Assignment 5 in ModSim

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Task 1

a)

We know that p_2 is dependent upon p_1. Furthermore the x, y and z coordinates of a spherical pendulum is given by $Lsin(\theta)cos(\phi)$, $Lsin(\theta)sin(\phi)$ and $L(1 - cos(\theta))$, respectively.

This results in the expression:

- $p_2 = p_1 + L \begin{bmatrix} sin(\theta)cos(\phi) \\ sin(\theta)sin(\phi) \\ 1 - cos(\theta) \end{bmatrix}$

Q is given by the external force $u \in IR^3$ on p_1. The first element of Q is therefore $Q_{p_1} = u$. Since there is no external force on p_2 the second element is 0. This yields:

- $Q = \begin{bmatrix} u \\ 0 \end{bmatrix} \in IR^5$

The kinetic energy of the system is given by:

- $T(q, \dot{q}) = \frac{1}{2} m_1 \frac{\partial p_1}{\partial q}^T \frac{\partial p_1}{\partial q} + \frac{1}{2} m_2 \frac{\partial p_2}{\partial q}^T \frac{\partial p_2}{\partial q}$

The potential energy of the system i given by:

- $V(q) = m_1 \cdot g \cdot p_{1y} + m_2 \cdot g \cdot p_{2y}$

where p_{1y} and p_{2y} are the y coordinates of m_1 and m_2, respectively. The lagrangian is given by:

- $\mathcal{L}(q, \dot{q}) = T(q, \dot{q}) - V(q)$

We found M and b using MATLAB code:

From HoveringMassUnconstraintTemplate.m

```
clear all

% Parameters
syms m1 m2 L g real
% Force
u = sym('u',[3,1]);
```

```
% Position point mass 1
pm1 = sym('p1',[3,1]);
dpm1 = sym('dp1',[3,1]);
ddpm1 = sym('d2p1',[3,1]);
% Angles for point mass 2
a  = sym('a',[2,1]);
da = sym('da',[2,1]);
dda = sym('d2a',[2,1]);
% Generalized coordinates
q  = [pm1;a];
dq = [dpm1;da];
ddq = [ddpm1;dda];

% Position of point mass 2
pm2  = pm1 + L*[sin(a(1))*cos(a(2));
                sin(a(1))*sin(a(2));
                1 - cos(a(1))];
% Velocity of point mass 2
dpm2 = jacobian(pm2,q)*dq;
% Generalized forces
Q = [u; 0; 0];
% Kinetic energy
T = 0.5*m1*dpm1'*dpm1 + 0.5*m2*dpm2'*dpm2;
T = simplify(T);
% Potential energy
V = g*(m1*pm1(3) + m2*pm2(3));
% Lagrangian
Lag = T - V;

% Derivatives of the Lagrangian
Lag_q = simplify(jacobian(Lag,q)).';
Lag_qdq = simplify(jacobian(Lag_q.',dq));
Lag_dq = simplify(jacobian(Lag,dq)).';
Lag_dqdq = simplify(jacobian(Lag_dq.',dq)); % W

% Matrices for problem 1
M = Lag_dqdq
```

```
M =
```

$$\left(\begin{array}{cc} m_1 + m_2 + \frac{2\,L\,m_2\,\delta(\mathrm{dp}_{11})\,(\mathrm{da}_1\,\cos(a_1)\,\cos(a_2) - \mathrm{da}_2\,\sin(a_1)\,\sin(a_2))}{\mathrm{sign}(\mathrm{dp}_{11})^3} & 0 \\ 0 & m_1 + m_2 + \frac{2\,L\,m_2\,\delta(\mathrm{dp}_{12})\,(\mathrm{da}_1\,\cos(a_1)\,\sin(a_2)}{\mathrm{sign}(\mathrm{dp}_{12})^3} \\ 0 & 0 \\ \frac{L\,m_2\,\cos(\bar{a_1})\,\cos(\bar{a_2})\,(\sigma_1 - 1)}{\sigma_1} + \frac{L\,m_2\,\cos(a_1)\,\cos(a_2)\,(\sigma_{10} - 1)}{\sigma_{10}} & \frac{L\,m_2\,\cos(\bar{a_1})\,\sin(\bar{a_2})\,(\sigma_1 - 1)}{\sigma_1} + \frac{L\,m_2\,\cos(a_1}{} \\ -\frac{L\,m_2\,\sin(a_1)\,\sin(a_2)}{\sigma_{10}} - \frac{L\,m_2\,\sin(\bar{a_1})\,\sin(\bar{a_2})\,(\sigma_2 - 1)}{\sigma_2} & \frac{L\,m_2\,\cos(a_2)\,\sin(a_1)}{\sigma_9} + \frac{L\,m_2\,\cos(\bar{a_2})\,\sin}{\sigma_2} \end{array} \right.$$

where

$$\sigma_1 = 2\,\mathrm{sign}\left(\mathrm{da}_1\right)^2$$

$$\sigma_2 = 2\,\mathrm{sign}\left(\mathrm{da}_2\right)^2$$

$$\sigma_3 = 2\,\sin\left(\bar{a_1}\right)\,\cos\left(a_1\right)\,\mathrm{sign}\left(\mathrm{da}_1\right)^2\,\mathrm{sign}\left(\mathrm{da}_2\right)^2$$

$$\sigma_4 = \mathrm{dp}_{13}\,\mathrm{sign}\left(\mathrm{da}_1\right)^2 + L\,\mathrm{da}_1\,\sin\left(\bar{a_1}\right)\,\mathrm{sign}\left(\mathrm{dp}_{13}\right)^2$$

$$\sigma_5 = \mathrm{dp}_{13} + L\,\mathrm{da}_1\,\sin\left(a_1\right)$$

$$\sigma_6 = 2\,L\,\mathrm{da}_1\,\sin\left(\bar{a_1}\right)\,\mathrm{sign}\left(\mathrm{dp}_{13}\right)^2 + \mathrm{dp}_{13}$$

$$\sigma_7 = \cos\left(\bar{a_1}\right)\,\sin\left(a_1\right)\,\mathrm{sign}\left(\mathrm{da}_2\right)^2$$

$$\sigma_8 = \sin\left(\bar{a_1}\right)\,\cos\left(a_1\right)\,\mathrm{sign}\left(\mathrm{da}_1\right)^2$$

$$\sigma_9 = 2\,\mathrm{sign}\left(\mathrm{dp}_{12}\right)^2$$

$$\sigma_{10} = 2\,\mathrm{sign}\left(\mathrm{dp}_{11}\right)^2$$

$$\sigma_{11} = 2\,\mathrm{sign}\left(\mathrm{da}_1\right)^2\,\mathrm{sign}\left(\mathrm{da}_2\right)^2$$

$$\sigma_{12} = \frac{1}{\mathrm{sign}(\mathrm{dp}_{12})^2} - 2$$

$$\sigma_{13} = \frac{1}{\mathrm{sign}(\mathrm{dp}_{11})^2} - 2$$

$$\sigma_{14} = L\,m_2\,\sin\left(\bar{a_1}\right)$$

$$\sigma_{15} = L\,\mathrm{da}_2\,\sin\left(a_1\right)\,\sin\left(a_2\right)$$

$$\sigma_{16} = L\,\mathrm{da}_2\,\cos\left(a_2\right)\,\sin\left(a_1\right)$$

$$\sigma_{17} = \left(\mathrm{sign}\left(\mathrm{da}_1\right)^2 - 1\right)^2$$

$$\sigma_{18} = \mathrm{da}_1\,\mathrm{sign}\left(\mathrm{da}_1\right)^2$$

$$\sigma_{19} = \sin\left(a_2 - \bar{a_2}\right)$$

$$\sigma_{20} = \frac{\sigma_{21}}{2\,\mathrm{sign}(\mathrm{dp}_{13})^2}$$

$$\sigma_{21} = L\,m_2\,\sin\left(a_1\right)$$

```
b = Q + simplify(Lag_q - Lag_qdq*dq)
```

```
b =
```

$$\left(\frac{\sigma_{35}}{\sigma_1} - \sigma_{35} - \sigma_{18} - \sigma_{17} - \sigma_{36} - \sigma_{25} - L\, g\, m_2\, \sin\left(a_1\right) + \sigma_{23} + \sigma_{22} + \sigma_{20} - \sigma_{33} + \frac{\sigma_{18}}{\sigma_1} + \frac{\sigma_{17}}{\sigma_1} + \sigma_{31} + \sigma_{30} + \right.$$

where

$$\sigma_1 = 2\,\mathrm{sign}\left(\mathrm{da}_1\right)^2$$

$$\sigma_2 = 2\,\mathrm{sign}\left(\mathrm{da}_2\right)^2$$

$$\sigma_3 = L^2\, \mathrm{da}_1\, \mathrm{da}_2\, m_2\, \cos\left(\bar{a_1}\right)\, \sin\left(\bar{a_2}\right)\, \cos\left(a_1\right)\, \cos\left(a_2\right)$$

$$\sigma_4 = L^2\, \mathrm{da}_1\, \mathrm{da}_2\, m_2\, \cos\left(\bar{a_1}\right)\, \cos\left(\bar{a_2}\right)\, \cos\left(a_1\right)\, \sin\left(a_2\right)$$

$$\sigma_5 = L^2\, {\mathrm{da}_2}^2\, m_2\, \sin\left(\bar{a_1}\right)\, \sin\left(\bar{a_2}\right)\, \cos\left(a_1\right)\, \sin\left(a_2\right)$$

$$\sigma_6 = L^2\, {\mathrm{da}_2}^2\, m_2\, \cos\left(\bar{a_1}\right)\, \sin\left(\bar{a_2}\right)\, \sin\left(a_1\right)\, \sin\left(a_2\right)$$

$$\sigma_7 = L^2\, {\mathrm{da}_1}^2\, m_2\, \sin\left(\bar{a_1}\right)\, \sin\left(\bar{a_2}\right)\, \cos\left(a_1\right)\, \sin\left(a_2\right)$$

$$\sigma_8 = L^2\, {\mathrm{da}_1}^2\, m_2\, \cos\left(\bar{a_1}\right)\, \sin\left(\bar{a_2}\right)\, \sin\left(a_1\right)\, \sin\left(a_2\right)$$

$$\sigma_9 = L^2\, {\mathrm{da}_2}^2\, m_2\, \cos\left(\bar{a_2}\right)\, \sin\left(\bar{a_1}\right)\, \cos\left(a_1\right)\, \cos\left(a_2\right)$$

$$\sigma_{10} = L^2\, {\mathrm{da}_2}^2\, m_2\, \cos\left(\bar{a_1}\right)\, \cos\left(\bar{a_2}\right)\, \cos\left(a_2\right)\, \sin\left(a_1\right)$$

$$\sigma_{11} = L^2\, {\mathrm{da}_1}^2\, m_2\, \cos\left(\bar{a_2}\right)\, \sin\left(\bar{a_1}\right)\, \cos\left(a_1\right)\, \cos\left(a_2\right)$$

$$\sigma_{12} = L^2\, {\mathrm{da}_1}^2\, m_2\, \cos\left(\bar{a_1}\right)\, \cos\left(\bar{a_2}\right)\, \cos\left(a_2\right)\, \sin\left(a_1\right)$$

$$\sigma_{13} = L^2\, \mathrm{da}_1\, \mathrm{da}_2\, m_2\, \sin\left(\bar{a_1}\right)\, \sin\left(\bar{a_2}\right)\, \cos\left(a_2\right)\, \sin\left(a_1\right)$$

$$\sigma_{14} = L^2\, \mathrm{da}_1\, \mathrm{da}_2\, m_2\, \cos\left(\bar{a_2}\right)\, \sin\left(\bar{a_1}\right)\, \sin\left(a_1\right)\, \sin\left(a_2\right)$$

$$\sigma_{15} = 2\,\mathrm{sign}\left(\mathrm{dp}_{12}\right)^2$$

$$\sigma_{16} = 2\,\mathrm{sign}\left(\mathrm{dp}_{11}\right)^2$$

$$\sigma_{17} = L^2\, {\mathrm{da}_1}^2\, m_2\, \sin\left(\bar{a_1}\right)\, \cos\left(a_1\right)$$

$$\sigma_{18} = L^2\, {\mathrm{da}_1}^2\, m_2\, \cos\left(\bar{a_1}\right)\, \sin\left(a_1\right)$$

$$\sigma_{19} = L\, \mathrm{da}_2\, \mathrm{dp}_{12}\, m_2\, \sin\left(\bar{a_1}\right)\, \sin\left(\bar{a_2}\right)$$

$$\sigma_{20} = L\, \mathrm{da}_1\, \mathrm{dp}_{12}\, m_2\, \sin\left(\bar{a_1}\right)\, \sin\left(\bar{a_2}\right)$$

$$\sigma_{21} = L\, \mathrm{da}_2\, \mathrm{dp}_{11}\, m_2\, \cos\left(\bar{a_2}\right)\, \sin\left(\bar{a_1}\right)$$

$$\sigma_{22} = L\, \mathrm{da}_2\, \mathrm{dp}_{11}\, m_2\, \cos\left(\bar{a_1}\right)\, \sin\left(\bar{a_2}\right)$$

$$\sigma_{23} = L\, \mathrm{da}_1\, \mathrm{dp}_{11}\, m_2\, \cos\left(\bar{a_2}\right)\, \sin\left(\bar{a_1}\right)$$

$$\sigma_{24} = L\, \mathrm{da}_1\, \mathrm{dp}_{11}\, m_2\, \cos\left(\bar{a_1}\right)\, \sin\left(\bar{a_2}\right)$$

The resulting matrices are as shown in the output of the system.

b)

We find the equations for $T(q, \dot{q})$ and $V(q)$ remain the same for the alternative cartesian system, but the Lagrangian changes:

- $\mathcal{L} = T(q, \dot{q}) - V(q) - z^T C(q)$
- $\mathcal{L} = \frac{1}{2} m_1 \frac{\partial p_1}{\partial q}^T \frac{\partial p_1}{\partial q} + \frac{1}{2} m_2 \frac{\partial p_2}{\partial q}^T \frac{\partial p_2}{\partial q} - m_1 \cdot g \cdot p_{1y} - m_2 \cdot g \cdot p_{2y} - \frac{1}{2} z^T (e^T e - L^2)$

where $e = p_1 - p_2$.

The system dynamics can then be derived using MATLAB.

From HoveringMassConstraintTemplate.m

```
% Parameters
syms m1 m2 L g real
% Force
u = sym('u',[3,1]);

% Positions of point masses
pm1 = sym('pm1',[3,1]);
pm2 = sym('pm2',[3,1]);
dpm1 = sym('dpm1',[3,1]);
dpm2 = sym('dpm2',[3,1]);
ddpm1 = sym('d2pm1',[3,1]);
ddpm2 = sym('d2pm2',[3,1]);
% Generalized coordinates
q  = [pm1;pm2];
dq = [dpm1;dpm2];
ddq = [ddpm1;ddpm2];
% Algebraic variable
z = sym('z');

% Generalized forces
Q = [u; 0; 0; 0];
% Kinetic energy (function of q and dq)
T = 0.5*m1*dpm1'*dpm1 + 0.5*m2*dpm2'*dpm2;
T = simplify(T);
% Potential energy
V = g*(m1*pm1(3) + m2*pm2(3));
% Lagrangian (function of q and dq)
```

```
Lag = T - V;
% Constraint
dpm  = pm1 - pm2; % difference of positions
C = 0.5*(dpm'*dpm - L^2);

% Derivatives of constrained Lagrangian
Lag_q = simplify(jacobian(Lag,q)).';
Lag_qdq = simplify(jacobian(Lag_q.',dq));
Lag_dq = simplify(jacobian(Lag,dq)).';
Lag_dqdq = simplify(jacobian(Lag_dq.',dq)); % W
C_q = simplify(jacobian(C,q)).';

% Matrices for problem 1b
M = Lag_dqdq
```

M =

$$\begin{pmatrix} m_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_2 \end{pmatrix}$$

```
b = Q + simplify(Lag_q - Lag_qdq*dq - z*C_q)
```

b =

$$\begin{pmatrix} u_1 - \sigma_2 \\ u_2 - \sigma_1 \\ u_3 - g\,m_1 - z\left(\frac{\mathrm{pm}_{13}}{2} - \frac{\mathrm{pm}_{23}}{2} + \frac{\overline{\mathrm{pm}_{13}}}{2} - \frac{\overline{\mathrm{pm}_{23}}}{2}\right) \\ \sigma_2 \\ \sigma_1 \\ z\left(\frac{\mathrm{pm}_{13}}{2} - \frac{\mathrm{pm}_{23}}{2} + \frac{\overline{\mathrm{pm}_{13}}}{2} - \frac{\overline{\mathrm{pm}_{23}}}{2}\right) - g\,m_2 \end{pmatrix}$$

where

$$\sigma_1 = z\left(\frac{\mathrm{pm}_{12}}{2} - \frac{\mathrm{pm}_{22}}{2} + \frac{|\mathrm{pm}_{12}|^2}{2\,\mathrm{pm}_{12}} - \frac{|\mathrm{pm}_{22}|^2}{2\,\mathrm{pm}_{22}}\right)$$

$$\sigma_2 = z\left(\frac{\mathrm{pm}_{11}}{2} - \frac{\mathrm{pm}_{21}}{2} + \frac{|\mathrm{pm}_{11}|^2}{2\,\mathrm{pm}_{11}} - \frac{|\mathrm{pm}_{21}|^2}{2\,\mathrm{pm}_{21}}\right)$$

We see by the output, that the expressions this gives are much simpler than in the previous subtask. It is clear that using this alternative representation has the advantage of making the model it self cleaner with less trigonometric complexities that are hard to calculate

Task 2

a)

From the lectures we have learned that the implicit system on matrix form is given by:

- $$\begin{bmatrix} M & C(q) \\ C(q)^T & 0 \end{bmatrix} \begin{bmatrix} \ddot{q} \\ z \end{bmatrix} = \begin{bmatrix} Q - \frac{\partial V}{\partial q}^T + \frac{\partial T}{\partial q}^T - \frac{\partial}{\partial q}[W(q)\dot{q}]\dot{q} \\ -\frac{\partial}{\partial q}\left[\frac{\partial C}{\partial q}\dot{q}\right]\dot{q} \end{bmatrix}$$

We then use this in matlab to get our full expressions for the two matrices.

Find the implicit model on Matrix form

```
a_q = C_q;
Mimplicit = [M, a_q; a_q', 0]
```

Mimplicit =

$$\begin{pmatrix} m_1 & 0 & 0 & 0 & 0 & 0 & \sigma_6 \\ 0 & m_1 & 0 & 0 & 0 & 0 & \sigma_4 \\ 0 & 0 & m_1 & 0 & 0 & 0 & \sigma_2 \\ 0 & 0 & 0 & m_2 & 0 & 0 & \sigma_5 \\ 0 & 0 & 0 & 0 & m_2 & 0 & \sigma_3 \\ 0 & 0 & 0 & 0 & 0 & m_2 & \sigma_1 \\ \sigma_6 & \sigma_4 & \sigma_2 & \sigma_5 & \sigma_3 & \sigma_1 & 0 \end{pmatrix}$$

where

$$\sigma_1 = \frac{\mathrm{pm}_{23}}{2} - \frac{\mathrm{pm}_{13}}{2} - \frac{\bar{\mathrm{pm}}_{13}}{2} + \frac{\bar{\mathrm{pm}}_{23}}{2}$$

$$\sigma_2 = \frac{\mathrm{pm}_{13}}{2} - \frac{\mathrm{pm}_{23}}{2} + \frac{\bar{\mathrm{pm}}_{13}}{2} - \frac{\bar{\mathrm{pm}}_{23}}{2}$$

$$\sigma_3 = \frac{\mathrm{pm}_{22}}{2} - \frac{\mathrm{pm}_{12}}{2} - \frac{\bar{\mathrm{pm}}_{12}}{2} + \frac{\bar{\mathrm{pm}}_{22}}{2}$$

$$\sigma_4 = \frac{\mathrm{pm}_{12}}{2} - \frac{\mathrm{pm}_{22}}{2} + \frac{\bar{\mathrm{pm}}_{12}}{2} - \frac{\bar{\mathrm{pm}}_{22}}{2}$$

$$\sigma_5 = \frac{\mathrm{pm}_{21}}{2} - \frac{\mathrm{pm}_{11}}{2} - \frac{\bar{\mathrm{pm}}_{11}}{2} + \frac{\bar{\mathrm{pm}}_{21}}{2}$$

$$\sigma_6 = \frac{\mathrm{pm}_{11}}{2} - \frac{\mathrm{pm}_{21}}{2} + \frac{\bar{\mathrm{pm}}_{11}}{2} - \frac{\bar{\mathrm{pm}}_{21}}{2}$$

```
c = [Q + simplify(Lag_q - Lag_qdq*dq); - jacobian((C_q')*dq, q)*dq]
```

```
c =
```

$$\left(\begin{array}{c} u_1 \\ u_2 \\ u_3 - g\,m_1 \\ 0 \\ 0 \\ -g\,m_2 \\ \mathrm{dpm}_{21}\,(\mathrm{dpm}_{11} - \mathrm{dpm}_{21}) - \mathrm{dpm}_{12}\,(\mathrm{dpm}_{12} - \mathrm{dpm}_{22}) - \mathrm{dpm}_{13}\,(\mathrm{dpm}_{13} - \mathrm{dpm}_{23}) - \mathrm{dpm}_{11}\,(\mathrm{dpm}_{11} - \mathrm{dpm} \end{array}\right.$$

The matrecies are given by the matlab ouput.

b)

We then do then find the explicit model of the system by inverting our M matrix.

Find the explicit model

```
size(c)
```

```
ans = 1x2
     7     1
```

```
% Matrices for problem 2
Mexplicit = simplify(inv(Mimplicit))
```

```
Mexplicit =
```

$$\left(\frac{\mathrm{pm}_{11}{}^2\,\mathrm{pm}_{12}{}^2\,\mathrm{pm}_{13}{}^2\,\mathrm{pm}_{21}{}^2\,\mathrm{pm}_{22}{}^2\,\mathrm{pm}_{23}{}^2\,\left(m_1\,\mathrm{pm}_{11}{}^2+m_1\,\mathrm{pm}_{12}{}^2+m_1\,\mathrm{pm}_{13}{}^2+m_2\,\mathrm{pm}_{12}{}^2+m_2\,\mathrm{pm}_{13}{}^2+m_1\,\mathrm{pm}_{21}{}^2+m_1\,\mathrm{pm}_{22}{}^2+m\right.}{} \right.$$

where

$$\sigma_1 = \frac{2\,m_2\,|\mathrm{pm}_{13}|^2\,|\mathrm{pm}_{23}|^2}{\mathrm{pm}_{13}\,\mathrm{pm}_{23}}$$

$$\sigma_2 = \frac{2\,m_1\,|\mathrm{pm}_{13}|^2\,|\mathrm{pm}_{23}|^2}{\mathrm{pm}_{13}\,\mathrm{pm}_{23}}$$

$$\sigma_3 = \frac{2\,m_2\,|\mathrm{pm}_{12}|^2\,|\mathrm{pm}_{22}|^2}{\mathrm{pm}_{12}\,\mathrm{pm}_{22}}$$

$$\sigma_4 = \frac{2\,m_1\,|\mathrm{pm}_{12}|^2\,|\mathrm{pm}_{22}|^2}{\mathrm{pm}_{12}\,\mathrm{pm}_{22}}$$

$$\sigma_5 = \frac{2\,m_2\,|\mathrm{pm}_{11}|^2\,|\mathrm{pm}_{21}|^2}{\mathrm{pm}_{11}\,\mathrm{pm}_{21}}$$

$$\sigma_6 = \frac{2\,m_1\,|\mathrm{pm}_{11}|^2\,|\mathrm{pm}_{21}|^2}{\mathrm{pm}_{11}\,\mathrm{pm}_{21}}$$

$$\sigma_7 = 2\,m_2\,|\mathrm{pm}_{23}|^2$$

$$\sigma_8 = 2\,m_2\,|\mathrm{pm}_{22}|^2$$

$$\sigma_9 = 2\,m_1\,|\mathrm{pm}_{23}|^2$$

$$\sigma_{10} = 2\,m_2\,|\mathrm{pm}_{21}|^2$$

$$\sigma_{11} = 2\,m_1\,|\mathrm{pm}_{22}|^2$$

$$\sigma_{12} = 2\,m_1\,|\mathrm{pm}_{21}|^2$$

$$\sigma_{13} = 2\,m_2\,|\mathrm{pm}_{13}|^2$$

$$\sigma_{14} = 2\,m_2\,|\mathrm{pm}_{12}|^2$$

$$\sigma_{15} = 2\,m_1\,|\mathrm{pm}_{13}|^2$$

$$\sigma_{16} = 2\,m_2\,|\mathrm{pm}_{11}|^2$$

$$\sigma_{17} = 2\,m_1\,|\mathrm{pm}_{12}|^2$$

$$\sigma_{18} = 2\,m_1\,|\mathrm{pm}_{11}|^2$$

$$\sigma_{19} = \frac{m_2\,|\mathrm{pm}_{23}|^4}{\mathrm{pm}_{23}{}^2}$$

$$\sigma_{20} = \frac{m_1\,|\mathrm{pm}_{23}|^4}{}$$

```
rhs = simplify(Mexplicit*c) % Right hand side
```

rhs =

$$\left(-\frac{\mathrm{pm}_{11}{}^2\,\mathrm{pm}_{12}{}^2\,\mathrm{pm}_{13}{}^3\,\mathrm{pm}_{21}{}^3\,\mathrm{pm}_{22}{}^2\,\mathrm{pm}_{23}{}^2\,u_1-\mathrm{pm}_{11}{}^2\,\mathrm{pm}_{12}{}^2\,\mathrm{pm}_{21}{}^2\,\mathrm{pm}_{22}{}^2\,\mathrm{pm}_{23}{}^2\,u_3\,|\mathrm{pm}_{13}|^4-\mathrm{pm}_{11}{}^2\,\mathrm{pm}_{12}{}^2\,\mathrm{pm}_{13}{}^2\,\mathrm{pm}_{21}{}^3\,\mathrm{pm}_{22}}{} \right.$$

where

$$\sigma_1 = 2\,m_1\,\mathrm{dpm}_{11}{}^2\,\mathrm{pm}_{11}\,\mathrm{pm}_{12}\,\mathrm{pm}_{13}\,\mathrm{pm}_{21}\,\mathrm{pm}_{22}\,\mathrm{pm}_{23} - 4\,m_1\,\mathrm{dpm}_{11}\,\mathrm{dpm}_{21}\,\mathrm{pm}_{11}\,\mathrm{pm}_{12}\,\mathrm{pm}_{13}\,\mathrm{pm}_{21}\,\mathrm{pm}_{22}\,\mathrm{pm}$$

$$\sigma_2 = m_1\,(m_1+m_2)\,\sigma_3$$

$$\sigma_3 = \mathrm{pm}_{11}{}^4\,\mathrm{pm}_{12}{}^2\,\mathrm{pm}_{13}{}^2\,\mathrm{pm}_{21}{}^2\,\mathrm{pm}_{22}{}^2\,\mathrm{pm}_{23}{}^2 - 2\,\mathrm{pm}_{11}{}^3\,\mathrm{pm}_{12}{}^2\,\mathrm{pm}_{13}{}^2\,\mathrm{pm}_{21}{}^3\,\mathrm{pm}_{22}{}^2\,\mathrm{pm}_{23}{}^2 - 2\,\mathrm{pm}_{11}{}^3\,\mathrm{pm}_1$$

Taking the inverse of the M matrix is clearly a heavy task for MATLAB as it takes some time. The answer is also less elegant and harder to read. Solving a system implicitly is therefore often prefered.

<u>Task 3</u>

a)

In the case of parallel manipulators, such as the Δ-robot, the arms of the robot is mechanically constrained and does not consist of three free hanging pendulums. We also need the position of the nacelle in order to properly describe the system and this can be a difficult task given just the angles.

b)

As by the lecture notes we have that the kinetic energy of the nacelle is given by:

- $T(q,\dot{q}) = \dfrac{1}{2}m\dot{p}^T\dot{p} + \dfrac{1}{2}J\sum_{k=1}^{3}\dot{\alpha}_k^2$

Where $p \in IR^3$ is the vector describing the position of the nacelle, m is the mass of the nacelle, $\alpha_k,\ k = 1, 2, 3$ are the angles of the pivots and J is the inertia matrix governing the system.

The potential energy is given by:

- $V(q) = m \cdot g \cdot p_y + \frac{1}{2} \sum_{k=1}^{3} M \cdot g \cdot l \cdot sin(\alpha_k)$

where p_y is the y component of the position vector p and l is the length between the motor and the next pivot. And the constraints are given by the equation:

- $C(q) = \left(\sum_{k=1}^{3} ||p - p_k||^2 \right) - L^2$

The Lagrange function is then given by:

- $\mathcal{L} = \frac{1}{2} m \dot{p}^T \dot{p} + \sum_{k=1}^{3} \left(\frac{1}{2} J \dot{\alpha}_k^2 - mgp_y - \frac{1}{2} Mgl \cdot sin(\alpha_k) + z_k \left(||p - p_k||^2 - L^2 \right) \right)$

where z is the vector of Lagrange multipliers.

c)