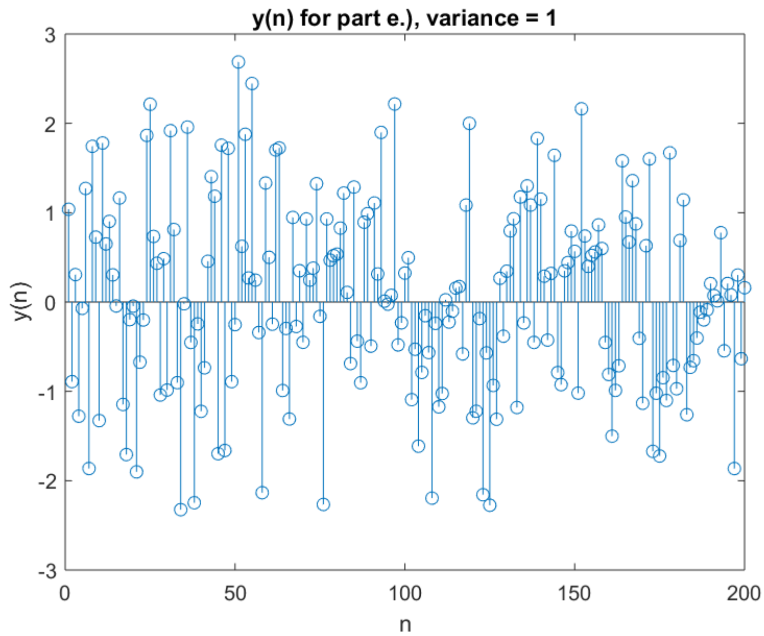

Figure 12: Plot of y(n) for sequence e with variance = 1

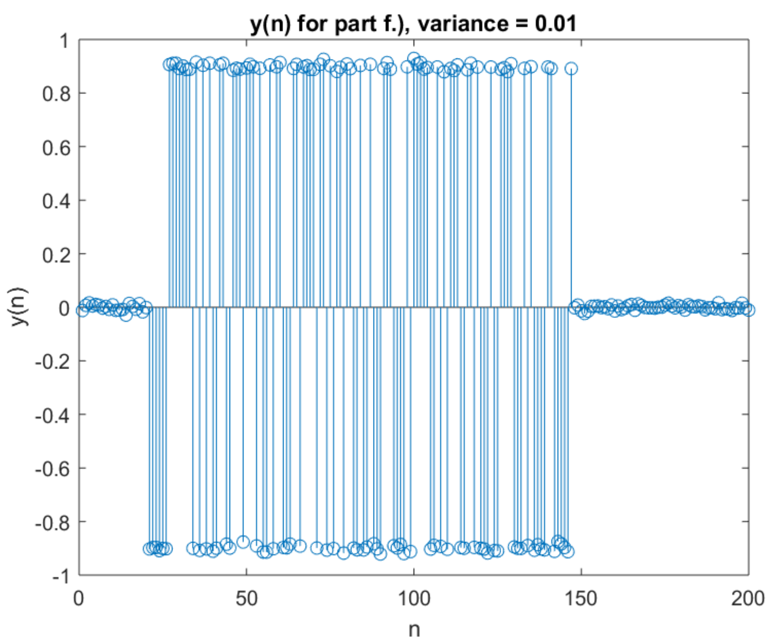

Figure 13: Plot of y(n) for sequence f with variance = 0.01

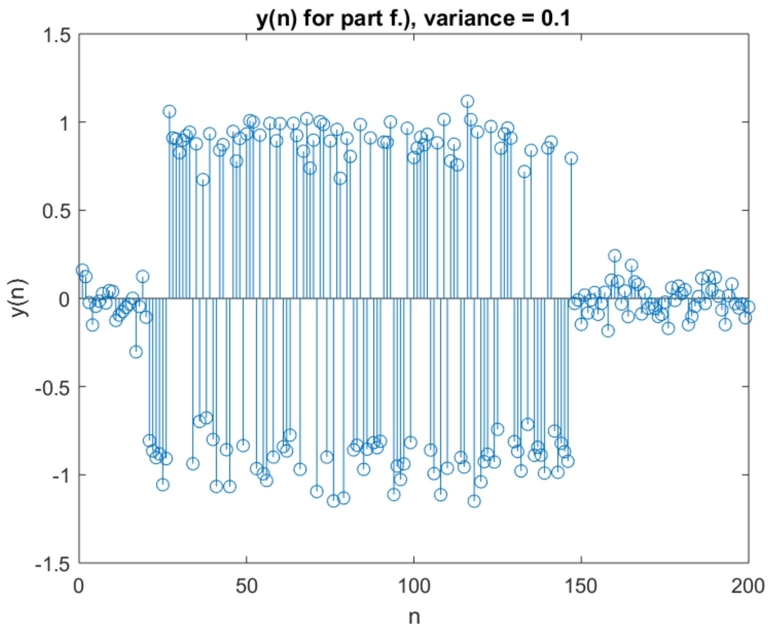

Figure 14: Plot of y(n) for sequence f with variance = 0.1

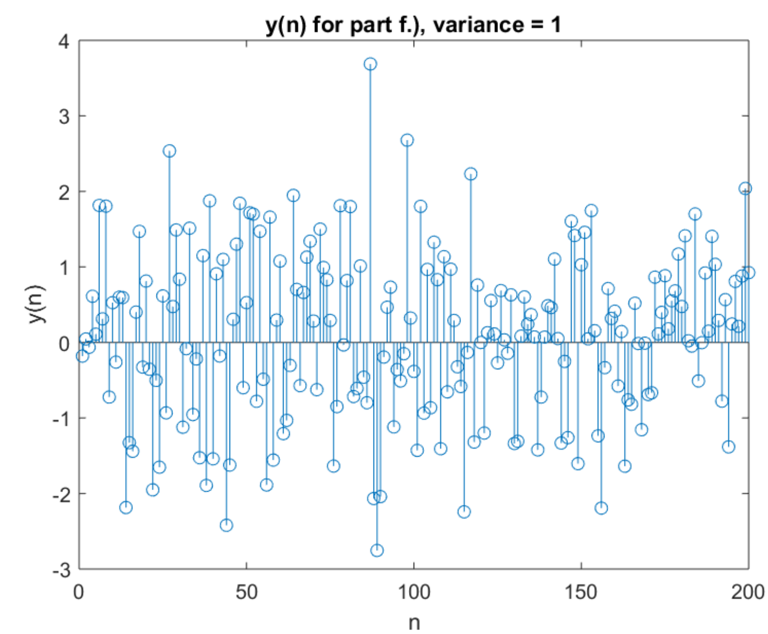

Figure 15: Plot of y(n) for sequence f with variance = 1

Code for y(n) plots:
y(n) associated with part b:

```
%% y(n) plot variance = 0.01 part b
xn = [1,1,1,1,1,-1,-1,1,1,-1,1,-1,1, zeros(1,187)];
d = 20; % delay value
a = 0.9; % A value as described

% variance = 0.01
% create 200 random values with specified mean and variance
yn001 = zeros(1,200);
for n = 1:200    % size of xn
    vn001 = normrnd(0,0.01); % generate random value
    if n-d >=1 && n-d <= 13
        yn001(n) = a*xn(n-d)+vn001;%add random value to y if xn
allows
    else
        yn001(n) = vn001;
    end
end
figure
stem(yn001)
title('y(n) for part b.), variance = 0.01');
xlabel('n'); ylabel('y(n)')


%% y(n) plot variance = 0.1 part b
d = 20; % delay value
a = 0.9; % A value as described

% variance = 0.1
% create 200 random values with specified mean and variance
yn01 = zeros(1,200);
for n = 1:200    % size of xn
    vn01 = normrnd(0,0.1);
    if n-d >=1 && n-d <= 13
        yn01(n) = a*xn(n-d)+vn01;
    else
        yn01(n) = vn01;
    end
end
figure
stem(yn01)
title('y(n) for part b.), variance = 0.1');
xlabel('n'); ylabel('y(n)')
```

```
%% y(n) plot variance = 1 plot b
d = 20; % delay value
a = 0.9; % A value as described
% variance = 1 part b
% create 200 random values with specified mean and variance
% vn001 = normrnd(0,1);
yn1 = zeros(1,200);
for n = 1:200    % size of xn
    vn1 = normrnd(0,1);
    if n-d >=1 && n-d <= 13
        yn1(n) = a*xn(n-d)+vn1;
    else
        yn1(n) = vn1;
    end
end
figure
stem(yn1)
title('y(n) for part b.), variance = 1');
xlabel('n'); ylabel('y(n)')
```

y(n) associated with part e:

```
%% y(n) plot variance = 0.01
xn = [-1,-1,-1,1,1,1,1,-1,1,-1,1,1,-1,-1,1, zeros(1,185)];
d = 20; % delay value
a = 0.9; % A value as described

% variance = 0.01
% create 200 random values with specified mean and variance
yn001 = zeros(1,200);
for n = 1:200    % size of xn
    vn001 = normrnd(0,0.01);
    if n-d >=1 && n-d <= 15
        yn001(n) = a*xn(n-d)+vn001;
    else
        yn001(n) = vn001;
    end
end
figure
stem(yn001)
title('y(n) for part e.), variance = 0.01');
xlabel('n'); ylabel('y(n)')
```

```
%% y(n) plot variance = 0.1
d = 20; % delay value
a = 0.9; % A value as described
% variance = 0.1
% create 200 random values with specified mean and variance
yn01 = zeros(1,200);
for n = 1:200    % size of xn
    vn01 = normrnd(0,0.1);
    if n-d >=1 && n-d <= 15
        yn01(n) = a*xn(n-d)+vn01;
    else
        yn01(n) = vn01;
    end
end
figure
stem(yn01)
title('y(n) for part e.), variance = 0.1');
xlabel('n'); ylabel('y(n)')


%% y(n) plot variance = 1
d = 20; % delay value
a = 0.9; % A value as described
% variance = 1
% create 200 random values with specified mean and variance
%vn001 = normrnd(0,0.01);
yn1 = zeros(1,200);
for n = 1:200    % size of xn
    vn1 = normrnd(0,1);
    if n-d >=1 && n-d <= 15
        yn1(n) = a*xn(n-d)+vn1;
    else
        yn1(n) = vn1;
    end
end
figure
stem(yn1)
title('y(n) for part e.), variance = 1');
xlabel('n'); ylabel('y(n)')
```

y(n) associated with part f:

```
%% y(n) plot
prex = LFSR([1,0,0,0,0,0,0],[1,7]); %using Feedback Shift Register
Function to create all 1s
prex = strrep(prex, 0, -1); %replace all zeros with -1 to build
completed sequence
xn3 = [prex, zeros(1,73)] %replace the rest of the sequence with
zeros
d = 20; % delay value
a = 0.9; % A value as described
% variance = 0.01
% create 200 random values with specified mean and variance
yn001 = zeros(1,200);
for n = 1:200    % size of xn
    vn001 = normrnd(0,0.01);
    if n-d >=1 && n-d <= 127
        yn001(n) = a*xn3(n-d)+vn001;
    else
        yn001(n) = vn001;
    end
end
figure
stem(yn001)
title('y(n) for part f.), variance = 0.01');
xlabel('n'); ylabel('y(n)')


%% y(n) plot variance = 0.1
d = 20; % delay value
a = 0.9; % A value as described
% variance = 0.1
% create 200 random values with specified mean and variance
yn01 = zeros(1,200);
for n = 1:200    % size of xn
    vn01 = normrnd(0,0.1);
    if n-d >=1 && n-d <= 127
        yn01(n) = a*xn3(n-d)+vn01;
    else
        yn01(n) = vn01;
    end
end
figure
stem(yn01)
```

```
title('y(n) for part f.), variance = 0.1');
xlabel('n'); ylabel('y(n)')
%% y(n) plot variance = 1
d = 20; % delay value
a = 0.9; % A value as described
% variance = 1
% create 200 random values with specified mean and variance
%vn001 = normrnd(0,1);
yn1 = zeros(1,200);
for n = 1:200    % size of xn
    vn1 = normrnd(0,1);
    if n-d >=1 && n-d <= 127
        yn1(n) = a*xn3(n-d)+vn1;
    else
        yn1(n) = vn1;
    end
end
figure
stem(yn1)
title('y(n) for part f.), variance = 1');
xlabel('n'); ylabel('y(n)')
```

Autocorrelation Plots:

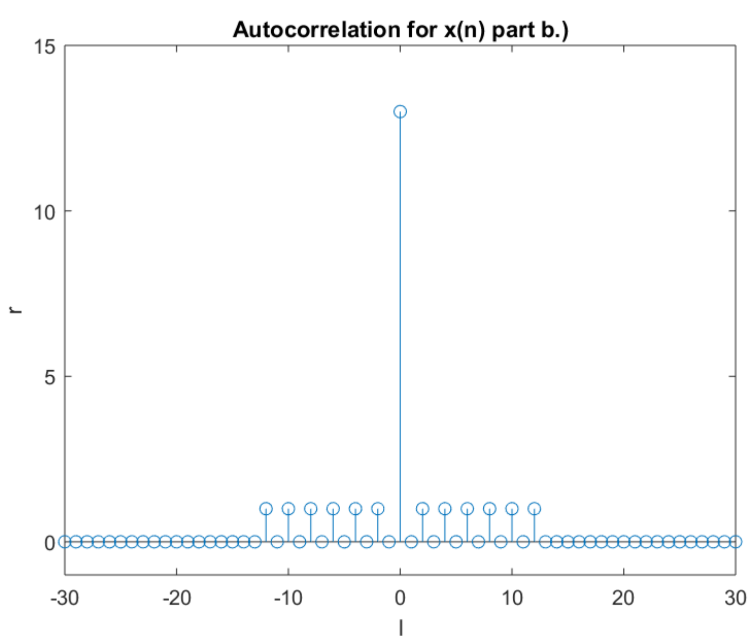

Figure 4: Autocorrelation of x(n) for sequence b

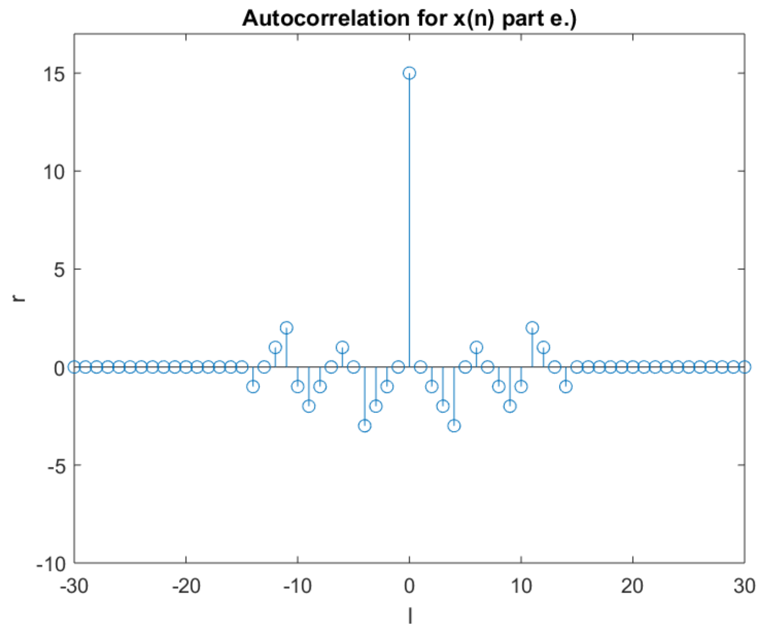

Figure 5: Autocorrelation of x(n) for sequence e

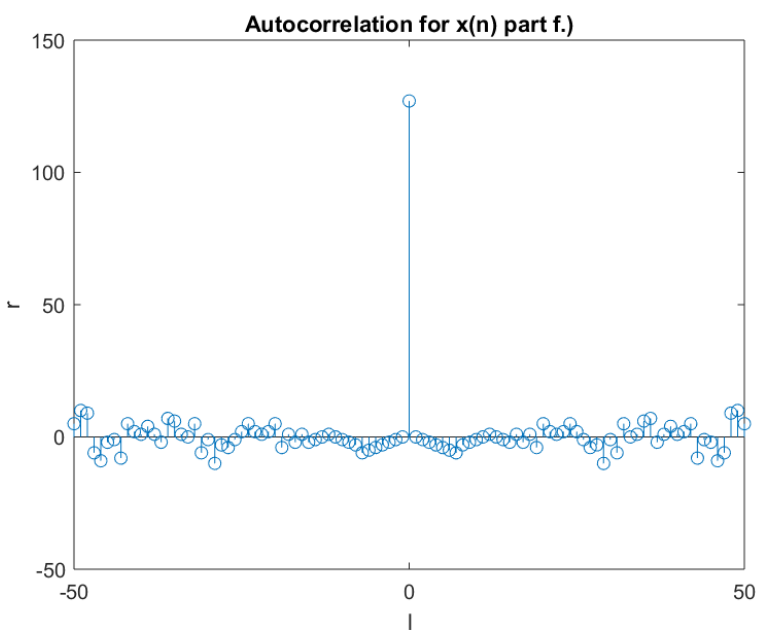

Figure 6: Autocorrelation of x(n) for sequence f

Code for Autocorrelation:

```
%% Autocorrelation for plot b
xn = [1,1,1,1,1,-1,-1,1,1,-1,1,-1,1, zeros(1,187)]; nx = [0:199];
[xa,nxa] = sigfold(xn,nx) % obtain x(-n)
[r,nr] = conv_m(xn,nx,xa,nxa); % auto-correlation
stem(nr,r)
axis([-30,30,-1,15])
title('Autocorrelation for x(n) part b.)')
xlabel('l'); ylabel('Autocorrelation')


%% Autocorrelation for plot e
xn = [-1,-1,-1,1,1,1,1,-1,1,-1,1,1,-1,-1,1, zeros(1,185)]; nx =
[0:199];
[xna,nxa] = sigfold(xn,nx); % obtain x(-n)
[r,nr] = conv_m(xn,nx,xna,nxa); % auto-correlation
stem(nr,r)
axis([-30,30,-10,17])
title('Autocorrelation for x(n) part e.)')
xlabel('l'); ylabel('Autocorrelation')


%% Autocorrelation for plot f
prex = LFSR([1,0,0,0,0,0,0],[1,7]); %using Feedback Shift Register
Function to create all 1s
prex = strrep(prex, 0, -1); %replace all zeros with -1 to build
completed sequence
xn3 = [prex, zeros(1,73)]; nx = [0:199];
[xna,nxa] = sigfold(xn3,nx); % obtain x(-n)
[r,nr] = conv_m(xn3,nx,xna,nxa); % auto-correlation
stem(nr,r)
axis([-50,50,-50,150])
title('Autocorrelation for x(n) part f.)')
xlabel('l'); ylabel('Autocorrelation')
```

Cross-correlation of x(n) and y(n) plots:

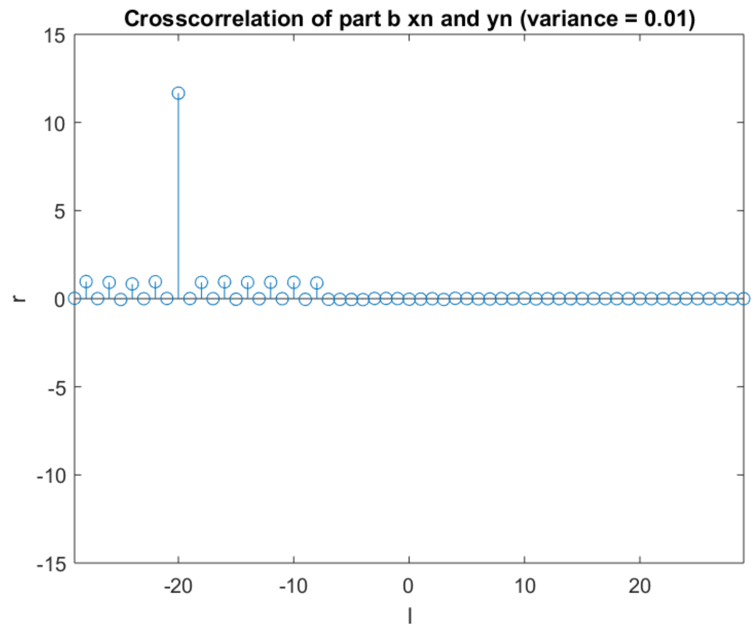

Figure 16: Cross-correlation of part b x(n) and y(n) with variance = 0.01

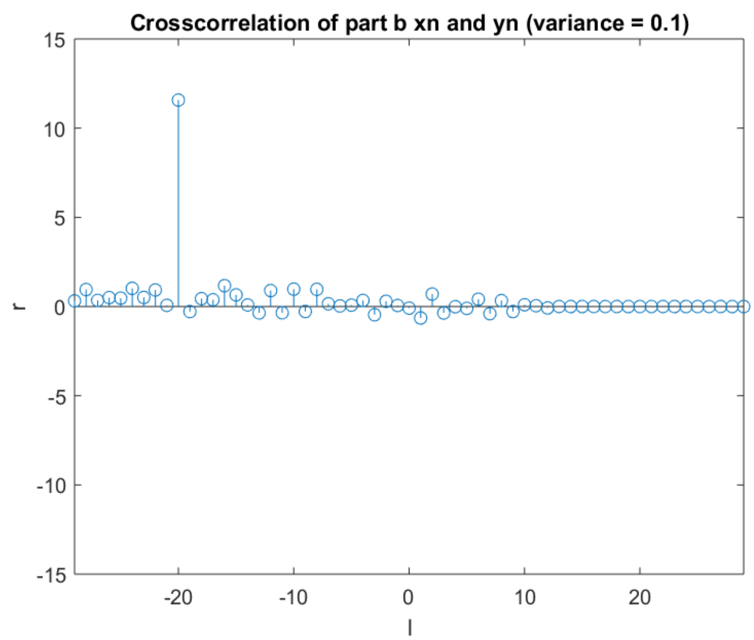

Figure 17: Cross-correlation of part b x(n) and y(n) with variance = 0.1

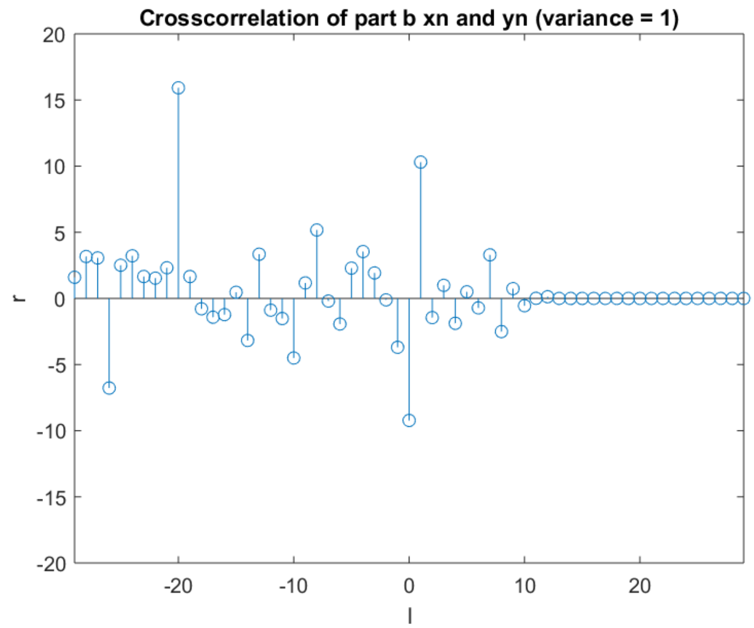

Figure 18: Cross-correlation of part b x(n) and y(n) with variance = 1

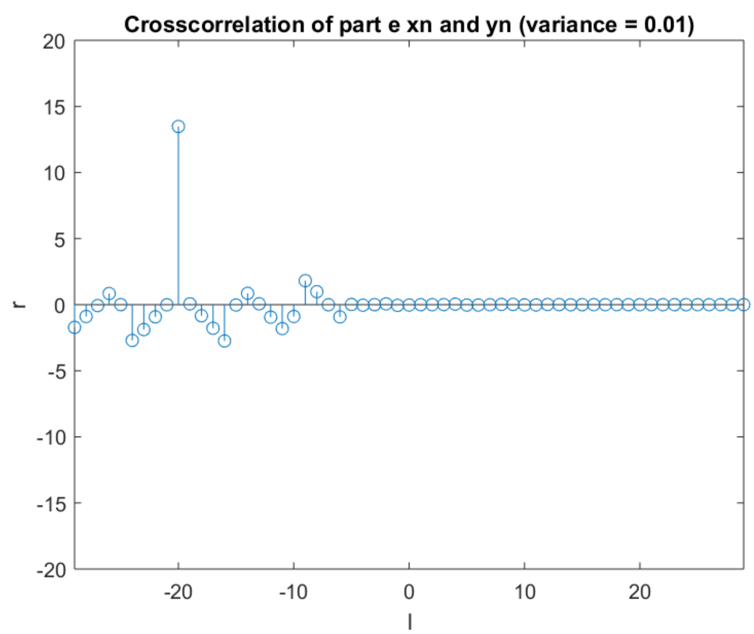

Figure 19: Cross-correlation of part e x(n) and y(n) with variance = 0.01

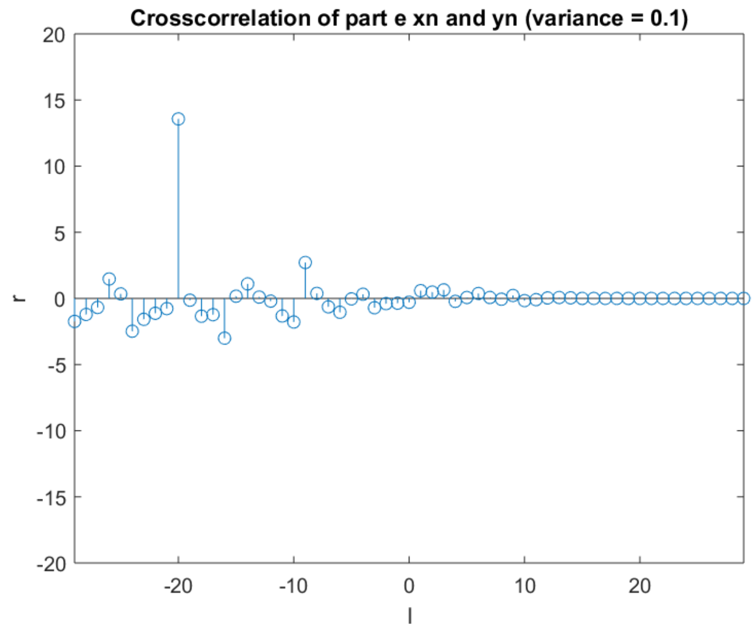

Figure 20: Cross-correlation of part e x(n) and y(n) with variance = 0.1

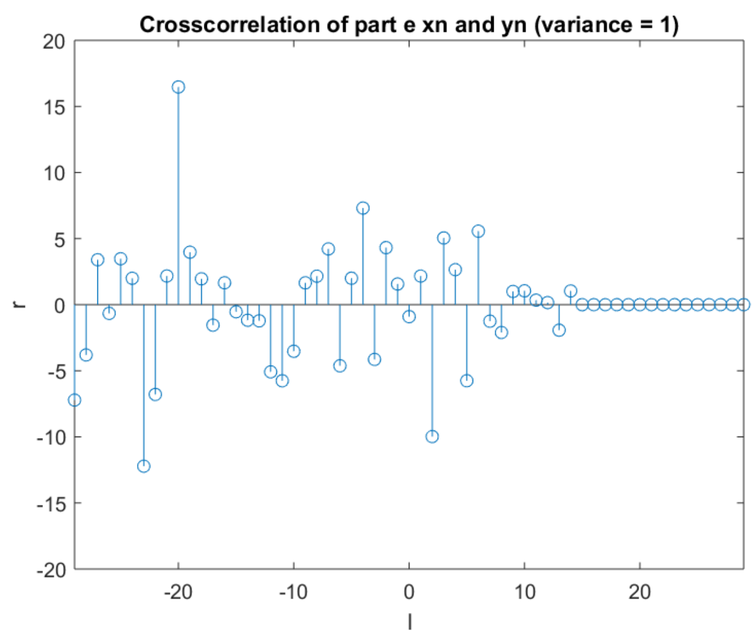

Figure 21: Cross-correlation of part e x(n) and y(n) with variance = 1

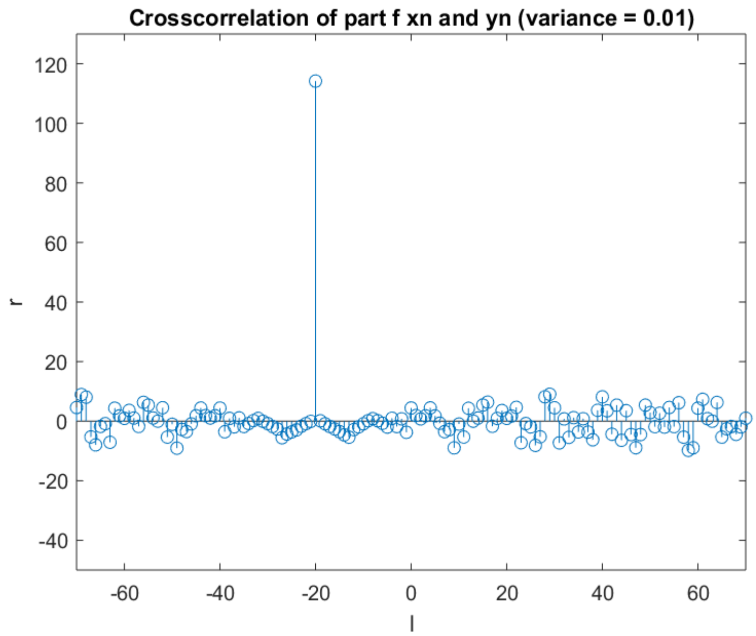

Figure 22: Cross-correlation of part f x(n) and y(n) with variance = 0.01

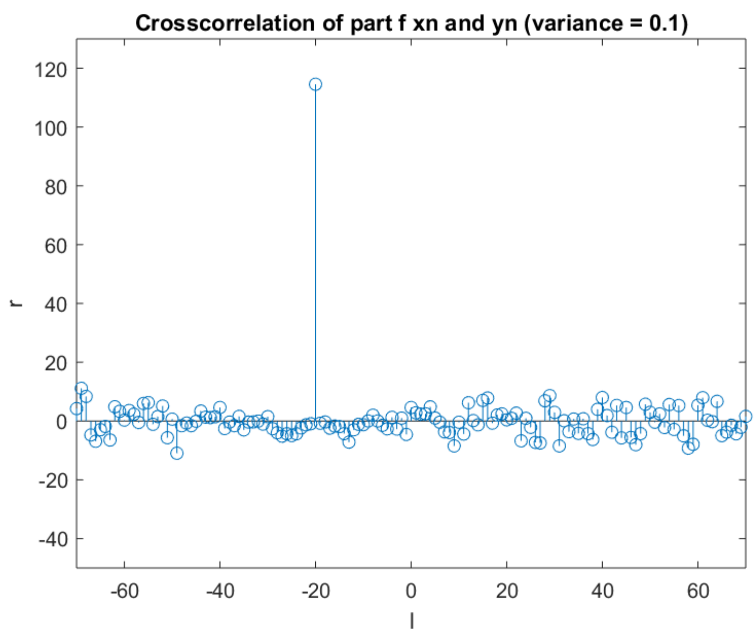

Figure 23: Cross-correlation of part f x(n) and y(n) with variance = 0.1

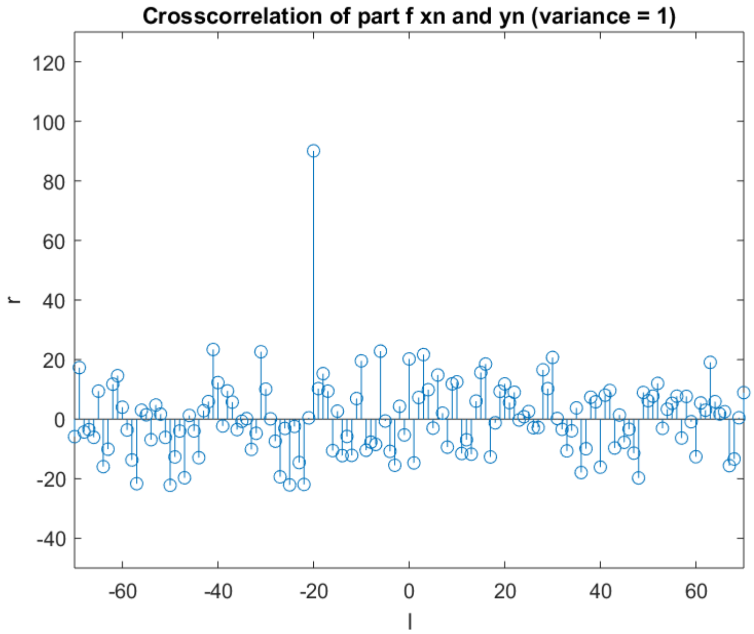

Figure 24: Cross-correlation of part f x(n) and y(n) with variance = 1

Code for Cross-correlation:

```
%% Crosscorrelation rxy(l) for plot b
% variance = 0.01
[h,nh] = sigfold(yn001,[0:199]); % obtain h(-n)
[r,nr] = conv_m(xn,[0:199],h,nh); % cross-correlation

% plot crosscorrelation
figure
stem(nr,r);
axis([-29,29,-15,15]);
title('Crosscorrelation of part b xn and yn (variance = 0.01)');
xlabel('l');ylabel('r');

% variance = 0.1
[h,nh] = sigfold(yn01,[0:199]); % obtain h(-n)
[r,nr] = conv_m(xn,[0:199],h,nh); % cross-correlation

% plot crosscorrelation
figure
stem(nr,r);
axis([-29,29,-15,15]);
title('Crosscorrelation of part b xn and yn (variance = 0.1)');
xlabel('l');ylabel('r');
% variance = 1
```

```
[h,nh] = sigfold(yn1,[0:199]); % obtain h(-n)
[r,nr] = conv_m(xn,[0:199],h,nh); % cross-correlation

% plot crosscorrelation
figure
stem(nr,r);
axis([-29,29,-20,20]);
title('Crosscorrelation of part b xn and yn (variance = 1)');
xlabel('l');ylabel('r');



%% Crosscorrreleation rxy(l) for plot e
% variance = 0.01
[h,nh] = sigfold(yn001,[0:199]); % obtain h(-n)
[r,nr] = conv_m(xn,[0:199],h,nh); % cross-correlation

% plot crosscorrelation
figure
stem(nr,r);
axis([-29,29,-20,20]);
title('Crosscorrelation of part e xn and yn (variance = 0.01)');
xlabel('l');ylabel('r');

% variance = 0.01
[h,nh] = sigfold(yn01,[0:199]); % obtain h(-n)
[r,nr] = conv_m(xn,[0:199],h,nh); % cross-correlation

% plot crosscorrelation
figure
stem(nr,r);
axis([-29,29,-20,20]);
title('Crosscorrelation of part e xn and yn (variance = 0.1)');
xlabel('l');ylabel('r');

% variance = 1
[h,nh] = sigfold(yn1,[0:199]); % obtain h(-n)
[r,nr] = conv_m(xn,[0:199],h,nh); % cross-correlation

% plot crosscorrelation
figure
stem(nr,r);
axis([-29,29,-20,20]);
title('Crosscorrelation of part e xn and yn (variance = 1)');
xlabel('l');ylabel('r');
```

```
%% Crosscorrreleation rxy(l) for plot f
% variance = 0.01
[h,nh] = sigfold(yn001,[0:199]); % obtain h(-n)
[r,nr] = conv_m(xn3,[0:199],h,nh); % cross-correlation

% plot crosscorrelation
figure
stem(nr,r);
axis([-70,70,-50,130]);
title('Crosscorrelation of part f xn and yn (variance = 0.01)');
xlabel('l');ylabel('r');


% variance = 0.01
[h,nh] = sigfold(yn01,[0:199]); % obtain h(-n)
[r,nr] = conv_m(xn3,[0:199],h,nh); % cross-correlation

% plot crosscorrelation
figure
stem(nr,r);
axis([-70,70,-50,130]);
title('Crosscorrelation of part f xn and yn (variance = 0.1)');
xlabel('l');ylabel('r');

% variance = 1
[h,nh] = sigfold(yn1,[0:199]); % obtain h(-n)
[r,nr] = conv_m(xn3,[0:199],h,nh); % cross-correlation

% plot crosscorrelation
figure
stem(nr,r);
axis([-70,70,-50,130]);
title('Crosscorrelation of part f xn and yn (variance = 1)');
xlabel('l');ylabel('r');
```

Discussion:

a. *Describe the three autocorrelation sequences. For each sequence, compare the values for $l = 0$ and $l \neq 0$. Do you notice any trend as the sequence length increases?*

In all three autocorrelation plots, $r_{xx}(l = 0)$ is the sample with the largest magnitude. For $r_{xx}(l \neq 0)$ there are N-1 samples on either side of $r_{xx}(l = 0)$, where N is the number of non-zero values in the sequence. Due to the padded zeros after N-1 samples on either side, all samples are 0 until $r_{xx}(| l | = 200)$. All three autocorrelation plots are symmetrical because $r_{xx}(l) = r_{xx}(-l)$. As the sequence length increases the noise becomes greater and more obvious but the signal remains symmetrical.

b. *The cross-correlation $r_{xy}(l)$ can be expressed as a sum of two terms: one term is due to noise and the other is shifted autocorrelation $r_{xx}(l)$. Specify the shift of the autocorrelation in the second term. Explain how the delay D can be estimated from the cross-correlation sequence. (Use observations made in part (a) in your explanation.)*

The following was the given equation for y(n):

$$y(n) = a*x_a(n - D) + v(n)$$

v(n) represents the noise added to the signal while $a*x_a(n - D)$ represents the shifted input sequence x(n). The cross correlation or $r_{xy}(l)$ is shifted left by 20 due to the delay D. This can be seen as the maximum magnitude of the signal, which was previously located at $l = 0$ in the autocorrelation plots, is now at $l = -20$. Due to the added noise, the cross correlation is asymmetrical.

c. *Explain why the three sequences x(n) were chosen for this application. Relate your answer to (a,b).*

The three sequences were chosen since they all vary in length, starting from 13 values in sequence b up to 127 values in sequence f. Having three sequences of varying lengths lets us see how the autocorrelation and crosscorrelation differs for sequences of different sizes. All three of the sequences also only have values of one and negative one, which simplifies the calculations and allows us to estimate, observe and understand the end result from correlations better.

Since all of the values are positive one or negative one, we can get more consistent results when computing autocorrelation and cross correlation. The values being the same and absolute value of each other also helps us see where the autocorrelation plot will go towards zero. The size of x(n) also helps us see approximately where the autocorrelation plot goes to zero. The larger the length of x(n) is, the more values we see further out from the center.

The cross correlation plots show us a distinct peak at -20 and then noise everywhere else. The cross correlations of x(n) and y(n) generated with higher variance have larger peaks apart from the main peak at -20 compared to the cross correlations of smaller variance. Another thing we can see is that the more values your x(n) function has, the more distinct the peak is regardless of the variance. We saw a similar pattern with the autocorrelation where for x(n) in part f (with 127 values) has a more distinct peak and the rest of the values are not nearly as high or noticeable. Since the cross correlation is also the convolution of the impulse response with the autocorrelation, it makes sense that this pattern is seen in the cross correlation as well.

d. *Make the following comparisons. Relate to answers in (a-c).*
   i. *Compare the accuracy of the estimation as the noise variance increases.*

As noise variance increases, the randomness of noise becomes more apparent. The magnitude of the resulting noise increases. The magnitude begins to approach the max magnitude found at l=-20. While the maximum is still obvious and indicative of the shift discussed in part b, the delay estimation is not as obvious as noise increases. This is somewhat mitigated by having a longer sequence because having more values dilutes the overall impact that noise has on the signal.

   ii. *Which sequence is likely to provide the most reliable estimate of the delay in the presence of significant noise? Explain.*

The sequence in part f is most likely to provide the most reliable estimate of the delay in the presence of significant noise. This is because the sequence in part f contains the most amount of values and therefore noise has less of an impact on the overall signal.

e. *Write a short conclusion based on your observations. Summarize results in (a-d) in a few sentences.*

Three x(n) sequences of various lengths were chosen to perform autocorrelations and cross correlations to examine the results. All three x(n) functions only have values of 1 and negative 1, which allows the correlations to have a more noticeable peak that we can observe.

The autocorrelation plots all have a noticeable peak at 0 and are also all symmetric since autocorrelation is nothing but a cross correlation with the signal itself and so $r_{xx}(l)$ will be equal to $r_{xx}(-l)$. We can see that the autocorrelation plots have more variation of values and stretch out for longer out from the center depending on the length of the x(n) sequence used. With a longer x(n), you can see that there are more noticeable values further away from the peak at the origin.

Nine different y(n) sequences were created, three for each x(n) sequence, as specified by the following equation: $y(n) = a*x_a(n - D) + v(n)$. Three different variances were used for creating the different y(n) sequences for the corresponding x(n) sequences. Based on the equation, we can see that the y(n) sequence is the result of the x(n) sequence being scaled by a value a, and the addition of noise generated based on the normal distribution with mean of 0 and variances of 0.01, 0.1, and 1. The y(n) sequences also mirrored the x(n) sequences shape with a delay of 20 and also multiplied with a random value so they are not all the same value as x(n) (this can be seen by the equation for y(n) as well).

The most noticeable change with the changing of variance is that the random values get larger as the variance is increased. For y(n) sequences generated with variances of 0.01 and 0.1, you can noticeably see where the delayed x(n) influence is. For the y(n) plots with variance of 1, it is harder to tell since the values are larger and so most of the values are similar and grouped together (there is no distinct set of larger values to stand apart).

As we have mentioned before, the cross correlation is the convolution of the impulse response with the autocorrelation of a function and we have found it here by cross correlating the x(n) sequences with their respective y(n) sequences for the three different variances. The biggest noticeable feature of all the cross correlations is that they all have a distinct peak at l = -20. This is because we delayed the y(n) sequence by 20 values and this delay is reflected by having the peak 20 units left from the origin. Unlike the autocorrelation, the cross correlation plots are asymmetrical since we have added noise in the form of the y(n) sequences. You can also see the cross correlation created using the y(n) sequences with variance = 1 have the smallest peak of the rest of the cross correlations and also have larger values surrounding the peak in general due to the nature of variance of 1 creating larger values in general.