**CSE 5050: Graph Algorithms Handout**

*Sample problem and solution*

<u>Question</u> (From the book "Introduction to Algorithms" by Cormen, Rivest, Leiserson, and Stein) There are two types of professional wrestlers: "babyfaces" (good guys) and "heels" (bad guys). Between any pair of professional wrestlers, there may or may not be a rivalry. Suppose we have $n$ professional wrestlers and a list of $r$ pairs of wrestlers for which there are rivalries. Give an $O(n + r)$-time algorithm that determines whether it is possible to designate some of the wrestlers as babyfaces and the remainder as heels such that each rivalry is between a babyface and a heel. If it is possible to perform such a designation, your algorithm should produce it.

<u>Solution</u>

We will model this problem as a graph problem. Create a graph $G$ where each node (i.e., vertex) represents a wrestler and each edge represents a rivalry. This graph will contain $n$ nodes and $r$ edges.

We can now use BFS to determine if $G$ is bipartite or not. (Note that you may need to run BSF starting at multiple source nodes if $G$ has more than one connected component.) If $G$ is bipartite then report "true" and output the bipartition $(A, B)$ of $V$ computed via BFS (as discussed in video lecture M3_L4; i.e., all nodes at even distance from their source are in one partition and all nodes at odd distance are in the other), designating all nodes in $A$ as "babyfaces" and all nodes in $B$ as "heels".

We will now show that this algorithm returns true if and only if a valid designation is possible. Specifically, it suffices to prove the following claim.

**Claim.** A valid designation exists if and only if $G$ is bipartite.

*Proof.* Suppose a valid designation exists. Consider any valid designation $D$. The designation $D$ defines a bipartition of the nodes of $G$ such that one partition consists of babyfaces and the other of heels and each edge (rivalry) is between a babyface and a heel. $G$ must thus be a bipartite graph.

Conversely, suppose $G$ is bipartite. Then, we can label one partition as babyfaces and the other as heels and get a valid designation. Thus, $G$ must have a valid designation. $\square$

*Complexity analysis.* This solution would require $O(n+r)$ time to determine if $G$ is bipartite using BFS, and a further $O(n)$ time to consider all node distances from the source (as computed by BFS) and output the two partitions if $G$ is bipartite. Thus, the total time complexity of the algorithm is $O(n + r)$, as desired.