$$(A - \lambda I)x = 0 \iff \det(A - \lambda I) = 0$$

Soluciones no triviales

$$A = \begin{pmatrix} 0 & 1 & -1 \\ 1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{pmatrix}$$

$$A = \begin{pmatrix} 8 & 3 \\ 2 & 7 \end{pmatrix}$$

$$\begin{array}{c} + \\ \\ \\ \\ - \end{array} \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{vmatrix}$$

Corroborar que $\lambda = -5$ es valor propio de $A = \begin{pmatrix} -4 & 2 \\ 3 & 1 \end{pmatrix}$

$\lambda = 1, 2, 3$

$$A = \begin{pmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}$$

$$A x = \lambda x.$$

Sistema dinámico discreto
Modelo depredador-presa

$$\to \; X_k = A X_{k-1}$$

- X = # de buhos en una población
- y = miles de ratones.

$$X_{t+1} = A X_t$$

$$X_k = c_1 \lambda_1^k v_1 + c_2 \lambda_2^k v_2$$

$$X_{t+1} = (0.5) X_t + (0.4) y_t$$

$$y_{t+1} = (0.104) X_t + (1.1) y_t$$

$$X_{t+1} = \underbrace{\begin{pmatrix} 0.5 & 0.4 \\ -0.104 & 1.1 \end{pmatrix}}_{} X_t$$

* $X_0 = c_1 v_1 + c_2 v_2$

$$X_1 = A X_0 = A[c_1 v_1 + c_2 v_2]$$
$$= c_1 \lambda_1 v_1 + c_2 \lambda_2 v_2$$

$$X_2 = A X_1 = A[c_1 \lambda_1 v_1 + c_2 \lambda_2 v_2]$$
$$= c_1 \lambda_1^2 v_1 + c_2 \lambda_2^2 v_2$$

$\vdots$

$\lambda_1 = \frac{51}{50}$ $v_1 = \begin{pmatrix} 10/13 \\ 1 \end{pmatrix}$

$\lambda_2 = \frac{29}{50}$ $v_2 = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$

$$X_k = C_1 \left(\frac{51}{50}\right)^k v_1 + C_2 \left(\frac{29}{50}\right)^k v_2$$

$k \to \infty$

$$X_k = \underline{C_1 \left(\frac{51}{50}\right)^k} \begin{pmatrix} 10/13 \\ 1 \end{pmatrix}$$

* Cadena de Markov.

* $(x_1, x_2, \ldots, x_k \ldots) \in \underline{X}^k$

$$\boxed{\epsilon_k = P\ \epsilon_{k-1}}$$

$$\epsilon_0 \longrightarrow P^k \epsilon_0 = \epsilon_k$$

$$P^{-1} = \frac{1}{10}\begin{pmatrix} 7 & 7 \\ -7 & 3 \end{pmatrix}$$

$$\underbrace{X_{t+1} = \begin{pmatrix} 0.65 & 0.15 \\ 0.35 & 0.85 \end{pmatrix} \begin{pmatrix} x_t \\ y_t \end{pmatrix}}_{A}$$

X_{t+1} cuando $t \to \infty$

$$A = PDP^{-1}$$

$$A^k = (PDP^{-1})(PDP^{-1}) \cdots$$

$$= PD^kP^{-1}$$

$\lambda_1 = 1 \quad v_1 = \begin{pmatrix} 3/7 \\ 1 \end{pmatrix}$

$\lambda_2 = 1/2 \quad v_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$P = \begin{pmatrix} 3/7 & -1 \\ 1 & 1 \end{pmatrix} \quad D = \begin{pmatrix} 1 & 0 \\ 0 & 1/2 \end{pmatrix}$

→ 65% Posibilidades que una P. f. Siga fumando

→ 85% Prob. N.f. Siga sin fumar

$$X_t = \begin{pmatrix} x_t \\ y_t \end{pmatrix} \begin{matrix} \to \text{Prop. de Personas f.} \\ \to \text{Prop. de Pesonas n.f} \end{matrix} \Bigg\} \text{en Per. } t.$$

Supongamos que actualmente

$$\bar{X}_0 = \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix} \qquad \bar{X}_1 = A\,\bar{X}_0$$

$$A = PDP^{-1}$$

$$= \begin{pmatrix} 3/7 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1/2 \end{pmatrix} \begin{pmatrix} 7/10 & 7/10 \\ -7/10 & 3/10 \end{pmatrix}$$

$\bar{X}_k$ cuando $k \to \infty$

$$\bar{X}_k = A^k \bar{X}_0$$

$$A^k = \begin{pmatrix} 3/7 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & \cancelto{0}{(1/2)^k} \end{pmatrix} \begin{pmatrix} 7/10 & 7/10 \\ -7/10 & 3/10 \end{pmatrix}$$

$$= \begin{pmatrix} 3/10 & 3/10 \\ 7/10 & 7/10 \end{pmatrix}$$

Sistemas de ecuaciones diferenciales Ordinarias con coef. Constantes.

$$\dot{X} = A X.$$

donde

$$X = \begin{pmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{pmatrix} \quad y \quad \dot{X} = \begin{pmatrix} \frac{dx_1}{dt} \\ \vdots \\ \frac{dx_n}{dt} \end{pmatrix}$$

$$X = C_1 V_1 e^{\lambda_1 t} + C_2 V_2 e^{\lambda_2 t} + \cdots + C_n V_n e^{\lambda_n t}$$

$\lambda_1, \ldots, \lambda_n$ V.P.

$V_1, \ldots, V_n$ Vec. P.

Análisis de componentes Principales

$$\underbrace{y}_{CP} = A x \longrightarrow \Sigma \rightarrow \text{Cov. } x.$$

$$\rightarrow y_1 = a_{11} x_1 + a_{12} x_2 + \dots + a_{1n} x_n$$

$$\rightarrow y_2 = a_{21} x_1 + \dots + a_{2n} x_n$$

$$\vdots$$

$$\rightarrow y_n = a_{n1} + \dots + a_{nn} x_n.$$

$$A_i^t A_i = 1 \qquad A_i^t A_j = 0 \quad \forall\, i \neq j$$

$$\max \ \mathrm{Var}(y_1) = A_1^t \Sigma A_1$$

s.a

$$A_1^t A_1 = 1$$

$$L(A_1) = A_1^t \Sigma A_1 + \lambda (A_1^t A_1 - 1)$$

$$(\Sigma - \lambda I) A_1 = 0$$

Solución λ_i y v_i tal que $\lambda_i = \max\limits_{\forall j = 1:n} (\lambda_j)$

$$\max \ \mathrm{Var}(y_2) = A_2^t \Sigma A_2$$

$$A_2^t A_2 = 1$$

$$A_1^t A_2 = 0$$

$$Var(y) = \begin{pmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_n \end{pmatrix}$$

$$\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4 \geq \cdots \geq \lambda_n$$

$$\underbrace{(\Sigma - \lambda I)x = 0}$$

Ordenar de manera decreciente los valores propios.

$$\langle x, y \rangle = 1 + 0 = 1$$

$x, y \in \mathbb{R}^2$.

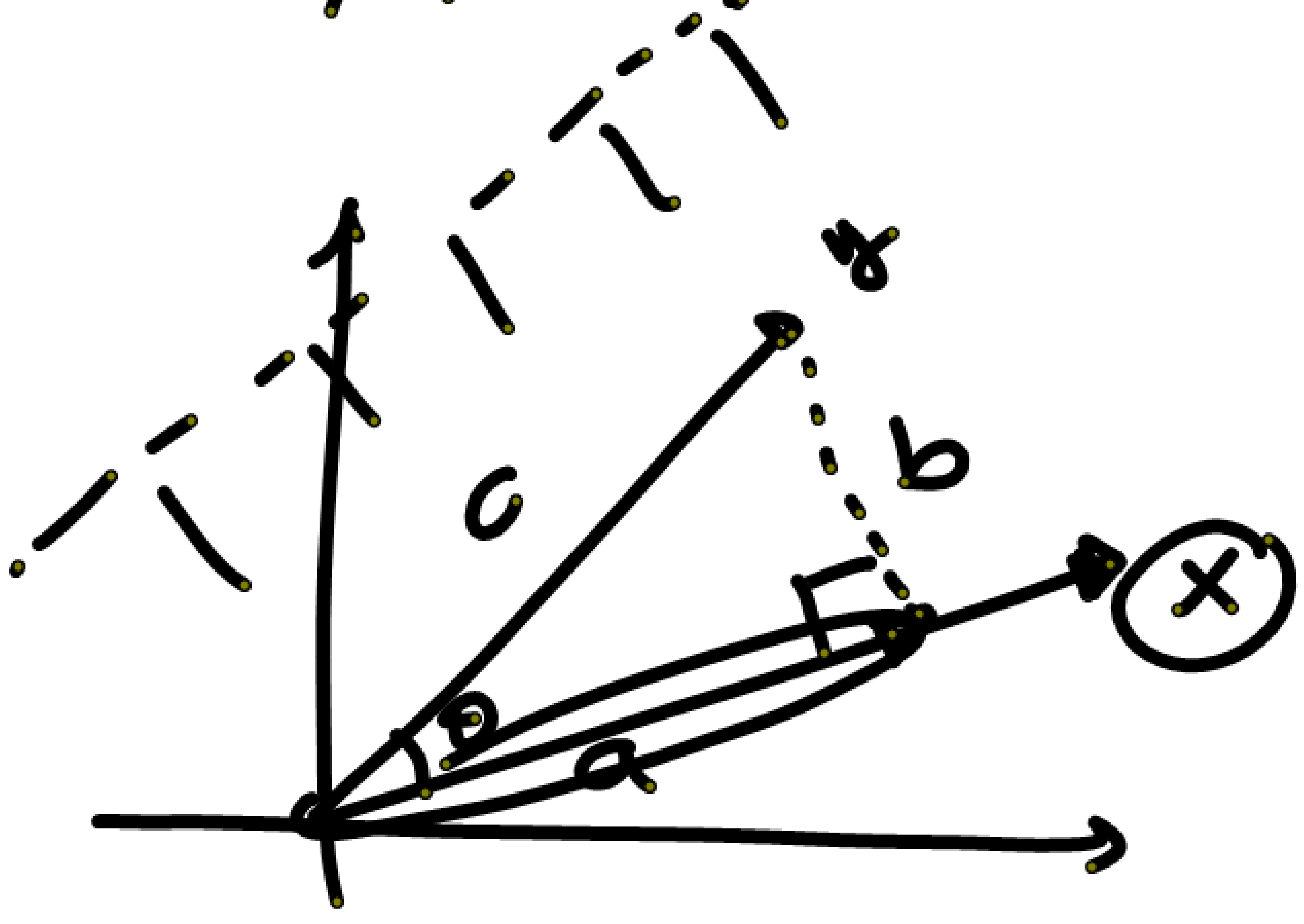
$$x = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad y = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

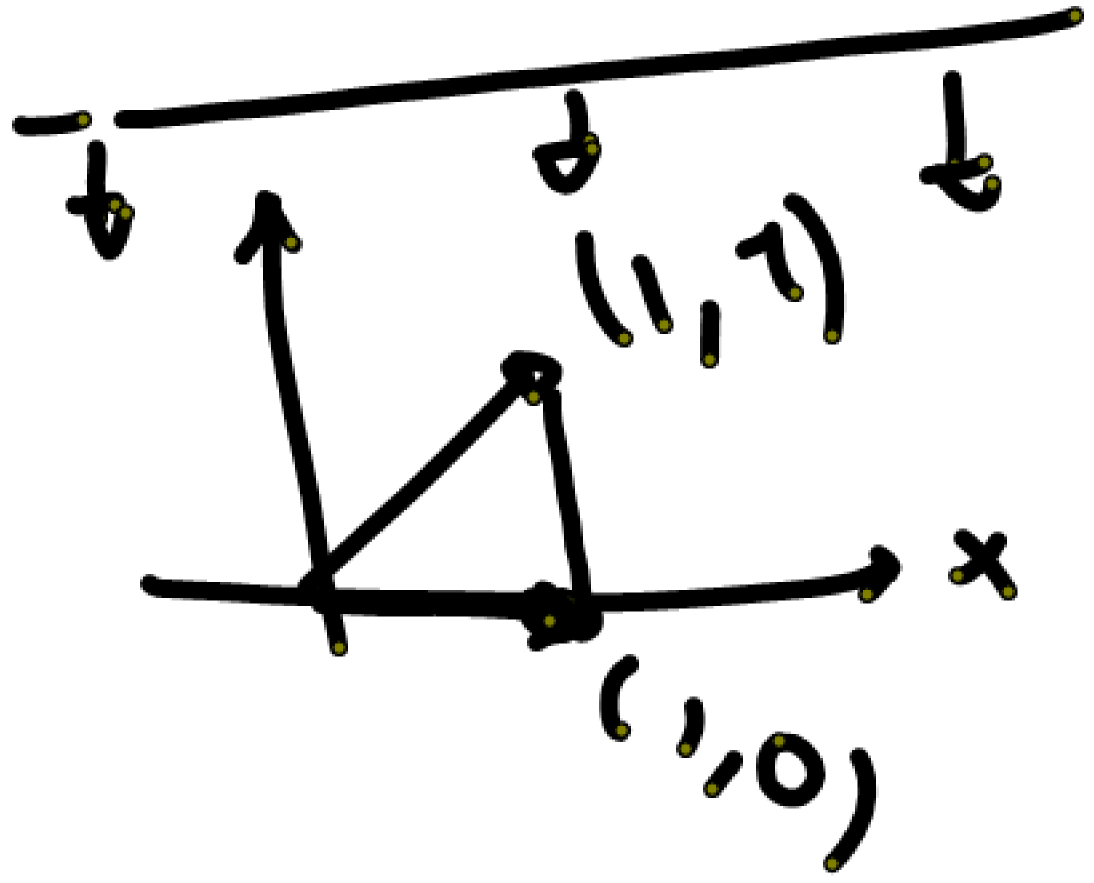
$$\|x\| = \sqrt{\langle x, x \rangle} = \sqrt{x_1^2 + x_2^2} = \sqrt{1 + 0} = 1$$

$$\langle x, y \rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} = 0 + 0 = 0$$

$$x = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad y = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$x, y \in \mathbb{R}^2$





$$\cos(\theta) = \frac{a}{c} = \frac{\|\text{Proy}_x\, y\|}{\|y\|}$$

$$\cos(\theta) = \frac{\langle x, y \rangle}{\|x\|\,\|y\|}$$

$$\frac{\|\text{Proy}_x\, y\|}{\|y\|} = \frac{x \cdot y}{\|x\|\,\|y\|}$$

$$\|\text{Proy}_x\, y\| = \frac{x \cdot y}{\|x\|}$$

$$\frac{x}{\|x\|} \longrightarrow \text{Proy}_x\, y = \frac{x \cdot y}{\|x\|^2} \cdot x$$