Lab 4: Lists and loops

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GitHub Classroom

[Lab 4 – Lists and loops]

Vector and matrix manipulation

Now that you know lists and loops you can start working on exciting linear algebra problems

Today, we will address two topics:

1. Dot product computation
2. Matrix averaging

Task 1: Dot product computation

The dot product is a mathematical operation that takes two vectors and produces a scalar value.

Dot product is everywhere

Machine learning

Computer Vision

Physics

Maybe you are already familiar with the dot product?

- Do you know what a correlation is?
- Have you ever heard of attention in AI?
- Have you ever checked the orthogonality between two vectors?

Task 1: Dot product computation

How to compute the dot product?

The dot product of two vectors of same length $a = [a_1 \quad a_2 \quad \cdots \quad a_n]$ and $b = [b_1 \quad b_2 \quad \cdots \quad b_n]$ can be written:

$$a.b = \sum_{i=1}^{n} a_i b_i = a_1 b_1 + a_2 b_2 + \cdots + a_n b_n$$

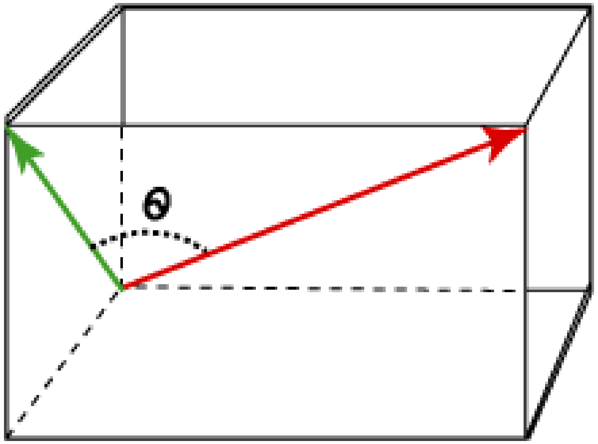
A concrete example

$$[1 \quad 3 \quad -5].[4 \quad -2 \quad -1] = (1 \times 4) + (3 \times -2) + (-5 \times -1) = 3$$

Task 1: Dot product computation

Let's compute the angle between two 3D vectors

$$a = [a_x \quad a_y \quad a_z]$$
$$b = [b_x \quad b_y \quad b_z]$$
$$\cos(\theta) = \frac{a.b}{|a||b|}$$



Let's solve it in three steps

1. Compute the magnitude $|a|$ and $|b|$ of both vectors $|a| = \sqrt{a_x^2 + a_y^2 + a_z^2}$ ⟸ In `magnitude(a, b)`
2. Compute the dot product between the vectors $a.b$ ⟸ In `dot(a, b)`
3. Compute the cosine distance (angle θ) with $\theta = acos\left(\frac{a.b}{|a||b|}\right)$ ⟸ In `cos_distance(a, b)`

Use acos from the module math!

Task 2: Matrix averaging

Matrix averaging, normalization or standardization is essential for many applications

1. **Numerical stability**: Normalizing matrices can prevent numerical errors when performing operations such as inversion, eigenvalue calculation, and matrix decomposition.

2. **Comparability**: Normalizing matrices can make it easier to compare them.

3. **Regularization**: Normalizing matrices can help prevent overfitting in machine learning models.

4. **Preprocessing**: Normalizing matrices is often an important preprocessing step in machine learning to improve performance.

Task 2: Matrix averaging

Now let's average a matrix using **nested loops**!

`Matrix averaging`

Matrix:

$$a = \begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,m} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n,1} & a_{n,2} & \cdots & a_{n,m} \end{bmatrix}$$

We will address the problem of averaging in two stages:

1. Compute the average of the matrix $\bar{a} = \frac{\sum_{i=1}^{n} \sum_{j=1}^{m} a_{i,j}}{n.m}$ ⟵ In `matrix_average(a)`

2. Normalize each element by the average $a_{averaged} = \begin{bmatrix} {}^{a_{1,1}}/_{\bar{a}} & {}^{a_{1,2}}/_{\bar{a}} & \cdots & {}^{a_{1,m}}/_{\bar{a}} \\ {}^{a_{1,1}}/_{\bar{a}} & {}^{a_{1,1}}/_{\bar{a}} & \cdots & {}^{a_{1,1}}/_{\bar{a}} \\ \vdots & \vdots & \ddots & \vdots \\ {}^{a_{1,1}}/_{\bar{a}} & {}^{a_{1,1}}/_{\bar{a}} & \cdots & {}^{a_{1,1}}/_{\bar{a}} \end{bmatrix}$

In `normalize_matrix(a)`