Método de la función acumulada

Ejemplo: $X \sim Bin(n,p)$ $\quad P(X=x) = f_X(x) = \binom{n}{x} p^x (1-p)^{n-x} I_{\{0,\dots,n\}}(x)$, $\quad p \in (0,1)$, $n \in \mathbb{N}$

$$\begin{cases} 2 & \text{si sol} \\ -2 & \text{si águila} \end{cases} \qquad Y = 2\underbrace{X}_{\#\text{soles}} - 2\underbrace{(n-X)}_{\#\text{águilas}} = 4X - 2n$$

$$f_Y(y) = P(Y=y) = P(4X-2n=y) = P\left(X = \frac{y+2n}{4}\right) = \binom{n}{\frac{y+2n}{4}} p^{\frac{y+2n}{4}} (1-p)^{n-\frac{y+2n}{4}}$$

$$I_{\{0-2n,\, 4-2n,\, \dots,\, 4n-2n\}}(y), \quad p \in (0,1),\ n \in \mathbb{N}$$

X v.a. continua   ent $f_X(x)$ $\cancel{\text{Es}}$ $f_X(x) = \frac{d}{dx} F_X(x) = \frac{d}{dx} P(X \leq x)$

función de densidad

ent. podemos obtener $F_Y(y)$ y a partir de ésta, a $f_Y(y)$

Ejemplo: $X \sim \exp(\theta)$ ent! $f(x) = \frac{1}{\theta} e^{-x/\theta} I_{(0,\infty)}(x), \quad \theta > 0$

Sea $Y = aX$ con $a > 0$

$$F_X(x) = \int_{-\infty}^{x} \frac{1}{\theta} e^{-u/\theta} du \, I_{(0,\infty)}(u) = \int_0^x \frac{1}{\theta} e^{-u/\theta} du = -e^{-u/\theta}\Big|_0^x = -e^{-x/\theta} - \left(-e^{-0/\theta}\right)$$

$$= 1 - e^{-x/\theta}$$

$$F_Y(y) = P(Y \le y) = P(aX \le y) = P\left(X \le \frac{y}{a}\right)$$

$$= F_X\left(\frac{y}{a}\right) = 1 - e^{-\frac{y}{a\theta}}$$

$$f_Y(y) = \frac{1}{a\theta} e^{-y/a\theta} I_{(0,\infty)}(y), \quad \begin{matrix} a>0 \\ \theta>0 \end{matrix} \quad (a\theta > 0)$$

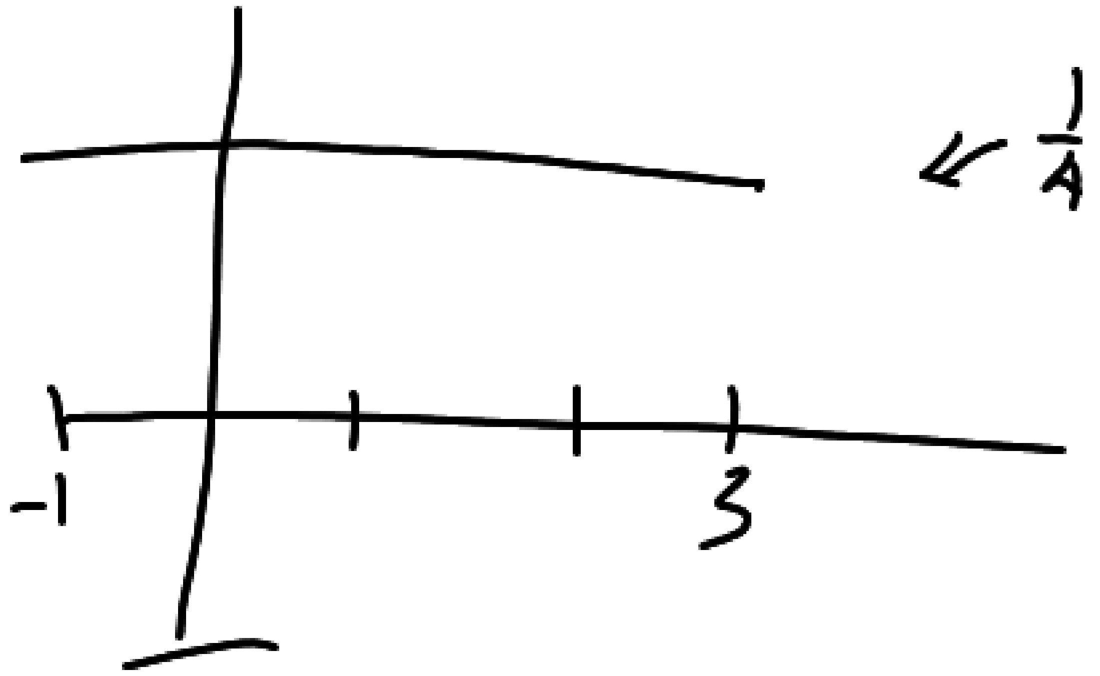
$$Y \sim exp(a\theta)$$

Si $a<0$

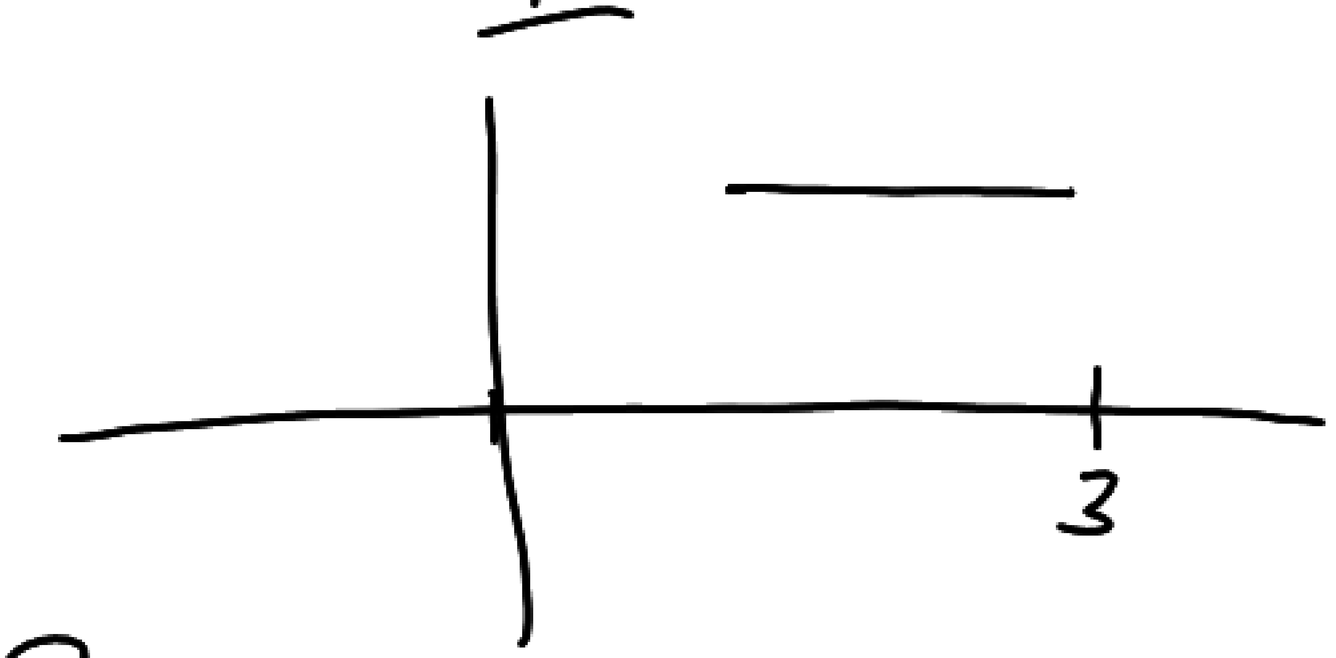
$$P(aX \le y) = P\left(X \ge \frac{y}{a}\right)$$

$$= 1 - F_X\left(\frac{y}{a}\right)$$

$X \sim \mathrm{Unif}(-1,3)$

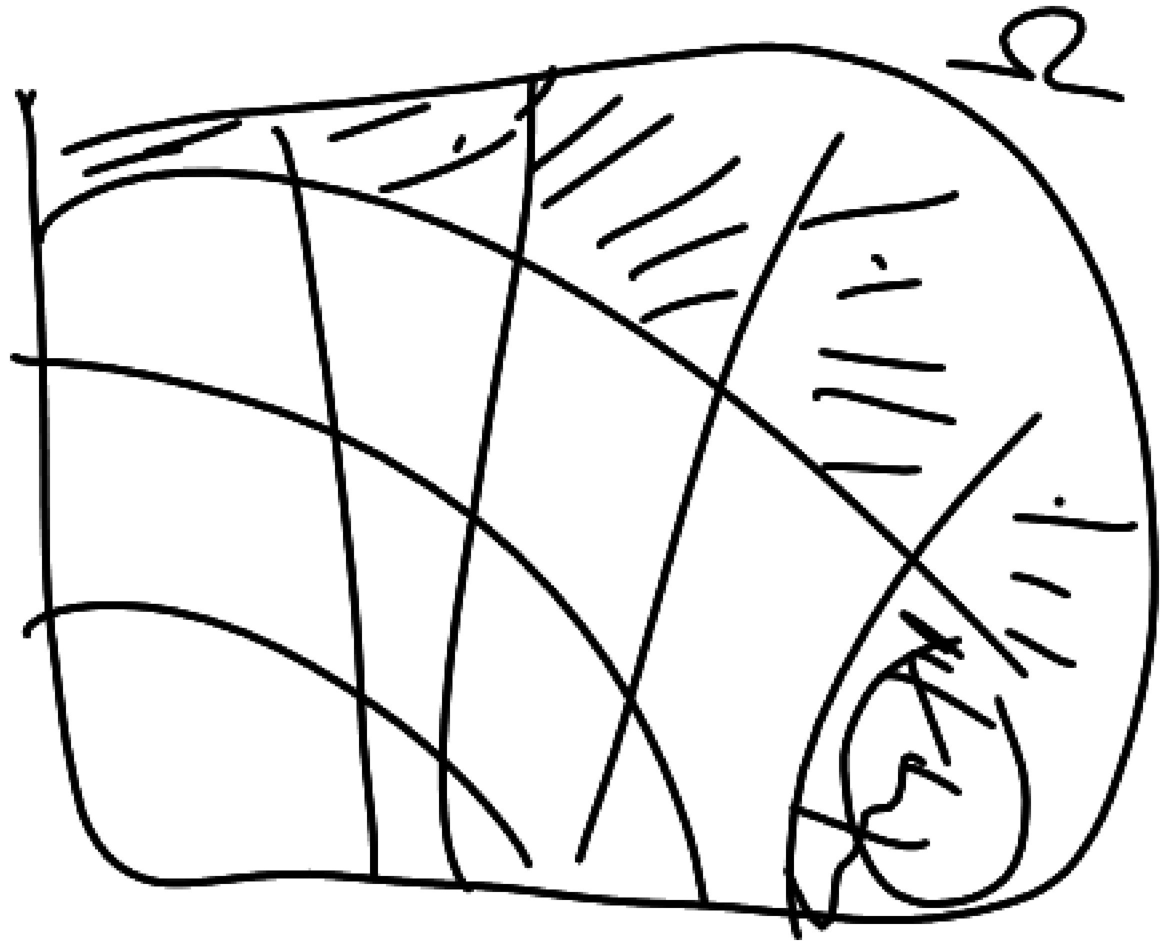


$Y = |X|$

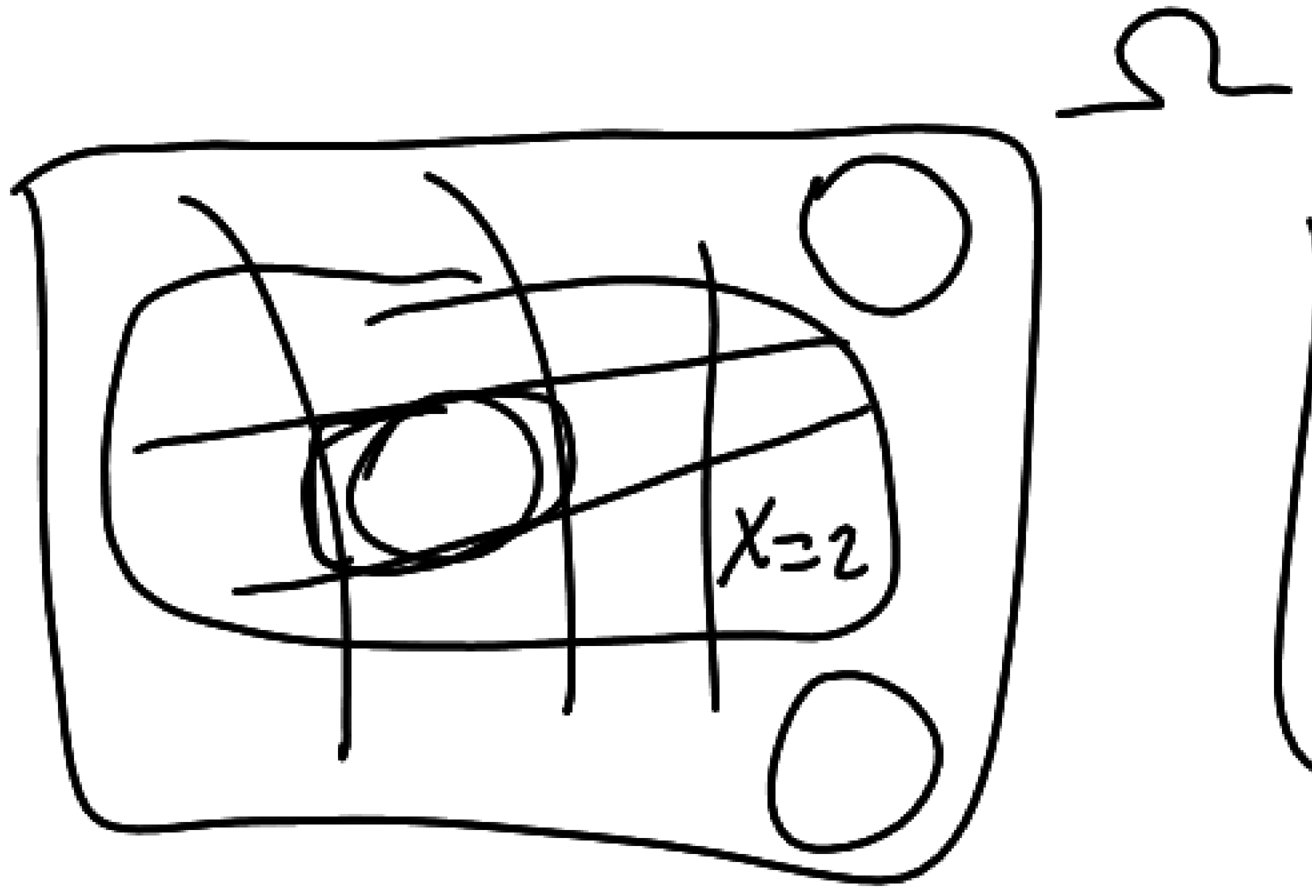


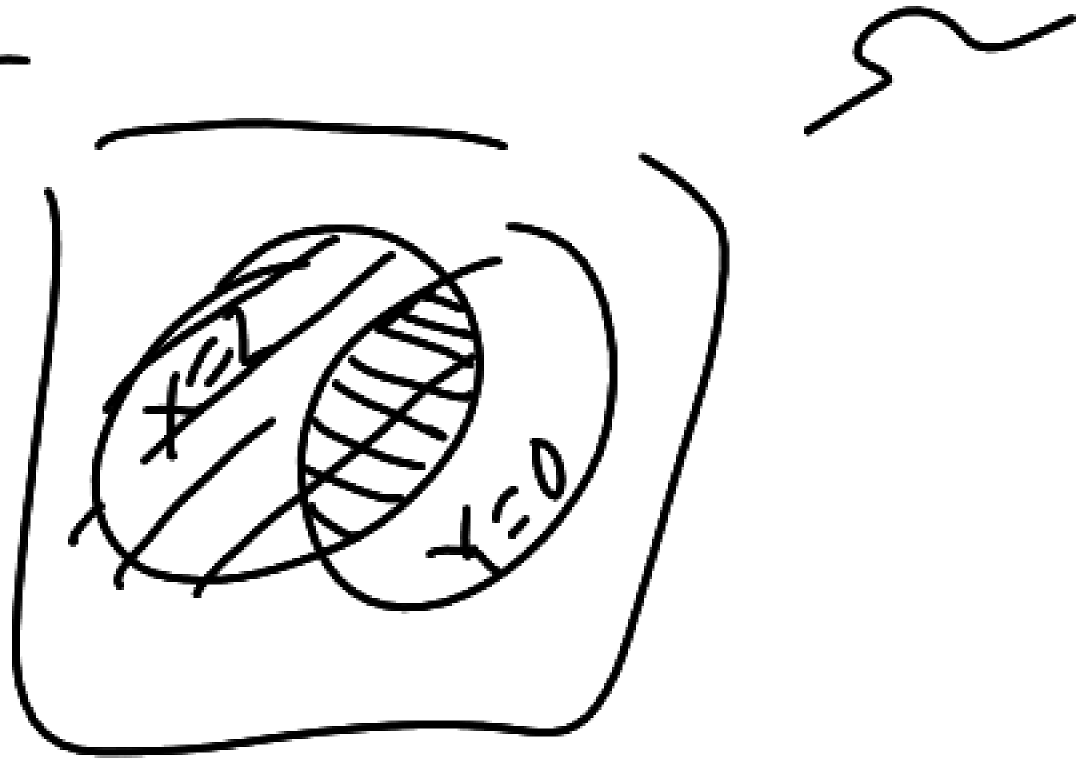
$P(Y=0, X=0)$

$P(Y=0, X=1)$ . . .

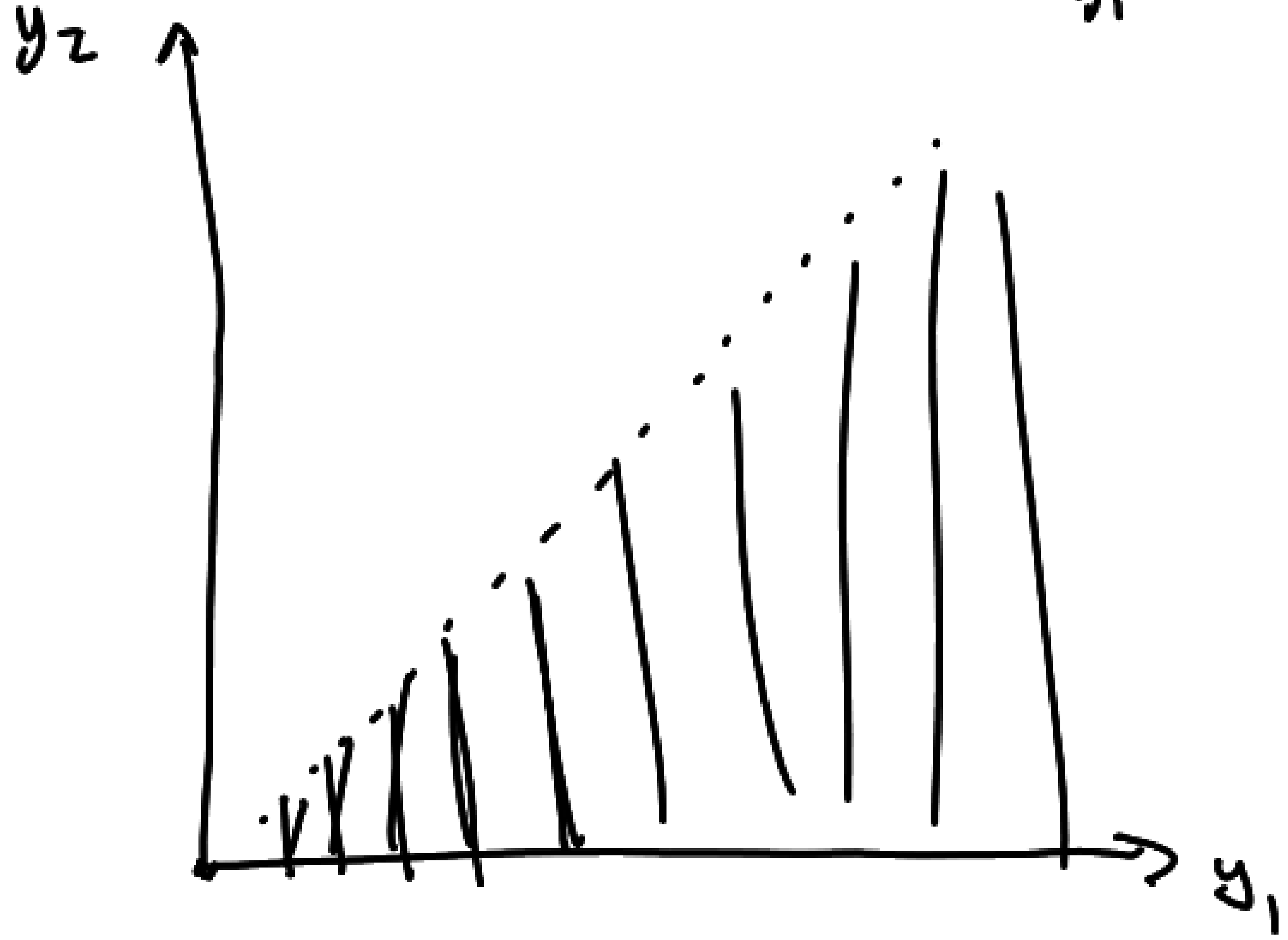


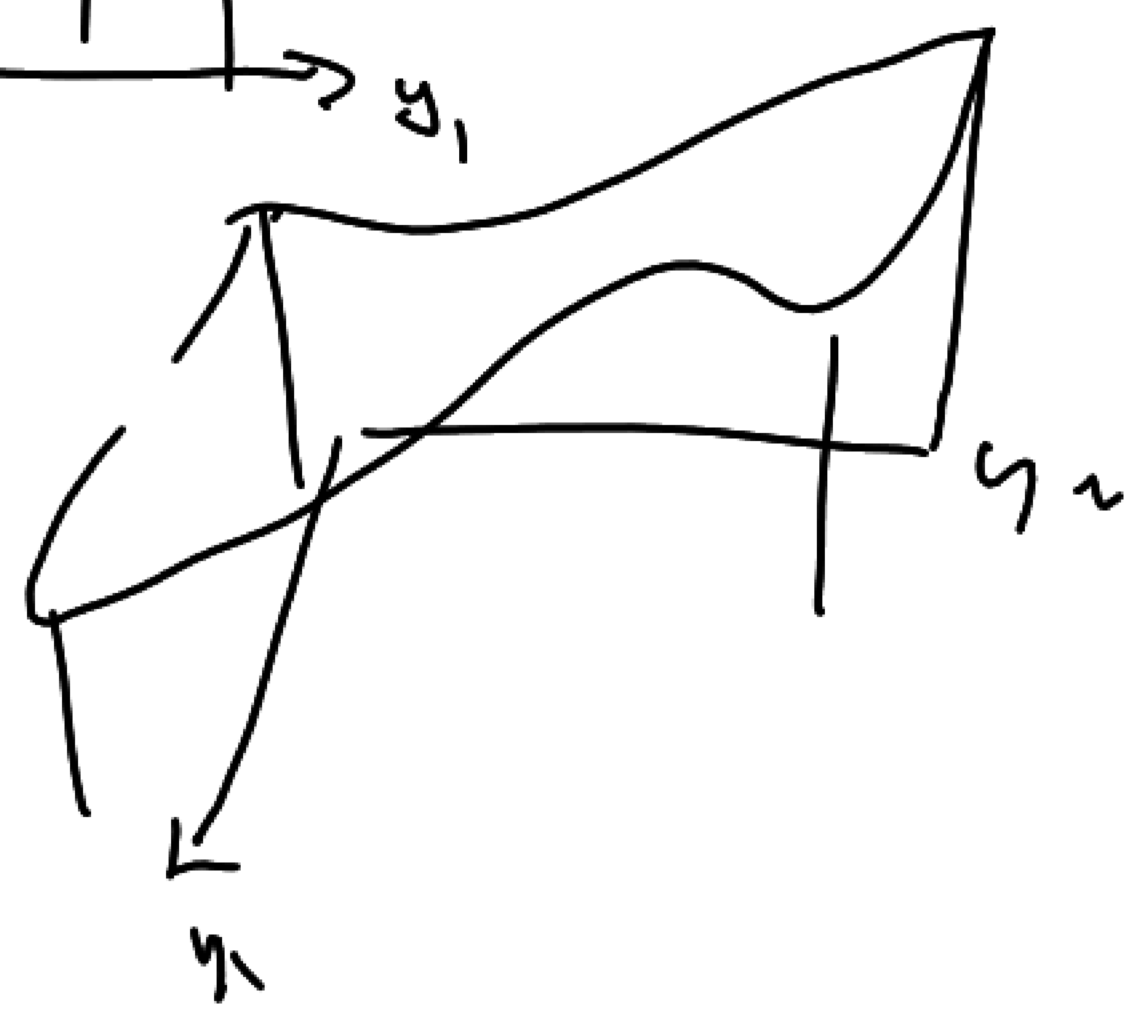
$P(\Omega) = 1$



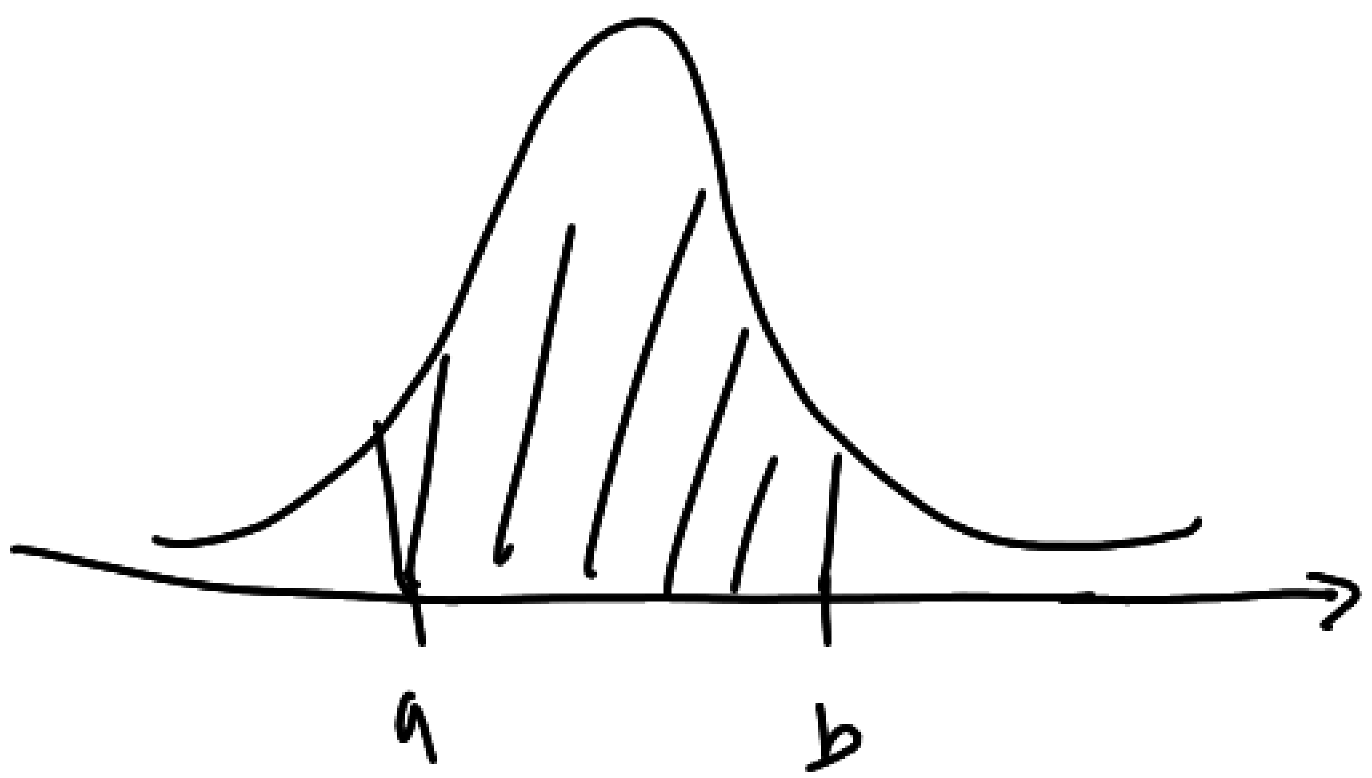


$$f(y_1, y_2) = e^{-y_1} I_{\{0 \le y_2 \le y_1 \le \infty\}}(y_1, y_2)$$





X con fd $f_X(x)$



$X \sim Gama(\alpha, \beta)$

ssi $f(x) = \frac{1}{\Gamma(\alpha)\beta^{\alpha}} x^{\alpha-1} e^{-x/\beta} I_{(0,\infty)}(x)$

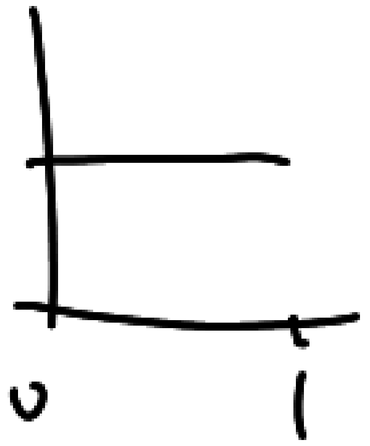
$\alpha, \beta > 0$

$$E(X+Y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(x+y)\, f_{X,Y}(x,y)\,dx\,dy = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x\, f_{X,Y}(x,y)\,dx\,dy + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} y\, f_{X,Y}(x,y)\,dx\,dy$$

$$= \int_{-\infty}^{\infty} x \underbrace{\int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dy}\,dx + \int_{-\infty}^{\infty} y \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dx\,dy$$

$$f_X(x) = \frac{1}{\theta} e^{-x/\theta}\, I_{(0,\infty)}(x)$$

$$= \int_{-\infty}^{\infty} x\, f_X(x)\,dx + \int_{-\infty}^{\infty} y\, f_Y(y)\,dy$$

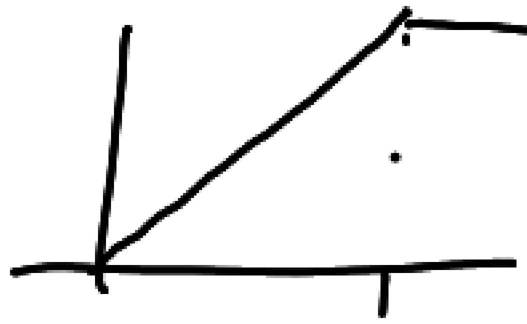


$$f(x) = I_{(0,1)}(x)$$

$$e^{\lambda} = \sum_{x=0}^{\infty} \frac{\lambda^x}{x!}$$

$$e^{-\lambda} = \lim_{x\to\infty}\left(1-\frac{\lambda}{x}\right)^x$$

$$= E(X) + E(Y)$$



$F(x)$

$$e^{x+y} = e^x e^y \qquad e^x = \exp(x)$$

$$Cov(X,Y) = E[(X-\mu_x)(Y-\mu_y)] = E[XY - \mu_x Y - X\mu_y + \mu_x\mu_y]$$

$$E[X-Y] = E[X] + E[-Y] = E(X) - E(Y)$$

$$= E(XY) - E(\mu_x Y) - E(X\mu_y) + E(\mu_x\mu_y)$$

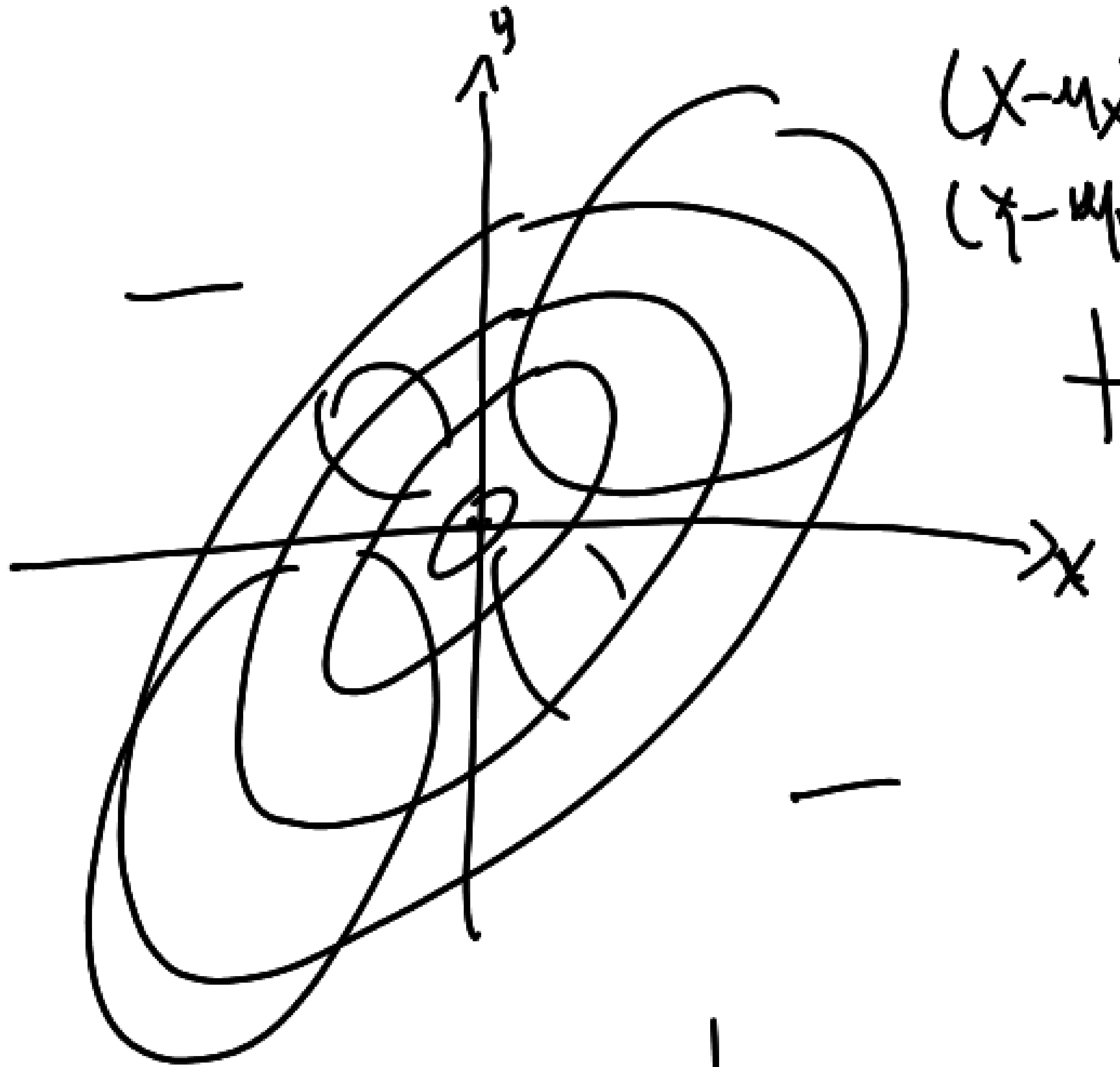
$$= E(XY) - \mu_x E(Y) - \mu_y E(X) + \mu_x\mu_y$$

$$= E(XY) - \mu_x\mu_y - \cancel{\mu_y\mu_x} + \cancel{\mu_x\mu_y} = E(XY) - \mu_x\mu_y$$

$Cov(X,X) = Var(X)$

Si X y Y son indep $E(XY) = E(X)E(Y) = \mu_x\mu_y$

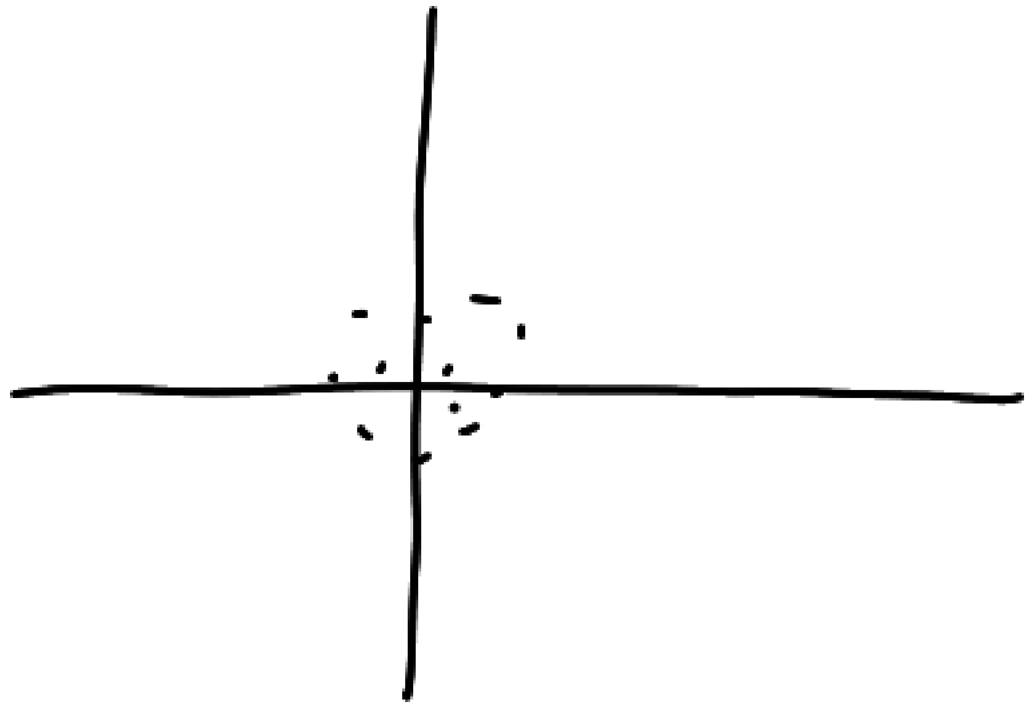
ent. $Cov(X,Y) = \mu_x\mu_x - \mu_x\mu_y = 0$



$(X-\mu_x)>0$
$(Y-\mu_y)>0$

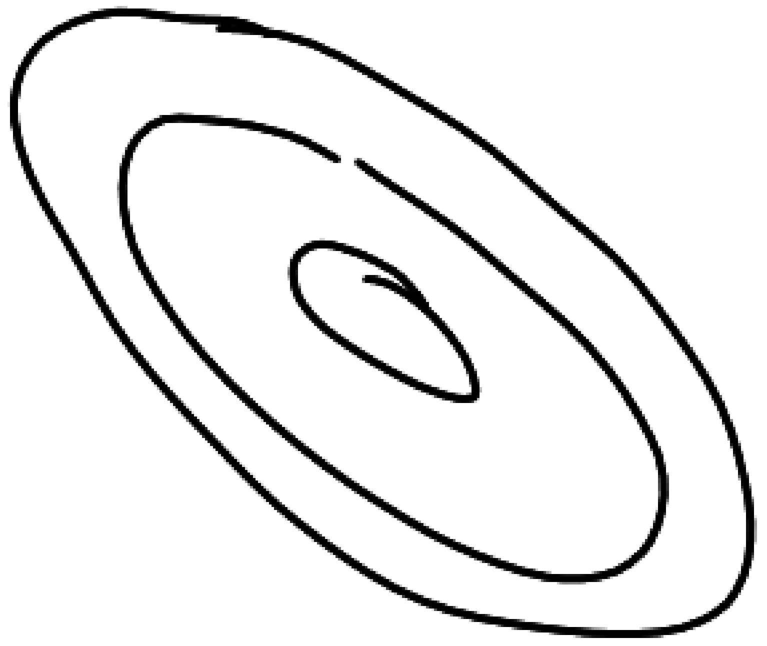
+

$$Cov(X,Y) = E\left((X-\mu_x)(Y-\mu_y)\right) > 0$$

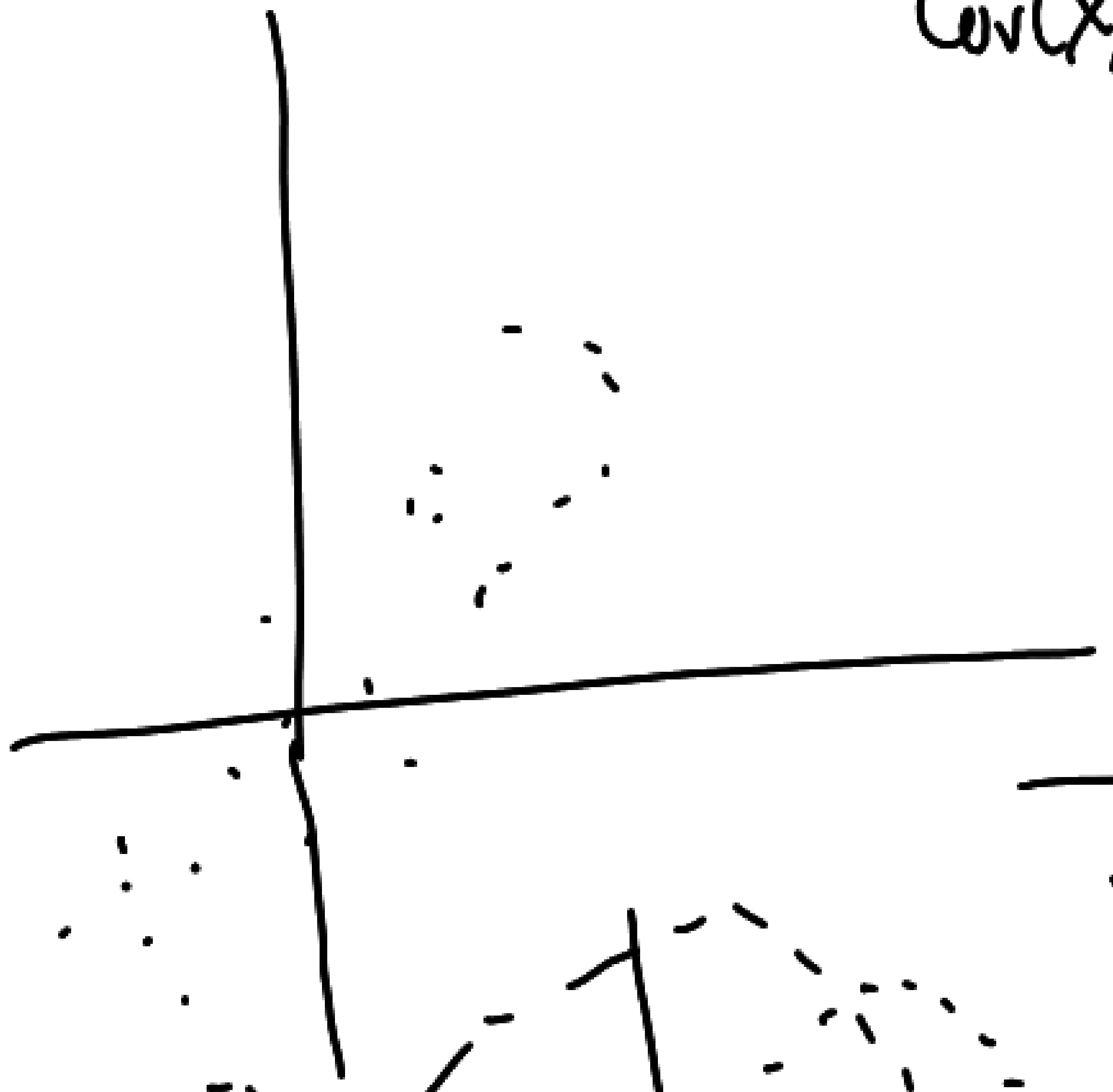


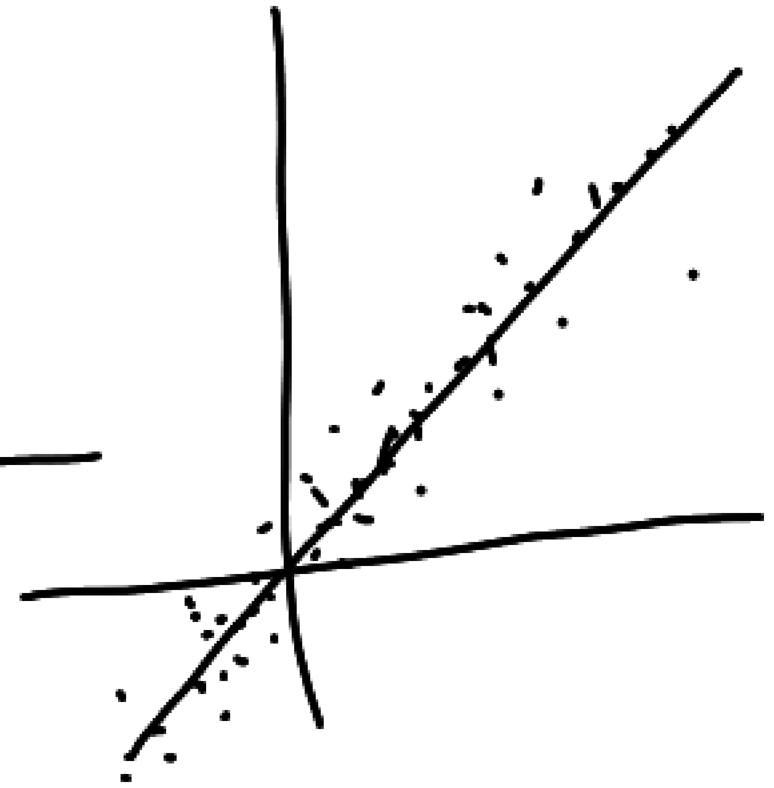
$$S_x = \sqrt{\frac{\sum (X_i-\bar{X})^2}{n-1}}$$

$$cov(X,Y) < 0$$



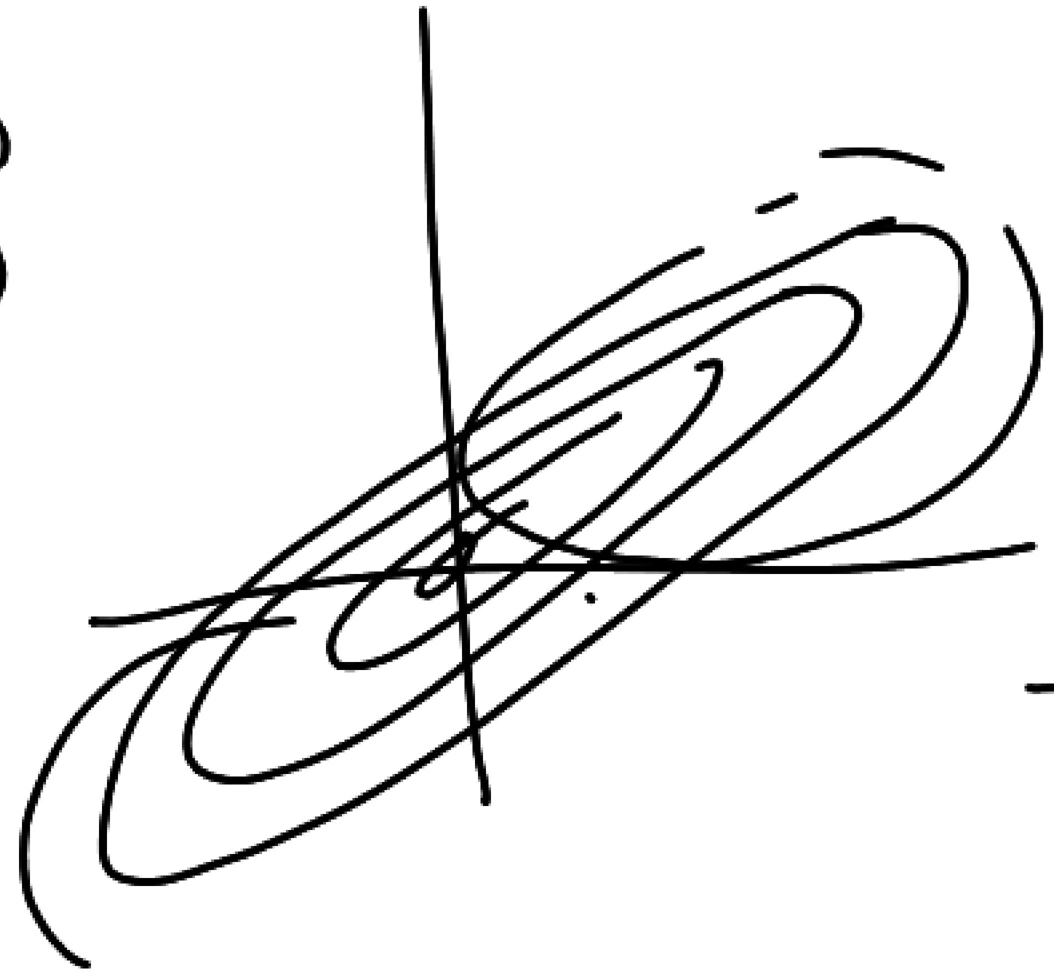
$$\widehat{Cov}(X,X) = \frac{\sum (X-\bar{X})(Y-\bar{Y})}{n-1}$$

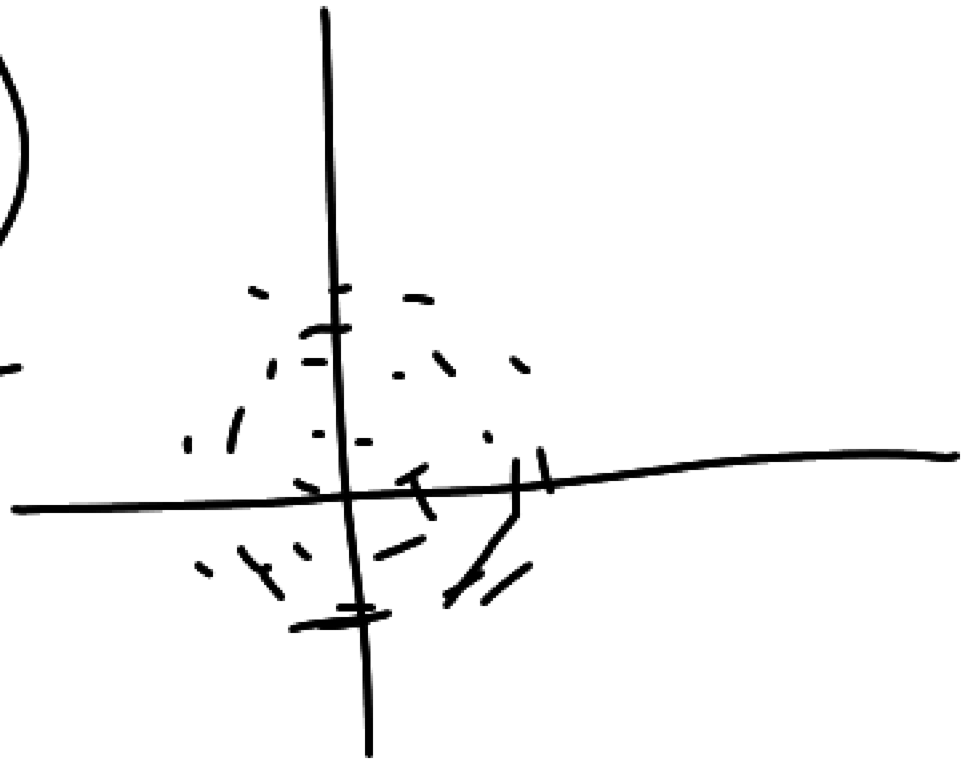


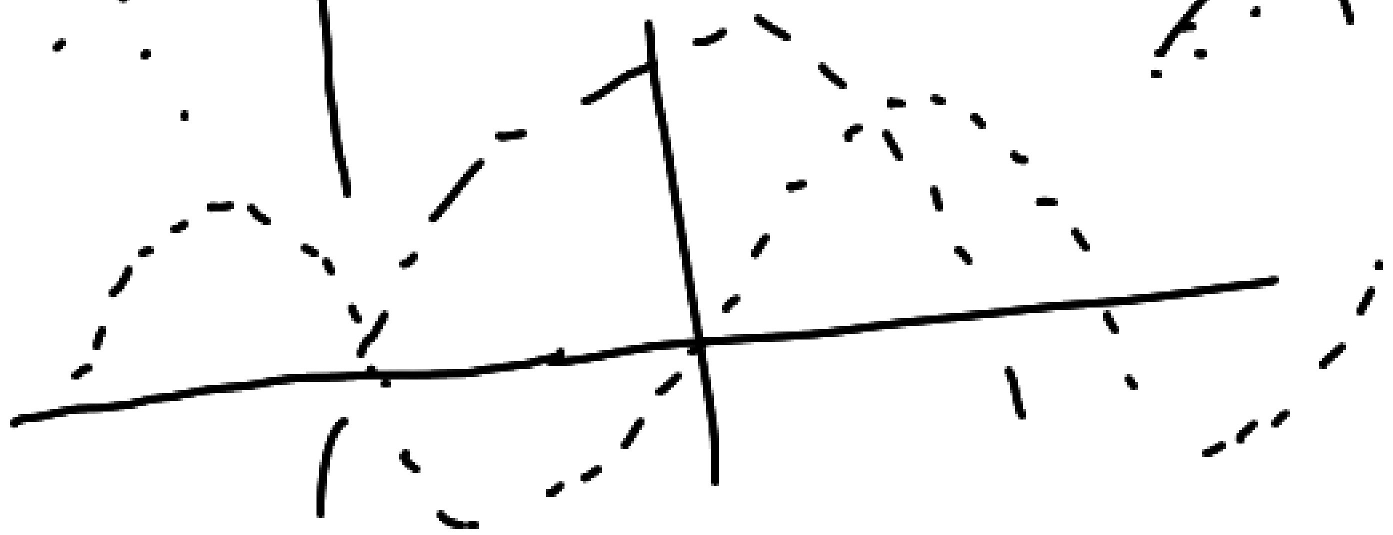


$(X-\mu_x)<0$
$(Y-\mu_y)<0$

+







Desig. de Chebyshev

Si X es una v.a no negativa y $f: \mathbb{R} \to \mathbb{R}$ creciente t.q. $E(f(X)) < \infty$

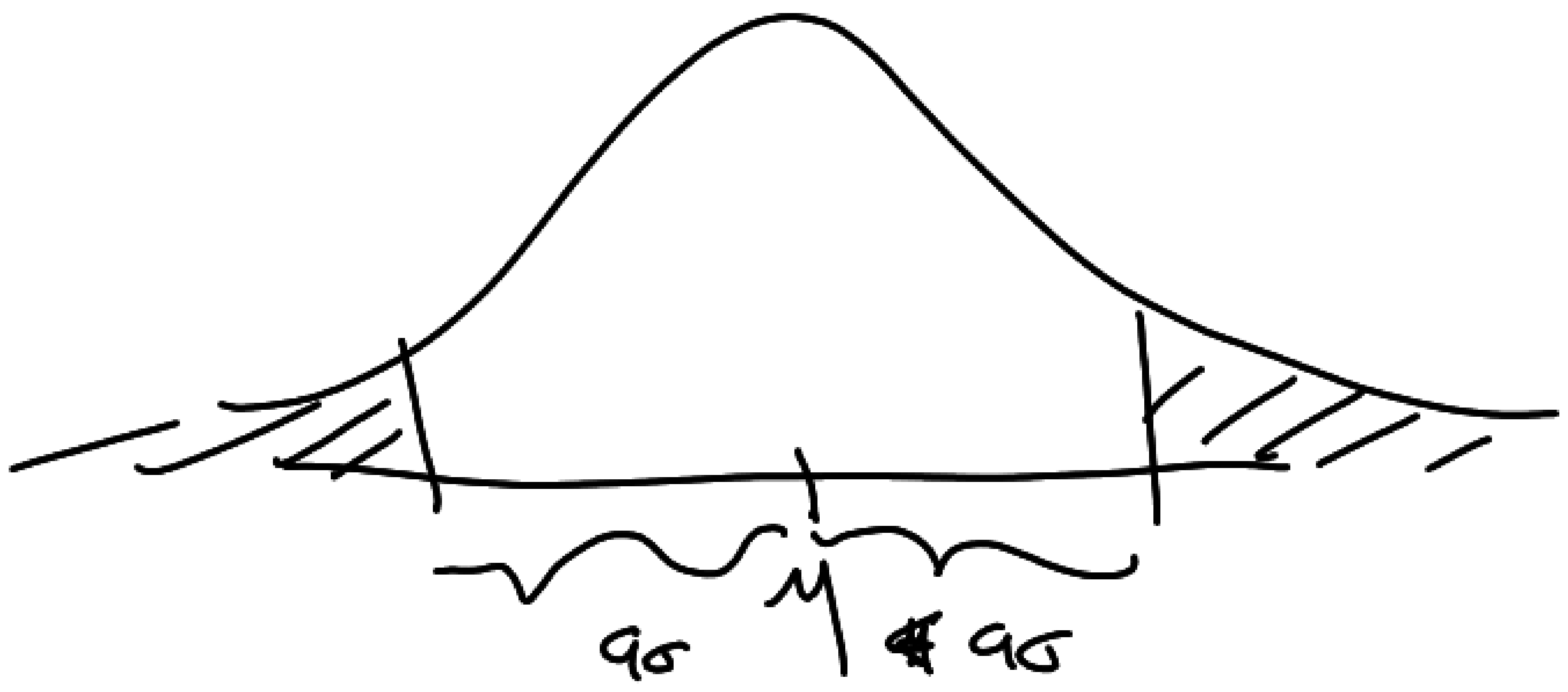
ent $\forall a \in \mathbb{R}$ se tiene que $f(a)\, P(X > a) \leq E(f(X))$

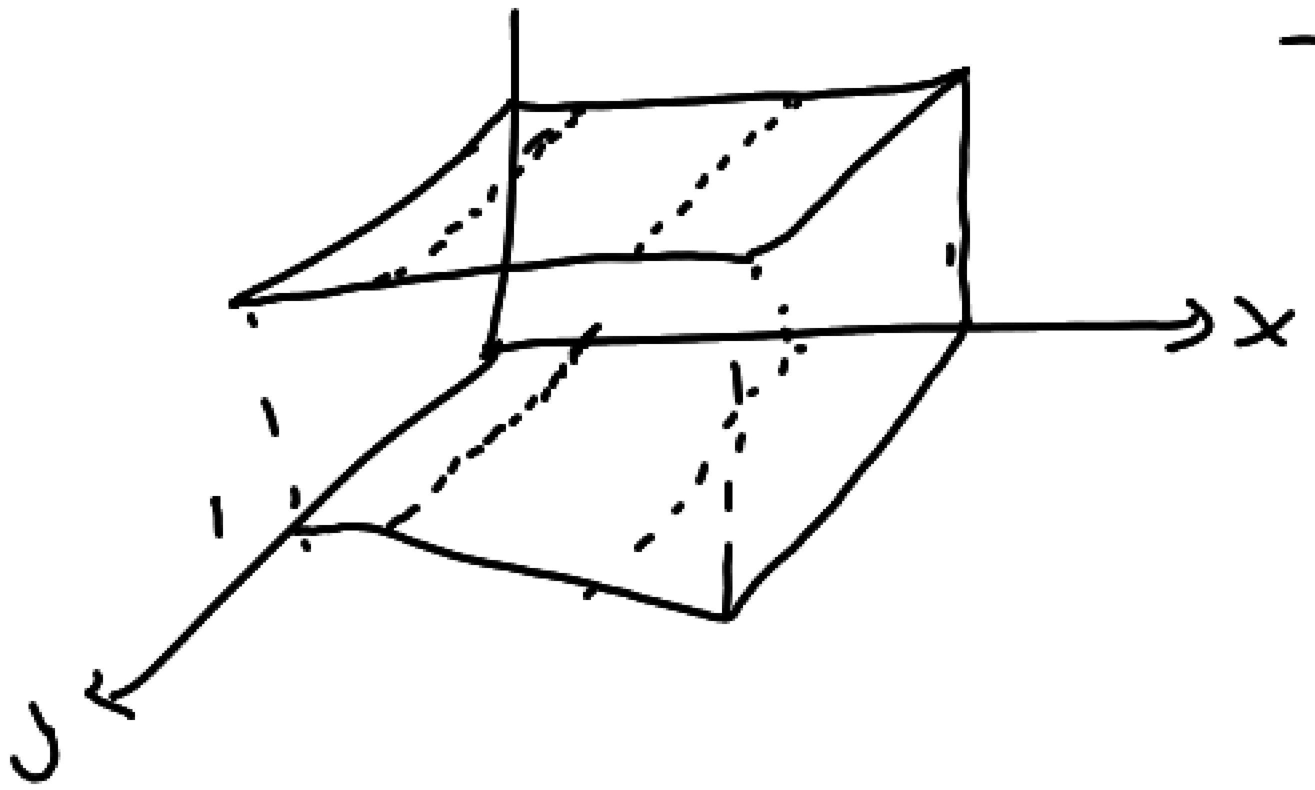
Corolario

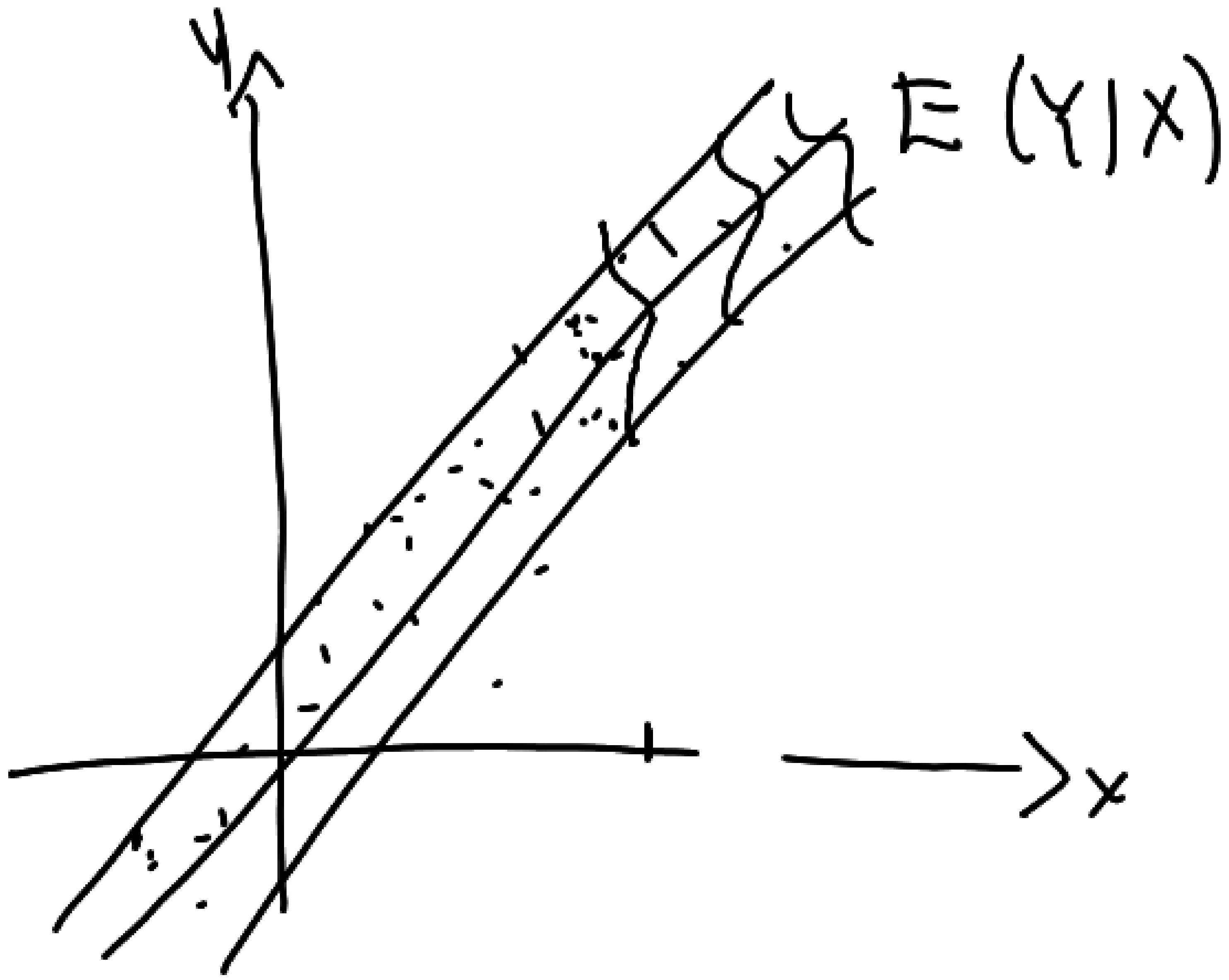
$$P(|X-\mu| > a\sigma) \leq \frac{Var(X)}{a^2 Var(X)} = \frac{1}{a^2}$$

(El teorema con $f(X) = (X-\mu)^2$

$$f_{X,Y}(x,y) = I_{(0,1)}(x)\, I_{(0,1)}(y)$$







$$y^{(x)} = \underbrace{E(Y|x)} + \varepsilon \qquad \varepsilon \text{ es ruido} \quad \begin{pmatrix} E(\varepsilon)=0 \\ var(\varepsilon)=\sigma^2 \end{pmatrix}$$

$$E(Y(x)) = E(Y|x)$$