Out: January 27, 2022
Due: February 11, 2022

NTNU – Trondheim
Norwegian University of Science and Technology

TTK4130 Modeling and Simulation

Assignment 3

Introduction

The objectives of this assignment are:

- To understand the Newton-Euler equations, and apply them to simple mechanical systems.
- To learn how to select convenient SO(3) representations and reference frames in order to simplify the associated Newton-Euler equations.
- To understand and apply the parallel axis theorem, also known as Huygens–Steiner theorem.

Problem 1 (Satellite)

In this task, we will consider a satellite orbiting Earth. We define an inertial reference frame with its origin at Earth's center and with an arbitrary and fixed orientation.

We will consider two cases:

1. The satellite is a cube of uniform, unitary density, having an edge of 50cm.
2. The satellite is the cube mentioned above, with the addition of a punctual mass of $m_0 = 0.1$kg placed at one of the cube's corners.

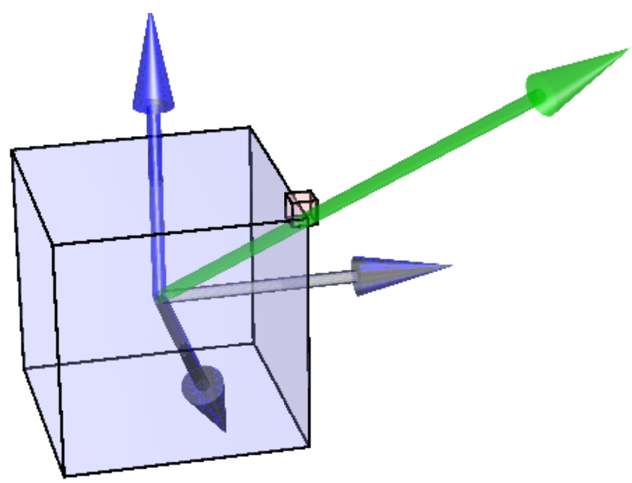

Figure 1: Schematic of the satellite.

We will assume that the force of gravity is given by Newton's law of universal gravitation:

$$\vec{F} = -\frac{G\, m_{\mathrm{T}}\, m}{\|\vec{r}_{\mathrm{c}}\|^2} \cdot \frac{\vec{r}_{\mathrm{c}}}{\|\vec{r}_{\mathrm{c}}\|} \tag{1}$$

The inertia matrix in the reference frame attached to the cube with its origin at the cube's center of mass and with the axes going through the center of the cube's faces is given by

$$\frac{1}{6} m l^2 I \tag{2}$$

where m is the mass, l is the length of the sides and I is the 3-by-3 identity matrix.

For task (a)-(c) consider the case without the added mass (case 1 above)

(a) Considering a frame at the center of mass, apply the Newton-Euler equations to describe the satellite's motion (position and orientation). What is the resulting state-space model?

(b) Complete the function `SatelliteDynamics.m` in the delivered code, and add it to your answer.

(c) Simulate the satellite system using the Matlab ODE integration function `ode45` (seen in previous assignments).

What do you observe? Are the results reasonable?

(d) Now consider the added mass (case 2 above). The added mass will shift the center of mass of the system. Calculate the inertia matrix around this new center of mass and repeat the simulation in (c).

What do you observe? Are the results reasonable? What is the main difference between the two cases? Explain, and comment on both rotation and translation.

Hint: Use the parallel axis theorem

An example code is provided on Blackboard. There you will also find the means to make a 3D animation of your simulation.

Hints:

- *You will find code templates / examples on Blackboard to help you get started. Further hints are provided therein. See `Satellite3DTemplate.m` for a template on how to build the simulation, and `Satellite3DExample.m` for tools to do 3D animations. These animations will allow you to assess your simulations, and describe the different motions.*
- *For parameters and initial values that are not given, you are free to choose reasonable numerical values. For example, Earth's radius is 6356 km and its orbital height is 36 Mm. The numerical solver will try to capture all dynamics at a certain precision. If the dynamics are "fast", the simulation can not run for long, since the step size will be small. However, we are interested in illustrating the difference of the two cases. What can be different? The thing we want to illustrate must therefore seem fast enough.*

Problem 2 (Spinner)

In this task, we will consider a "spinner", i.e. a disk of uniform density, radius $R = 1\,\text{m}$ and mass $m = 1\,\text{kg}$ mounted on a massless rod of length $L = 2\,\text{m}$ connected to a free joint. See Figure 2.

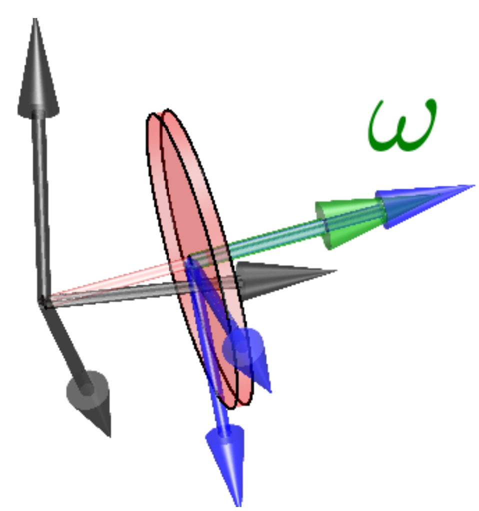

Figure 2: Schematic of the spinner.

We assume that the thickness of the disk is zero. Hence, the inertia matrix of the disk taken in a frame attached at its center with the radial symmetry axis as the third axis, is given by

$$M_c^b = \frac{mR^2}{4}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \tag{3}$$

We will consider that the force of gravity is given by

$$\vec{F} = -m\vec{g} \tag{4}$$

- **Write down the equations:** Select a frame for the spinner and a representation of the SO(3) Lie group (orientation of the spinner). Then apply correctly the Newton-Euler equations to describe the motion of the spinner.

  What is your resulting state-space model?
- **Implement** your model and simulate it for different angular velocities $\boldsymbol{\omega}$, where

  $$\boldsymbol{\omega}_{ab}^b = \omega\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \tag{5}$$

  i.e. $\boldsymbol{\omega}$ is aligned with the radial symmetry axis of the disk. Test e.g. $\omega = \pi, 2\pi, 4\pi, 6\pi\,\mathrm{rad\,s^{-1}}$. When implementing it may be useful to reuse some code that you have already made, and create a function returning the derivative of the states, as we have done before.
- What do you observe? Are the results reasonable? Explain.

Hints:

- *The selection of a convenient body frame and* SO(3) *representation will make the problem much easier.*
- *You may need the parallel axes theorem.*
- *You will find code templates / examples on Blackboard to help you. See* `Gyropscope3DExample.m` *and previously delivered codes for tools to do 3D animations. These animations will allow you to assess your simulations.*