CHAPTER-VIII

APPLICATIONS OF THE INTEGRALS

2 MARK QUESTIONS

1.Using integration, find the area of triangle whose vertices are (2, 3), (3, 5) and (4, 4).

Answer:

32 sq units.

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2.Using integration, find the area of region bounded by the triangle whose vertices are (- 2, 1), (0, 4) and (2, 3).

Answer:

4 sq units

3.Using the method of integration, find the area of the ΔABC, coordinates of whose vertices are A (4, 1), B(6, 6) and C (8, 4).

Answer:

7 sq units

4.Using integration, find the area of the triangular region whose vertices are (2, – 2), (4, 3) and (1, 2).

Answer:

132 sq units

5.Using integration, find the area of the region in the first quadrant enclosed by the X-axis, the line $y = x$ and the circle $x^2 + y^2 = 18$.

Answer:

9π4 sq units

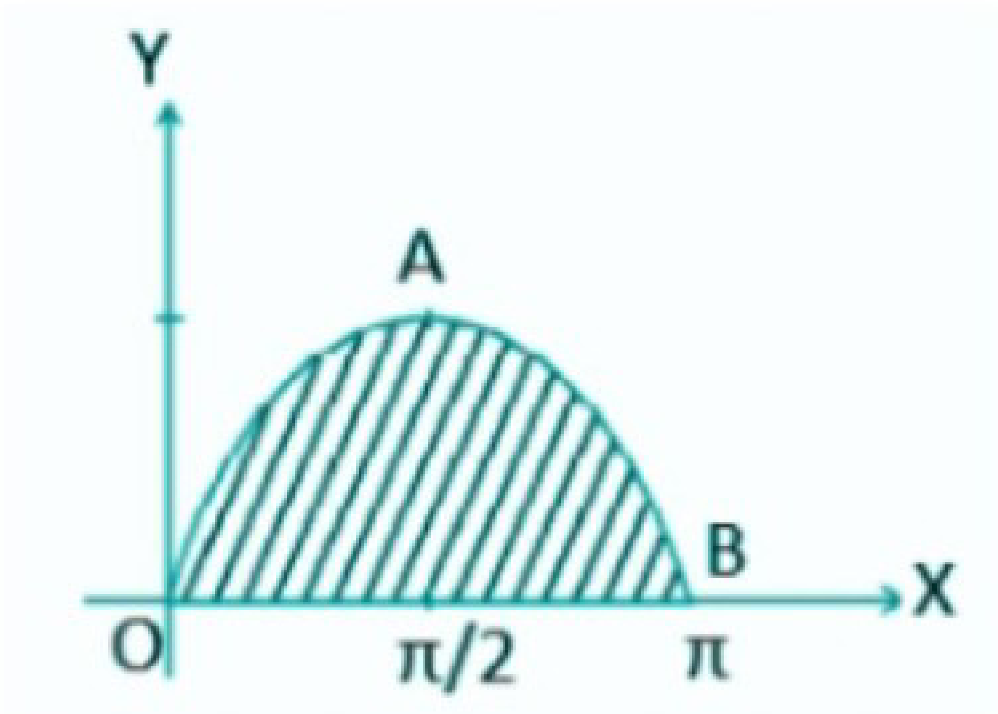
4 MARK QUESTIONS

1: Find the area of the curve y = sin x between 0 and π.

Solution:

Given,

y = sin x



Area of OAB

$$= \int_0^{\pi} y\, dx$$
$$= \int_0^{\pi} \sin x\, dx$$
$$= [-\cos x]_0^{\pi}$$

= − [cos π − cos 0]

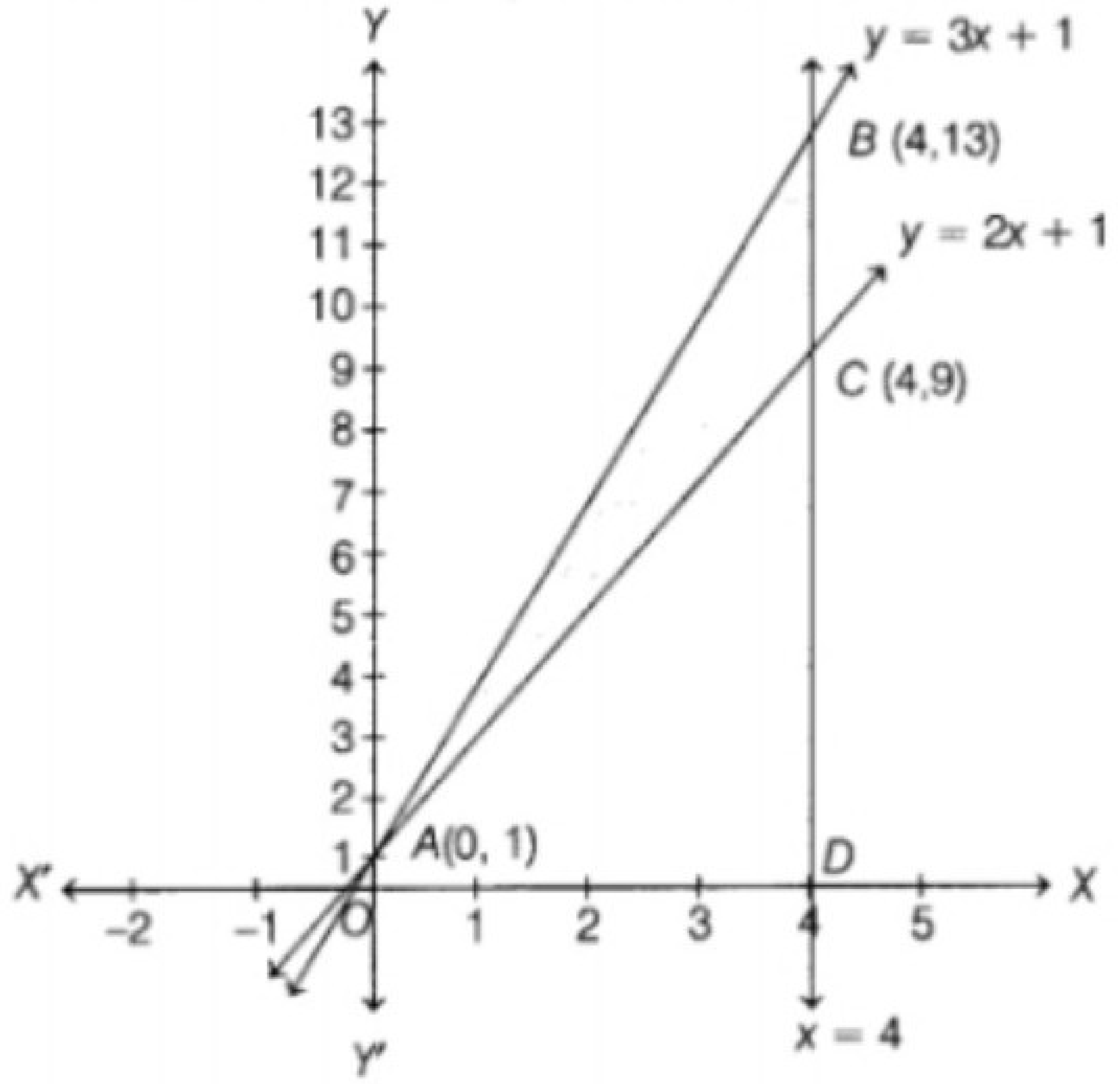
= -(-1 -1)

= 2 sq. units

2.Using integration, find the area of the triangular region whose sides have the equations y = 2x + 1, y = 3x + 1 and x = 4.

Answer:

Given, equation of sides are
$y = 2x + 1$, $y = 3x + 1$ and $x = 4$
On drawing the graph of these equations, we get the following triangular region



By solving these equations we get the vertices of triangle as A(0, 1), B(4, 13) and C(4, 9).

∴ Required area = Area (OABDO) – area (OACDO)

$$= \int_0^4 (3x + 1)\,dx - \int_0^4 (2x + 1)\,dx$$

$$= \left[\frac{3x^2}{2} + x\right]_0^4 - [x^2 + x]_0^4$$

$$= \frac{3 \times 4^2}{2} + 4 - 0 - (4^2 + 4 - 0)$$

$$= 24 + 4 - 20$$

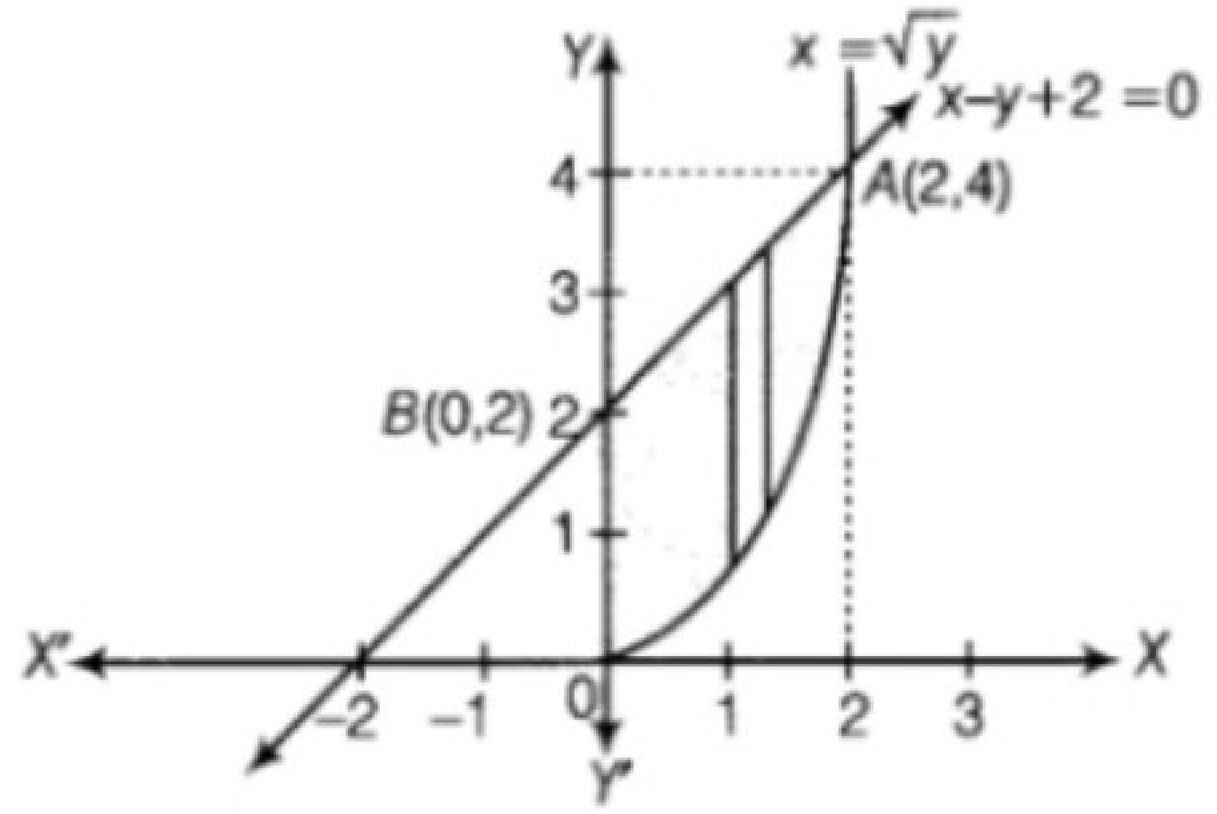
$$= 8 \text{ sq units}$$

3.Using integration, find the area of the region bounded by the line $x - y + 2 = 0$, the curve $x = \sqrt{y}$ and Y-axis.

Answer:

Given curves are
$x - y + 2 = 0$ ..... (i)
and $x = \sqrt{y}$ ..... (ii)
Consider $x = \sqrt{y} \Rightarrow x^2 = y$, which represents the parabola whose vertex is (0, 0) and axis is Y-axis.
Now, the point of intersection of Eqs.(i) and (ii)
is given by x = x+2----√
$\Rightarrow x^2 = x + 2$
$\Rightarrow x^2 - x - 2 = 0$
$\Rightarrow (x - 2)(x + 1) = 0$
$\Rightarrow x = -1, 2$
But $x = -1$ does not satisfy the Eq. (ii).
$\therefore x = 2$
Now, putting $x = 2$ in Eq. (ii), we get
$2 = \sqrt{y} \Rightarrow y = 4$
Hence, the point of intersection is (2, 4).
But actual equation of given parabola is $x = \sqrt{y}$, it means a semi-parabola which is on right side of Y – axis.

The graph of given curves are shown below:



Clearly, area of bounded region

$$= \text{Area of region } OABO$$

$$= \int_0^2 [y_{(\text{line})} - y_{(\text{parabola})}]dx$$

$$= \int_0^2 (x+2)dx - \int_0^2 x^2 dx$$

$$= \left[\frac{x^2}{2} + 2x\right]_0^2 - \left[\frac{x^3}{3}\right]_0^2 = \left[\frac{4}{2} + 4 - 0\right] - \left[\frac{8}{3} - 0\right]$$

$$= 6 - \frac{8}{3} = \frac{18-8}{3} = \frac{10}{3} \text{ sq units}$$

4.Find the area of the region $\{(x, y): y^2 \le 4x, 4x^2 + 4y^2 \le 9\}$, using method of integration.

Answer:

First, find the intersection points of given curves and then draw a rough diagram to represent the required area. If it is symmetrical about X-axis or Y-axis, then we first find area of only one portion , from them and then required area is twice of that area.

Given curves are

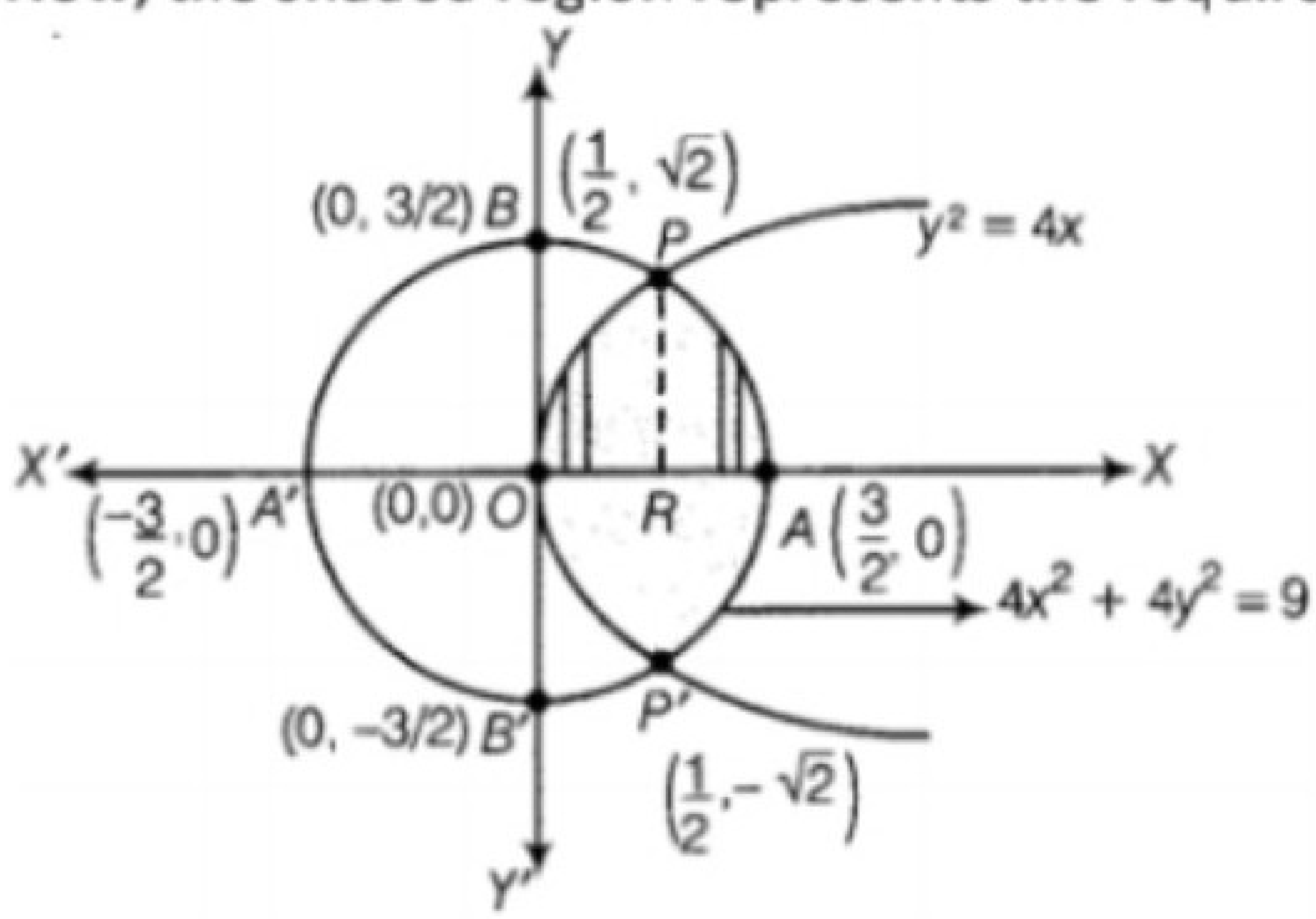
$y^2 = 4x$ ...... (i)

and $4x^2 + 4y^2 = 9$

$\Rightarrow x^2 + y^2 = 94$ ...... (ii)

Eq. (i) represents a parabola having vertex (0, 0) and axis is X-axis and Eq. (ii)

represents a circle having centre (0, 0) and radius 32.
On substituting $y^2 = 4x$ in Eq. (ii), we get
$x^2 + 4x = 94$
$\Rightarrow 4x^2 + 18x - 2x - 9 = 0$
$\Rightarrow 2x(2x + 9) - 1\,(2x + 9) = 0$
$\Rightarrow (2x + 9)\,(2x - 1) = 0$
$\Rightarrow x = 12, -92$
On putting $x = 12$ in Eq. (i), we get
$y = \pm\sqrt{2}$
At $x = -92$, y have imaginary values.
So, intersection points are p(12,2−√) and p(12,−2−√).
Now, the shaded region represents the required region as shown below:



∴ Required area = 2[Area of the region ORPO + Area of the region RAPR]

$$= 2\left[\int_0^{1/2} y_{(\text{parabola})}\,dx + \int_{1/2}^{3/2} y_{(\text{circle})}\,dx\right]$$

$$= 2\left[\int_0^{1/2} \sqrt{4x}\,dx + \int_{1/2}^{3/2}\sqrt{\frac{9}{4} - x^2}\,dx\right]$$

$$= 2\left\{\left[2\left(\frac{2}{3}x^{3/2}\right)\right]_0^{1/2} + \left[\frac{x}{2}\sqrt{\frac{9}{4} - x^2} + \frac{9}{8}\sin^{-1}\left(\frac{2x}{3}\right)\right]_{1/2}^{3/2}\right\}$$

$$\left[\because \int\sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]$$

$$= 2\left[\frac{4}{3}\left\{\left(\frac{1}{2}\right)^{3/2} - 0\right\} + \left\{0 + \frac{9}{8}\sin^{-1}(1) - \frac{1}{4}\cdot\sqrt{2} - \frac{9}{8}\sin^{-1}\frac{1}{3}\right\}\right]$$

$$= 2\left[\frac{4}{3}\times\frac{1}{2\sqrt{2}} + \frac{9}{8}\times\frac{\pi}{2} - \frac{\sqrt{2}}{4} - \frac{9}{8}\sin^{-1}\frac{1}{3}\right]$$

$$= 2\left[\frac{2}{3\sqrt{2}} + \frac{9\pi}{16} - \frac{1}{2\sqrt{2}} - \frac{9}{8}\sin^{-1}\frac{1}{3}\right]$$

$$= 2\left[\frac{4-3}{6\sqrt{2}} + \frac{9\pi}{16} - \frac{9}{8}\sin^{-1}\frac{1}{3}\right]$$

$$= 2\left[\frac{1}{6\sqrt{2}} + \frac{9\pi}{16} - \frac{9}{8}\sin^{-1}\frac{1}{3}\right]$$

$$= \left[\frac{1}{3\sqrt{2}} + \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3}\right] \text{sq units}$$

7 MARK QUESTIONS

1: Find the area enclosed by the ellipse $x^2/a^2 + y^2/b^2 = 1$.

Solution:

Given,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{y^2}{b^2} = 1 - \frac{x^2}{a^2}$$

$$\frac{y^2}{b^2} = \frac{a^2 - x^2}{a^2}$$

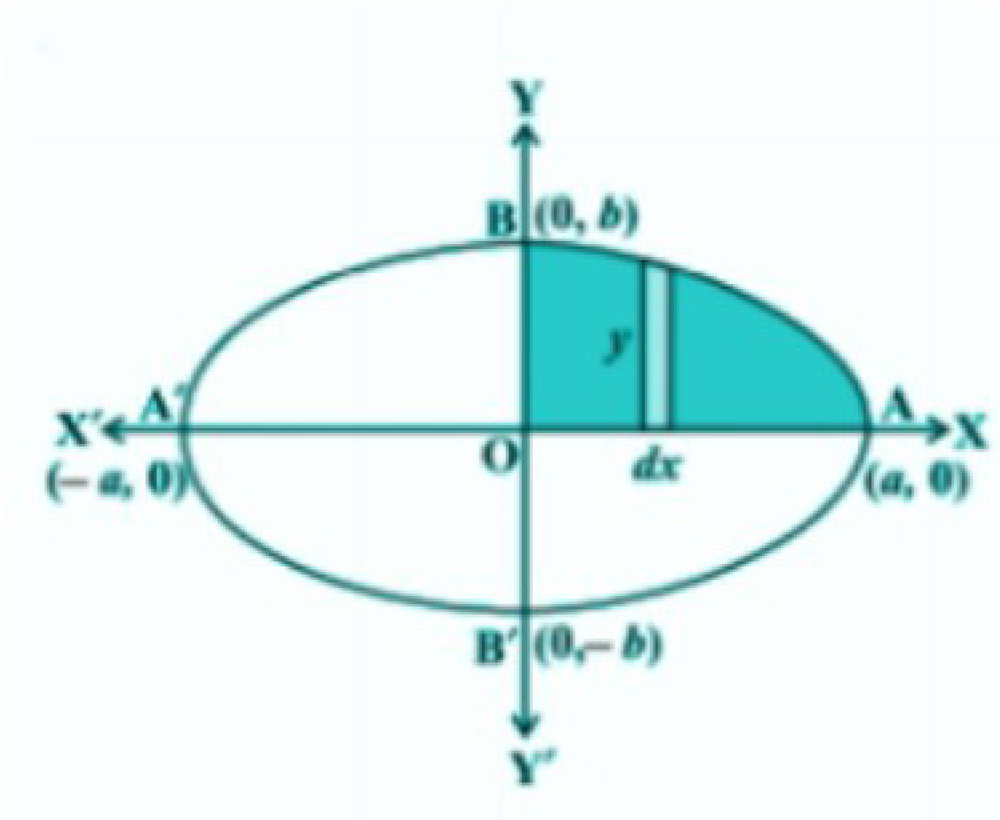
$$y^2 = \frac{b^2}{a^2}(a^2 - x^2)$$

$$y = \pm\sqrt{\frac{b^2}{a^2}(a^2 - x^2)}$$

$$y = \pm\frac{b}{a}\sqrt{(a^2 - x^2)}$$

We know that,

Ellipse is symmetrical about both x-axis and y-axis.



Area of ellipse = 4 × Area of AOB

$= 4 \times \int_0^a y\, dx$

Substituting the positive value of y in the above expression since OAB lies in the first quadrant.

$$= 4\int_0^a \frac{b}{a}\sqrt{a^2 - x^2}dx$$

$$= \frac{4b}{a}\int_0^a \sqrt{a^2 - x^2}dx$$

$$= \frac{4b}{a}\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$$

$$= \frac{4b}{a}\left[\left(\frac{a}{2}\sqrt{a^2 - a^2} + \frac{a^2}{2}\sin^{-1}\frac{a}{a}\right) - \left(\frac{0}{2}\sqrt{a^2 - 0} - \frac{a^2}{2}\sin^{-1}(0)\right)\right]$$

$$= \frac{4b}{a}\left[0 + \frac{a^2}{2}\sin^{-1}(1) - 0 - 0\right]$$

$$= \frac{4b}{a} \times \frac{a^2}{2}\sin^{-1}(1)$$

$= 2ab \times \sin^{-1}(1)$

$= 2ab \times \pi/2$

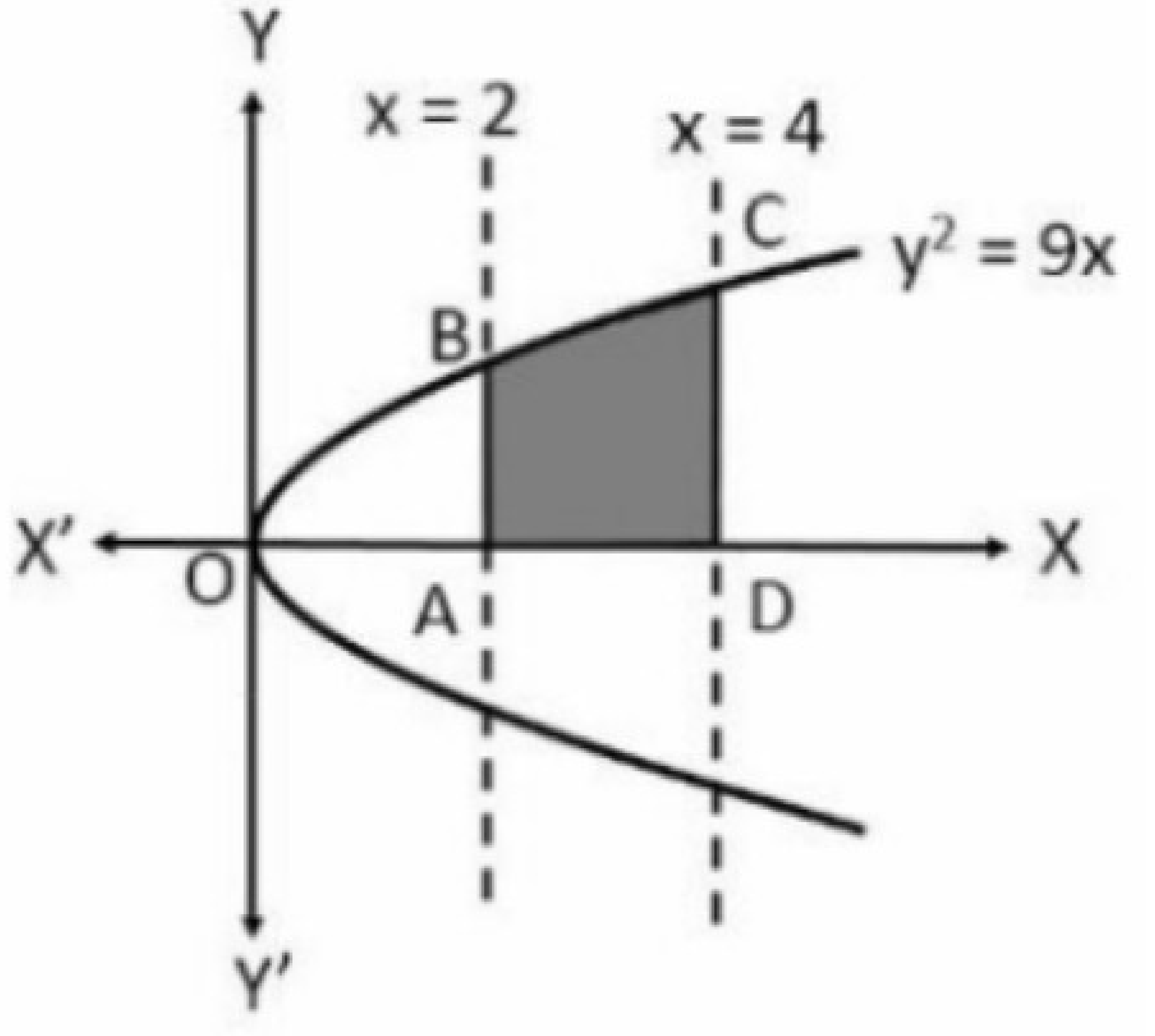
$= \pi ab$

Hence, the required area is πab sq.units.

2: Find the area of the region bounded by $y^2 = 9x$, $x = 2$, $x = 4$ and the x-axis in the first quadrant.

Solution:

We can draw the figure of $y^2 = 9x$; $x = 2$, $x = 4$ and the x-axis in the first curve as below.



$y^2 = 9x$

$y = \pm\sqrt{(9x)}$

$y = \pm 3\sqrt{x}$

We can consider the positive value of y since the required area is in the first quadrant.

The required area is the shaded region enclosed by ABCD.

$= \int_2^4 y \, . dx$

$= 3\int_2^4 \sqrt{x}\, dx$

$= 3\int_2^4 (x)^{\frac{1}{2}} dx$

$= 3\left[\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]_2^4$

$= 3\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_2^4$

$= 3 \times \frac{2}{3}\left[x^{\frac{3}{2}}\right]_2^4$

$= 2\left[(4)^{\frac{3}{2}} - (2)^{\frac{3}{2}}\right]$

$= 2\left[\left((4)^{\frac{1}{2}}\right)^3 - \left((2)^{\frac{1}{2}}\right)^3\right]$

$= 2\, [(2)^3 - (\sqrt{2})^3]$

$= 2[8 - 2\sqrt{2}]$

$= 16 - 4\sqrt{2}$

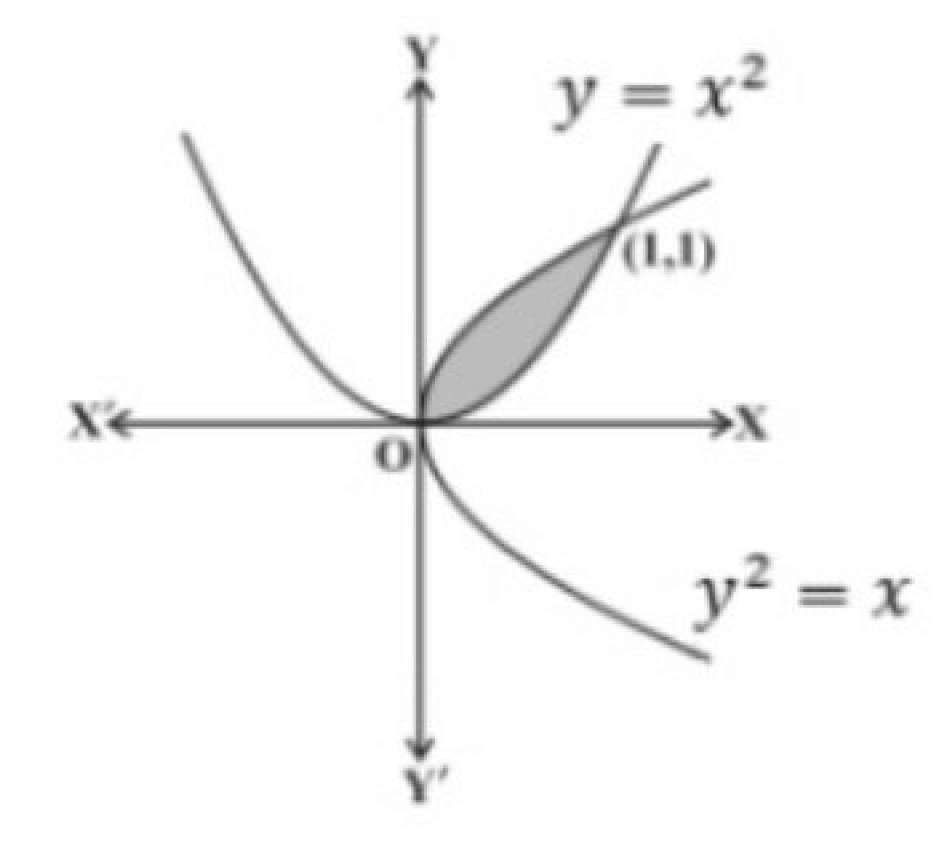
Hence, the required area is $16 - 4\sqrt{2}$ sq.units.

3: Find the area of the region bounded by the two parabolas $y = x^2$ and $y^2 = x$.

Solution:

Given two parabolas are $y = x^2$ and $y^2 = x$.

The point of intersection of these two parabolas is O (0, 0) and A (1, 1) as shown in the below figure.



Now,

$y^2 = x$

$y = \sqrt{x} = f(x)$

$y = x^2 = g(x)$, where, $f(x) \geq g(x)$ in $[0, 1]$.

Area of the shaded region

$$= \int_0^1 [f(x) - g(x)]dx$$

$$= \int_0^1 \left[\sqrt{x} - x^2\right]dx$$

$$= \left[\frac{2}{3}x^{\frac{3}{2}} - \frac{x^3}{3}\right]_0^1$$

$= (2/3) - (1/3)$

$= 1/3$

Hence, the required area is $1/3$ sq.units.

4: Smaller area enclosed by the circle $x^2 + y^2 = 4$ and the line $x + y = 2$ is

(A) $2(\pi - 2)$

(B) $\pi - 2$

(C) $2\pi - 1$

(D) $2(\pi + 2)$

Solution:

Option (B) is the correct answer.

Explanation:

Given,

Equation of circle is $x^2 + y^2 = 4$……….(i)

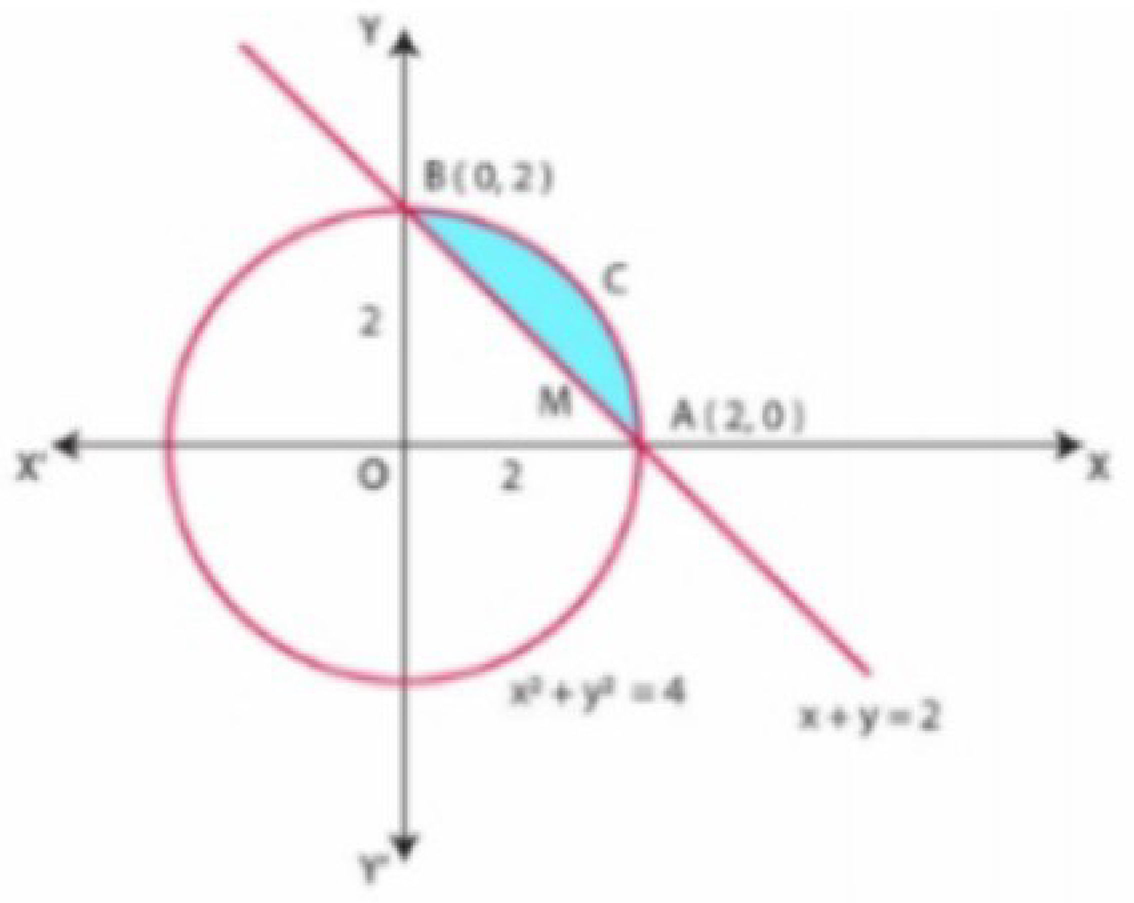
$x^2 + y^2 = 2^2$

$y = \sqrt{(2^2 - x^2)}$ …………(ii)

Equation of a lines is $x + y = 2$ ………(iii)

$y = 2 - x$

X	0	2
Y	2	0



Therefore, the graph of equation (iii) is the straight line joining the points (0, 2) and (2, 0).

From the graph of a circle (i) and straight-line (iii), it is clear that points of intersections of circle

(i) and the straight line (iii) is A (2, 0) and B (0.2).

Area of OACB, bounded by the circle and the coordinate axes is

$$= \int_0^2 \sqrt{2^2 - x^2}\, dx$$

$$= \left[\frac{x}{2}\sqrt{2^2 - x^2} + \frac{2^2}{2}\sin^{-1}\frac{x}{2}\right]_0^2$$

$$= \left[\frac{x}{2}\sqrt{4 - x^2} + 2\sin^{-1}\frac{x}{2}\right]_0^2$$

$$= \left[\frac{2}{2}\sqrt{4 - 2^2} + 2\sin^{-1}\frac{2}{2}\right] - \left[\frac{0}{2}\sqrt{4 - 0^2} + 2\sin^{-1}\frac{0}{2}\right]$$

$= [\ 1 \times \sqrt{0} + 2\sin^{-1}(1) - 0\sqrt{4} - 2 \times 0]$

$= 2\sin^{-1}(1)$

$= 2 \times \pi/2$

$= \pi$ sq. units

Area of triangle OAB, bounded by the straight line and the coordinate axes is

$= \int_0^2 y\, dx$

$= \int_0^2 (2-x) dx$

$= \left[2x - \frac{x^2}{2}\right]_0^2$

$= 2 \times 2 - \frac{2^2}{2} - \left[2 \times 0 - \frac{0}{2}\right]$

$= 4 - 2 - 0 + 0$

= 2 sq.units

Hence, the required area = Area of OACB − Area of triangle OAB

= $(\pi - 2)$ sq.units

5.Find the area of the region lying above X-axis and included between the circle $x^2 + y^2 = 8x$ and inside the parabola $y^2 = 4x$

Answer:

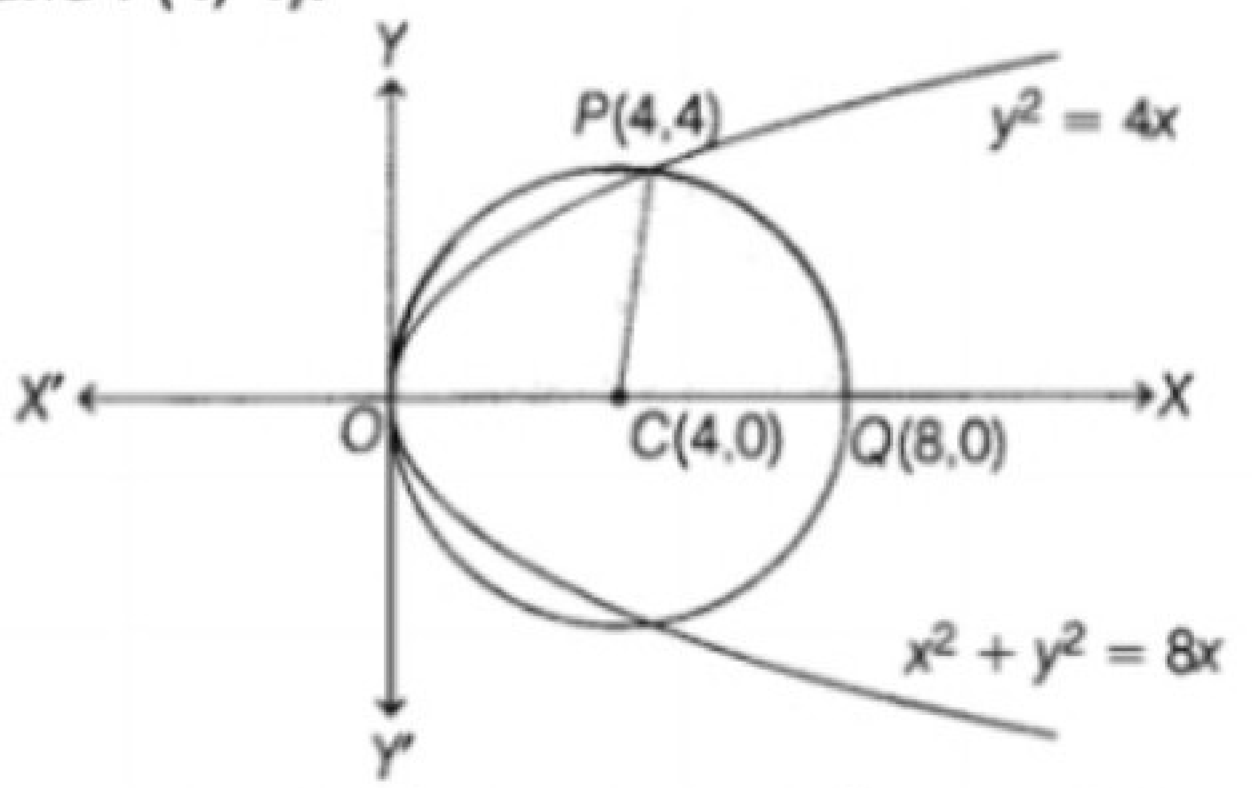
The equation of circle is
$x^2 + y^2 = 8x$ ...... (i)
and the equation of parabola is
$y^2 = 4x$ ...... (ii)
Eq. (i) can be written as
$(x^2 - 8x) + y^2 = 0$
$\Rightarrow (x^2 - 8x + 16) + y2 = 16$
$\Rightarrow (x - 4)^2 + y^2 = (4)^2$ ...... (iii)
which is a circle with centre C(4, 0) and radius = 4.
From Eqs. (i) and (ii), we get
$x^2 + 4x = 8x$
$\Rightarrow x^2 - 4x = 0$
$\Rightarrow x(x - 4) = 0$

$\Rightarrow x = 0, 4$

Now, from Eq. (ii), we get

$y = 0, 4$

$\therefore$ Points of intersection of circle (i) and parabola (ii), above the A-axis, are O(0, 0) and P(4, 4).



Now, required area = area of region OPQCO = (area of region OCPQ + (area of region PCQP)

$$= \int_0^4 y(\text{Parabola})\, dx + \int_4^8 y(\text{Circle})\, dx$$

$$= 2\int_0^4 \sqrt{x}\, dx + \int_4^8 \sqrt{(4)^2 - (x-4)^2}\, dx$$

[from Eqs. (ii), (iii)]

$$= 2\left[\frac{x^{3/2}}{3/2}\right]_0^4$$

$$+ \left[\frac{(x-4)}{2}\sqrt{(4)^2 - (x-4)^2} + \frac{(4)^2}{2}\cdot \sin^{-1}\frac{x-4}{4}\right]_4^8$$

$$\left[\because \int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + C\right]$$

$$= \frac{4}{3}[x^{3/2}]_0^4$$

$$+ \left\{\left[\left(\frac{8-4}{2}\right)\sqrt{16-16} + 8\sin^{-1}(1)\right] - [0 + 8\sin^{-1} 0]\right\}$$

$$= \frac{4}{3}[(4)^{3/2} - 0] + \left[0 + 8 \times \frac{\pi}{2}\right] - [0 + 0]$$

$$= \frac{4}{3} \times 8 + 4\pi = \frac{32}{3} + 4\pi = \frac{4}{3}(8 + 3\pi) \text{ sq units}$$

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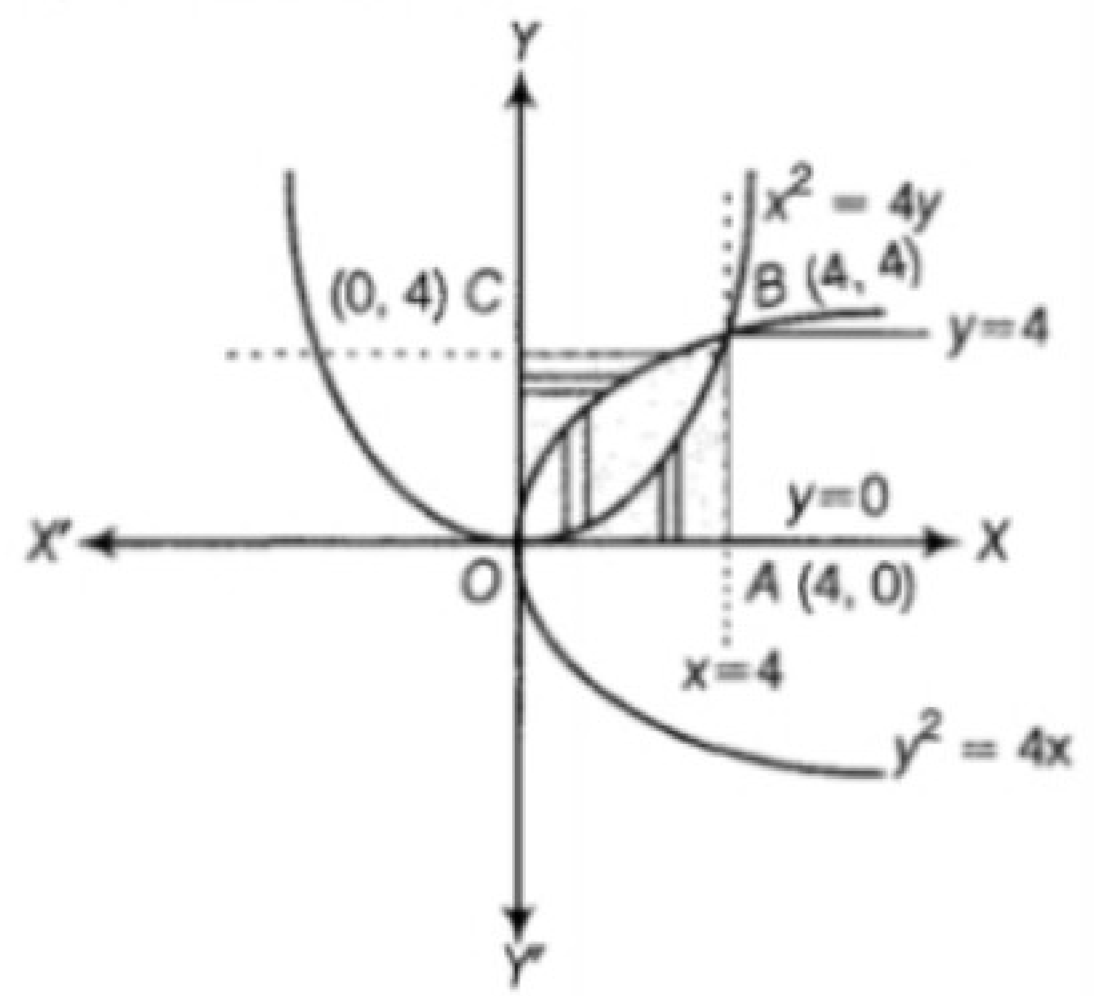
6.Using integration, prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of the square bounded by $x = 0$, $x = 4$, $y = 4$ and $y = 0$ into three equal parts.

Answer:

First, we draw a square formed by the lines $x = 0$, $x = 4$, $y = 4$, and $y = 0$ and after that, we draw given parabolas which intersect each other on the square such that the whole region divided into three parts. Now, we find separately area of each part and show that area of each part is equal.

Let OABC be the square whose sides are represented by following equations
Equation of OA is $y = 0$
Equation of AB is $x = 4$
Equation of BC is $y = 4$
Equation of CO is $x = 0$



On solving equations $y^2 = 4x$ and $x^2 = 4y$, we get A(0, 0) and B(4, 4) as their points of intersection. Now, area bounded by these curves is given by.

$$\int_0^4 [y_{(\text{parabola } y^2=4x)} - y_{(\text{parabola} x^2=4y)}]\, dx$$

$$= \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4}\right) dx = \left[2 \cdot \frac{2}{3} x^{3/2} - \frac{x^3}{12}\right]_0^4$$

$$= \left[\frac{4}{3} x^{3/2} - \frac{x^3}{12}\right]_0^4 = \frac{4}{3} \cdot (4)^{3/2} - \frac{64}{12}$$

$$= \frac{4}{3} \cdot (2^2)^{3/2} - \frac{64}{12} = \frac{4}{3} \cdot (2)^3 - \frac{64}{12}$$

$$= \frac{32}{3} - \frac{16}{3} = \frac{16}{3} \text{ sq units}$$

Hence, area bounded by curves $y^2 = 4x$ and $x^2 = 4y$ is \frac{16}{3} sq units .... (i)
Now, area bounded by curve $x^2 = 4y$ and the lines $x = 0$, $x = 4$ and X-axis

$$= \int_0^4 y_{(\text{parabola } x^2=4y)}\, dx = \int_0^4 \frac{x^2}{4}\, dx$$

$$= \left[\frac{x^3}{12}\right]_0^4 = \frac{64}{12} = \frac{16}{3} \text{ sq units} \quad \ldots\text{(ii)}$$

Similarly, the area bounded by curve $y^2 = 4x$, the lines $y = 0$, $y = 4$ and Y-axis

$$= \int_0^4 x_{(\text{parabola } y^2=4x)}\, dy = \int_0^4 \frac{y^2}{4}\, dy$$

$$= \left[\frac{y^3}{12}\right]_0^4 = \frac{64}{12} = \frac{16}{3} \text{ sq units} \quad \ldots\text{(iii)}$$

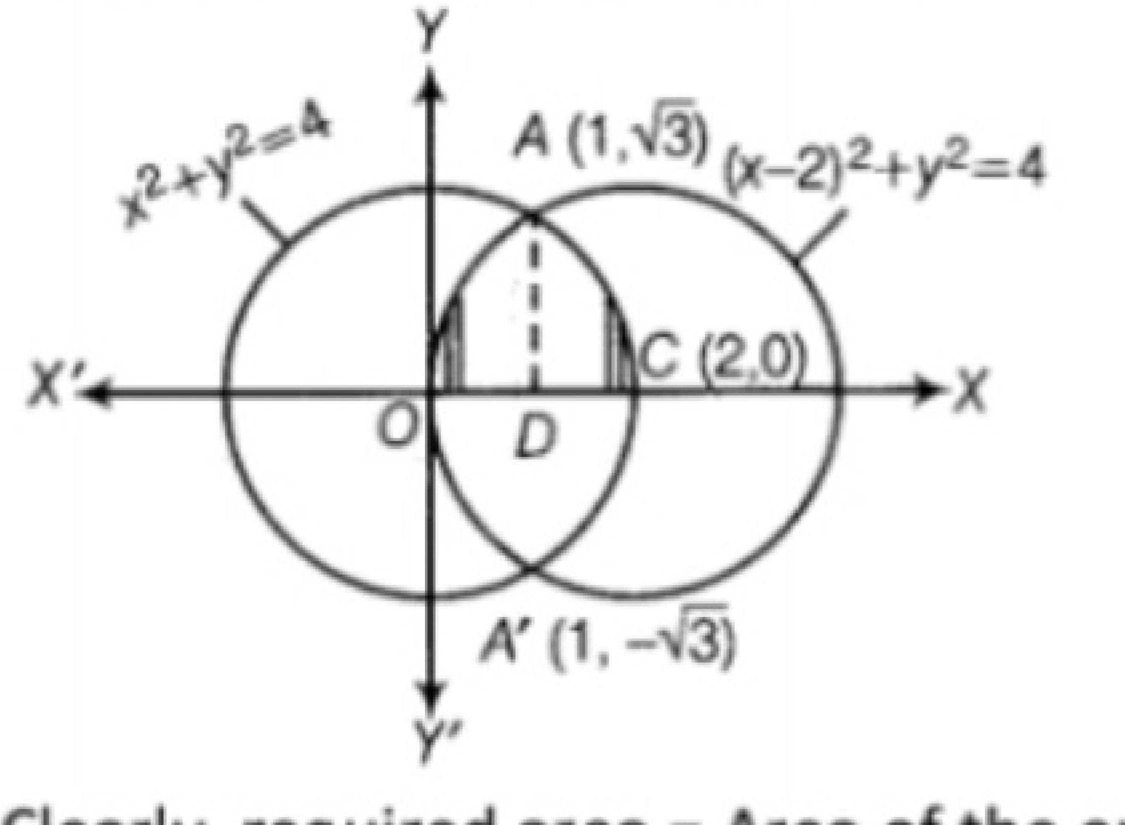
From Eqs. (i), (ii) and (iii), it is clear that area bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$ divides the area of square into three equal parts.
Hence proved.

7.Using integration, find the area of the region enclosed between the two circles $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$.

Answer:

First, find the intersecting points of two circles and then draw a rough sketch of these two circles. The common shaded region is symmetrical about X-axis. So, we find area of one part only, i.e. upper part of X-axis.
After that required area is twice of that area.
Given circles are
$x^2 + y^2 = 4$ .... (i)
and $(x - 2)^2 + y^2 = 4$ ...... (ii)

Eq. (i) is a circle with centre origin and radius 2, Eq. (ii) is a circle with centre C (2, 0) and radius 2.
On solving Eqs. (i) and (ii), we get
$(x - 2)^2 + y^2 = x^2 + y^2$
$\Rightarrow x^2 - 4x + 4 + y^2 = x^2 + y^2$
$\Rightarrow x = 1$
On putting $x = 1$ in Eq. (i), we get
$y = \pm \sqrt{3}$
Thus, the points of intersection of the given circles are A $(1, \sqrt{3})$ and A'$(1, -\sqrt{3})$ as shown in the figure given below:



Clearly, required area = Area of the enclosed region OACA'O between circles
= 2 [Area of the region ODCAO]

= 2 [Area of the region ODAO + Area of the region DCAD]

$$=2\left[\int_0^1 y_2\,dx+\int_1^2 y_1\,dx\right]$$

[where, $y_1=\sqrt{4-x^2}$ and $y_2=\sqrt{4-(x-2)^2}$]

$$=2\left[\int_0^1\sqrt{4-(x-2)^2}\,dx+\int_1^2\sqrt{4-x^2}\,dx\right]$$

$$=2\left[\frac{1}{2}(x-2)\sqrt{4-(x-2)^2}+\frac{1}{2}\times 4\sin^{-1}\left(\frac{x-2}{2}\right)\right]_0^1$$

$$+2\left[\frac{1}{2}x\sqrt{4-x^2}+\frac{1}{2}\times 4\sin^{-1}\frac{x}{2}\right]_1^2$$

$$\left[\because \int\sqrt{a^2-x^2}\,dx=\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]$$

$$=\left[(x-2)\sqrt{4-(x-2)^2}+4\sin^{-1}\left(\frac{x-2}{2}\right)\right]_0^1$$

$$+\left[x\sqrt{4-x^2}+4\sin^{-1}\frac{x}{2}\right]_1^2$$

$$=\left[\left\{-\sqrt{3}+4\sin^{-1}\left(\frac{-1}{2}\right)\right\}-0-4\sin^{-1}(-1)\right]$$

$$+\left[0+4\sin^{-1}1-\sqrt{3}-4\sin^{-1}\frac{1}{2}\right]$$

$$=\left[\left(-\sqrt{3}-4\times\frac{\pi}{6}\right)+4\times\frac{\pi}{2}\right]+\left[4\times\frac{\pi}{2}-\sqrt{3}-4\times\frac{\pi}{6}\right]$$

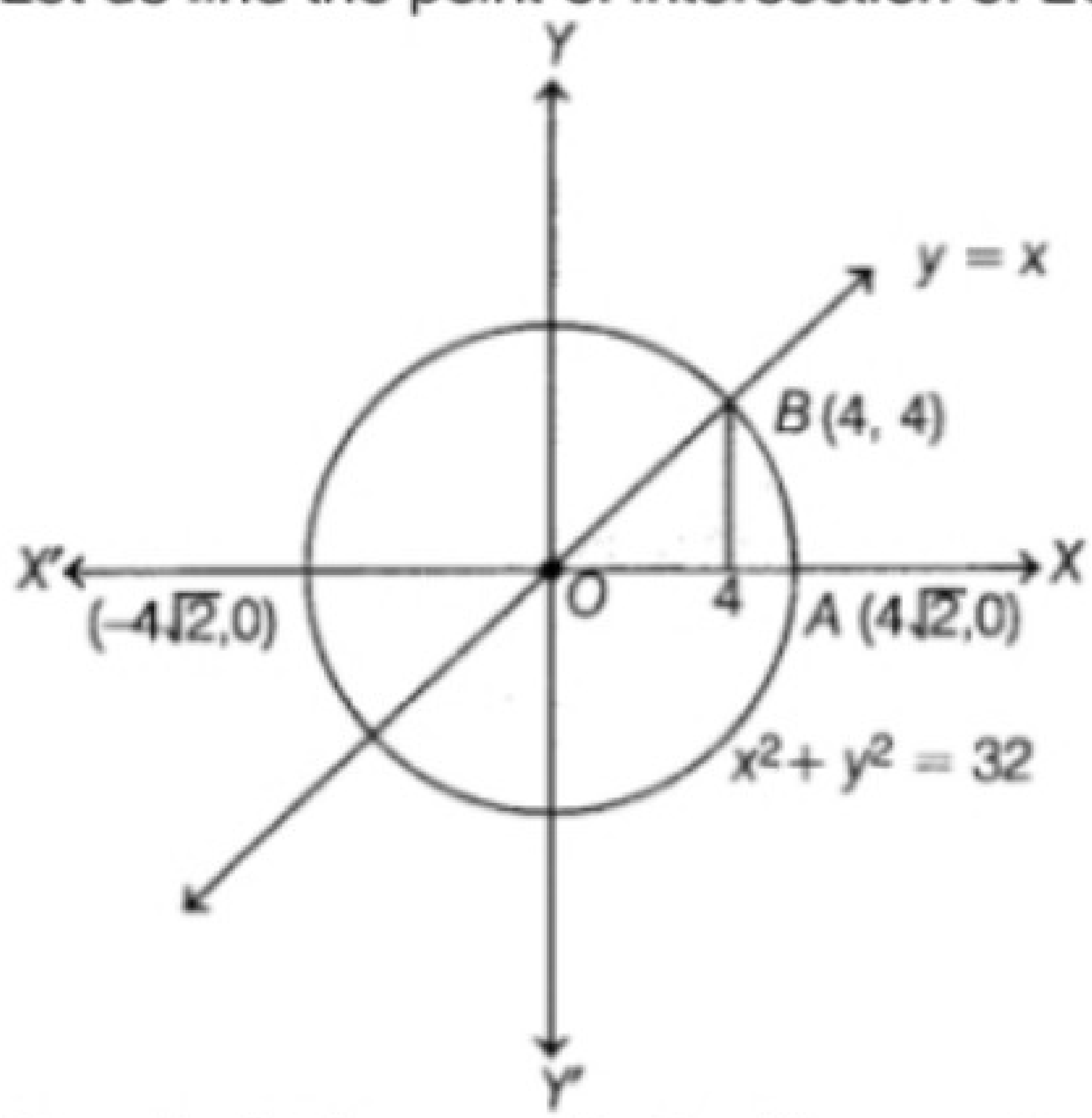
$$=\left(-\sqrt{3}-\frac{2\pi}{3}+2\pi\right)+\left(2\pi-\sqrt{3}-\frac{2\pi}{3}\right)$$

$$=\frac{8\pi}{3}-2\sqrt{3}\text{ sq units}$$

8.Find the area of the region in the first quadrant enclosed by the X-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$.

Answer:

Given the circle $x^2 + y^2 = 3^2$ ..... (i)
having centre (0, 0) and radius $4\sqrt{2}$ and the line $y = x$ ..... (ii)
Let us find the point of intersection of Eqs. (i) and (ii).



On substituting $y = x$ in Eq. (i), we get
$x^2 + x^2 = 32$
$\Rightarrow 2x^2 = 32$
$\Rightarrow x^2 = 16$
$\Rightarrow x = \pm 4$
Thus, the points of intersection are (4, 4) and (-4, -4). [$\because y = x$]
Clearly, the required area

= Area of shaded region OABO

$= \int_0^4 y(\text{line})\, dx + \int_4^{4\sqrt{2}} y(\text{circle})\, dx$

$= \int_0^4 x\, dx + \int_4^{4\sqrt{2}} \sqrt{32 - x^2}\, dx$

$[\because x^2 + y^2 = 32 \Rightarrow y = \pm\sqrt{32 - x^2} \text{ and } y > 0]$

$= \left[\frac{x^2}{2}\right]_0^4 + \int_4^{4\sqrt{2}} \sqrt{(4\sqrt{2})^2 - x^2}\, dx$

$= \frac{1}{2}[16 - 0] + \frac{1}{2}\left[x\sqrt{(4\sqrt{2})^2 - x^2}\right.$

$\left. + (4\sqrt{2})^2 \sin^{-1}\left(\frac{x}{4\sqrt{2}}\right)\right]_4^{4\sqrt{2}}$

$= 8 + \frac{1}{2}[(0 + 32\sin^{-1}(1)) -$

$\left.\left(4\sqrt{32-16} + 32\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)\right)\right]$

$= 8 + \frac{1}{2}\left[32\sin^{-1}(1) - 16 - 32\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)\right]$

$= 8 + \frac{1}{2}\left[32 \cdot \frac{\pi}{2} - 16 - 32 \cdot \frac{\pi}{4}\right]$

$= 8 + \frac{1}{2}[16\pi - 16 - 8\pi]$

$= 8 + \frac{1}{2}[8\pi - 16] = 8 + 4\pi - 8$

$= 4\pi$ sq units