Problem set 3.2

Question: 1

Solution : To prove that $G=T$ when T is both DFS tree and BFS tree rooted at same vertex 'u', we will proceed by Contradiction.

We have assumptions that (i) G is a Connected Graph.

(ii) DFS & BFS of G obtain Tree 'T' includes all nodes.

Proof by Contradiction:-

(i) Assume there is an edge $e=(x,y)$ in G that is not in 'T'.

(ii) As per properties, DFS and BFS span all the vertices of the Graph (since T includes all nodes of G) and are acyclic, as they are trees.

Contradiction from DFS:-

(iii) In DFS, if there exists any edge(x,y) that is not part of the tree, this means both 'x' & 'y' were already visited when edge was considered.

(iv) However, DFS would have added this edge if it were leading to a new vertex, meaning this edge forms a cycle with rest of tree if it's not included in T. "implies G has cycle"

Contradiction with BFS:-

↳(v) Here, we explore all neighbors of vertex 'x', when we reach vertex 'x', before moving on to next level.

(vi) if e = (x, y) was not included in the BFS tree, then 'y' must have already been visited before, meaning e should have created a cycle if it were added to tree.

(vii) therefore, if G contains e = (x, y) but e is not part of 'T'. it implies that G contains cycle contradicting the assumption that 'T' is Tree.

So, Both BFS & DFS trees are acyclic by definition (since they are trees), and if there were any edge e = (x, y) that is not in T, it would create a cycle, which contradicts the tree structure.

Thus, there cannot be any edge in 'G' that is not in 'T' implying "G = T".

Question: 2

Answer: We have 3 Containers (4 Gallon, 7 Gallon, 10 Gallon)

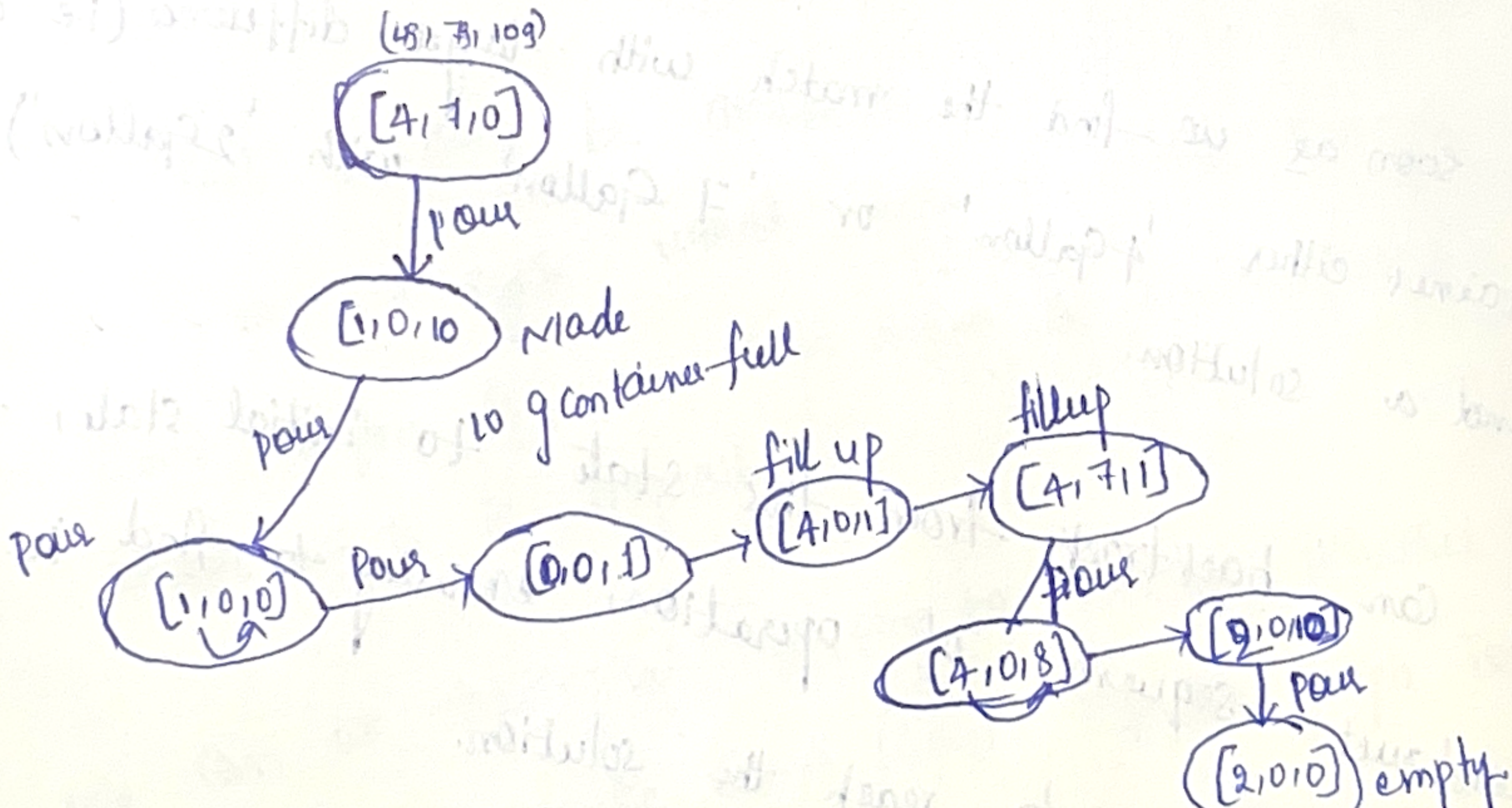
(i) Let's keep them in a list [4, 7, 0] as 10 litres is empty initially (We kept only state of container as we

Consider it as vertex)

Let's start pouring one to another

~~[4,0,7] → [1,0,10]~~ ~~Container full~~

[4, 7, 0] ⇒ pouring 7 Gallons content to 10 gallon container



Now, we have 2 Gallon in 4- Gallon Container

[2, 0, 0]

Graph Modelling

(i) We can describe containers of nodes

let's say (4, 7, 0) is initial state

(ii) operations as edges:- An edgy between two nodes can be different types of operation we perform (fill, pour, empty)

Graph structure:- This is a directed graph where edges are labelled with the specific operation we perform (fill, pour, empty).

proof:- Using BFS Algorithm, start explore the graph of container states starting from initial state [4, 7, 0], BFS explores all possible operations while maintaing a queue of states of visit.

As soon as we find the match with target difference (i.e any container either '4 Gallon' or '7 Gallon' with '2 Gallon'), we found a solution.

We can backtrack from this state to initial state, to reconstruct sequence of operation, ensuring to find the shortest sequence to reach the solution.

Question:3

Algorithm :

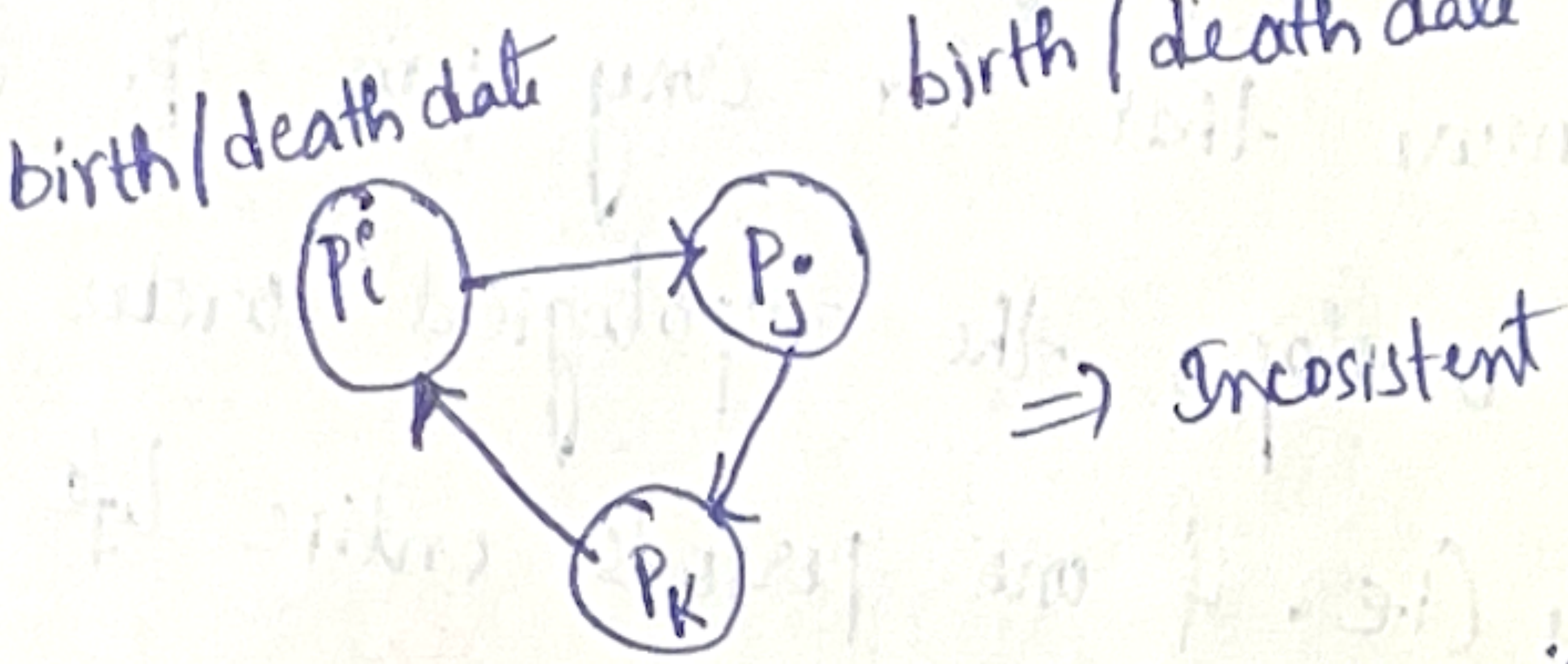
Graph Construction:- (one node)

(i) Creating a directed 'G' with 'n' vertices, each for person P_i

(ii) For each fact of type 1 (" P_i died before P_j was born), we add a directed edge from P_i to P_j, indicating that 'P_i' must precede 'P_j' in timeline.

(iii) for each fact type 2 (if living time is overlapped), we will ensure that no directed edge between P_i' and P_j'. (or) from P_j' to P_i'.

(iv)



$\Rightarrow$ Inconsistent

The facts can only be consistent, if there is no directed cycle in the graph. A cycle would imply that, for example, P_i must lived before P_j, who lived before P_K, who lived before P_i, ~~before~~ which is impossible.

We can use topological sorting, to detect whether the graph is directed Acyclic Graph (DAG). If the graph has a cycle. The facts are inconsistent.

Topological sorting

(v) If the graph is Acyclic (i.e no cycles found), Perform topological sort. The result of sorting gives consistent ordering of people's lives such that all facts of type 1 are satisfied.

→ for each person 'P_i' we can assign hypothetical dates of birth and death based on their position in topological order.

(v) After obtaining the topological order, check for consistency with type 2 facts. Ensure that for any pair 'P_i' and 'P_j' where their life spans overlap, the topological order does not violate this. if it does (i.e. if one person's entire life span comes after the other's), then the facts are inconsistent.