$$\underline{A} = \begin{pmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{pmatrix}_{3\times 2}$$

$$A = U \Sigma V^t$$

$\rightarrow U \rightarrow 3\times 3 \quad V \rightarrow 2\times 2$

$\rightarrow \Sigma \rightarrow 3\times 2$

$$* A^t A = \begin{pmatrix} 9 & -9 \\ -9 & 9 \end{pmatrix}$$

$A \cdot v_i \neq 0$

$$\Sigma = \begin{pmatrix} \sqrt{18} & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$\lambda_1 = 18 \quad \lambda_2 = 0$

$$V = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

$$\underline{v_1} = \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix} \qquad \underline{v_2} = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$$

$\sigma_1 = \sqrt{18}$

$$u_1 = \begin{pmatrix} 2/\sqrt{2} \\ -4/\sqrt{2} \\ 4/\sqrt{2} \end{pmatrix}$$

$U_2 = \bar{0}$

$$U_1 = \begin{pmatrix} 1/3 \\ -2/3 \\ 2/3 \end{pmatrix}$$

$$U_1 = \begin{pmatrix} 1/3 \\ -2/3 \\ 2/3 \end{pmatrix}$$

$$U_1 \cdot \frac{X}{?} = 0$$

$$\begin{pmatrix} 1/3 \\ -2/3 \\ 2/3 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$1/3\,x_1 - 2/3\,x_2 + 2/3\,x_3 = 0$$

$$\Leftrightarrow$$

$$\boxed{x_1 - 2x_2 + 2x_3 = 0}$$

$$x_1 = 2x_2 - 2x_3$$

$$\begin{pmatrix} 2x_2 - 2x_3 \\ x_2 \\ x_3 \end{pmatrix} = \underbrace{\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}}_{w_1} x_2 + \underbrace{\begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}}_{w_2} x_3$$

$$U_2 = \begin{pmatrix} 2/\sqrt{5} \\ 1/\sqrt{5} \\ 0 \end{pmatrix}$$

$$\overline{U_3} = \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix} - \frac{\begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 2/\sqrt{5} \\ 1/\sqrt{5} \\ 0 \end{pmatrix}}{\begin{pmatrix} 2/\sqrt{5} \\ 1/\sqrt{5} \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 2/\sqrt{5} \\ 1/\sqrt{5} \\ 0 \end{pmatrix}} \cdot U_2$$

$$\overline{U_3} = \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix} + 4/5 \begin{pmatrix} 2/\sqrt{5} \\ 1/\sqrt{5} \\ 0 \end{pmatrix} = \begin{pmatrix} -2/5 \\ 4/5 \\ 1 \end{pmatrix}$$

$$\|\overline{U_3}\| = \frac{\sqrt{45}}{5}$$

$$U_3 = \begin{pmatrix} -2/\sqrt{45} \\ 4/\sqrt{45} \\ 5/\sqrt{45} \end{pmatrix}$$

$$U = \begin{pmatrix} 1/3 & 2/\sqrt{5} & -2/\sqrt{45} \\ -2/3 & 1/\sqrt{5} & 4/\sqrt{45} \\ 2/3 & 0 & 5/\sqrt{45} \end{pmatrix}$$

$$\Sigma = \begin{pmatrix} \sqrt{18} & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$V = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

$$A = U \Sigma V^t \qquad A = \begin{pmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{pmatrix}$$

$$U_r = \begin{pmatrix} 1/3 \\ -2/3 \\ 2/3 \end{pmatrix}$$

$$\Sigma_r = \sqrt{18} = 3\sqrt{2}$$

$$V_r = \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}$$

$$A = U_r \Sigma_r U_r^t$$

$$= \begin{pmatrix} \sqrt{18}/3 \\ -2\sqrt{18}/3 \\ 2\sqrt{18}/3 \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix}$$

$$A = \begin{pmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{pmatrix} \qquad b = \begin{pmatrix} 2 \\ 0 \\ 11 \end{pmatrix}$$

$A^{+} = V_r \Sigma_r^{-1} U_r^{+}$

$U \Rightarrow 3\times3$ $\quad \Sigma \Rightarrow 3\times2$ $\quad V \Rightarrow 2\times2$ $\quad \rightarrow$ $\quad U_r \Rightarrow 3\times2$ $\quad \Sigma_r \Rightarrow 2\times2$ $\quad V_r \Rightarrow 2\times2$.

$Ax = b$

$$U_r = \begin{pmatrix} -0.9649 & 0.1988 \\ -0.0399 & -0.8987 \\ -0.2611 & -0.4122 \end{pmatrix}$$

$$\Sigma_r = \begin{pmatrix} \sqrt{11+\sqrt{37}} & 0 \\ 6 & \sqrt{11-\sqrt{37}} \end{pmatrix}$$

$$A^t A = \begin{pmatrix} 4 & 0 & 1 \\ 0 & 2 & 1 \end{pmatrix} \begin{pmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 17 & 1 \\ 1 & 5 \end{pmatrix}$$

$$V_r = \begin{pmatrix} \frac{V_1}{\|V_1\|} & \frac{V_2}{\|V_2\|} \end{pmatrix}$$

$\rightarrow$

$\lambda_1 = 11+\sqrt{37} \qquad \lambda = 11-\sqrt{37}$

$$V_1 \begin{pmatrix} 6+\sqrt{37} \\ 1 \end{pmatrix} \qquad V_2 = \begin{pmatrix} 6-\sqrt{37} \\ 1 \end{pmatrix}$$

$$V_r = \begin{pmatrix} -0.9966 & 0.0825 \\ -0.0825 & -0.9966 \end{pmatrix}$$