$$a = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}}\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} 2a_{11} & 2a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = e \qquad ¿e^{-1}?$$

$$e^{-1} = \begin{pmatrix} 1/2 & 0 \\ 0 & 1 \end{pmatrix}$$

Si e es una matriz elemental de cualquier tipo, entonces

$e^{-1} \longrightarrow$ deshacer los cambios de e.

$$\begin{aligned} x_1 + x_2 - x_3 &= 0 \\ 4x_1 - x_2 + 5x_3 &= 0 \\ 6x_1 + x_2 + 3x_3 &= 0 \end{aligned} \longrightarrow \begin{aligned} x_1 + 0.8\,x_3 &= 0 \\ x_2 - 1.8\,x_3 &= 0 \end{aligned}$$

$x_3 = 1$
$x_1 = -0.8$
$x_2 = 1.8$

$$\begin{pmatrix} -0.8 \\ 1.8 \\ 1 \end{pmatrix}$$

$$\alpha(V) = x_3 \begin{pmatrix} -0.8 \\ 1.8 \\ 1 \end{pmatrix} = \begin{pmatrix} -0.8x_3 \\ 1.8x_3 \\ x_3 \end{pmatrix}$$

$$2x_1 - 3x_2 - 4x_3 + 5x_4 = 0$$

$$7x_2 + 3x_4 = 0$$

$$\underline{x_1} = \frac{3}{2} x_2 + 2\underline{x_3} - \frac{5}{2}\underline{x_4} \equiv \frac{3}{2}\left(-\frac{3}{7}x_4\right) + 2x_3 - \frac{5}{2}\underline{x_4}$$

$$-\frac{9}{14}$$

$$\underline{x_2} = \underline{-\frac{3}{7}x_4}$$

$$= -\frac{44}{14}x_4 + 2x_3$$

$$= -\frac{22}{7}x_4 + 2x_3$$

$$\begin{pmatrix} -\frac{22}{7}x_4 + 2x_3 \\ -3/7\, x_4 \\ x_3 \\ x_4 \end{pmatrix}$$

① $x_3 = 1$, $x_4 = 0$ $\begin{pmatrix} 2 \\ 0 \\ 1 \\ 0 \end{pmatrix}$

② $x_4 = 1$, $x_3 = 0$ $\begin{pmatrix} -22/7 \\ -3/7 \\ 0 \\ 1 \end{pmatrix}$

$$X = \alpha_1 \begin{pmatrix} 2 \\ 0 \\ 1 \\ 0 \end{pmatrix} + \alpha_2 \begin{pmatrix} -22/7 \\ -3/7 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} -\frac{22}{7}x_4 + \underline{2x_3} \\ -3/7\, x_4 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 2x_3 \\ 0 \\ x_3 \\ 0 \end{pmatrix} + \begin{pmatrix} -\frac{22}{7}x_4 \\ -3/7\, x_4 \\ 0 \\ x_4 \end{pmatrix}$$

$$= \underline{x_3} \begin{pmatrix} 2 \\ 0 \\ 1 \\ 0 \end{pmatrix} + \underline{x_4} \begin{pmatrix} -\frac{22}{7} \\ -3/7 \\ 0 \\ 1 \end{pmatrix}$$

$x = x_p + x_h$.

. donde x_h Solución general homogeneo

x_p Part. no homogeneo

$$A(x_p + x_h) = \underbrace{A x_p}_{=b} + \underbrace{A x_h}_{=0}$$

$S \to Ax = b$

$\Downarrow$

$AS = b$

① $x_1 - 3x_2 + 2x_3 = 0$

$3x_1 + 6x_2 - 3x_3 = 0$

② $x_1 + x_2 - x_3 = 0$

$2x_1 - 4x_2 + 3x_3 = 0$

$3x_1 + 7x_2 - x_3 = 0$

Solución única

③ $x_1 - 2x_2 + x_3 + x_4 = 0$

$3x_1 + 2x_3 - 2x_4 = 0$

$4x_2 - x_3 - x_4 = 0$

$5x_1 + 3x_3 - x_4 = 0$

④ $2x_1 + 6x_2 - 3x_3 + x_4 = 0$

① $\begin{pmatrix} -0.2x_3 \\ 0.6x_3 \\ x_3 \end{pmatrix} = \begin{pmatrix} -0.2 \\ 0.6 \\ 1 \end{pmatrix} x_3$

③ $\begin{pmatrix} -4x_4 \\ 2x_4 \\ 7x_4 \\ x_4 \end{pmatrix} = \begin{pmatrix} -4 \\ 2 \\ 7 \\ 1 \end{pmatrix} x_4$

$$\rightarrow 2x_1 + 6x_2 - 3x_3 + x_4 = 0$$

$$\underline{x_1} = -3\underline{x_2} + \frac{3}{2}\underline{x_3} - \frac{1}{2}\underline{x_4}$$

$$\begin{pmatrix} -3x_2 + \frac{3}{2}x_3 - \frac{1}{2}x_4 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -3x_2 \\ x_2 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \frac{3}{2}x_3 \\ 0 \\ x_3 \\ 0 \end{pmatrix} + \begin{pmatrix} -\frac{1}{2}x_4 \\ 0 \\ 0 \\ x_4 \end{pmatrix}$$

$$= x_2 \begin{pmatrix} -3 \\ 1 \\ 0 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} \frac{3}{2} \\ 0 \\ 1 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} -\frac{1}{2} \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Si A es invertible.

$$A^{-1} = \frac{1}{\det(A)} \text{Adj}(A)$$

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}_{2\times 3}$$

$$B = \begin{pmatrix} 1 & -1 \\ 0 & 1 \\ 0 & -1 \end{pmatrix}_{3\times 2}$$

$$A \cdot B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I_2$$

$$B \times A \neq I_3$$

$$(\underline{A}\mid I) \Rightarrow \underbrace{e_k \cdots e_2 e_1}(A \mid I)$$

$$= E(\underline{A} \mid I)$$

$$= (EA \mid E)$$

$$= (I \mid E)$$

$$A^{-1} = e_k \cdot e_{k-1} \cdots e_1$$

$$(A^{-1})^{-1} = e_1^{-1} \cdot e_2^{-1} \cdots e_{k-1}^{-1} \cdot e_k^{-1}$$

$$= A$$

$$(AB)^{-1} = B^{-1} A^{-1}$$

$$(A(BC))^{-1} = (BC)^{-1} \cdot A^{-1}$$

$$= C^{-1} B^{-1} A^{-1}$$

$$(A^{-1})^{-1} = \underline{A}$$

$$A \rightsquigarrow D \Rightarrow U$$

GAUSS

$$\underbrace{(e_k \cdots e_2 e_1)}_{E} \cdot A \rightarrow U$$

$$A = \underbrace{E^{-1}}_{L} U$$

$$\begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 6 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 & 2 \\ 4 & 1 & 5 \\ 2 & -1 & 7 \end{pmatrix} \xrightarrow[\cdot R_3 \leftarrow R_3 - R_1]{R_2 \leftarrow R_2 - 2R_1} \begin{pmatrix} 2 & 1 & 2 \\ 0 & -1 & 1 \\ 0 & -2 & 5 \end{pmatrix} \xrightarrow{\cdot R_3 \leftarrow R_3 - 2R_2} \underbrace{\begin{pmatrix} 2 & 1 & 2 \\ 0 & -1 & 1 \\ 0 & 0 & 3 \end{pmatrix}}_{U}$$

$$\boxed{L = E^{-1}.}$$

$$e_2^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \qquad e_1^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\overset{e_3}{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix}} \overset{e_2}{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}} \overset{e_1}{\begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}} A = U$$

$$A = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 2 & 1 \end{pmatrix}}_{L} \underbrace{\begin{pmatrix} 2 & 1 & 2 \\ 0 & -1 & 1 \\ 0 & 0 & 3 \end{pmatrix}}_{U}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix} \qquad L = E^{-1} = e_1^{-1} e_2^{-1} e_3^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 2 & 1 \end{pmatrix}$$

$Ax = b \longrightarrow LUx = b$

$L(Ux) = b$

$\boxed{y = Ux}$

Lu.decomposition(A)

$Ly = b$

$Ux = y$

$A = \begin{pmatrix} 0 & 1 \\ 1 & 9 \end{pmatrix}$

$\underbrace{e_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}$

$e_1^{-1} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$U = \begin{pmatrix} 1 & 9 \\ 0 & 1 \end{pmatrix}$

"L" $= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$$A = \begin{pmatrix} 2 & 3 & 2 & 4 \\ 4 & 10 & -4 & 0 \\ -3 & -2 & -5 & -2 \\ -2 & 4 & 4 & -7 \end{pmatrix} \quad L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -3/2 & 5/8 & 1 & 0 \\ -1 & 7/4 & \frac{20}{3} & 1 \end{pmatrix} \quad U = \begin{pmatrix} 2 & 3 & 2 & 4 \\ 0 & 4 & -8 & -8 \\ 0 & 0 & 3 & 9 \\ 0 & 0 & 0 & -49 \end{pmatrix}$$

$$b = \begin{pmatrix} 4 \\ -8 \\ -4 \\ -1 \end{pmatrix} \quad e_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$e_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 3/2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad e_6 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{20}{3} & 1 \end{pmatrix}$$

$$e_4 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -5/8 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad e_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

$$e_5 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -7/4 & 0 & 1 \end{pmatrix}$$

* Operaciones

$R_2 \leftarrow -2R_1 + R_2$

$R_3 \leftarrow 3/2\, R_1 + R_3$

$R_4 \leftarrow R_1 + R_4$

$R_3 \leftarrow -5/8\, R_2 + R_3$

$R_4 \leftarrow -7/4\, R_2 + R_4$

$R_4 \leftarrow -\frac{20}{3} R_3 + R_4$

$$A = LU = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -3/2 & 5/8 & 1 & 0 \\ -1 & 7/4 & 20/3 & 1 \end{pmatrix} \begin{pmatrix} 2 & 3 & 2 & 4 \\ 0 & 4 & -8 & -8 \\ 0 & 0 & 3 & 9 \\ 0 & 0 & 0 & -49 \end{pmatrix}$$

$$AX = \begin{pmatrix} 4 \\ -8 \\ -4 \\ -1 \end{pmatrix}$$

$$L(Ux) \longrightarrow Ly = b$$

$$Ux = y$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -3/2 & 5/8 & 1 & 0 \\ -1 & 7/4 & 20/3 & 1 \end{pmatrix} y = \begin{pmatrix} 4 \\ -8 \\ -4 \\ -1 \end{pmatrix}$$

$$\begin{cases} y_1 = 4 \\ y_2 = -16 \\ y_3 = 12 \\ y_4 = -49 \end{cases}$$

$$x_1 = -1$$
$$x_2 = 0$$
$$x_3 = 1$$
$$x_4 = 1$$

$$\begin{array}{r} -4 \\ -28 \\ +80 \\ \hline 48 \end{array}$$

$$Ux = y$$

$$\begin{pmatrix} 2 & 3 & 2 & 4 \\ 0 & 4 & -8 & -8 \\ 0 & 0 & 3 & 9 \\ 0 & 0 & 0 & -49 \end{pmatrix} x = \begin{pmatrix} 4 \\ -16 \\ 12 \\ -49 \end{pmatrix}$$