Assignment 3 in Modsim

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Task 1

a)

The only force acting on the satellite and it's reference frame is gravity. We want to desctive the rotation, translation, velocity and angular velocity of the satellite in relation to the earth. The world frame is given by $\{w\}$, the center of mass of the satellite is denoted by $\{s\}$ and the frame of the satelite sides is given by $\{b\}$.

The change in distance to earth is given by:

- $\dot{r}_s^w = v_s^w$

where r_s^w is the dirstance between the two bodies. The change in velocity given by the gravitational force is:

- $m\dot{v}_s^w = F^w = -\frac{Gm_T m}{||\vec{r}_s||^2} \cdot \frac{\vec{r}_s}{||\vec{r}_s||}$
- $\dot{v}_s^w = \frac{F^w}{m} = -\frac{Gm_T}{||\vec{r}_s||^2} \cdot \frac{\vec{r}_s}{||\vec{r}_s||}$

The rotation of the box represented in the world frame is given by:

- $\dot{R}_b^w = R_b^w(\omega_{wb}^b)^\times$

Finally the change in angular velocity is given by:

- $M_s^b\dot{\omega}_{wb}^b = -(\omega_{wb}^b)^\times M_s^b \omega_{wb}^b$
- $\dot{\omega}_{wb}^b = -(M_s^b)^{-1}(\omega_{wb}^b)^\times M_s^b \omega_{wb}^b$

where M_s^b is the inertia matrix of the satellite.

b)

```
% SatelliteDynamics.m

pos = x(1:3); % Position of the satellite in x, y, z coordinates
vel = x(13:15); % Velocity of the satellite in x, y, z coordinates
omega = x(16:18); % Angular velocity around the x, y, z axes
R = reshape(x(4:12), [3, 3]); % Rotational matrix
omega_skew = [0, -omega(3), omega(2);
               omega(3), 0, -omega(1);
              -omega(2), omega(1), 0];
```

```
G = parameters(1);
m_T = parameters(2);
norm_of_r = parameters(3);
M = parameters;

gravity = - G * m_T / norm_of_r^3;
omega_b_wb = -M\omega_skew*M*omega;


state_dot = [vel;
             reshape(R*omega_skew, 9, 1);
             gravity;
             omega_b_wb];
```

c)

Implementation of SatelliteDynamics.m

```
clear all;

time_final = 20;
init_state = zeros(18, 1)
gravity = -9.81
m_T = 5.972*exp(24)
norm_of_r = 35*exp(4)
parameters = [gravity; m_T; norm_of_r]

[t, state] = ode45(@(t, x) SatelliteDynamics(t, x, parameters), [0, time_final], init_state)
```

Task 2

a)

We choose the origin of the body as our frame. We can then choose our states as R_b^w and ω_{wb}^b. This yields:

- $\dot{R}_b^w = R_b^w(\omega_{wb}^b)^\times$ and $M_0^b\dot{\omega}_{wb}^b = -(\omega_{wb}^b)^\times M_0^b\omega_{wb}^b + r_s^b \times F^b$

I didn't really make time for this one, I will do better next time!