Assignment 6 in ModSim

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Task 1

a)

It's a DAE because the variable z is not part of the equation with as a time derivative. The equation is therefore a DAE.

b)

In order to find the order of the DAE we differentiate the semi-expicit DAE $g = 0 = \frac{1}{2}\left(x_1^2 + x_2^2 - 1\right)$.

We see that by differentiating twice we get:

- $g^{''} = \dot{x}_1(x_1 + x_2 + z) + \dot{x}_2(z + u) + x_1(\dot{x}_1 + \dot{x}_2 + \dot{z}) + x_2(\dot{z} + \dot{u})$

This yields the differential equation:

- $\dot{z} = \dfrac{-\dot{x}(z + u) - x_1\dot{x}_2 - x_1\dot{x}_1 - \dot{x}(x_1 + x_2 + z) - x_2\dot{u}}{x_1 + x_2}$

This holds because we know that $x_1 + x_2 = 0$.

c)

Since we have DAE of 2nd order, we only have to differentiate once to reduce the index, this yields:

- $g^{'} = x_1^2 + x_1x_2 + x_2u + (x_1 + x_2)z$

Task 2

a)

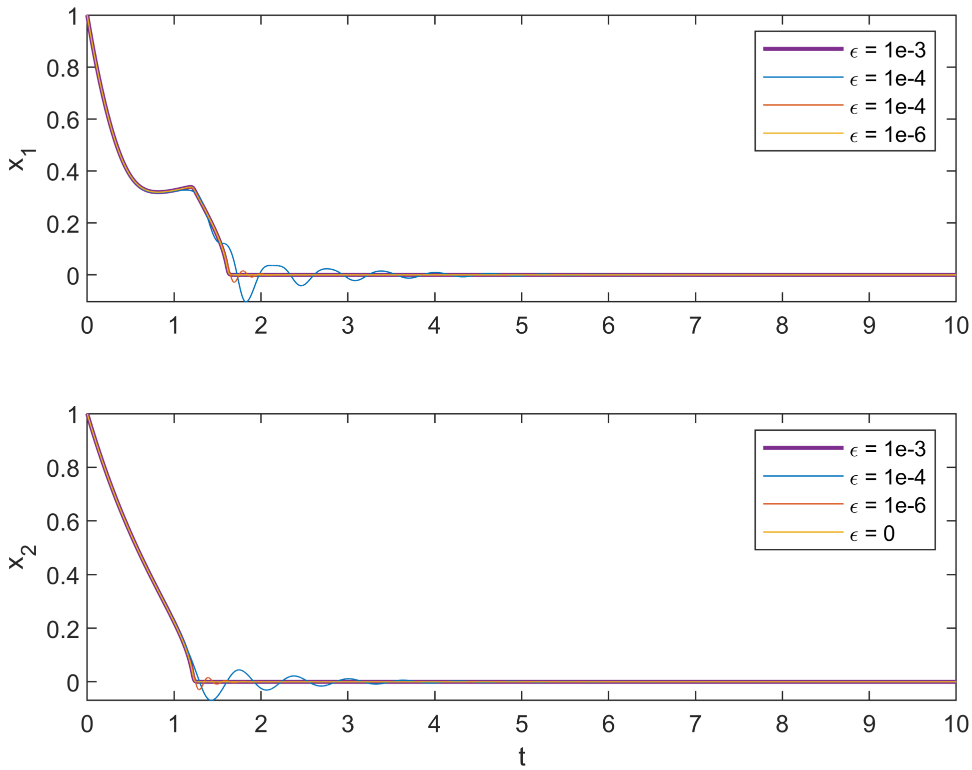
We can clearly see that given that if $\epsilon = 0$, the equation is a DAE, because z has no derivative in any of the expressions. Conversely, it is an ODE if $z > 0$.

b)

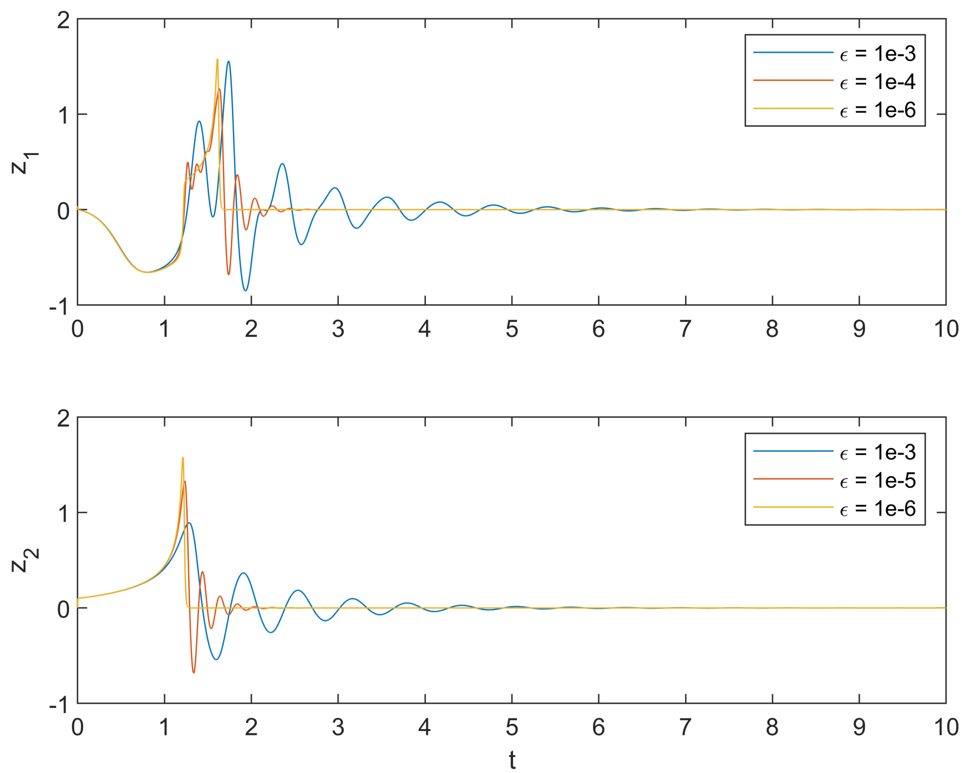
```
% Initializing model
clear
syms x1 x2 z1 z2 u epsilon alpha real;
M = - [1 1;
       0 1];
x = [x1;
     x2];
z = [z1;
     z2];
dx = M*x-z;
```

```
A = [x1^2 x2;
     0 x2^2]+  alpha*eye(2);
dz = simplify(1/10*x-A*z)/epsilon;
z_dae = simplify((1/10)*x-A*z);


% Initializin paramters
epsilon =1e-3;
alpha = 1e-3;
init = [1 1 0 0];
tf = 10;
tspan = [0 tf];
param = [epsilon,alpha];
setings = odeset('RelTol',1e-12,'AbsTol',1e-16);

dae_sol = solve(z_dae==0,z);
dae_sol = simplify([dae_sol.z1;dae_sol.z2]);


[t_0,x_0] = ode15s(@(t,x)ODEfunc(t,x,param),tspan,init,setings);
epsilon =1e-4;
param = [epsilon,alpha];
[t_1,x_1] = ode15s(@(t,x)ODEfunc(t,x,param),tspan,init,setings);

epsilon =1e-6;
param = [epsilon,alpha];
[t_2,x_2] = ode15s(@(t,x)ODEfunc(t,x,param),tspan,init,setings);
epsilon =0;
param = [epsilon,alpha];
[t_3,x_3] = ode15s(@(t,x)DAEfunc(t,x,param),tspan,[1 1],setings);
figure()
subplot(2,1,1)
plot(t_0,x_0(:,1))
hold on
plot(t_1,x_1(:,1))
plot(t_2,x_2(:,1))
d = plot(t_3,x_3(:,1),'linewidth',1.5);

uistack(d,'bottom')
hold off
legend('\epsilon = 1e-3','\epsilon = 1e-4','\epsilon = 1e-4','\epsilon = 1e-6')
ylabel('x_1')
subplot(2,1,2)
plot(t_0,x_0(:,2))
hold on

plot(t_1,x_1(:,2))
plot(t_2,x_2(:,2))
d = plot(t_3,x_3(:,2),'linewidth',1.5);

uistack(d,'bottom')
hold off
legend('\epsilon = 1e-3','\epsilon = 1e-4','\epsilon = 1e-6','\epsilon = 0')
ylabel('x_2')
```

```
xlabel('t')
```




```
figure()
subplot(2,1,1)
plot(t_0,x_0(:,3))
hold on

plot(t_1,x_1(:,3))
plot(t_2,x_2(:,3))
hold off
legend('\epsilon = 1e-3','\epsilon = 1e-4','\epsilon = 1e-6')
ylabel('z_1')
subplot(2,1,2)
plot(t_0,x_0(:,4))
hold on

plot(t_1,x_1(:,4))
plot(t_2,x_2(:,4))
hold off
legend('\epsilon = 1e-3','\epsilon = 1e-5','\epsilon = 1e-6')
ylabel('z_2')
xlabel('t')
```



c)

```
% New alpha
alpha = 0;

param = [epsilon,alpha];
[t_0,x_0] = ode15s(@(t,x)ODEfunc(t,x,param),tspan,init,setings);
```

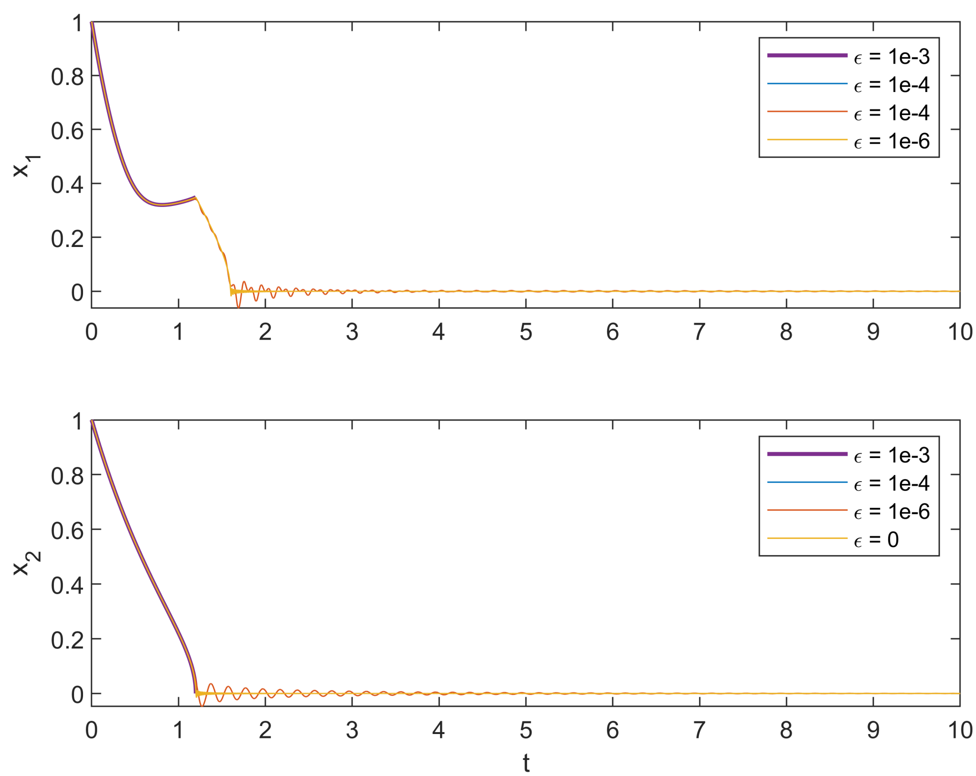
```
Warning: Failure at t=0.000000e+00.  Unable to meet integration tolerances without reducing the step size
below the smallest value allowed (7.905050e-323) at time t.
```

```
epsilon =1e-4;
param = [epsilon,alpha];
[t_1,x_1] = ode15s(@(t,x)ODEfunc(t,x,param),tspan,init,setings);

epsilon =1e-6;
param = [epsilon,alpha];
[t_2,x_2] = ode15s(@(t,x)ODEfunc(t,x,param),tspan,init,setings);
epsilon =0;
param = [epsilon,alpha];
[t_3,x_3] = ode15s(@(t,x)DAEfunc(t,x,param),tspan,[1 1],setings);
```

```
Warning: Failure at t=1.198948e+00.  Unable to meet integration tolerances without reducing the step size
below the smallest value allowed (3.552714e-15) at time t.
```

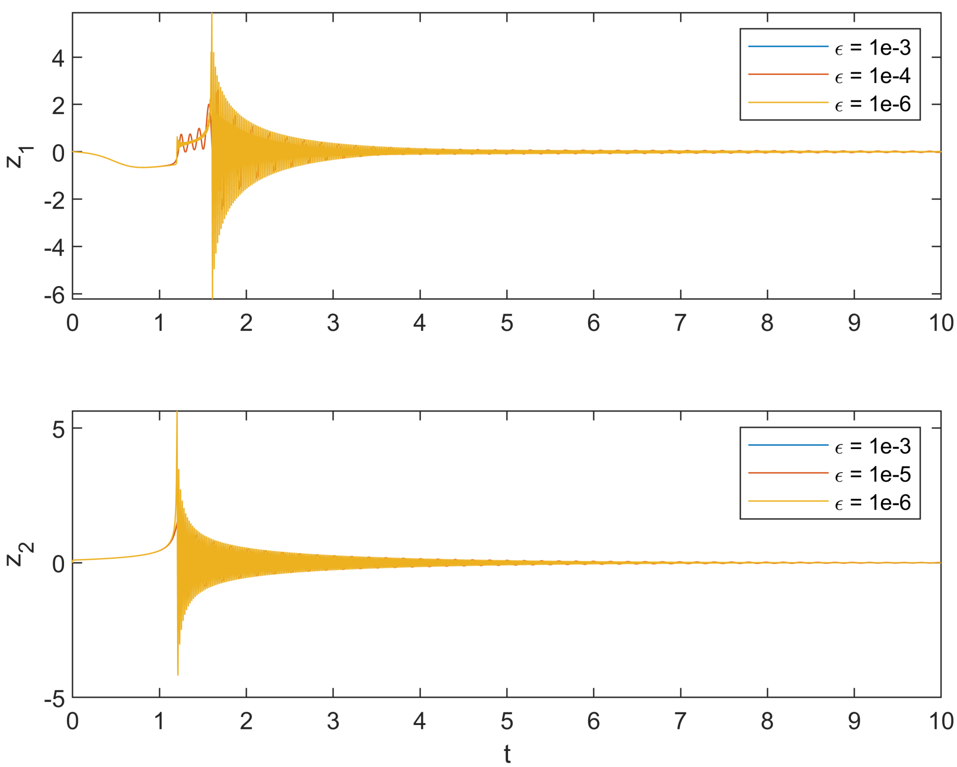
```
figure()
```

```
subplot(2,1,1)
plot(t_0,x_0(:,1))
hold on
plot(t_1,x_1(:,1))
plot(t_2,x_2(:,1))
d = plot(t_3,x_3(:,1),'linewidth',1.5);

uistack(d,'bottom')
hold off
legend('\epsilon = 1e-3','\epsilon = 1e-4','\epsilon = 1e-4','\epsilon = 1e-6')
ylabel('x_1')
subplot(2,1,2)
plot(t_0,x_0(:,2))
hold on

plot(t_1,x_1(:,2))
plot(t_2,x_2(:,2))
d = plot(t_3,x_3(:,2),'linewidth',1.5);

uistack(d,'bottom')
hold off
legend('\epsilon = 1e-3','\epsilon = 1e-4','\epsilon = 1e-6','\epsilon = 0')
ylabel('x_2')
xlabel('t')
```




```
figure()
subplot(2,1,1)
plot(t_0,x_0(:,3))
hold on
```

```
plot(t_1,x_1(:,3))
plot(t_2,x_2(:,3))
hold off
legend('\epsilon = 1e-3','\epsilon = 1e-4','\epsilon = 1e-6')
ylabel('z_1')
subplot(2,1,2)
plot(t_0,x_0(:,4))
hold on

plot(t_1,x_1(:,4))
plot(t_2,x_2(:,4))
hold off
legend('\epsilon = 1e-3','\epsilon = 1e-5','\epsilon = 1e-6')
ylabel('z_2')
xlabel('t')
```



b)

Task 3

a)

We clearly see by the equations that u has no derivatives, given this is a variable of it's own right we can say that the this is a DAE. Using u can cause som confusion tho. To avoid this we should use an other name.

b)

The same goes for this equation. It is not an ODE because u has no derivative here either. However, to check if it is a DAE we check if the jacobian matrix of the system is of full rank:

```
clear
syms x1 x2 dx1 dx2 u real;
x = [x1;
     x2];
dx = [dx1;
      dx2];
f = [u+x.'*dx;
     u*x.'*dx]
```

f =

$$\begin{pmatrix} u+\mathrm{dx}_1\,x_1+\mathrm{dx}_2\,x_2 \\ \mathrm{dx}_1\,u\,x_1+\mathrm{dx}_2\,u\,x_2 \end{pmatrix}$$

```
df = jacobian(f, x)
```

df =

$$\begin{pmatrix} \mathrm{dx}_1 & \mathrm{dx}_2 \\ \mathrm{dx}_1\,u & \mathrm{dx}_2\,u \end{pmatrix}$$

```
det(df)
```

ans = 0

We see the determinant of the jacobian is 0 and system is there for not full rank and not a DAE

<u>Task 4</u>

a)

No, trigonometric functions cannot be semi-explicit.

```
function state_dot = ODEfunc(t,states,param)
    x1 = states(1);
    x2 = states(2);
    z1 = states(3);
    z2 = states(4);
    M = - [1 1;0 1];
    x= [x1;x2];
    z= [z1;z2];
    A = [x1^2 x2; 0 x2^2]+param(2)*eye(2);
    dx = M*x-z;
    dz = (1/10*x-A*z)/param(1);
    state_dot = [dx;dz];
```

```
end

function state_dot = DAEfunc(t,states,param)
    x1 = states(1);
    x2 = states(2);
    x= [x1;
        x2];
    alpha = param(2);
    z = [(alpha*x1+x1*x2^2-x2^2)/(10*(x1^2+alpha)*(x2^2+alpha));
        x2/(10*(x2^2+alpha))];
    M = - [1 1;0 1];
    dx = M*x-z;
    state_dot = [dx];
end
```