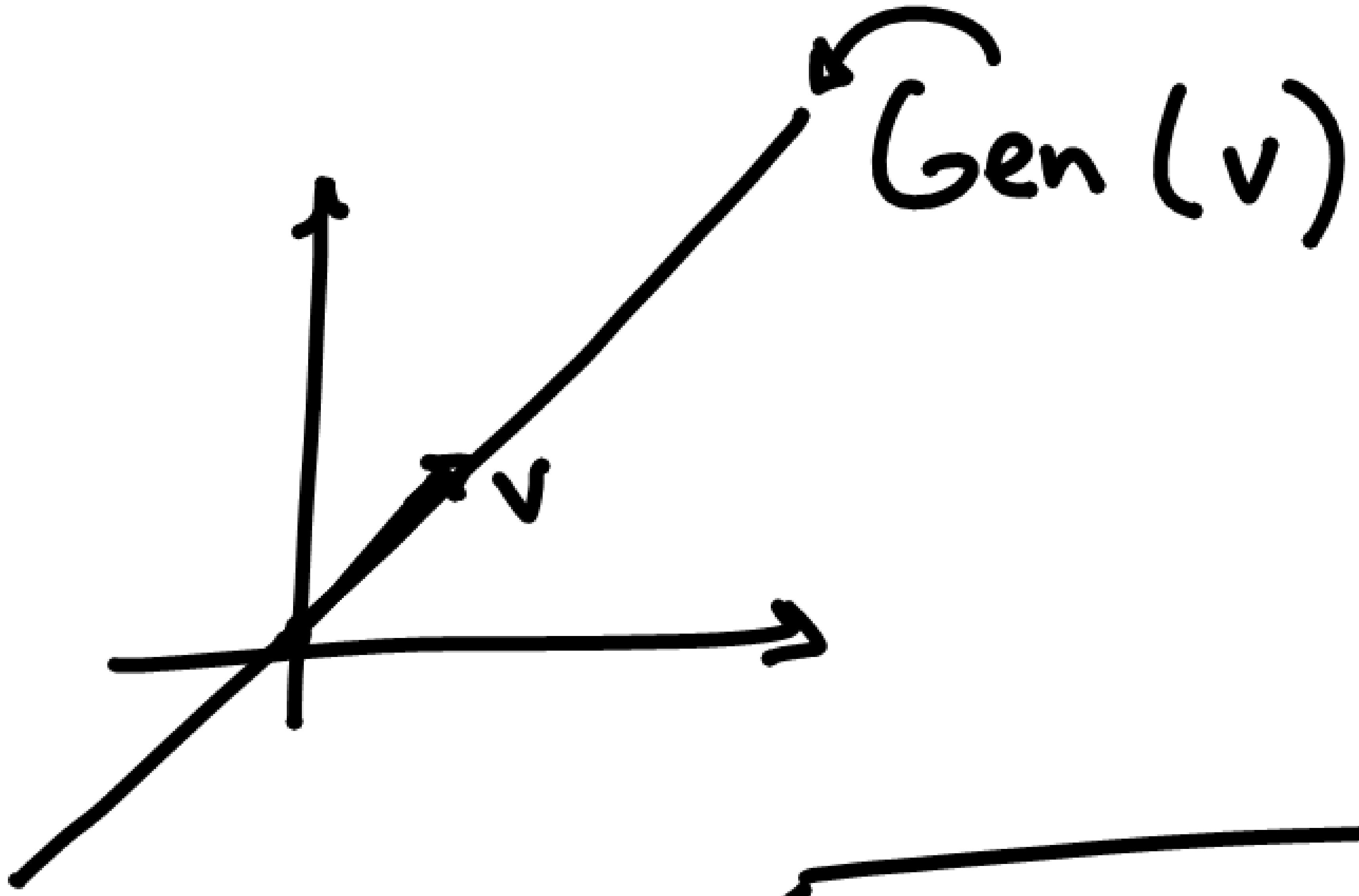
$W \to$ espacio vect.

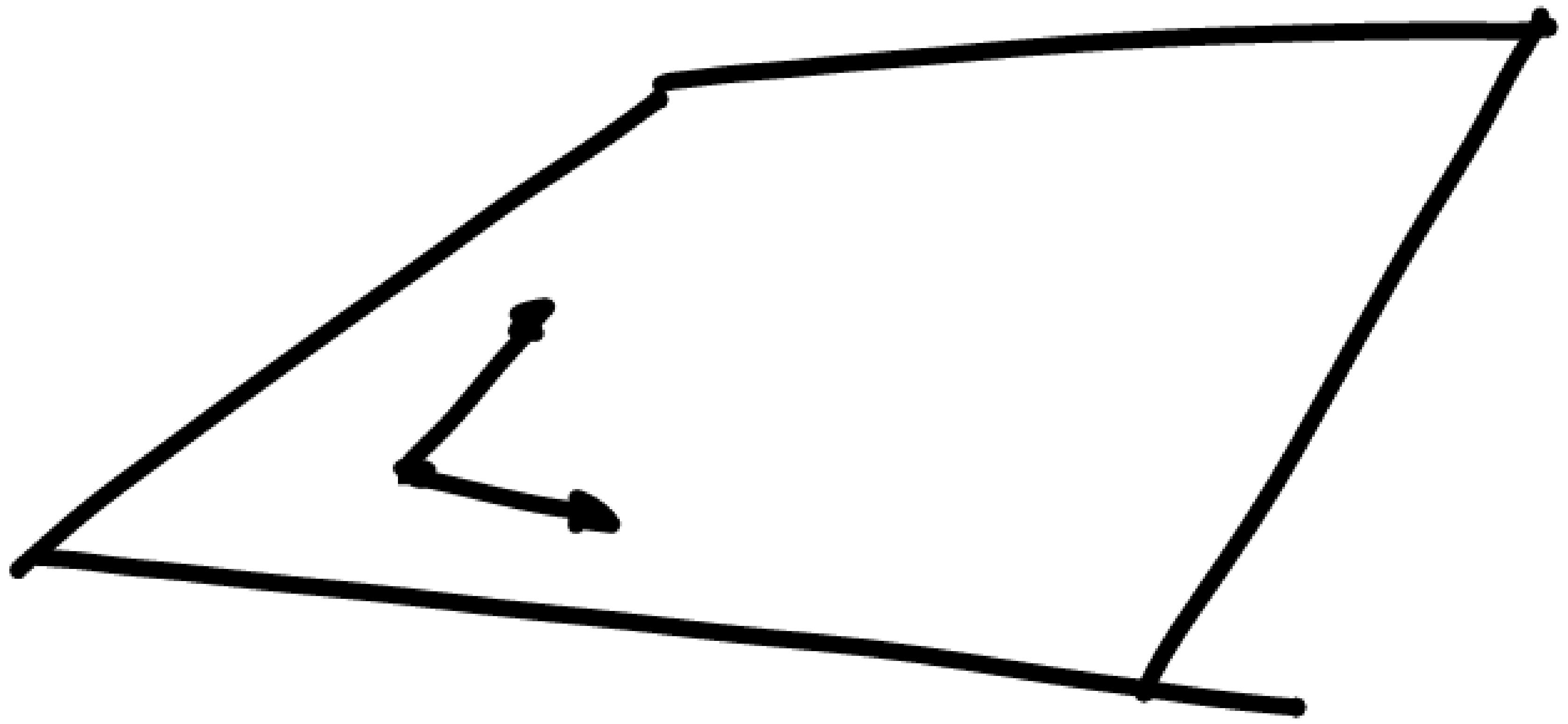
$B_1 = \{ w_1, w_2, \dots, w_p \}$

$B_2 = \{ u_1, u_2, \dots, u_p \}$.
donde $u_i \perp u_j \quad \forall\, i \neq j$
que

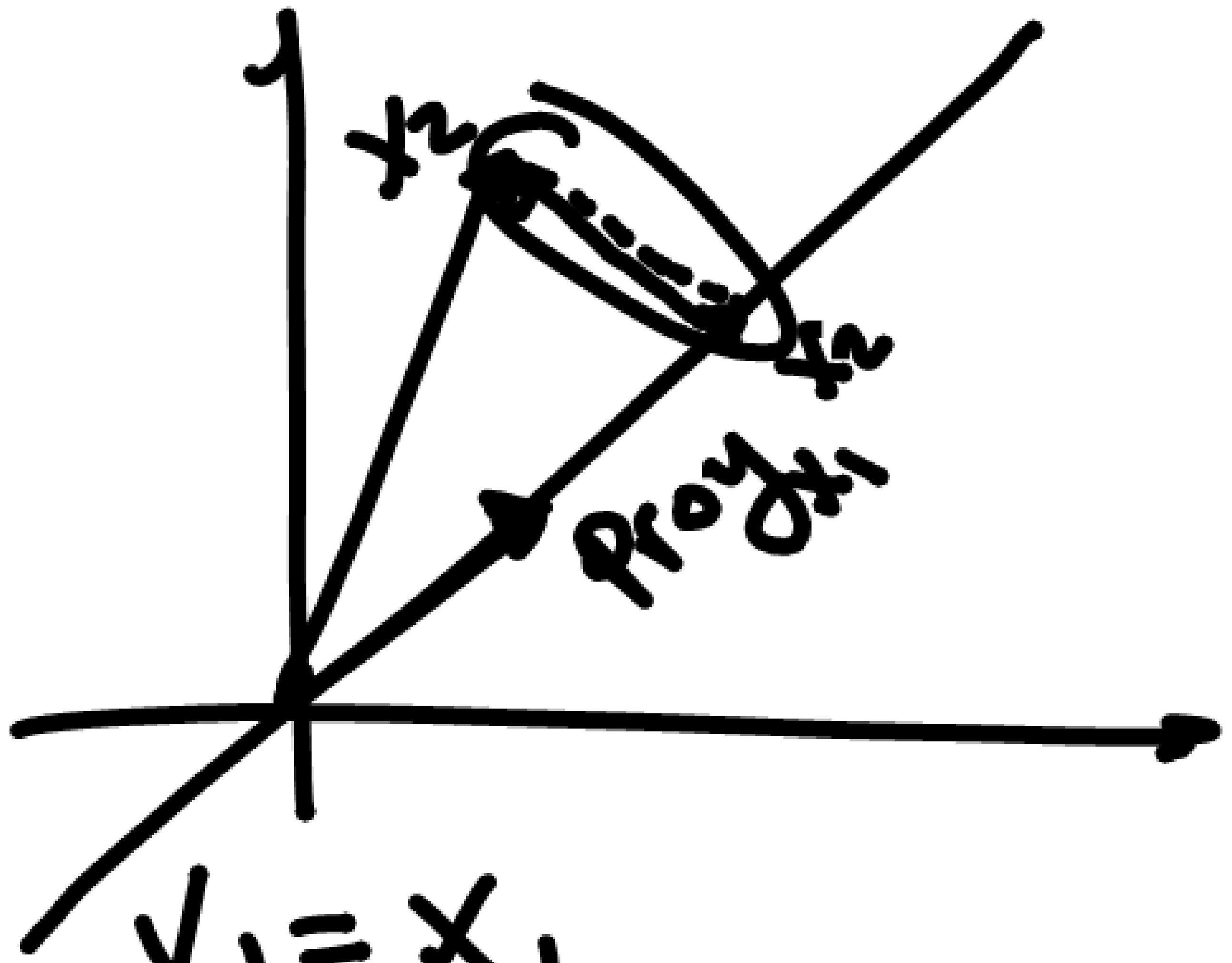
$$Gen(B_1) = Gen(B_2).$$

Proceso Gram-Schmitd





$$B = \{x_1, x_2, x_3, \ldots, x_p\}.$$



$$v_1 = x_1$$

$$v_2 = x_2 - \text{Proy}_{x_1} x_2$$

$$v_2 = x_2 - \frac{x_2 \cdot x_1}{x_1 \cdot x_1} \cdot x_1$$

* $x_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$, $x_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix}$, $x_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}$

- $V_2' = \begin{pmatrix} -3 \\ 1 \\ 1 \\ 1 \end{pmatrix} = 4 v_2$

- $V_1 = x_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$

- $V_2 = x_2 - \dfrac{x_2 \cdot v_1}{v_1 \cdot v_1} v_1 = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix} - \dfrac{\begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}}{\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix} - \dfrac{3}{4} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} -3/4 \\ 1/4 \\ 1/4 \\ 1/4 \end{pmatrix}.$

$V_3 = X_3 - \dfrac{x_3 \cdot v_1}{V_1 \cdot V_1} \cdot V_1 - \dfrac{x_3 \cdot V_2'}{V_2' \cdot V_2'} V_2' = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix} - \dfrac{2}{4} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} - \dfrac{2}{12} \begin{pmatrix} -3 \\ 1 \\ 1 \\ 1 \end{pmatrix}$

$V_3 = \begin{pmatrix} 0 \\ -2/3 \\ 1/3 \\ 1/3 \end{pmatrix}$ v_3

$B = \{ v_1, v_2, v_3 \} \to$ Base ortogonal

$B' = \left\{ \frac{v_1}{\|v_1\|}, \frac{v_2}{\|v_2\|}, \frac{v_3}{\|v_3\|} \right\} \to$ Base ortonomal.

Para $v_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$, $v_2 = \begin{pmatrix} -3 \\ 1 \\ 1 \\ 1 \end{pmatrix}$, $v_3 = \begin{pmatrix} 0 \\ -2/3 \\ 1/3 \\ 1/3 \end{pmatrix}$

$$\|v_1\| = \sqrt{v_1 \cdot v_1} = \sqrt{4} = 2.$$

$$\|v_2\| = \sqrt{v_2 \cdot v_2} = \sqrt{12} = 2\sqrt{3}$$

$$\|v_3\| = \sqrt{v_3 \cdot v_3} = \sqrt{\frac{6}{9}} = \frac{\sqrt{6}}{3}$$

$v_1, \dots, v_p$ ortonomales.

$A = (v_1 \; v_2 \; \cdots \; v_p)$

$A^t = \begin{pmatrix} v_1^t \\ v_2^t \\ \vdots \\ v_p^t \end{pmatrix}$ $\quad A^t \cdot A = \begin{pmatrix} v_1^t \cdot v_1 & v_1^t v_2 & \cdots \\ \vdots & \ddots & \end{pmatrix}$

$$\underline{A} = QR$$

$$Q^tA = \underbrace{Q^tQ}_{I}\, R = Q^tA$$

$$\underbrace{\begin{pmatrix} q_1^t \\ \vdots \\ q_m^t \end{pmatrix}}_{m\times n} \cdot \underbrace{\left(q_1, q_2, \dots, q_m\right)}_{n\times m} = \underbrace{\begin{pmatrix} \underline{q_1^t q_1} & q_1^t q_2 & \cdots \\ q_2^t q_1 & \underline{q_2^t q_2} & \cdots \\ \vdots & \vdots & \ddots \\ q_m^t q_1 & q_m^t q_2 & \cdots \ \underline{q_m^t q_m} \end{pmatrix}}_{m\times m}$$

$$A = \begin{array}{c} \begin{matrix} x_1 & x_2 & x_3 \end{matrix} \\ \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \end{array}$$

$$\underbrace{\alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 = 0}_{\text{Sol. trivial.}}$$

$$R = Q^t A.$$

$$= \begin{pmatrix} 1/2 & 1/2 & 1/2 & 1/2 \\ -\sqrt{3}/2 & \sqrt{3}/6 & \sqrt{3}/6 & \sqrt{3}/6 \\ 0 & -2\sqrt{6}/6 & \sqrt{6}/6 & \sqrt{6}/6 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

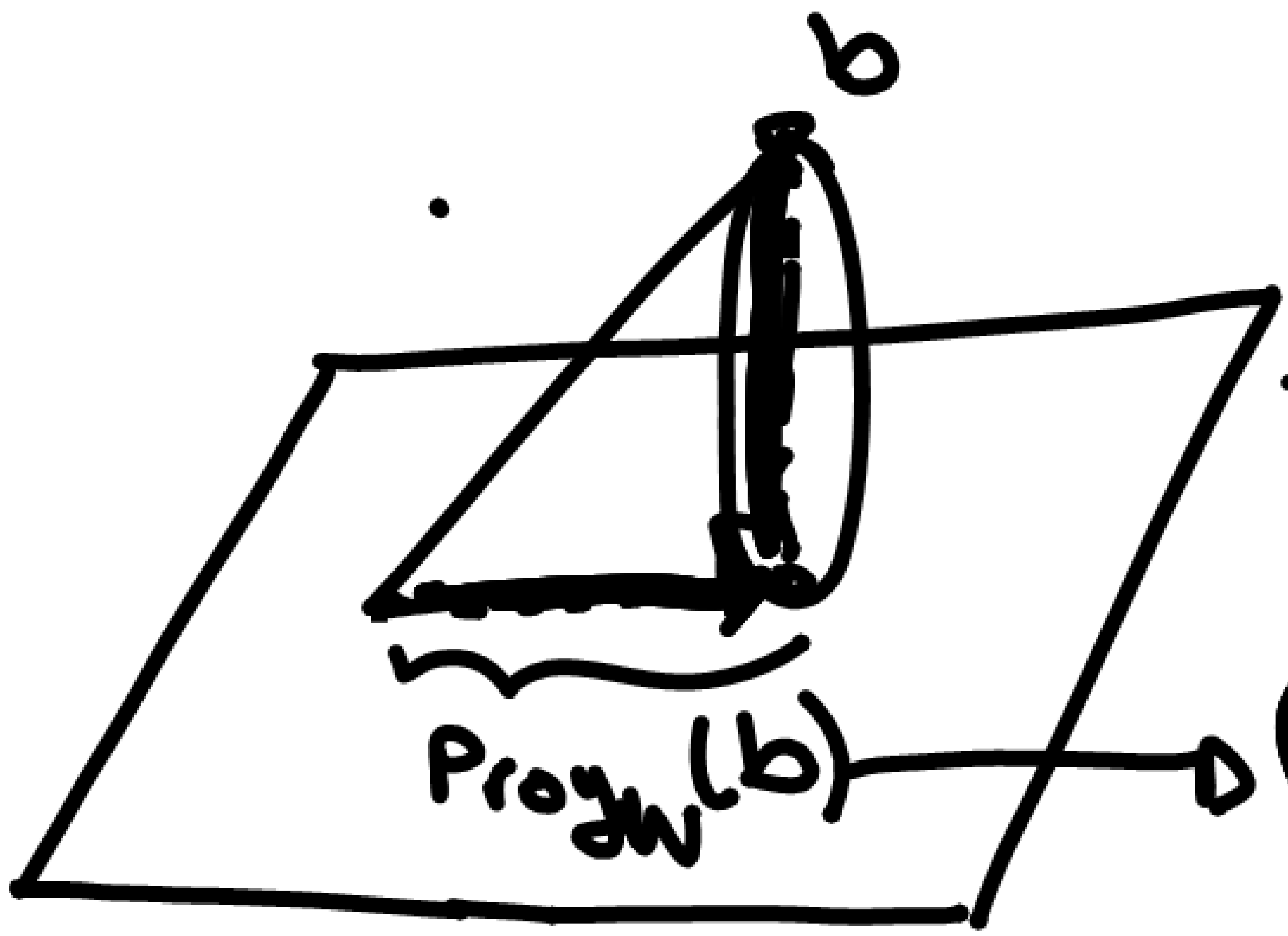
$$R = \begin{pmatrix} 2 & 3/2 & 1 \\ 0 & \sqrt{3}/2 & \sqrt{3}/3 \\ 0 & 0 & \sqrt{6}/3 \end{pmatrix}$$

$$\times \quad v_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \qquad v_2 = \begin{pmatrix} -3 \\ 1 \\ 1 \\ 1 \end{pmatrix} \qquad v_3 = \begin{pmatrix} 0 \\ -2 \\ 1 \\ 1 \end{pmatrix}$$

$$\|V\| = 2 \qquad \|v_2\| = 2\sqrt{3} \qquad \|v_3\| = \sqrt{6}$$

$$Q = \begin{pmatrix} \frac{v_1}{\|V\|} & \frac{v_2}{\|v_2\|} & \frac{v_3}{\|v_3\|} \end{pmatrix} = \begin{pmatrix} 1/2 & -\frac{\sqrt{3}}{2} & 0 \\ 1/2 & \sqrt{3}/6 & -\frac{2\sqrt{6}}{6} \\ 1/2 & \sqrt{3}/6 & \frac{\sqrt{6}}{6} \\ 1/2 & \sqrt{3}/6 & \sqrt{6}/6 \end{pmatrix}$$

$Ax = b$



$\underline{W} = \mathrm{Gen}(a_1, \dots, a_m)$

$\underline{Ax̂ = \hat{b}}$

$(b - \hat{b})$ perpendicular a $w \in W$

$(b - A\hat{x})$

$A^t(b - A\hat{x}) = 0$

$A^t b - A^t A \hat{x} = 0$

$A^t A \hat{x} = A^t b$

$A = \begin{pmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{pmatrix} \quad b = \begin{pmatrix} 2 \\ 0 \\ 11 \end{pmatrix} \quad Ax = \underline{b}$

$\underbrace{(-2, -4, 8)}$

$\|b - \hat{b}\| = \sqrt{84}$

* Solución con mínimos cuadrados.

* $A^t A x = A^t b$

$$\overset{A^t}{\begin{pmatrix} 4 & 0 & 1 \\ 0 & 2 & 1 \end{pmatrix}} \overset{A}{\begin{pmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{pmatrix}} = \begin{pmatrix} 17 & 1 \\ 1 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 4 & 0 & 1 \\ 0 & 2 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \\ 11 \end{pmatrix} = \begin{pmatrix} 19 \\ 11 \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} 17 & 1 \\ 1 & 5 \end{pmatrix} x = \begin{pmatrix} 19 \\ 11 \end{pmatrix}}$$

Consistente

$\hat{x} \to$ min. Cuad.

$$\hat{x} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$A\hat{x} = \begin{pmatrix} 4 \\ 4 \\ 3 \end{pmatrix} = \underline{\hat{b}}$$

$$A = \begin{pmatrix} 1 & 3 & 5 \\ 1 & 1 & 0 \\ 1 & 1 & 2 \\ 1 & 3 & 3 \end{pmatrix} \text{ y } b = \begin{pmatrix} 3 \\ 5 \\ 7 \\ -3 \end{pmatrix} \qquad Q = \begin{pmatrix} 1/2 & 1/2 & 1/2 \\ 1/2 & -1/2 & -1/2 \\ 1/2 & -1/2 & 1/2 \\ 1/2 & 1/2 & -1/2 \end{pmatrix} \quad R = Q^t A.$$

$$V_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$$=$$

$$V_2 = \begin{pmatrix} 3 \\ 1 \\ 1 \\ 3 \end{pmatrix} - \frac{\begin{pmatrix} 3 \\ 1 \\ 1 \\ 3 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}}{\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 1 \\ 3 \end{pmatrix} - 2 \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix}$$

$$R = \begin{pmatrix} 2 & 4 & 5 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{pmatrix}$$

$$V_3 = \begin{pmatrix} 5 \\ 0 \\ 2 \\ 3 \end{pmatrix} - \frac{10}{4} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} - \frac{6}{4} \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}$$

$$A = \begin{pmatrix} 1/2 & 1/2 & 1/2 \\ 1/2 & -1/2 & -1/2 \\ 1/2 & -1/2 & 1/2 \\ 1/2 & 1/2 & -1/2 \end{pmatrix} \begin{pmatrix} 2 & 4 & 5 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{pmatrix}$$

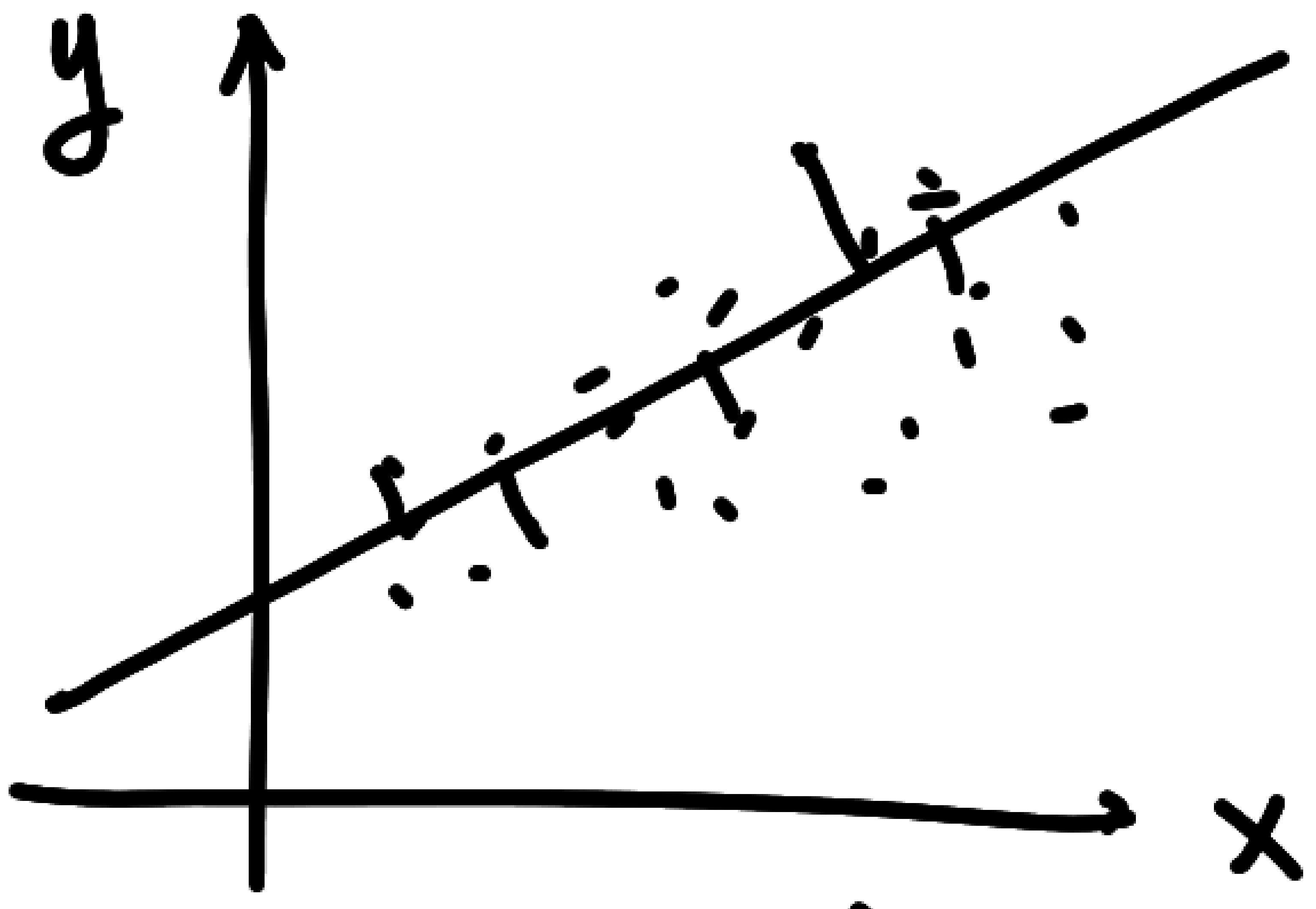
$$\hat{x} = R^{-1} Q^t b$$

$$R\hat{x} = \underbrace{R R^{-1}}_{I} Q^t b$$

$$\begin{pmatrix} 2 & 4 & 5 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{pmatrix} \hat{x} = \begin{pmatrix} 1/2 & 1/2 & 1/2 & 1/2 \\ 1/2 & -1/2 & -1/2 & 1/2 \\ 1/2 & -1/2 & 1/2 & -1/2 \end{pmatrix} \begin{pmatrix} 3 \\ 5 \\ 7 \\ -3 \end{pmatrix} = \begin{pmatrix} 6 \\ -6 \\ 4 \end{pmatrix}$$

$$\hat{x}_3 = 2$$
$$\hat{x}_2 = -6$$
$$\hat{x}_1 = 10$$

Ajuste lineal por mínimos Cuadrados.



$\hat{y} = \beta_1 x + \beta_0$

$y \longleftrightarrow \hat{y}$ min. dist.

$\min \|y - \hat{y}\|$

x	y	$\hat{y}$
x_1	y_1	$\underline{\beta_1} x_1 + \underline{\beta_0}$
$\vdots$	$\vdots$	$\ddots$
x_n	y_n	$\beta_1 x_n + \beta_0$

$$\underbrace{\overset{X}{\begin{pmatrix} 1 & x_1 \\ 1 & x_2 \\ 1 & x_3 \\ \vdots & \vdots \\ 1 & x_n \end{pmatrix} \begin{pmatrix} \underline{\beta_0} \\ \underline{\beta_1} \end{pmatrix}}} = y$$

$\mathrm{Proy}_{C(\underline{X})}(y) \to$ Sol. min. Cuad.

x	y	$\hat{y}$
2	1	$\beta_1 2 + \beta_0$
5	2	$5\beta_1 + \beta_0$
7	3	$7\beta_1 + \beta_0$
8	3	$8\beta_1 + \beta_0$

$$\overset{X}{\begin{pmatrix} 1 & 2 \\ 1 & 5 \\ 1 & 7 \\ 1 & 8 \end{pmatrix}} \begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 3 \end{pmatrix} \to \text{No Solución.}$$

$$\boxed{X^t X \beta = X^t y}$$

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 5 & 7 & 8 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 1 & 5 \\ 1 & 7 \\ 1 & 8 \end{pmatrix} = \begin{pmatrix} 4 & 22 \\ 22 & 142 \end{pmatrix}$$

$$X^t b = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 5 & 7 & 8 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 9 \\ 57 \end{pmatrix}$$

$$\begin{pmatrix} 4 & 22 \\ 22 & 142 \end{pmatrix} \hat{\beta} = \begin{pmatrix} 9 \\ 57 \end{pmatrix}$$

$$\hat{\beta} = \begin{pmatrix} 2/7 \\ 5/14 \end{pmatrix}$$

$$\hat{y} = 5/14\, x + 2/7$$

$$\underset{1\times 2}{(x_1, x_2)}\ \underset{2\times 2}{\overset{A}{\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}}}\ \underset{2\times 1}{\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}} = \begin{pmatrix} \underline{x_1} + 3x_2 & \underline{2x_1} + 4x_2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$= x_1^2 + 3x_2x_1 + 2x_1x_2 + 4x_2^2$$

$$= \underline{x_1^2} + \underline{5x_1x_2} + \underline{4x_2^2}$$

$$(x_1 \quad x_2) \begin{pmatrix} 1 & 2.5 \\ 2.5 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} =$$

$A = PDP^t$

$P \to$ ortogonal

$D \to$ mat. diagonal.

$$Q(x) = x^t A x = \underbrace{x^t P}_{y^t} \underline{D} \underbrace{P^t x}_{y}$$

$$y = P^{-1} x = \underline{\underline{P^t x}}$$

$$y^t = x^t P$$

$A \to m \times n$

$\Sigma \to m \times n$

Val. Sing.

$U \to m \times m$ Ortogonal

$V \to n \times n$ ortogonal

- $A^t A \to$ Calcular val. Sing.
  - $\to$ Vectores propios de $A^t A$
    Vectores Son ortogonales.

- $\Sigma \to$ diag. de Val. Sing. $\sigma_1 \geq \sigma_2 \geq \sigma_3 \geq \dots \geq 0$
- $V \to$ Vectores propios de A como Vectores columna
- $U \to$ Vectores columna Av_i

$$A=\begin{pmatrix} 4 & 11 & 14 \\ 8 & 7 & -2 \end{pmatrix}_{2\times 3} \Rightarrow U_{2\times 2}\ \Sigma_{2\times 3}\ V_{3\times 3}.$$

- $$A^tA=\begin{pmatrix} 4 & 8 \\ 11 & 7 \\ 14 & -2 \end{pmatrix}\begin{pmatrix} 4 & 11 & 14 \\ 8 & 7 & -2 \end{pmatrix}=\begin{pmatrix} 80 & 100 & 40 \\ 100 & 170 & 140 \\ 40 & 140 & 200 \end{pmatrix}$$

- $\lambda_1=0$, $v_1=\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$ $\qquad \lambda_2=90$, $v_2=\begin{pmatrix} -2 \\ -1 \\ 2 \end{pmatrix}$ $\qquad \lambda_3=360$, $v_3=\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$

Normalizando

$$U=\begin{pmatrix} \dfrac{Av_3}{\|Av_3\|} & \dfrac{Av_2}{\|Av_2\|} & \cancel{\dfrac{Av_1}{\|Av_1\|}} \end{pmatrix} \qquad Av_3 = \begin{pmatrix} 18 \\ 6 \end{pmatrix} \qquad Av_2 = \begin{pmatrix} 3 \\ -9 \end{pmatrix}$$

$$=\begin{pmatrix} 18/\sqrt{360} & 3/\sqrt{90} \\ 6/\sqrt{360} & -9/\sqrt{90} \end{pmatrix}$$

- $$\Sigma=\begin{pmatrix} \sqrt{360} & 0 & 0 \\ 0 & \sqrt{90} & 0 \end{pmatrix}$$

- $$V=\begin{pmatrix} 1/3 & -2/3 & 2/3 \\ 2/3 & -1/3 & -2/3 \\ 2/3 & 2/3 & 1/3 \end{pmatrix}$$

$$A=U\Sigma V^t$$