Tarea 1 Algebra Matricial y Optimización.

(1) $A = \begin{pmatrix} 1 & -3 \\ -3 & 5 \end{pmatrix}$ $\quad AB = \begin{pmatrix} -3 & -11 \\ 1 & 17 \end{pmatrix}$ $\quad$ ¿B?

Solución:

$$A^{-1} \cdot AB = I \cdot B = B$$

Calculando A^{-1}:

$$\left(\begin{array}{cc|cc} 1 & -3 & 1 & 0 \\ -3 & 5 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & -3 & 1 & 0 \\ 0 & -4 & 3 & 1 \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & -3 & 1 & 0 \\ 0 & 1 & -\frac{3}{4} & -\frac{1}{4} \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & 0 & -5/4 & -3/4 \\ 0 & 1 & -3/4 & -1/4 \end{array}\right)$$

$$\Longrightarrow \quad A^{-1} = \begin{pmatrix} -5/4 & -3/4 \\ -3/4 & -1/4 \end{pmatrix}$$

$$\Longrightarrow \quad B = \begin{pmatrix} -5/4 & -3/4 \\ -3/4 & -1/4 \end{pmatrix} \begin{pmatrix} -3 & -11 \\ 1 & 17 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix}$$

(3) Muestre que $\begin{pmatrix} I & 0 \\ A & I \end{pmatrix}$ es invertible y encuentre su inversa.

$\begin{pmatrix} I & 0 \\ A & I \end{pmatrix}$ es triangular $\Longrightarrow \det\left[\begin{pmatrix} I & 0 \\ A & I \end{pmatrix}\right] = \prod_i a_{ii}$

$\Longrightarrow \det\left[\begin{pmatrix} I & 0 \\ A & I \end{pmatrix}\right] = \prod_i 1 = 1 \neq 0 \Longrightarrow \begin{pmatrix} I & 0 \\ A & I \end{pmatrix}$ es invertible

Sea $X = \begin{pmatrix} I & 0 \\ A & I \end{pmatrix}$, $\quad Y = \begin{pmatrix} I & 0 \\ -A & I \end{pmatrix}$

$$X \cdot Y = \begin{pmatrix} I & 0 \\ A & I \end{pmatrix} \cdot \begin{pmatrix} I & 0 \\ -A & I \end{pmatrix} = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix} = I_n$$

$$Y \cdot X = \begin{pmatrix} I & 0 \\ -A & I \end{pmatrix} \begin{pmatrix} I & 0 \\ A & I \end{pmatrix} = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix} = I_n$$

$$\Longrightarrow \quad Y = X^{-1} = \begin{pmatrix} I & 0 \\ A & I \end{pmatrix}^{-1}$$

(4) $V_1 = \begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix}$ $V_2 = \begin{pmatrix} -2 \\ -3 \\ 7 \end{pmatrix}$ $w = \begin{pmatrix} -3 \\ -3 \\ 10 \end{pmatrix}$

¿ $W \in \text{gen}\{v_1, v_2\}$?

Si $w \in \text{gen}\{v_1, v_2\}$, entonces existen $\alpha, \beta \in \mathbb{R}$ tal que

$w = \alpha v_1 + \beta v_2$

$$\alpha \begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix} + \beta \begin{pmatrix} -2 \\ -3 \\ 7 \end{pmatrix} = \begin{pmatrix} -3 \\ -3 \\ 10 \end{pmatrix}$$

$$\Rightarrow \begin{array}{l} \alpha - 2\beta = -3 \Rightarrow \alpha = -3 + 2\beta \\ 3\alpha - 3\beta = -3 \Rightarrow \alpha = -1 + \beta \\ -4\alpha + 7\beta = 10 \end{array} \quad \begin{array}{l} -3 + 2\beta = -1 + \beta \\ \Rightarrow \boxed{\beta = 2} \end{array}$$

$$-4\alpha + 7(2) = 10 \Rightarrow \boxed{\alpha = 1}$$

$\Rightarrow w = v_1 + 2v_2$

$\therefore w \in \text{gen}\{v_1, v_2\}$

(5) $b_1 = \begin{pmatrix} -3 \\ 2 \\ -4 \end{pmatrix}$ $b_2 = \begin{pmatrix} 7 \\ -3 \\ 5 \end{pmatrix}$ $X = \begin{pmatrix} 5 \\ 0 \\ -2 \end{pmatrix}$

$X \in H$, $H = \text{gen}\{b_1, b_2\}$

como $X \in H$, $\Rightarrow$ $X = \alpha b_1 + \beta b_2 = \alpha \begin{pmatrix} -3 \\ 2 \\ -4 \end{pmatrix} + \beta \begin{pmatrix} 7 \\ -3 \\ 5 \end{pmatrix}$

$$\Rightarrow \begin{array}{l} -3\alpha + 7\beta = 5 \\ 2\alpha - 3\beta = 0 \Rightarrow \alpha = 3/2\,\beta \\ -4\alpha + 5\beta = -2 \Rightarrow -4\left(3/2\right)\beta + 5\beta = 2 \Rightarrow -6\beta + 5\beta = -2 \end{array}$$

$\Rightarrow \beta = 2$

$\Rightarrow \alpha = \left(\frac{3}{2}\right)(2) = 3$

$$\therefore X = \begin{pmatrix} 5 \\ 0 \\ -2 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}_{\{b_1, b_2\}}$$

8.-

W : vectores de la forma $\begin{pmatrix} 2s+4t \\ 2s \\ 2s-3t \\ 5t \end{pmatrix}$

P.D W es un subespacio de $\mathbb{R}^4$

- Sean $x = \begin{pmatrix} 2s_1+4t_1 \\ 2s_1 \\ 2s_1-3t_1 \\ 5t_1 \end{pmatrix}$, $y = \begin{pmatrix} 2s_2+4t_2 \\ 2s_2 \\ 2s_2-3t_2 \\ 5t_2 \end{pmatrix} \in W$

- Cerradura bajo la suma

$$x+y = \begin{pmatrix} 2s_1+4t_1+2s_2+4t_2 \\ 2s_1+2s_2 \\ 2s_1-3t_1+2s_2-3t_2 \\ 5t_1+5t_2 \end{pmatrix} = \begin{pmatrix} 2(s_1+s_2)+4(t_1+t_2) \\ 2(s_1+s_2) \\ 2(s_1+s_2)-3(t_1+t_2) \\ 5(t_1+t_2) \end{pmatrix}$$

$\Rightarrow x+y \in W$

$\Rightarrow W$ es cerrado bajo la suma

- PD $\alpha x \in W$, $\alpha \in \mathbb{R}$, $x \in W$

Sea $\alpha \in \mathbb{R}$

$$\alpha x = \begin{pmatrix} 2(\alpha s_1)+4(\alpha t_1) \\ 2(\alpha s_1) \\ 2(\alpha s_1)-3(\alpha t_1) \\ 5(\alpha t_1) \end{pmatrix} \in W \text{ ya que } \alpha s_1, \alpha t_1 \in \mathbb{R}$$

$\therefore W$ es un subespacio de $\mathbb{R}^4$

$$W = s\begin{pmatrix} 2 \\ 2 \\ 2 \\ 5 \end{pmatrix} + t\begin{pmatrix} 4 \\ 0 \\ -3 \\ 5 \end{pmatrix} \Rightarrow W = \text{gen}\left\{\begin{pmatrix} 2 \\ 2 \\ 2 \\ 5 \end{pmatrix}, \begin{pmatrix} 4 \\ 0 \\ -3 \\ 5 \end{pmatrix}\right\}$$

Sea $\alpha\begin{pmatrix} 2 \\ 2 \\ 2 \\ 5 \end{pmatrix} + \beta\begin{pmatrix} 4 \\ 0 \\ -3 \\ 5 \end{pmatrix} = 0$

$$\Rightarrow \begin{array}{l} 2\alpha+4\beta=0 \\ 2\alpha=0 \\ 2\alpha-3\beta=0 \\ 5\alpha+5\beta=0 \end{array} \quad \begin{array}{l} \Rightarrow \alpha=0 \\ \Rightarrow \beta=0 \end{array}$$

$\Rightarrow v_1 = \begin{pmatrix} 2 \\ 2 \\ 2 \\ 5 \end{pmatrix}$ y $v_2 = \begin{pmatrix} 4 \\ 0 \\ -3 \\ 5 \end{pmatrix}$ son linealmente independientes

$\Rightarrow \{v_1, v_2\}$ forman una base de W

$\Rightarrow \dim(W) = 2$