Sup $Var(X)=\sigma^2$

$$Var(aX)=E\big([(aX)-\underbrace{E(aX)}_{aE(X)}]^2\big)=E\big(a^2(X-E(X))^2\big)=a^2E\big([X-E(X)]^2\big)=a^2\sigma^2$$

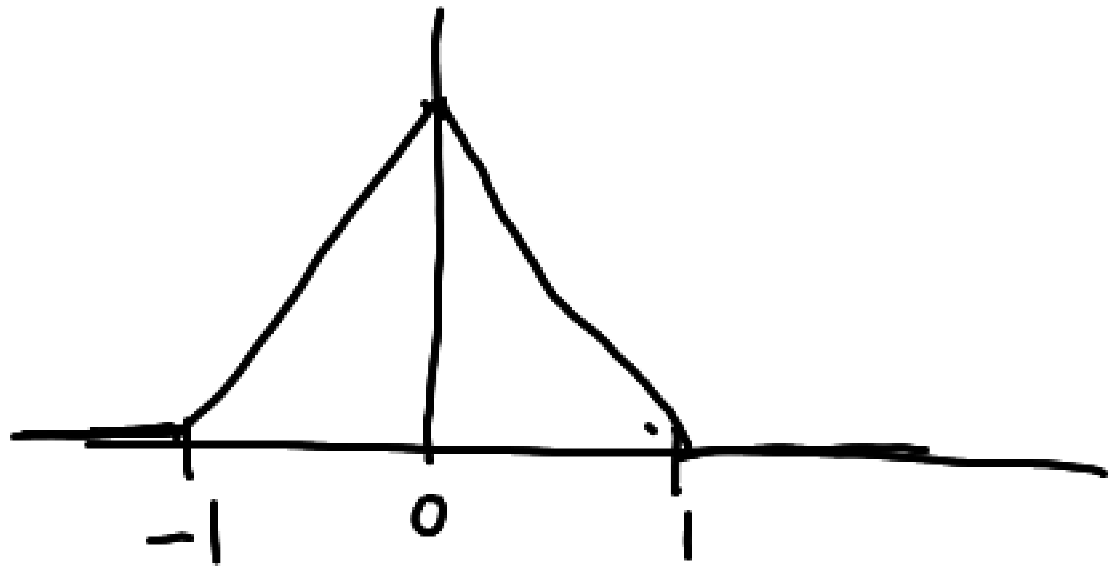
$X\sim N(\mu,\sigma^2)$

$E(e^{tX})=M_X(t)$

$n=2,\ \bar{X}=\frac{X_1+X_2}{n}$

$$M_{\bar{X}}(t)=E(e^{t\bar{X}})=E\left(e^{t\left(\frac{X_1+X_2}{2}\right)}\right)=E\left(e^{\frac{t}{2}(X_1)}e^{\frac{t}{2}(X_2)}\right)=M_{X_1}\left(\tfrac{t}{2}\right)M_{X_2}\left(\tfrac{t}{2}\right)$$

$$f(x)=\begin{cases}1+x & \text{si } x\in(-1,0)\\ 1-x & \text{si } x\in[0,1)\\ 0 & \text{eoc}\end{cases}$$



Queremos obtener simulaciones independientes de esta función de densidad

si $x\in(-1,0)$

$$u= F(x) = \int_{-1}^{x}(1+u)\,du = u+\frac{u^2}{2}\Big|_{-1}^{x} = x+\frac{x^2}{2}-\left(-1+\frac{(-1)^2}{2}\right) = x+\frac{x^2}{2}+\frac{1}{2}$$

Si $x\in[0,1)$

$$u= F(x) = \frac{1}{2}+\int_0^x (1-u)\,du = \frac{1}{2}+\left(u-\frac{u^2}{2}\right)\Big|_0^x = \frac{1}{2}+\left(x-\frac{x^2}{2}\right) = x-\frac{x^2}{2}+\frac{1}{2}$$

Calcular $F^{-1}(u)$ se obtiene en dos partes

Si $u\in(0,\frac{1}{2})$ proviene del caso $x\in(-1,0)$

$$u=F(x)=x+\frac{x^2}{2}+\frac{1}{2}$$

$$\Leftrightarrow \frac{1}{2}x^2+x-\frac{1}{2}-u=0$$

$$ax^2+bx+c=0$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-1\pm\sqrt{1-4\left(\frac{1}{2}\right)\left(\frac{1}{2}-u\right)}}{2\left(\frac{1}{2}\right)} = -1+\sqrt{2u}$$

~~Si $u \in (0, \frac{1}{2})$ proviene de $x \in (-1, 0$~~

Si $u \in [\frac{1}{2}, 1)$ proviene del caso $x \in [0,1)$

$$u = F(x) = x - \frac{x^2}{2} + \frac{1}{2}$$

$$\Leftrightarrow \frac{x^2}{2} - x + (u - \tfrac{1}{2}) = 0$$

$$x = \frac{1 \pm \sqrt{(-1)^2 - 4(\frac{1}{2})(u - \frac{1}{2})}}{2(\frac{1}{2})} = 1 \pm \sqrt{1 - 2(u - \tfrac{1}{2})} = 1 \pm \sqrt{1 - 2u + 1}$$

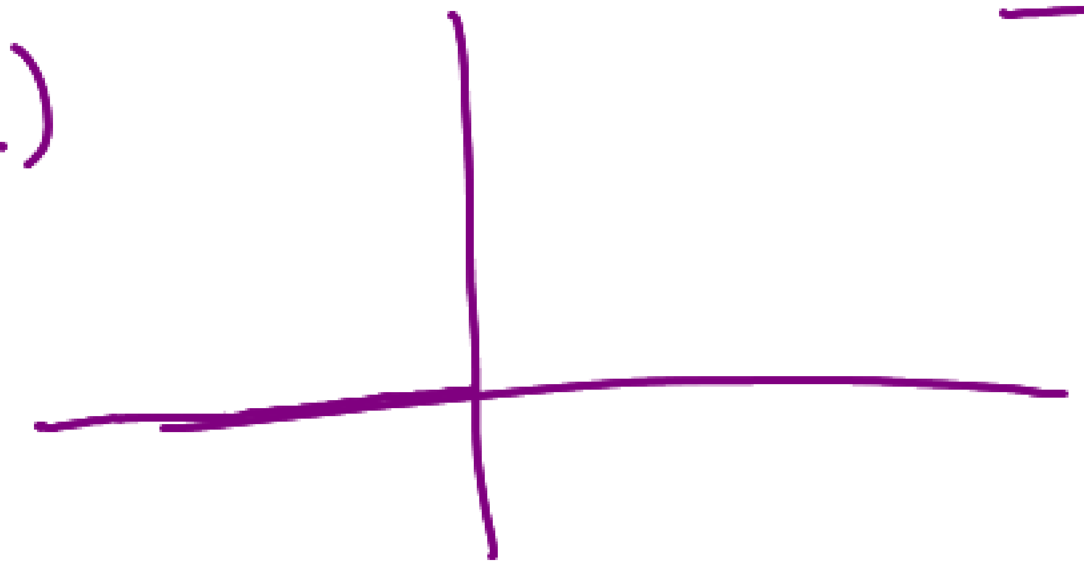
$$= 1 - \sqrt{2(1-u)}$$

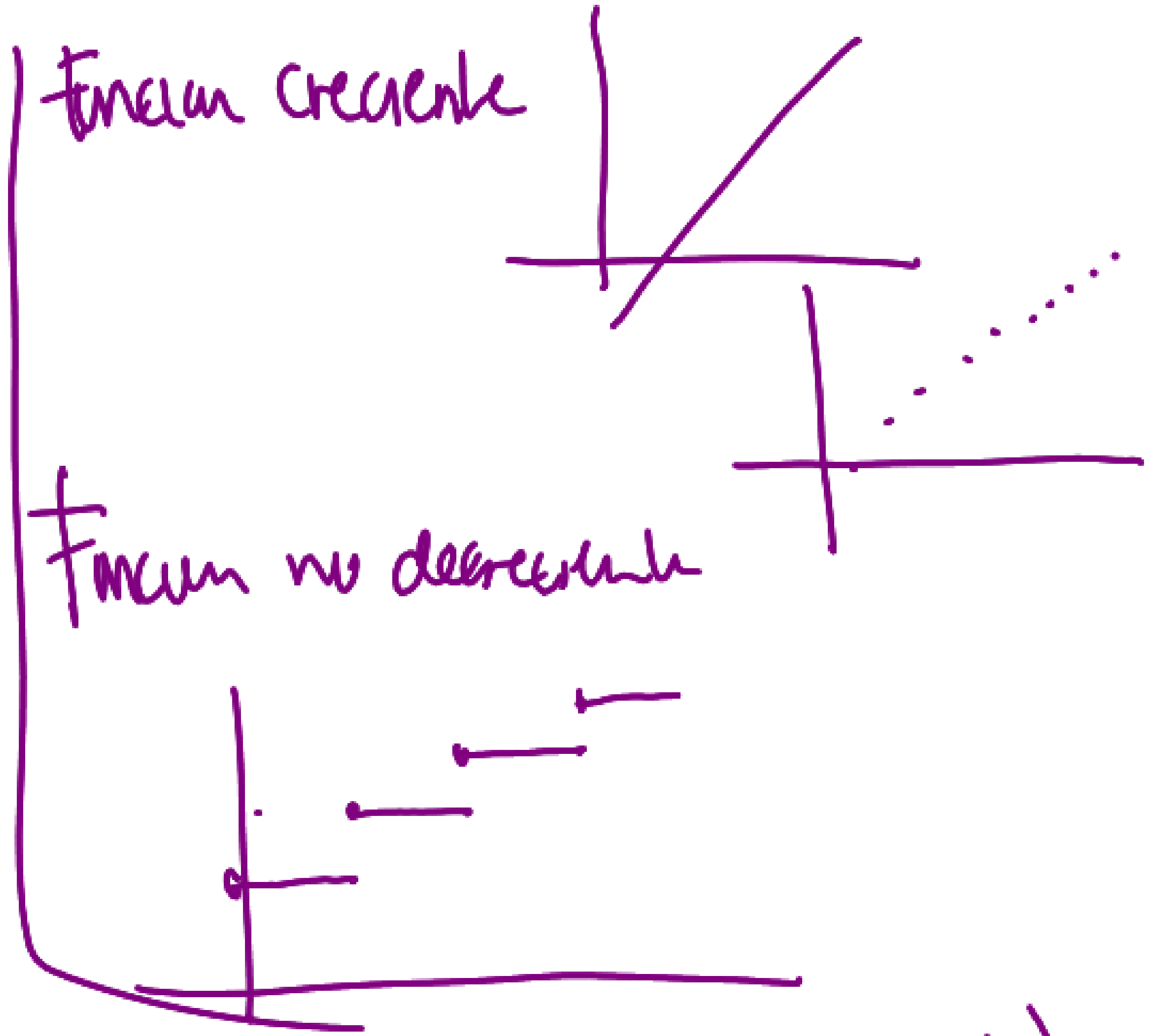
$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} du$$

$$\int_{-\infty}^{x} e^{-u^2/2} du = \sqrt{2\pi}\, \Phi(x)$$

$$\int_{-\infty}^{\infty} e^{-u^2/2} du = \sqrt{2\pi}$$

$F(x) = 2x^2 - x^4 \qquad x \in (0,1)$





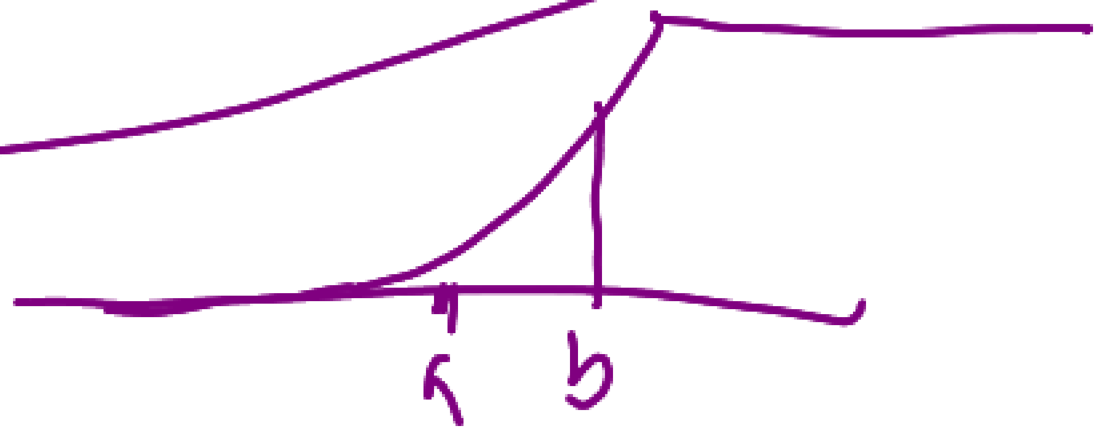
a) $\lim_{x \to \infty} F(x) = F(0) = 0$

b) $\lim_{x \to \infty} F(x) = F(1) = 2 - 1 = 1.$

c) $a < b \qquad F(a) = 2a^2 - a^4 = (2 - a^2)a^2$ .... una rta. (no tan fácil)

$F'(x) = 4x - 4x^3 = 4x(1 - x^2) > 0 \qquad \forall x \in (0,1)$

b) Calcular $P(\frac{1}{4} \le X \le \frac{3}{4})$ $\rightarrow 2(\frac{3}{4})^2 - (\frac{3}{4})^4 - \left(2(\frac{1}{4})^2 - (\frac{1}{4})^4\right)$



$P(a \le X \le b) = F(b) - F(a)$

Cuando X es continua $P(a < X < b) = P(a \le X \le b) = P(a \le X < b) = P(a < X \le b)$

$= F(b) - F(a)$

c) $f_X(x) = F_X'(x) = \frac{d}{dx} F_X(x) = \dot{F}_X(x)$

d) $P(e^X < 2) = P(X < \log(2)) = F(\log(2))$

e) $Y = e^X$

$F_Y(y) = P(Y \leq y) = P(e^X \leq y) = F_X(\log(y)) = 2(\log(y))^2 - (\log(y))^4$

$Y = X^2$

$F_Y(y) = P(Y \leq y) = P(X^2 \leq y) = F_X(X \leq \sqrt{y}) = 2(\sqrt{y})^2 - (\sqrt{y})^4$

$= 2y - y^2$

$f_Y(y) = F_Y'(y) = 2 - 2y = 2(1-y)\, I_{(0,1)}(y)$

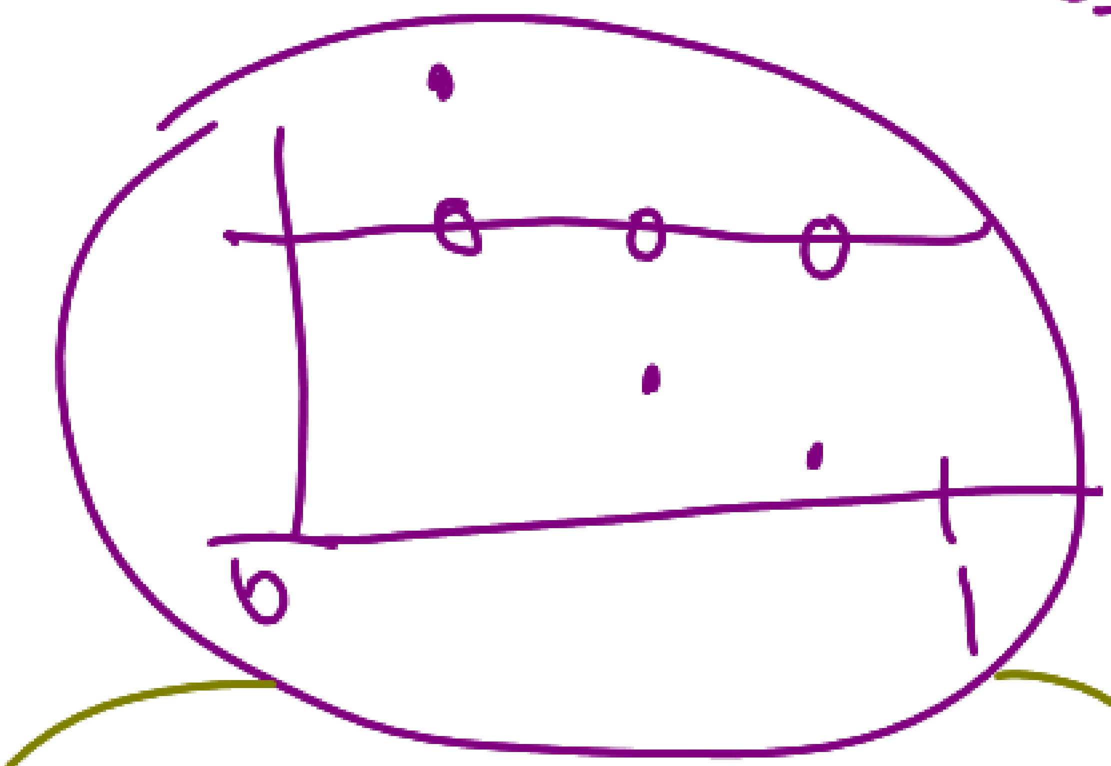
9: $f(x) = \frac{\theta}{x^{\theta+1}} I_{(1,\infty)}(x)$, $\theta = 0.5$

a) $f(x)$ tiene que ser no negativa. Sí porque $\theta > 0$ y $x^{\theta+1} > 0 \ \forall x \in (1,\infty)$

b) $\int_{-\infty}^{\infty} f(x)dx$ tiene que ser 1.

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_1^{\infty} \frac{\theta}{x^{\theta+1}}\,dx = -x^{-\theta}\Big|_1^{\infty} = -0-(-1) = 1$$

c) $F_X(x) = \begin{cases} \int_1^x \frac{\theta}{u^{\theta+1}}\,du = -u^{-\theta}\Big|_1^x = -x^{-\theta}+1 = 1-x^{-\theta} & \text{si } x \in (1,\infty) \\ 0 & \text{si } x \le 1 \end{cases}$



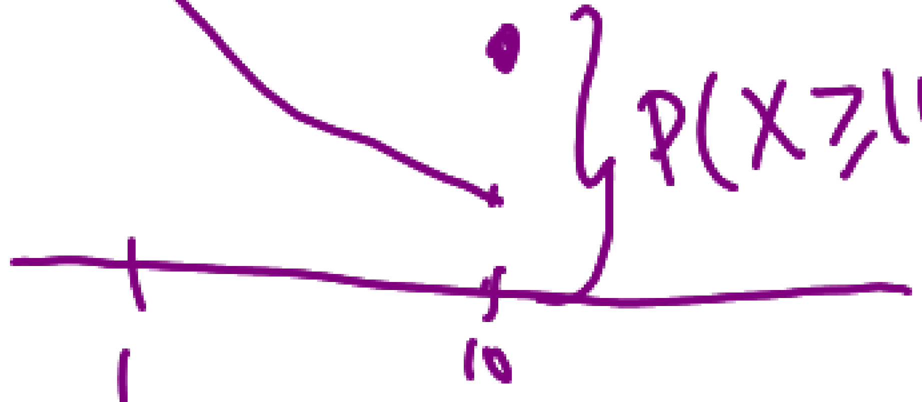
d) $P(X \le 3) = F_X(3) = p$

$W \sim Bin(10, p)$

$P(W \ge 2) = 1 - P(W=0) - P(W=1)$

e) $f(x) = f_X(x) I_{(1,10)}(x) + P(X \ge 10) I_{\{10\}}(x)$

$\} P(X \ge 10)$



$u = F_X(x) = 1 - \frac{1}{x^{\theta}}$

$\Leftrightarrow 1-u = x^{-\theta}$

$\Leftrightarrow \frac{1}{1-u} = x^{\theta}$

$\Leftrightarrow \ln\left(\frac{1}{1-u}\right) = \theta \ln(x)$

$\Leftrightarrow \ln(1) - \ln(1-u) = \theta \ln(x)$

histograma de frecuencias de $\{x_i\}_{i=1}^{n}$

Definimos una partición $\{c_i\}$ $c_i \in \mathbb{R}$ tq $c_i < c_j$ $i < j$

el histograma relativo es

$$(c_{i+1}-c_i)\, h((c_i, c_{i+1})) = \frac{\#\ \text{observaciones en } \{x_i\}_{i=1}^{n} \text{ que están entre } c_i \text{ y } c_{i+1}}{n}$$