$$F(x) = P(X \leq x) \qquad \forall x \in \mathbb{R}$$

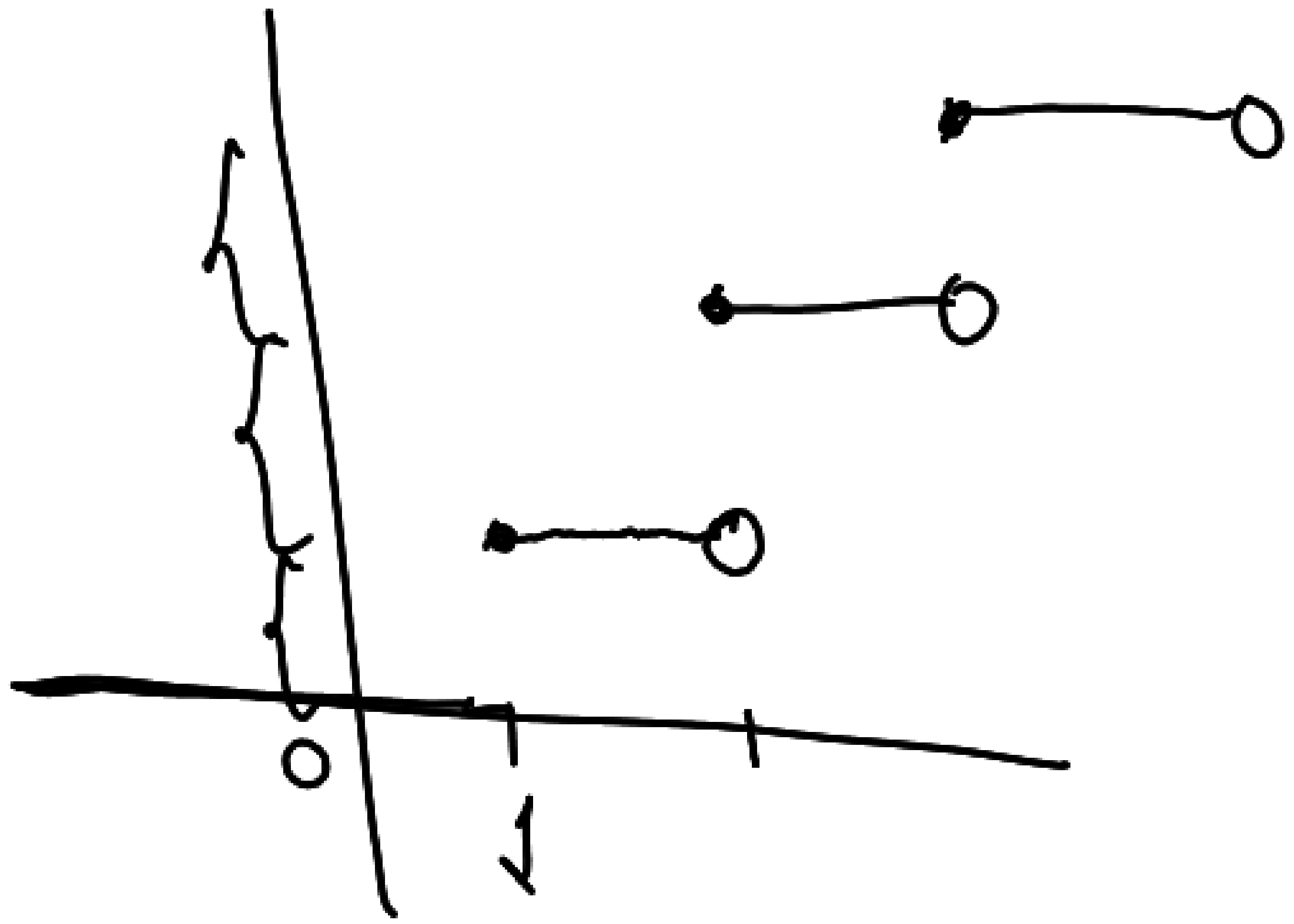
$$P(X=x) = \begin{cases} \frac{1}{5}, & x = 1,2,3,4,5 \\ 0 & \text{en otro caso} \end{cases} \qquad X \sim \text{Unif}(\{1,2,3,4,5\})$$

Función de distribución se def. en $\mathbb{R}$

$P(X \leq 0) = 0$

$P(X \leq 1) = \frac{1}{5}$

$P(X \leq 2) = P(X=1) + P(X=2)$



$X$ v.a. discreta que toma valores : $\{x_1, x_2, \dots\}$

Media

$$E(X) = \sum_{i=1}^{\infty} x_i P(X = x_i)$$

$$Var(X) = V(X) = E\left((X - E(X))^2\right)$$

$g : \mathbb{R} \to \mathbb{R}$

determinista

$$E(g(X)) = \sum_{i=1}^{\infty} g(x_i) P(X = x_i)$$

$$V(X) = \sum_{i=1}^{\infty} (x_i - E(X))^2 P(X = x_i) = E(X^2) - (E(X))^2$$

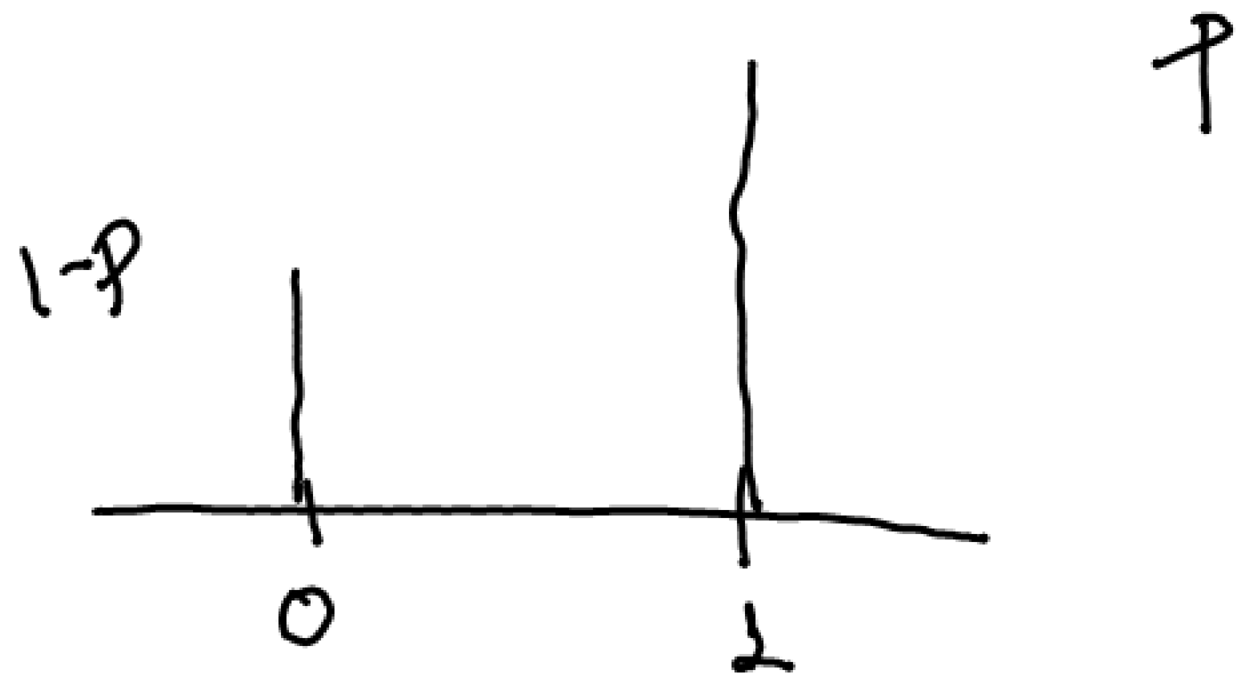
Datos

$X_1, \dots, X_n$

$$\overline{X} = \sum_{i=1}^{n} \frac{X_i}{n}$$

$X \sim \text{Bernoulli}(p)$

$$f(x) = P(X=x) = \begin{cases} p & \text{si } x=1 \\ 1-p & \text{si } x=0 \end{cases}$$




$X \sim \text{Bin}(n, p)$

$$P(X=x) = f(x) = \binom{n}{x} p^x (1-p)^{n-x} I_{\{0,1,\dots,n\}}(x) \quad , \quad p \in (0,1), n \in \mathbb{N}$$

$$\binom{n}{x} = \frac{n!}{(n-x)!\,x!}$$

$n=5, p=1/2$

$$[X=3] = \{(1,1,1,0,0), (1,1,0,1,0), (1,$$

$X \sim \text{Hipergeometrica}(N, k, n)$

$$P(X=x) = \frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}} = \frac{\frac{k!}{(k-x)!\,x!} \cdot \frac{(N-k)!}{(N-k-(n-x))!\,(n-x)!}}{\frac{N!}{(N-n)!\,n!}}$$

$$f(x)=P(X=x)=(1-p)^{x-1}p \; I_{\{1,2,\dots\}}(x), \qquad p\in(0,1)$$

$$\sum_{x=1}^{\infty} f(x) = 1 \qquad \sum f(x)=\sum_{x=1}^{\infty}(1-p)^{x-1}p = p\sum_{x=1}^{\infty}(1-p)^{x-1}$$

$$\sum_{r=0}^{\infty} a^r = \frac{1}{1-a} \quad |a|<1 \qquad y=x-1 \quad = p\sum_{y=0}^{\infty}(1-p)^y = \frac{p}{1-(1-p)} = 1$$

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$X\sim \text{Poiss}(\lambda)$ $\quad\lambda$ tasa en 1 unidad

$$P(X=x)=\frac{\lambda^x e^{-\lambda}}{x!}$$

$Y=$ # éxitos en $t$ unidades

$$P(Y=y)=\frac{(\lambda t)^x e^{-\lambda t}}{x!}$$

$X \sim Pois(\lambda) \qquad f(x) = \frac{\lambda^x e^{-\lambda}}{x!} I_{\{0,1,\dots\}}(x) \quad , \lambda > 0$

$$E(X) = \sum_{x=0}^{\infty} x \frac{\lambda^x e^{-\lambda}}{x!} = e^{-\lambda} \sum_{x=1}^{\infty} \frac{x \lambda^x}{x!} = e^{-\lambda} \sum_{x=1}^{\infty} \frac{\lambda \lambda^{x-1}}{(x-1)!}$$

$$= \lambda e^{-\lambda} \sum_{x=1}^{\infty} \frac{\lambda^{x-1}}{(x-1)!} \underset{y=x-1}{=} \lambda e^{-\lambda} \underbrace{\sum_{y=0}^{\infty} \frac{\lambda^y}{y!}}_{e^{\lambda}} = \lambda \cancel{e^{-\lambda}} \cancel{e^{\lambda}} = \lambda$$

$$E(X^2) = \sum_{x=0}^{\infty} x^2 \frac{\lambda^x e^{-\lambda}}{x!} = \sum_{x=1}^{\infty} \frac{x \lambda^x e^{-\lambda}}{(x-1)!} \underset{y=x-1}{=} \sum_{y=0}^{\infty} (y+1) \frac{\lambda^{y+1} e^{-\lambda}}{\cancel{(x-1)!}\; y!} =$$

$$= e^{-\lambda} \left( \sum_{y=0}^{\infty} y \frac{\lambda^{y+1}}{y!} + \sum_{y=0}^{\infty} \frac{\lambda^{y+1}}{y!} \right) =$$

$$\lambda e^{-\lambda}\left(\sum_{y=0}^{\infty}\frac{y\cdot\lambda^{y}}{y!}+\sum_{y=0}^{\infty}\frac{\lambda^{y}}{y!}\right)=\lambda e^{-\lambda}\left(\lambda e^{\lambda}+e^{\lambda}\right)=\lambda^{2}+\lambda$$

$$\mathrm{Var}(X)=\lambda$$

$$P(X=x)=\frac{\left(\int_a^b q(t)\,dt\right)^{x} e^{-\int_a^b q(t)\,dt}}{x!}$$