Y, X v.a. discretas $\qquad E(X|Y=y) = \sum_x x\, P(X=x|Y=y)$

Y, X v.a. continuas $\qquad E(X|Y=y) = \int_X x \underbrace{f_{X|Y}(x|y)}_{f(x|y)}$


$$f_{X|Y}(x|y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}$$

$$E(X) = E(E(X|Y))$$

$$E(E(X|Y)) = \int_Y \left[\int_X x\, f_{X|Y}(x|y)\, dx\right] f_Y(y)\, dy$$

$$f_{X,Y}(x,y) = f_{X|Y}(x|y)\, f_Y(y)$$

$$= \int_Y \int_X x\, f_{X|Y}(x|y)\, f_Y(y)\, dx\, dy = \int_X x \overbrace{\int_Y \underbrace{f_{X|Y}(x|y)\, f_Y(y)}_{f_{X,Y}(x,y)}\, dy}^{f_X(x)}\, dx$$

$$= \int_X x\, f_X(x)\, dx = E(X)$$

Como $\int_Y f_{X,Y}(x,y)\,dy = f_X(x)$

uni $E(E(X|Y)) = \int_X x f_X(x)\,dx = E(X)$

---

$X_1, X_2, \ldots$ v.a. Poisson$(\lambda)$ independientes

$N$ v.a. Poisson$(\lambda_0)$

$Y = X_1 + X_2 + \ldots + X_N$

como $Y|N \sim \text{Poisson}\left(\sum_{i=1}^{N} \lambda\right)$

Poisson$(N\lambda)$

Queremo $E(Y) = E(E(Y|N)) = E(N\lambda)$

$= \lambda E(N) = \lambda \lambda_0$

---

$X_i$ v.a indep.

$N$ v.a. entera e indep de $X_i$'s

$E\left(\sum_{i=1}^{N} X_i\right) = E(N)E(X_1)$

$Cov(X,Y) = E(XY) - E(X)E(Y)$

$$\boxed{Var(X) = E(Var(X|Y)) + Var(E(X|Y))}$$

$\vec{X} = \underline{X} = (X_1,\dots,X_n)$ $\quad X_i$ v.a.

función de prob. conjunta $\quad f_{X,Y}(x,y) = P(X=x, Y=y)$

función de densidad conjunta $\quad f_{X,Y}(x,y)$, $\quad P(X\in A, Y\in B) = \int_A\int_B f_{X,Y}(x,y)\,dy\,dx$

$f_{\underline{X}}(x_1,\dots,x_n) = P(X_1=x_1,\dots,X_n=x_n)$

Marginales $\quad f_X(x) = \int f_{X,Y}(x,y)\,dy$ $\quad\rightarrow\quad f_{X_i}(x_i) = \int_{x_1}\cdots\int_{x_{i-1}}\int_{x_{i+1}}\cdots\int_{x_n} f_{\underline{X}}(x_1,\dots,x_n)\,dx_1\dots dx_{i-1}\,dx_{i+1}\dots dx_n$

Condicional
$$f_{X|Y}(x|y) = \frac{f_{X,Y}(x,y)}{f_Y(y)} = \frac{P(X=x, Y=y)}{P(Y=y)} \quad \text{(discreto)}$$
$$f_{X|Y}(x|y) = \frac{f_{X,Y}(x,y)}{f_Y(y)} \leftarrow \text{densidad (continuo)} \quad P(X\in A|Y=y) = \int_A f_{X|Y}(x|y)\,dx$$

independencia $\quad X, Y$ son independientes ssi $\quad f_{X,Y}(x,y) = f_X(x)f_Y(y)\ \forall x,y$

$$E(X|Y) = \int x\, f_{X|Y}(x|y)\,dx$$

$$Var(X|y) = \int_X (x - E(X|y))^2 f_{X|Y}(x|y)\,dx$$

$$f_{X_{i_1},X_{i_2},\dots,X_{i_m}}(x_{i_1},\dots,x_{i_m}) = \int_{\{i\in I\,:\,i\notin J\}} f_{X_1,\dots,X_n}(x_1,\dots,x_n)\,dx_{\ell_1}\,dx_{\ell_2}\cdots dx_{\ell_p}$$

$\ell_k \notin J$

$i_j \in \{1,\dots,n\}$

$i_j \neq i_k \quad \forall j\neq k$

$\{i_j\}_{j=1}^{m} = J$

$$\mathrm{Cov}(X,Y) = E(XY) - E(X)E(Y) = \mathrm{Cov}(Y,X)$$

Condicional:

$$f_{X_i\mid \underline{X}_{-i}}(x_i\mid \cdot) = \frac{f_{X_1,\dots,X_n}(x_1,\dots,x_n)}{f_{X_1,\dots,X_{i-1},X_{i+1},\dots,X_n}(x_1,\dots,x_{i-1},x_{i+1},\dots,x_n)}$$

$$\underline{X}_{-i} = (X_1,\dots,X_{i-1},X_{i+1},\dots,X_n)$$

$$\underline{X} = \begin{pmatrix} X_1 \\ \vdots \\ X_n \end{pmatrix} \qquad E(\underline{X}) = \begin{pmatrix} E(X_1) \\ E(X_2) \\ \vdots \\ E(X_n) \end{pmatrix}$$

$$\mathrm{Var}(\underline{X}) = \begin{pmatrix} \mathrm{Var}(X_1) & \mathrm{Cov}(X_1,X_2) & \cdots & \mathrm{Cov}(X_1,X_n) \\ \mathrm{Cov}(X_2,X_1) & \mathrm{Var}(X_2) & & \\ & & \ddots & \\ \mathrm{Cov}(X_n,X_1) & & & \mathrm{Var}(X_n) \end{pmatrix}$$

matriz simétrica.

independencia

$\underline{X} = (X_1, X_2)^t$ $X_1$ y $X_2$ son indep ssi $f_{X,Y}(x,y) = f_X(x) f_Y(y)$ $\left( \Rightarrow f_{X|Y}(x|y) = f_X(x) \right.$

$\underline{X} = (X_1, \dots, X_n)^t$ $X_1, \dots, X_n$ son indep ssi

$$f_{X_1,\dots,X_n}(x_1,\dots,x_n) = f_{X_1}(x_1) \cdots f_{X_n}(x_n) \quad \forall x_1,\dots,x_n$$

y

$$f_{X_{i_1},\dots,X_{i_j}}(x_{i_1},\dots,x_{i_j}) = f_{X_{i_1}}(x_{i_1}) \cdots f_{X_{i_j}}(x_{i_j})$$

$\forall$ subconjunto de indices $i_1, \dots, i_j$ en el conjunto $\{1, \dots, n\}$

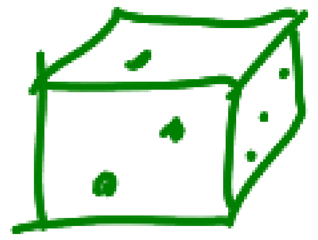
---

$(X_1, X_2, X_3)$ Pd $X_1, X_2, X_3$ son indep.

$$f_{X_1,X_2,X_3}(x_1,x_2,x_3) = \prod_{i=1}^{3} f_{X_i}(x_i) \quad \forall x_i$$

$$f_{X_1,X_2}(x_1,x_2) = f_{X_1}(x_1) f_{X_2}(x_2) \quad \forall x_1, x_2$$

$\vdots$

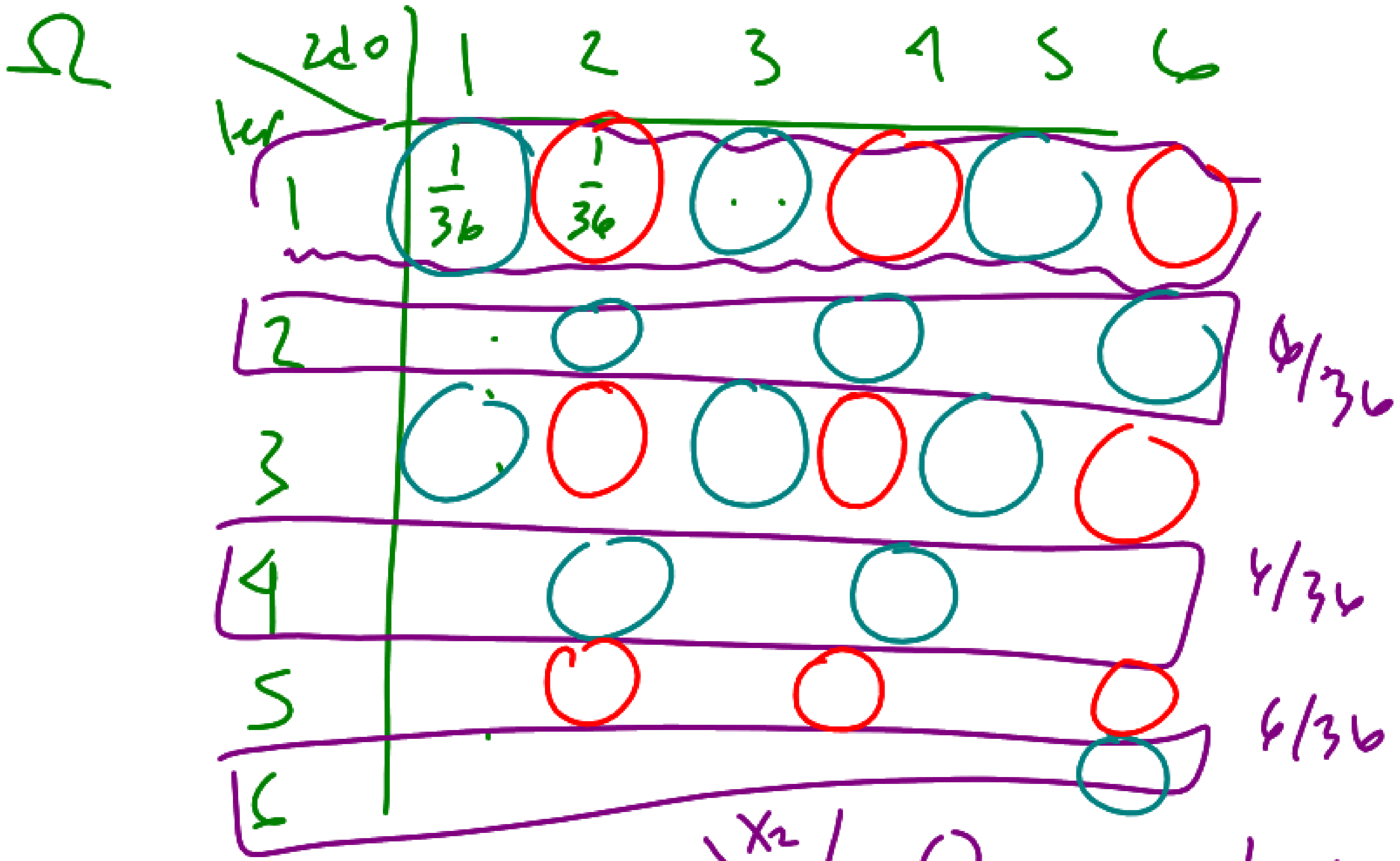


dado justo tiramos 2 veces

$$X_1 = \text{"Sale un par en la 1ra"} = \begin{cases} 0 & \text{fracaso} \\ 1 & \text{si si sale par} \end{cases}$$

$$X_2 = \text{"Suma de los dos tiros es par"} \begin{cases} 0 \\ 1 \end{cases}$$

- $P_{X_1,X_2}(x_1,x_2)$
- $P_{X_1}(x_1)$ , $P_{X_2}(x_2)$
- $P_{X_2|X_1}(x_2|x_1)$
- $E(X_2)$ , $E(X_2|X_1)$




$$X_1 = \begin{cases} 1 & \text{si par en el 1er tiro} \\ 0 & \text{eoc.} \end{cases}$$

$$X_2 = \cancel{\text{suma es par}} \begin{cases} 0 & \\ 1 & \text{si suma es par} \end{cases}$$

$f_{X_1,X_2}(x_1,x_2) \rightarrow$

| $x_1 \backslash x_2$ | 0 | 1 | $P(X_1 = x_1)$ |
|---|---|---|---|
| 0 | 9/36 | 9/36 | 18/36 |
| 1 | 9/36 | 9/36 | 18/36 |
| $P(X_2 = x_i)$ | 18/36 | 18/36 | 1 |

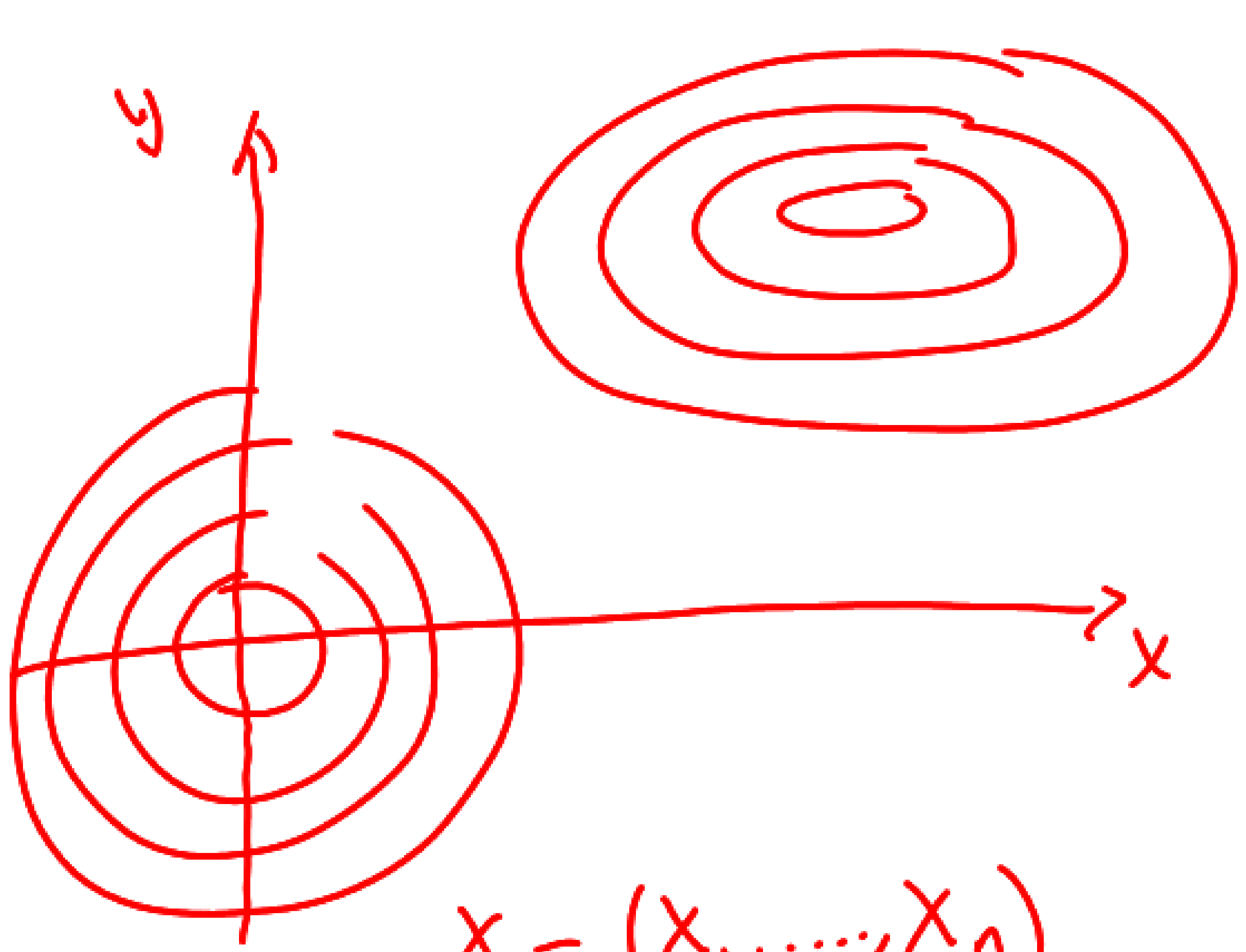
$\underbrace{P_{X_2|X_1}(x_2|x_1)}$

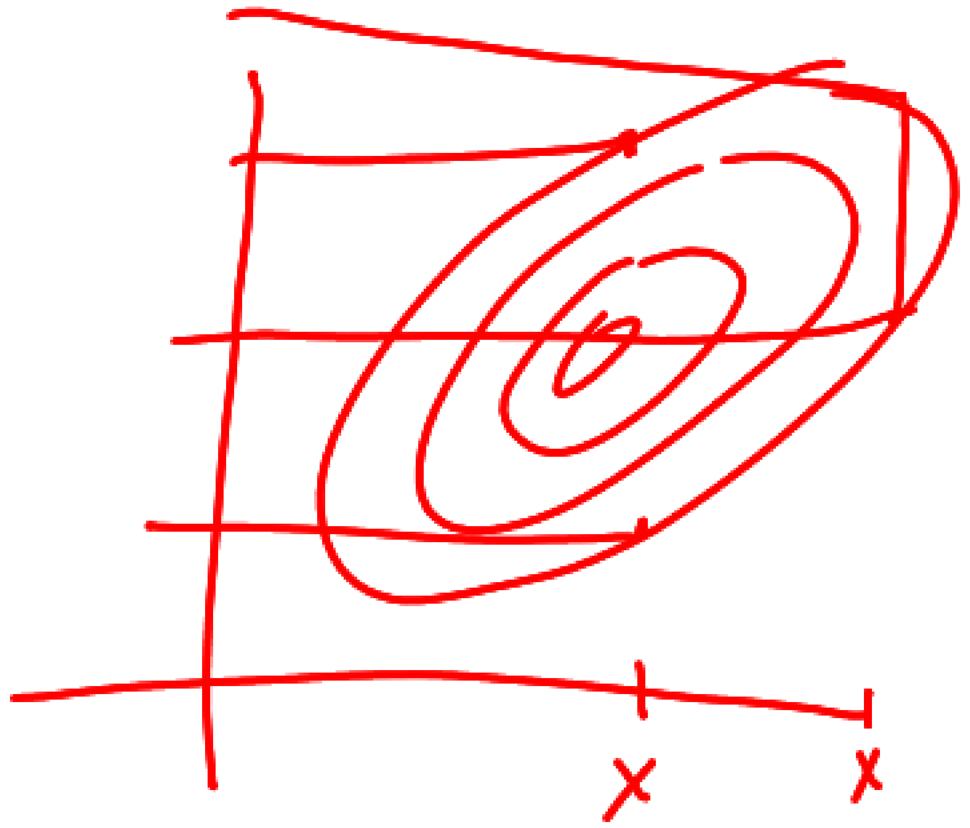
| $x_1 \backslash x_2$ | 0 | 1 | |
|---|---|---|---|
| 0 | 9/18 | 9/18 | 1 |
| 1 | 9/18 | 9/18 | 1 |

$$P(X_2=0|X_1=0) = \frac{P(X_1=0, X_2=0)}{P(X_2=0)} = \frac{9/36}{18/36} = \frac{9}{18}$$

$$E(X_2) = 0P(X_2=0) + 1P(X_2=1) = P(X_2=1) = 18/36$$

$$E(X_2|X_1) = \begin{cases} 9/18 & x_1 = 0 \\ 9/18 & x_1 = 1 \end{cases}$$




$\underline{X} = (X_1, \dots, X_n)$

$$f_{\underline{X}}(x_1, \dots, x_n) = \frac{1}{(2\pi)^{n/p} |\Sigma|^{1/2}} \exp\left\{ -\frac{1}{2} \underbrace{\underset{1\times n}{(\underline{x}-\underline{\mu})^t} \; \underset{n\times n}{\Sigma^{-1}} \; \underset{n\times 1}{(\underline{x}-\underline{\mu})}} \right\}$$

$\underline{X} \sim N_n(\underline{\mu}, \Sigma)$

M poblaciones

cada una con fd $f_i(x)$

em!.. la ~~distn~~ fd de la mezcla es

$$f_T(x) = a_1 f_1(x) + a_2 f_2(x) + \dots + a_M f_M(x)$$

dònde $a_1 + a_2 + \dots + a_M = 1$

---

$$F_T(x) = a_1 F_1(x) + a_2 F_2(x) + \dots + a_M F_M(x)$$

Pd. $\int_{-\infty}^{\infty} f_T(x) = 1 \qquad \lim_{x \to \infty} F_T(x) = 1$

```r
#poblacion 1:
mux<-70
sigmax<- 3

#poblacion 2:
muy<-90
sigmay<-4

#proporcion de personas de la poblacion 1 en el total
alfa<-0.2


#seleccionamos n personas al azar
n<-10000
a<-runif(n)
res<-rep(NA,n)
for(i in 1:n){
if(a[i]<0.2){res[i]<-rnorm(1,mux,sigmax)
}else{res[i]<-rnorm(1,muy,sigmay)}
}
hist(res,col="grey",border="white",freq=FALSE)

x<-seq(0,150,by=.1)
mezcla<-alfa*dnorm(x,mux,sigmax)+(1-alfa)*dnorm(x,muy,sigmay)

lines(x,mezcla,col="red")
```

Global Environment

| | |
|---|---|
| a | num [1:10000] 0.4989 0.10388 0.17475 0.00432 0.20626 ... |
| alfa | 0.2 |
| i | 10000L |
| mezcla | num [1:1501] 9.36e-112 1.64e-111 2.88e-111 5.04e-111 8.83e-111 ... |
| mux | 70 |
| muy | 90 |
| n | 10000 |
| res | num [1:10000] 92.4 69.4 73.7 66.6 90.4 ... |
| sigmax | 3 |
| sigmay | 4 |
| x | num [1:1501] 0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 ... |

```
> a<-runif(n)
> res<-rep(NA,n)
> for(i in 1:n){
+ if(a[i]<0.2){res[i]<-rnorm(1,mux,sigmax)
+ }else{res[i]<-rnorm(1,muy,sigmay)}
+ }
> hist(res,col="grey",border="white",freq=FALSE)
> x<-seq(0,150,by=.1)
> mezcla<-alfa*dnorm(x,mux,sigmax)+(1-alfa)*dnorm(x,muy,sigmay)
> lines(x,mezcla,col="red")
>
```