CS 377P Assignment 2

Joseph Bess, Krishna Ramdeep

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1 Iterative Solution of Linear Systems

a) The recurrence relation for the linear system

$$3x - 4y = -1$$

$$x + 2y = 3$$

can be expressed as follows:

$$x_{i+1} = x_i - \frac{1}{3}(3x_i - 4y_i + 1)$$

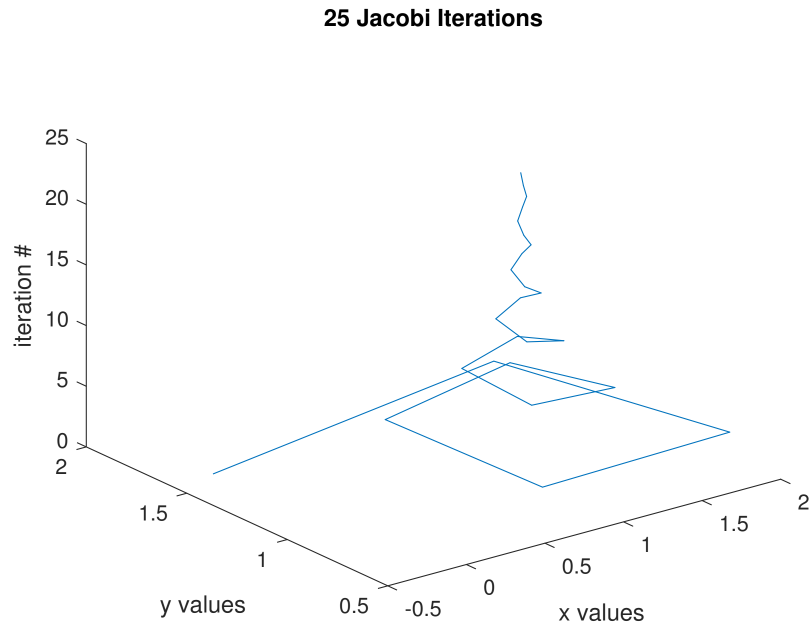
$$y_{i+1} = y_i - \frac{1}{2}(x_i + 2y_i - 3)$$

Expressed in terms of matrices and vectors, the recurrence becomes:

$$\begin{pmatrix} x_{i+1} \\ y_{i+1} \end{pmatrix} = \begin{pmatrix} x_i \\ y_i \end{pmatrix} - \begin{pmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 3x_i - 4y_i + 1 \\ x_i + 2y_i - 3 \end{pmatrix}$$

Where $\begin{pmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$ is the inverse of the diagonal coefficient matrix.

b) The plot:



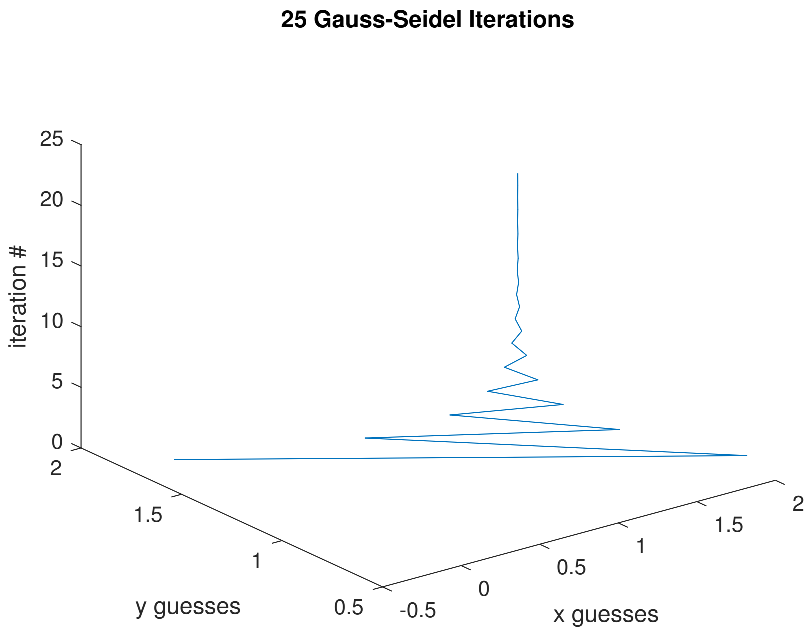
This plot looks as one would expect, as we can see that as the number of iterations increases (in the positive z-direction), the x, y values converge towards the actual solutions of the linear system $(x = 1, y = 1)$.

c) The recurrence relation is:

$$x_{i+1} = x_i - \frac{1}{3}(3x_i - 4y_i + 1)$$

$$y_{i+1} = y_i - \frac{1}{2}(x_{i+1} + 2y_i - 3)$$

The plot:



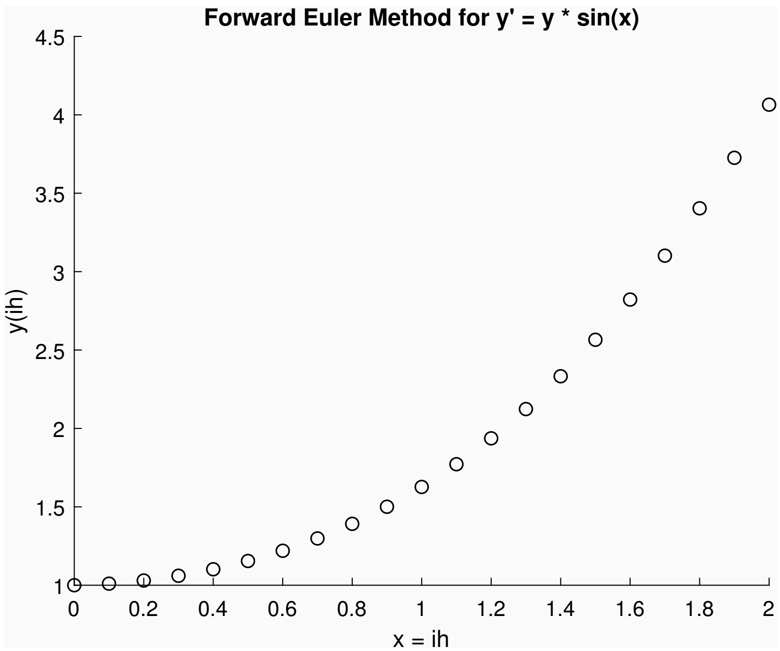
This plot looks as one would expect, but notice that Gauss-Seidel converges much faster for this system.

2 Ordinary Differential Equations

a) The recurrence relation for estimating the forward step:

$$y((i+1)*h) = y(ih)(h*sin(x)+1)$$

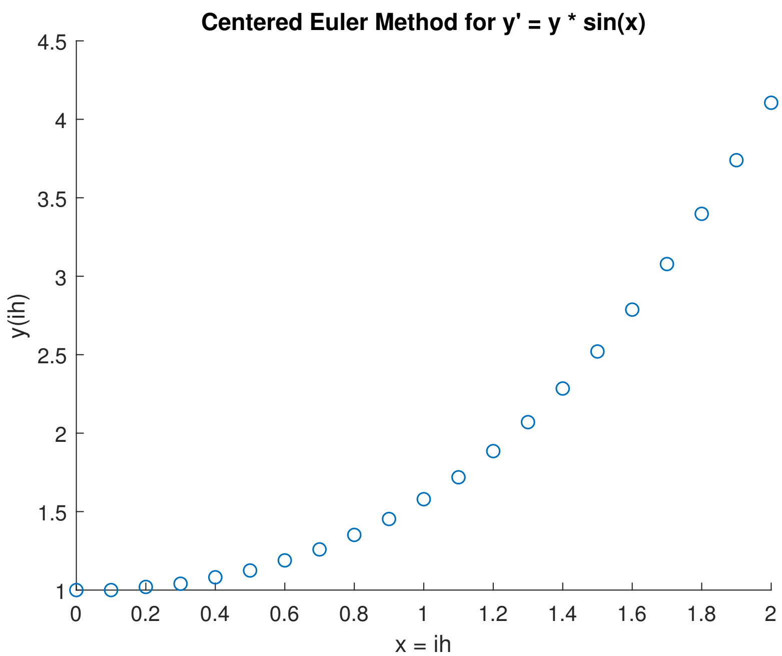
The graph is:



b) It makes sense that $y_c(h) = 1$ since $y_c(0) = 1$. Since the derivative at 0 is 0, there is no change. The recurrence relation is:

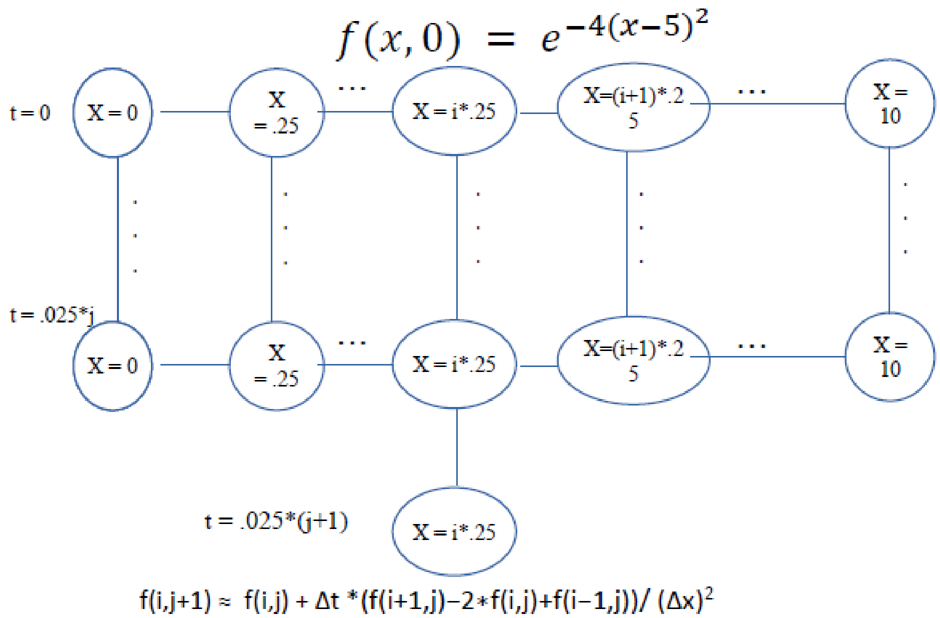
$$y((i+1)*h) = 2hy(ih)sin(x) + y((i-1)*h)$$

The graph is:



3 Partial Differential Equations

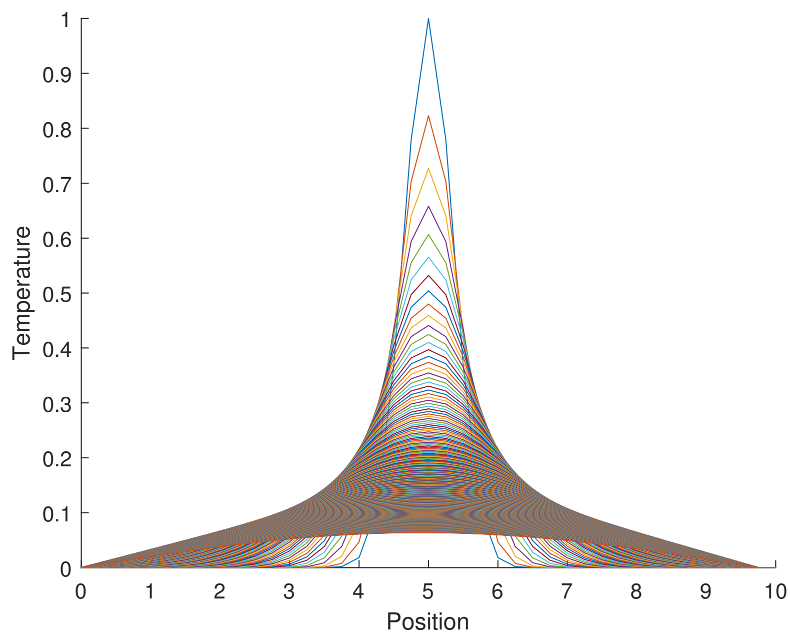
a) The stencil looks like this:



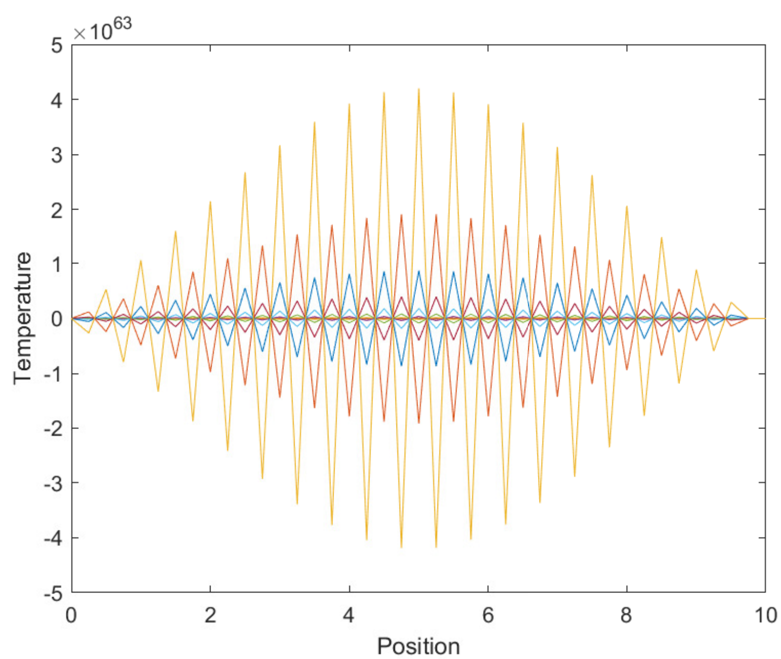
b) The recurrence relation is:

$$\hat{f}(i, j+1) = \hat{f}(i, j) + 0.4 * (\hat{f}(i+1, j) - 2\hat{f}(i, j) + \hat{f}(i-1, j))$$

The 0.4 is just a constant obtained from the term $\frac{\Delta t}{(\Delta x)^2}$. The graph is:



This does jive with our intuition.
c) This is the new graph:



Our best guess is that since we are using forward-euler to approximate $\frac{\delta f}{\delta t}$, we are seeing oscillation in the graph.