$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{pmatrix} = A$$

$$I_c = \{1, \dots, \underline{k}\}, \quad k < n$$

$$1 \leq k \leq n-1$$

$$I_c = \{2\} = I_R$$

$$A_S = \begin{pmatrix} 1 & 3 \\ 0 & 6 \end{pmatrix}$$

Submatriz Principal

$k=1$ $\quad I_c = \{1\}$

$$A_{S_1} = \begin{pmatrix} 4 & 5 \\ 0 & 6 \end{pmatrix}$$

$\underline{k=2}$ $\quad I_c = \{1,2\}$

$$A_{S_2} = (6)$$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ \vdots & \vdots & & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & & a_{nn} \end{pmatrix} = A_{n\times n}$$

$Ax = b$

$x \rightarrow$ Sol. única.

$\rightarrow A = LU \rightarrow$ Red. Gauss

$L \rightarrow$ inv. mat. elem.

$K = n-1 \qquad I_c = \{1, 2, 3, 4, \ldots, n-1\}.$

$\rightarrow A_{S_{n-1}} = (a_{nn}) \qquad \underline{a_{nn} \neq 0}$

$\rightarrow K = n-2 \qquad I_c = \{1, 2, 3, \ldots, n-2\}.$

$$\rightarrow A_{S_{n-2}} = \begin{pmatrix} a_{n-1,n-1} & a_{n-1,n} \\ a_{n,n-1} & a_{nn} \end{pmatrix}$$

$\vdots$

$$A = \begin{pmatrix} 0 & 2 & 3 \\ 2 & -4 & 7 \\ 1 & -2 & 5 \end{pmatrix}$$

$$P = \underbrace{P_2 \cdot P_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 2 & 3 \\ 2 & -4 & 7 \\ 1 & -2 & 5 \end{pmatrix} \xrightarrow{R_1 \leftrightarrows R_3} \begin{pmatrix} 1 & -2 & 5 \\ 2 & -4 & 7 \\ 0 & 2 & 3 \end{pmatrix} \xrightarrow{R_2 \leftarrow -2R_1 + R_2} \begin{pmatrix} 1 & -2 & 5 \\ 0 & 0 & -3 \\ 0 & 2 & 3 \end{pmatrix}$$

$$P_1 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$P_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad \xrightarrow{R_2 \leftrightarrows R_3} \underbrace{\begin{pmatrix} 1 & -2 & 5 \\ 0 & 2 & 3 \\ 0 & 0 & -3 \end{pmatrix}}_{U}$$

$$PA = LU$$

$$PA = \begin{pmatrix} 1 & -2 & 5 \\ 0 & 2 & 3 \\ 2 & -4 & 7 \end{pmatrix}$$

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}$$

$PA = LU$

L: triangular inferior con 1's en diag.

U: triangular superior.

$PA = LDU'$

$U = DU'$

D: matriz diag. piv. de U

U': matriz de dividir cada renglón por piv.

$$u_{11} \to \begin{pmatrix} \overset{1}{u_{11}} & \cdots & u_{1n} \\ & u_{22} & \vdots \\ 0 & \ddots & u_{nn} \end{pmatrix}$$

$$\begin{pmatrix} u_{11} & 0 & 0 \\ 0 & u_{22} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & \frac{u_{12}}{u_{11}} & \cdots & \frac{u_{1n}}{u_{11}} \\ & \ddots & & \vdots \\ 0 & & \ddots & u_{nn} \end{pmatrix}$$

$A \to$ simétrica y no sing.

$A = LDU = A^t$

$A^t = (LDU)^t = U^t D L^t$

$\underline{L}D\underline{U} = \underline{U}^t D \underline{L}^t$

$$\boxed{\begin{aligned} L &= U^t \\ U &= L^t \end{aligned}}$$

$A = LDL^t = U^t D U$

$D \to$ elementos positivos

$$D = \underbrace{\begin{pmatrix} \sqrt{d_{11}} & 0 & 0 \\ 0 & \sqrt{d_{22}} & 0 \\ 0 & 0 \ddots & \sqrt{d_{nn}} \end{pmatrix}}_{\sqrt{D}} * \begin{pmatrix} \sqrt{d_{11}} & 0 & 0 \\ 0 & \sqrt{d_{22}} & 0 \\ 0 & \vdots \; 0 \ddots & \sqrt{d_{nn}} \end{pmatrix}$$

$$A = \underbrace{(L\sqrt{D})}_{T} \cdot \underbrace{(\sqrt{D}L^t)}_{T^t} \to \text{Cholesky}$$

$$A = \begin{pmatrix} 3 & 4 & 3 \\ 4 & 8 & 6 \\ 3 & 6 & 9 \end{pmatrix} = LU = LD\underbrace{U'}_{\to L^t} = \underline{LDL^t} = \overbrace{(L\sqrt{D})}^{T}\overbrace{(\sqrt{D}L^t)}^{T^t}$$

$$\begin{pmatrix} 3 & 4 & 3 \\ 4 & 8 & 6 \\ 3 & 6 & 9 \end{pmatrix} \rightsquigarrow \underbrace{\begin{pmatrix} 3 & 4 & 3 \\ 0 & 8/3 & 2 \\ 0 & 0 & 9/2 \end{pmatrix}}_{U}$$

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 4/3 & 1 & 0 \\ 1 & 6/8 & 1 \end{pmatrix}$$

$$U = \underline{D}\,\underline{L^t} = \overbrace{\begin{pmatrix} 3 & 0 & 0 \\ 0 & 8/3 & 0 \\ 0 & 0 & 9/2 \end{pmatrix}}^{D}\overbrace{\begin{pmatrix} 1 & 4/3 & 1 \\ 0 & 1 & 6/8 \\ 0 & 0 & 1 \end{pmatrix}}^{L^t}$$

$\cdot\, T = L \cdot \sqrt{D}$

$\cdot\, T^t = \sqrt{D}\, L^t$

$$T = \begin{pmatrix} \sqrt{3} & 0 & 0 \\ 4/\sqrt{3} & \sqrt{\frac{8}{3}} & 0 \\ \sqrt{3} & \sqrt{\frac{8}{3}} \cdot 6/8 & \frac{3\sqrt{2}}{2} \end{pmatrix}$$

$$A x_t = x_{t-1}$$

$$x_0 = b$$

$$A = LU$$

$$Ax = b$$

$$L\underbrace{Ux}_{y} = b$$

$$Ly = b \qquad y = Ux$$

$\rightarrow v_1, v_2, v_3 \in \mathbb{R}^m$

$$x = \underline{\alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3}$$

$$\underbrace{\begin{pmatrix} v_1 & v_2 & v_3 \end{pmatrix}}_{A_{m \times 3}} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} = x$$

$$(A \alpha = x) \qquad \alpha_1, \alpha_2, \alpha_3 \in \mathbb{R}$$

$\rightarrow s_1, \dots, s_n \in \mathbb{R}^m$

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{pmatrix}$$

$$\alpha_1 s_1 + \alpha_2 s_2 + \dots + \alpha_n s_n = 0$$

Son l.i. si y sólo si $\alpha_1 = \alpha_2 = \dots = \alpha_n = 0$.

$$\operatorname{Ker}(A) = \{x \mid Ax = 0\}.$$

$$(s_1 \ \ s_2 \ \dots \ s_n)\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{pmatrix} = 0$$

$$\Updownarrow$$

$$\operatorname{Ker}(A) = \{\bar{0}\} \text{ l.i.}$$

$$\alpha = \bar{0}$$

$$A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix} \rightsquigarrow \begin{pmatrix} a'_{11} & \cdots & a'_{1n} \\ \vdots & \ddots & \vdots \\ a'_{r1} & \cdots & a'_{rn} \\ 0 & \cdots & 0 \end{pmatrix}$$

$$\text{Rango}(A) = r$$

$$Ax = 0$$

$Ax = 0$

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 3 & -3 & 2 \\ -1 & -11 & 6 \end{pmatrix} \rightsquigarrow \left(\begin{array}{ccc|c} 1 & 0 & -1/9 & 0 \\ 0 & 1 & -5/9 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$x_1 - 1/9\, x_3 = 0$

$x_2 - 5/9\, x_3 = 0$

$x_1 = 1/9\, x_3$

$x_2 = 5/9\, x_3$

$$\begin{pmatrix} 1/9\, x_3 \\ 5/9\, x_3 \\ x_3 \end{pmatrix} \xrightarrow{\text{Libre}} \begin{pmatrix} 1/9 \\ 5/9 \\ 1 \end{pmatrix}$$

Rango $(A) = 2$ ¿Dim. esp. Col.?

Nul $(A) = 1$
?

$$A x = y$$

$$\left(\begin{array}{ccc|c} 1 & 2 & -1 & y_1 \\ 3 & -3 & 2 & y_2 \\ -1 & -11 & 6 & y_3 \end{array}\right) \rightsquigarrow \left(\begin{array}{ccc|c} 1 & 2 & -1 & y_1 \\ 0 & -9 & 5 & -3y_1 + y_2 \\ 0 & 0 & 0 & 4y_1 - y_2 + y_3 \end{array}\right)$$

$$\text{Rango}(A) = 2 \qquad 4y_1 - y_2 + y_3 = 0$$

$$y_1 = \frac{y_2}{4} - \frac{y_3}{4}$$

$$\begin{pmatrix} y_2/4 - y_3/4 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 1/4 \\ 1 \\ 0 \end{pmatrix} y_2 + \begin{pmatrix} -1/4 \\ 0 \\ 1 \end{pmatrix} y_3$$

$$A = \begin{pmatrix} 10 & -18 \\ 6 & -11 \end{pmatrix}$$

$$V = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$A \cdot V = \begin{pmatrix} 10 & -18 \\ 6 & -11 \end{pmatrix} \underbrace{\begin{pmatrix} 2 \\ 1 \end{pmatrix}}_{\text{Vect. Propio}} = \overbrace{1}^{\text{Val. propio.}} \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$Av = \lambda v \Rightarrow Av = \lambda I_n v \Rightarrow (Av - \lambda I_n v) = \bar{0}$$

$$\hookrightarrow (A - \lambda I_n) \cdot V = 0$$

$(A-\lambda I)x = 0$ ó $\det(A-\lambda I) = 0$ *

Sol. No trivial.

$$A = \begin{pmatrix} 4 & 2 \\ 3 & 3 \end{pmatrix}$$

$$\det(A-\lambda I) = \det\left(\begin{pmatrix} 4 & 2 \\ 3 & 3 \end{pmatrix} - \overbrace{\begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}}^{\lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}} \right) = 0$$

$$= \det\begin{pmatrix} 4-\lambda & 2 \\ 3 & 3-\lambda \end{pmatrix} = (4-\lambda)(3-\lambda) - 6$$

$$= 12 - 7\lambda + \lambda^2 - 6$$

$$= \underline{6 - 7\lambda + \lambda^2 = 0}$$

$$= (\lambda-6)(\lambda-1) = 0$$

$\lambda_1 = 6$

$\lambda_2 = 1$

$\lambda_1 = 6$

$$\begin{pmatrix} 4-6 & 2 \\ 3 & 3-6 \end{pmatrix} = \begin{pmatrix} -2 & 2 \\ 3 & -3 \end{pmatrix}$$

$$(A - \lambda_1 I)x = 0$$

$$\rightarrow \left(\begin{array}{cc|c} -2 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right) \Rightarrow -2x_1 + 2x_2 = 0$$

$$\boxed{x_1 = x_2}$$

$$\begin{pmatrix} x_2 \\ x_2 \end{pmatrix} = \underbrace{\begin{pmatrix} 1 \\ 1 \end{pmatrix}}\, x_2$$

Vector Propio.

$\lambda_2 = 1$

$$\begin{pmatrix} 4-1 & 2 \\ 3 & 3-1 \end{pmatrix} = (A - \lambda_2 I) = \begin{pmatrix} 3 & 2 \\ 3 & 2 \end{pmatrix}$$

$$(A - \lambda_2 I)x = 0$$

$$\left(\begin{array}{cc|c} 3 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right) \to 3x_1 + 2x_2 = 0$$

$$x_1 = -\frac{2}{3}x_2 \qquad x_2 = -\frac{3}{2}x_1$$

$$\begin{pmatrix} -2/3\, x_2 \\ x_2 \end{pmatrix} = \begin{pmatrix} -2/3 \\ 1 \end{pmatrix} x_2 \to \text{vect. Propio}$$

Val. y vect. Propios de A.

$\lambda_1 = 6 \quad V_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\lambda_2 = 1 \quad V_2 = \begin{pmatrix} -2/3 \\ 1 \end{pmatrix}$