$X \sim Pois(\lambda)$

$$M_X(t) = E(e^{tX}) = \sum_{x=0}^{\infty} e^{tx} \frac{e^{-\lambda}\lambda^x}{x!} = e^{-\lambda} \sum_{x=0}^{\infty} \frac{(e^t\lambda)^x}{x!} = e^{-\lambda} e^{e^t\lambda}$$

$X_1 \sim exp(\lambda_1)$

$X_2 \sim exp(\lambda_2)$

$Y = X_1 + X_2$ $\quad$ X_1 y X_2 indep.

$$M_Y(t) = E(e^{tY}) = E(e^{t(X_1+X_2)}) = E(e^{tX_1} e^{tX_2}) \underset{\text{indp}}{=} E(e^{tX_1})E(e^{tX_2})$$

$$= M_{X_1}(t) M_{X_2}(t) = e^{-\lambda_1} e^{e^t\lambda_1} e^{-\lambda_2 + e^t\lambda_2} = e^{-(\lambda_1+\lambda_2)} e^{(\lambda_1+\lambda_2)e^t}$$