1.1) $m\{a+bx\} = m\{a\} + m\{bx\}$

$m\{a\}$ is a b/c the mean of a constant is the constant

$m\{bx\}$ is $b(m\{x\})$ using a property

Thus: $m\{a+bx\} = a + b\,(m\{x\})$

1.2) $$\mathrm{cov}(X, a+bY) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m\{x\})(a+bx_i) - m\{a+bY\}$$

From 1.1) $m\{a+bY\} = a + b\,m\{Y\}$

$$\mathrm{cov}(X, a+bY) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m\{x\})(a + by_i - a - b\,m\{Y\})$$

$$= \frac{1}{N}\sum_{i=1}^{N}(x_i - m\{x\})(b(y_i - m\{Y\}))$$

$$= b\left[\frac{1}{N}\sum_{i=1}^{N}(x_i - m\{x\})(y_i - m\{Y\})\right]$$

$$= (b)\ \mathrm{cov}(X, Y)$$

1.3) $$\mathrm{cov}(a+bX, a+bX) = (b)\ \mathrm{cov}(X, a+bX)$$ using 1.2)

$$= (b)\left[(b)\ \mathrm{cov}(X,X)\right]$$

$$= b^2\ \mathrm{cov}(X,X)$$

1.4) If g(x) is non-decreasing, the median of g(X) is the transformation of the median: median(g(X)) = g(median(X)). This happens because a non-decreasing function preserves the order of data points, ensuring the median maps correctly. Since the IQR is based on quartiles, if g(x) is strictly increasing, the transformed IQR remains proportional to the original IQR. The range is transformed as, range(g(X)) = g(max(X)) − g(min(X)). If g(x) is linear, the range transformation is proportional.

1.5) This is not necessarily true. The mean is affected by the distribution of values, while g(m(X)) is simply the function applied to the mean. If g(x) is linear, then m(g(X)) = g(m(X)). If g(x) is non-linear then m(g(X)) ≠ g(m(X)) because the transformation shifts values differently based on their distance from the mean.