MMA/MMAI 861
Analytical Decision Making

Decision Analysis (One)

Class Five

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Introduction

- People avoid or ignore uncertainty because it's too hard to think about.

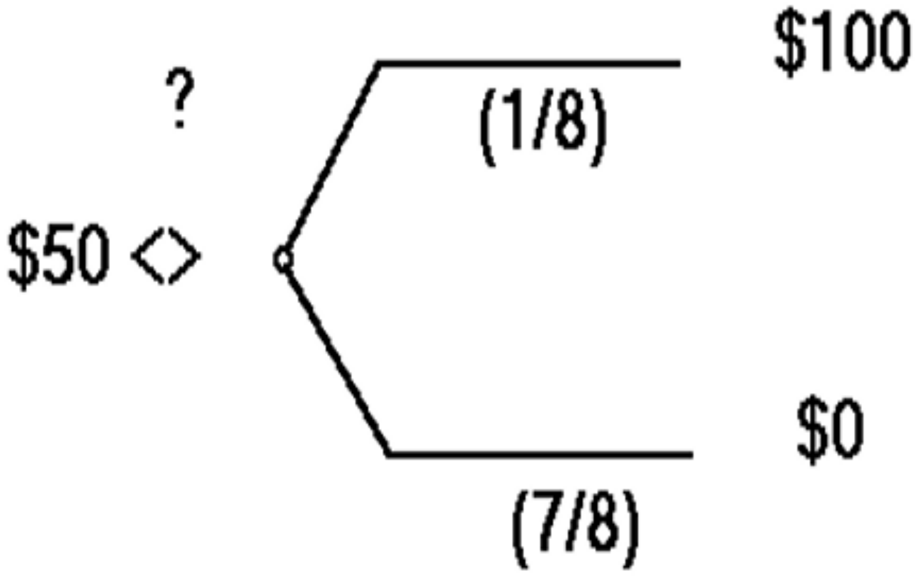
  "Send me a one-handed economist!" - Harry Truman

- We need ways to assess uncertainty and risk, and incorporate these assessments in decision making.

Peanuts by Charles Schulz for February 27, 1963

A Preliminary Observation

- Suppose you are offered <u>one</u> play of the game shown below for a price of $50. Would you play?
- Now suppose you did play the game and you won. Was this a good decision?



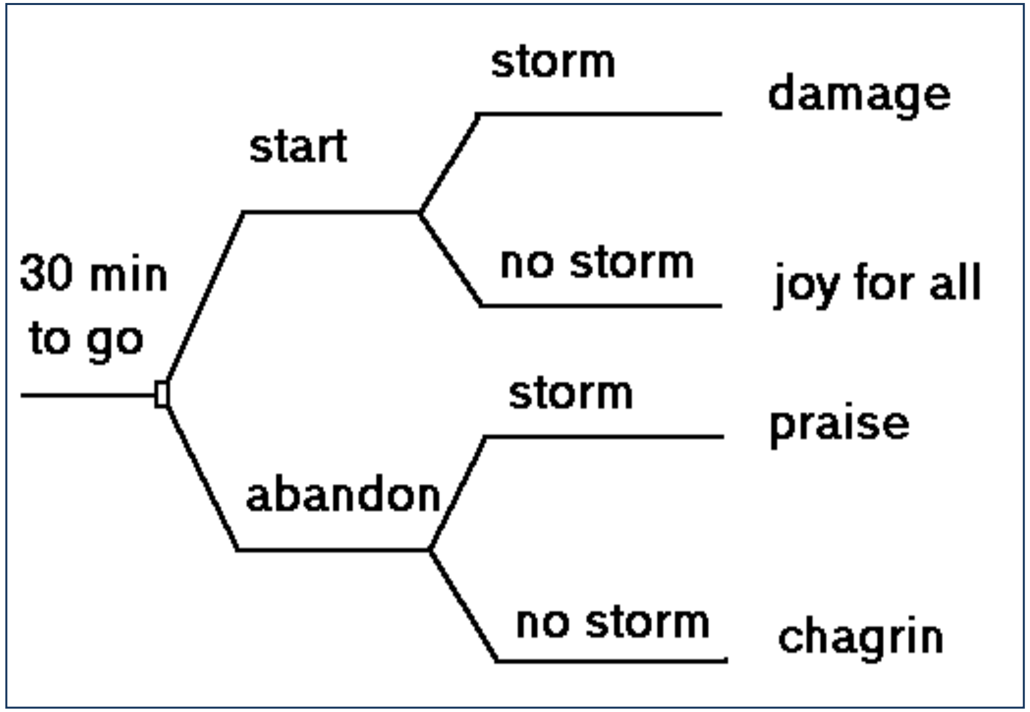
Motivations for Decision Analysis

- Psychological Comfort: To assuage worry, "Did I do the right thing?"
- Communicating: With others about the nature of the uncertainty and how to deal with it.
- Advocacy: Convincing others that you are right.
- Generating New Solutions: Decomposing a decision problem and recombining may reveal new opportunities for problem resolution or enhancement.

Many Complex Issues

- Value assessment
- Risk assessment
- Time frame
- Multiple (conflicting) objectives
- Group decisions
- Ethical issues

The Race Committee



- How to cope?
- Suppose a probability "*p*" of storm. How big does *p* have to be before the decision would switch?

Choice Criteria

- Dominated alternative
- Extremal methods
  - MAXIMIN
  - MAXIMAX
  - MINIMAX
- Pretty unsophisticated since all information is not used. How to include relative likelihood?
- What <u>is</u> probability?

Classical versus Bayesian Probability

Classical:

"Probability is an attribute of things. It is assessed by counting relative frequency of occurrence, and in the absence of such empirical evidence, has no meaning."

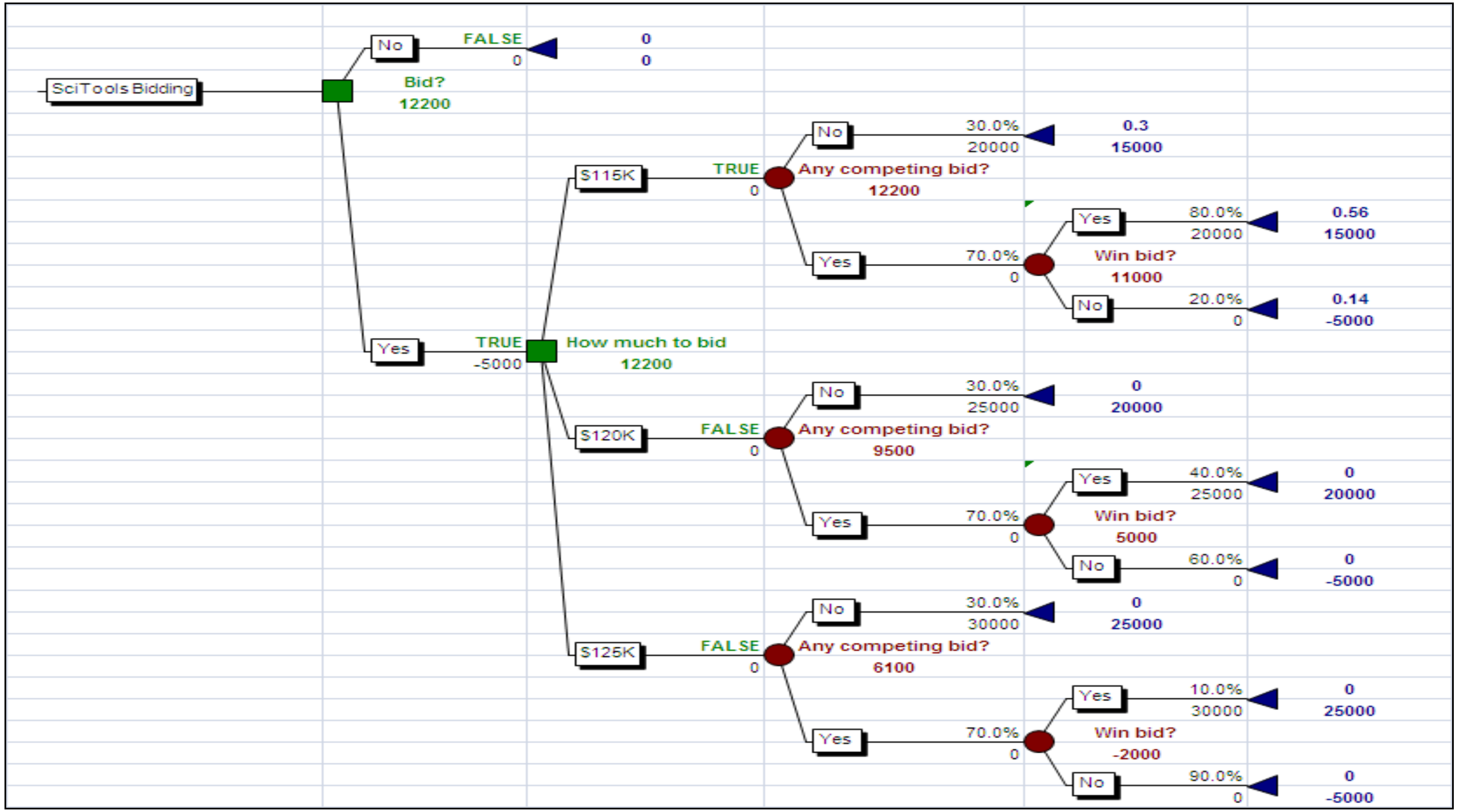
Bayesian:

"Experimentation is fine, but many interesting problems do not allow for any, and yet a decision must be made. Assert that probability is a state of mind; a statement about the belief that one has as to likelihood of occurrence."

So Who's Right?

- Classicists have the right answer to the wrong question.
- Bayesians have no answer to the right question.
- Bayes is compelling, but two people can have conflicting answers to a question, and they're both right! How do beliefs get formed and modified?

One Approach: A 'Decision Tree'



Decision Tree Elements – Nodes

- Tree depicts sequence of points in time (nodes) when decisions are made and chance events happen.
  - Important to keep in logical time order
- Nodes

Decision: **Point of choosing one out of several options**

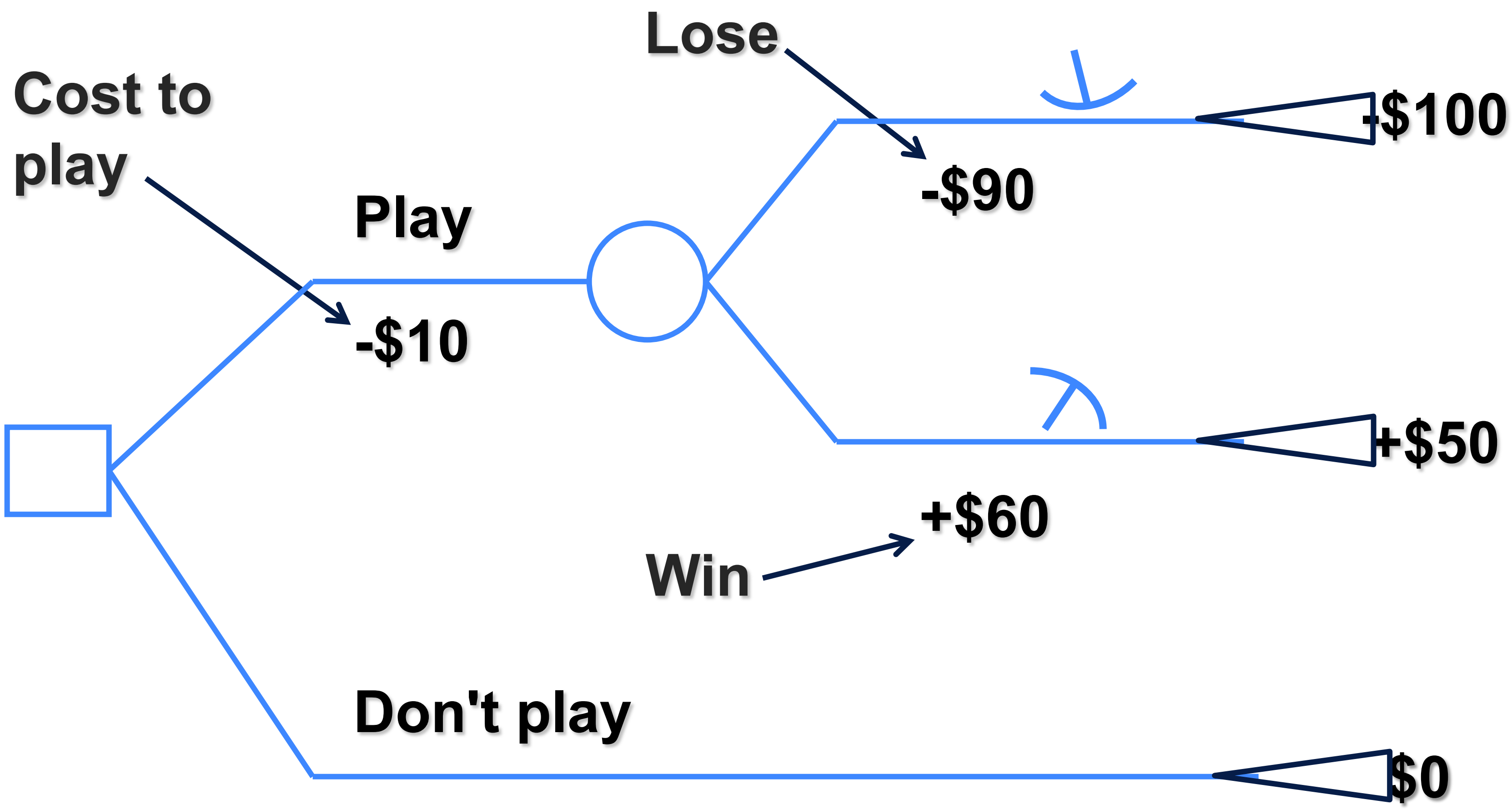
Event: **Point when the result of a chance event becomes known**

Terminal: **Point when the decision-making process is over along one path**

Decision Tree Elements – Arcs

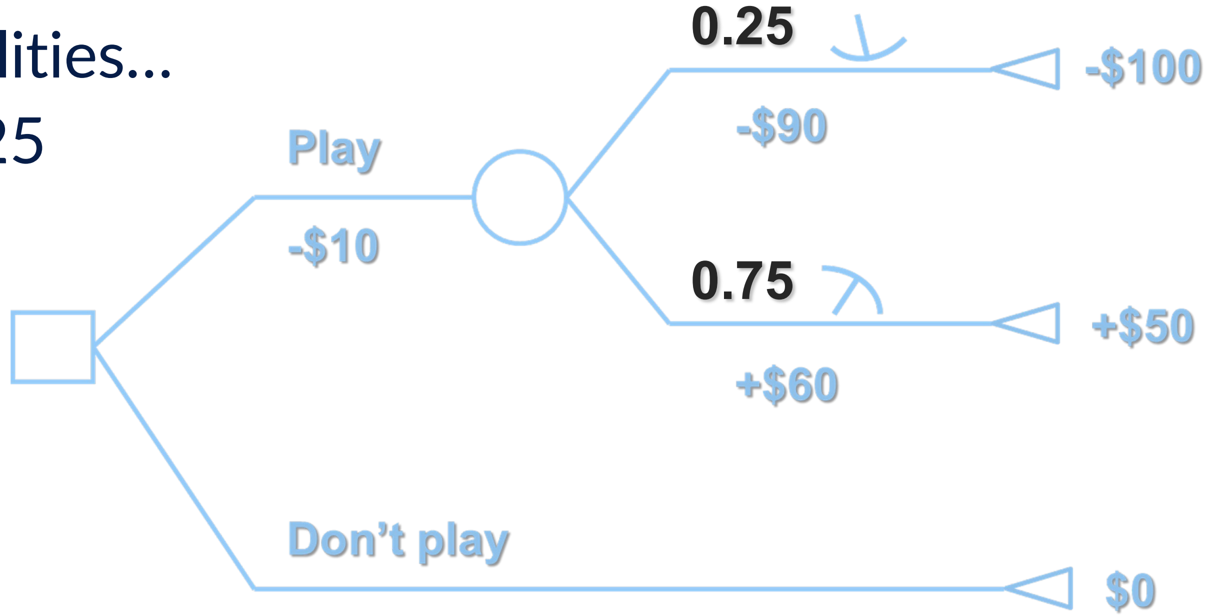
- **Decision Arc:**
  - Branch out of a decision node
  - Represents a particular decision
  - May have an associated $-value
- **Chance Arc:**
  - Branch out of a chance node
  - Represents a particular outcome of a chance event
  - Has a probability value (chance of occurrence)
  - May have an associated $-value

Example: The 'Thumbtack Game'



Thumbtack Game: Assessing Probabilities

If we assign probabilities...
- P(point up) = 0.25
- P(side) = 0.75



...then we can find the 'Expected Value' of the game.

EV = (0.25)(–$100) + (0.75)(+$50) = +$12.50

interpretation...

Expected Value Interpretation

- $12.50 is the long run average value if the same trials are repeated over and over
- Not the amount we expect to receive on one trial!
- Maximizing expected value is a good strategy for a long sequence of repeated gambles
- On average, over many plays, you would come out ahead by playing the game
- But is this a good rule for one play?

Bidding for a Contract

SciTools Inc., a company that specializes in scientific instruments, has been invited to make a bid on a government contract.

The bids will be sealed, so that no company knows what the others are bidding, and the low bid wins the contract.

Bidding for a Contract: Estimated Costs and Probabilities

SciTools estimates that it will cost $5,000 to prepare a bid and $95,000 to supply the instruments if it wins the contract.

On the basis of past contracts of this type, SciTools believes there is a 30% chance of no competing bids.

SciTools believes that the possible low bids from the competition, if there are any, and their associated probabilities are as shown below.

Bidding for a Contract: Possible Competition

Overall 70% chance of at least one competing bid, and…

Low Bid	Probability
< $115K	20%
($115K, $120K)	40%
($120K, $125K)	30%
> $125K	10%

Where do numbers like these come from?

Modeling Assumptions

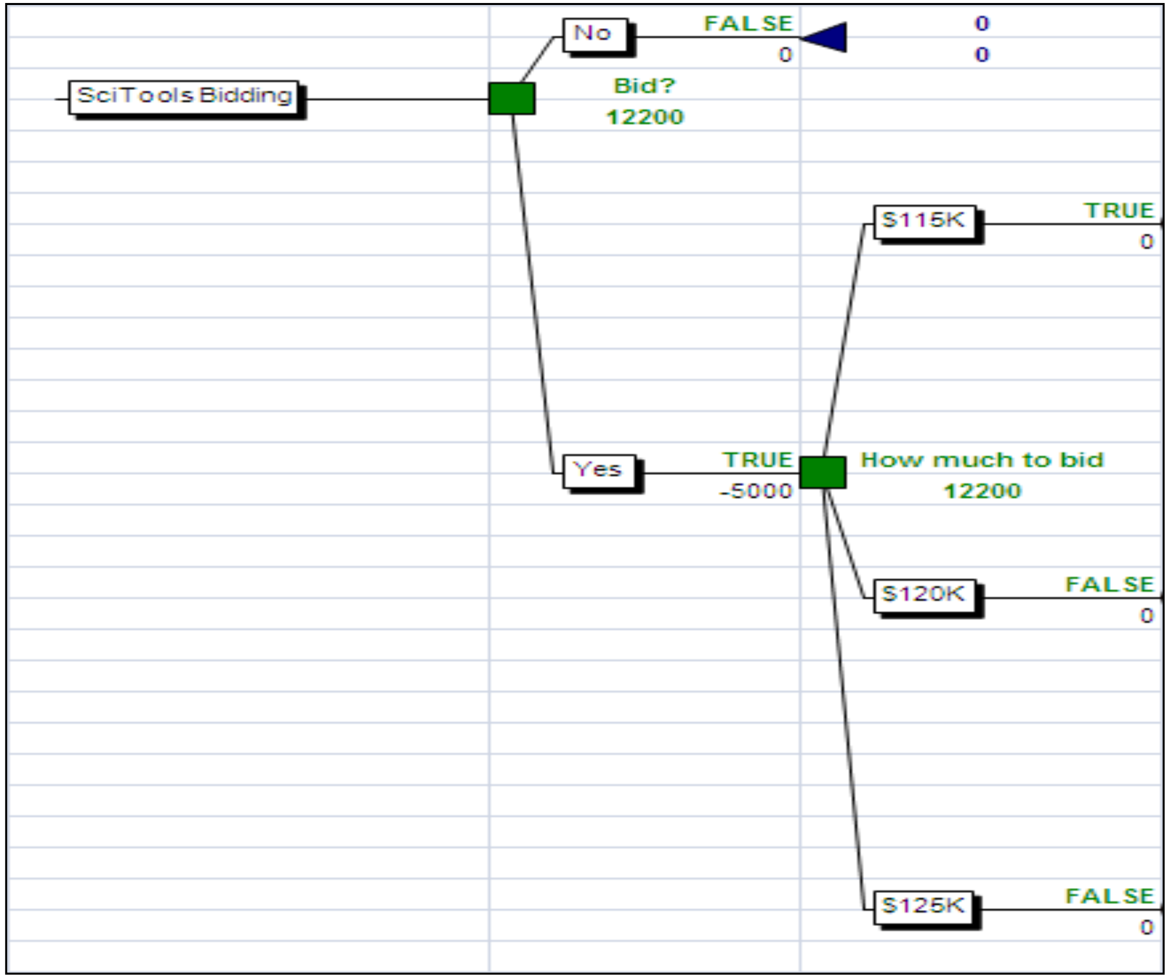
- Low bid will not depend on number of competing bidders
- Ignore possibility of tie bids
- Based on the current market conditions, and taking into account little chance that low bid is above $125K, do not bid above $125K
- Maximizing expected value is an acceptable decision criterion
- SciTools bids on many contracts

SciTools Decision Data

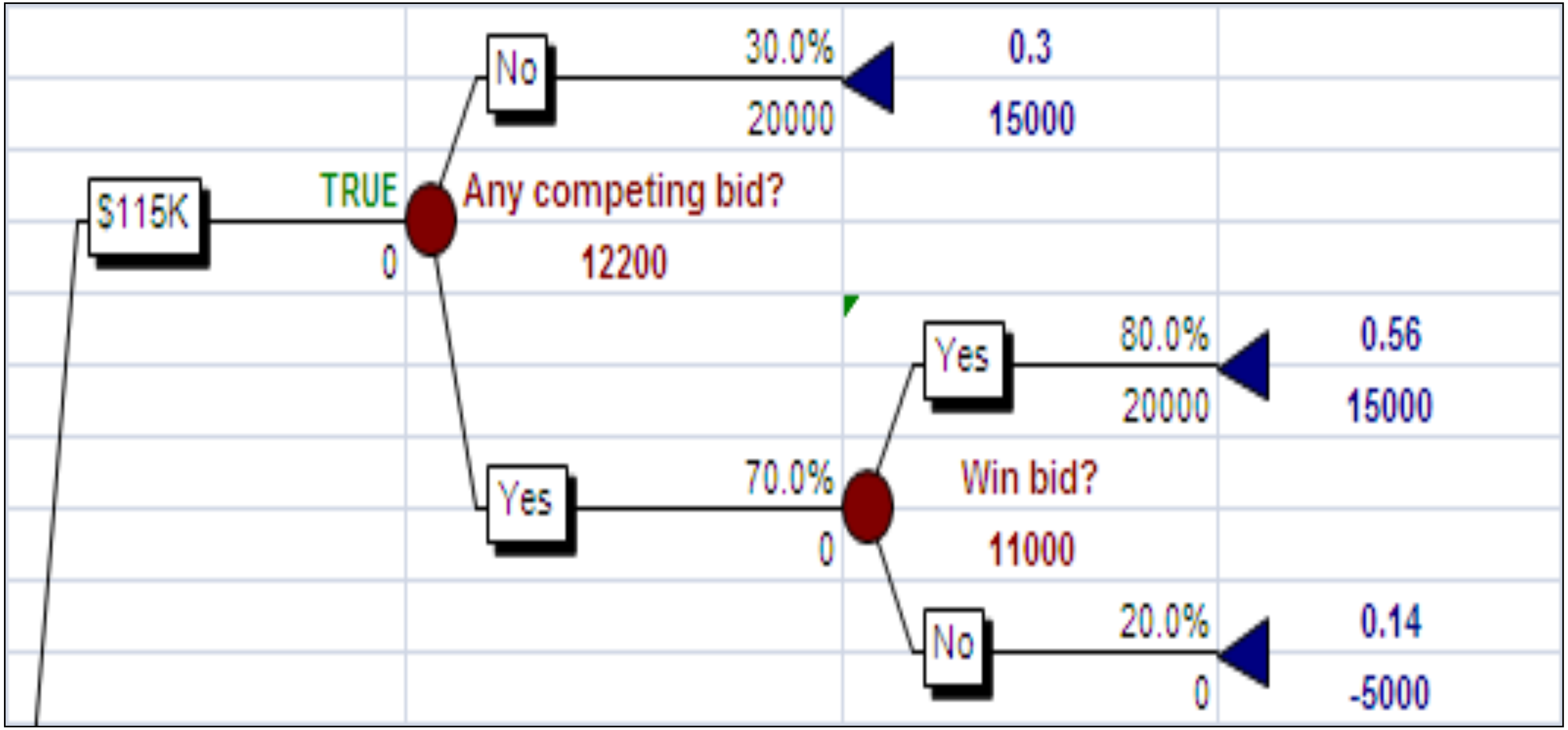
	A	B	C	D
1	SciTools Bidding Example			
2	Inputs			
3	Cost to prepare a bid	$5,000		
4	Cost to supply instruments	$95,000		
5				
6	Probability of no competing b	0.3		
7	Comp bid distribution (if any)			
8	<$115K	0.2		
9	$115K to < $120K	0.4	0.8	Win115
10	$120K to < $125K	0.3	0.4	Win120
11	>=$125K	0.1	0.1	Win125
12				

Range names
BidCost: B3
PrNoBid: B6
ProdCost: B4

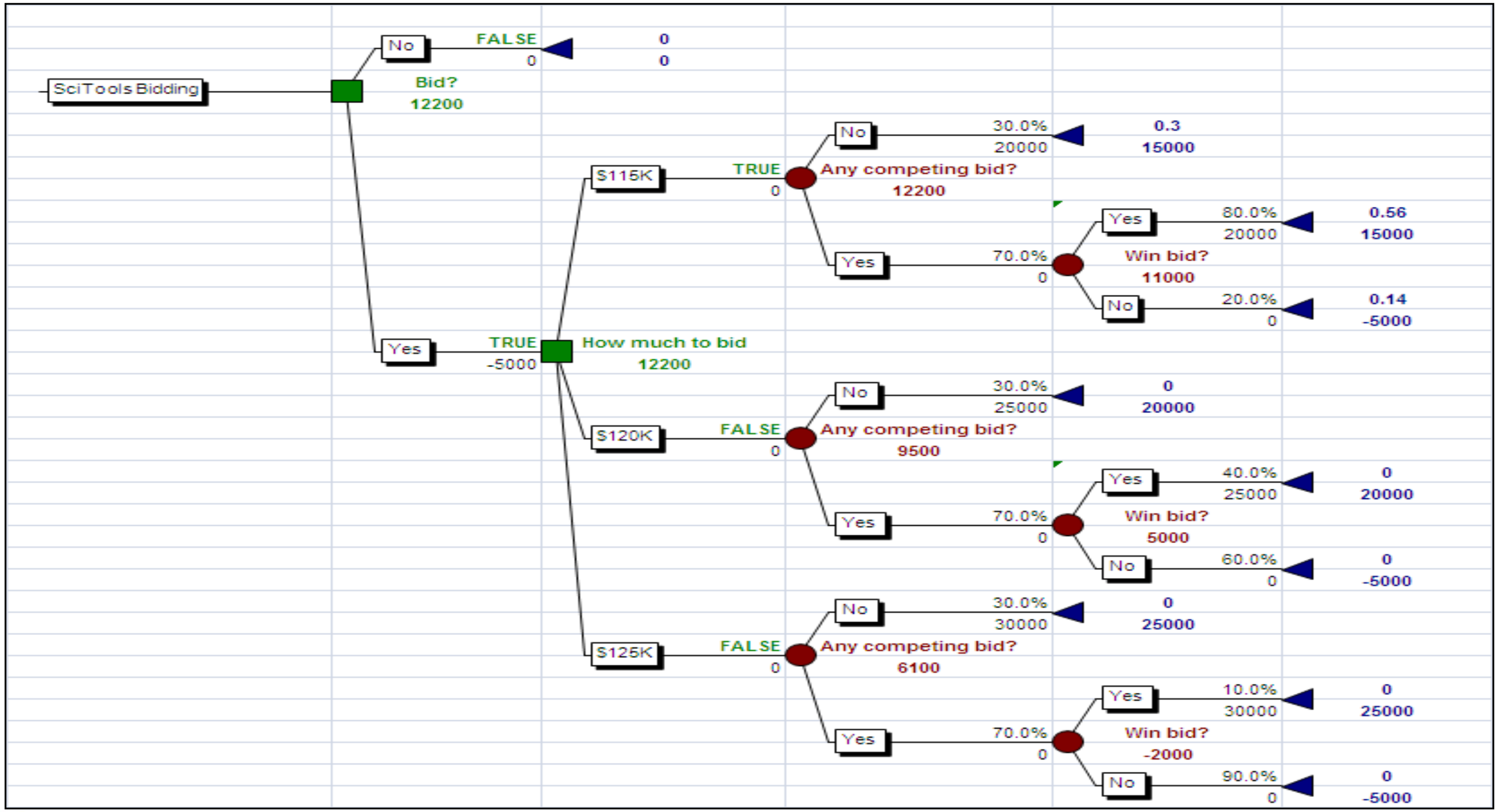
Bidding Decision Tree



Bidding Decision Tree (continued...)



Bidding Decision Tree (continued…)

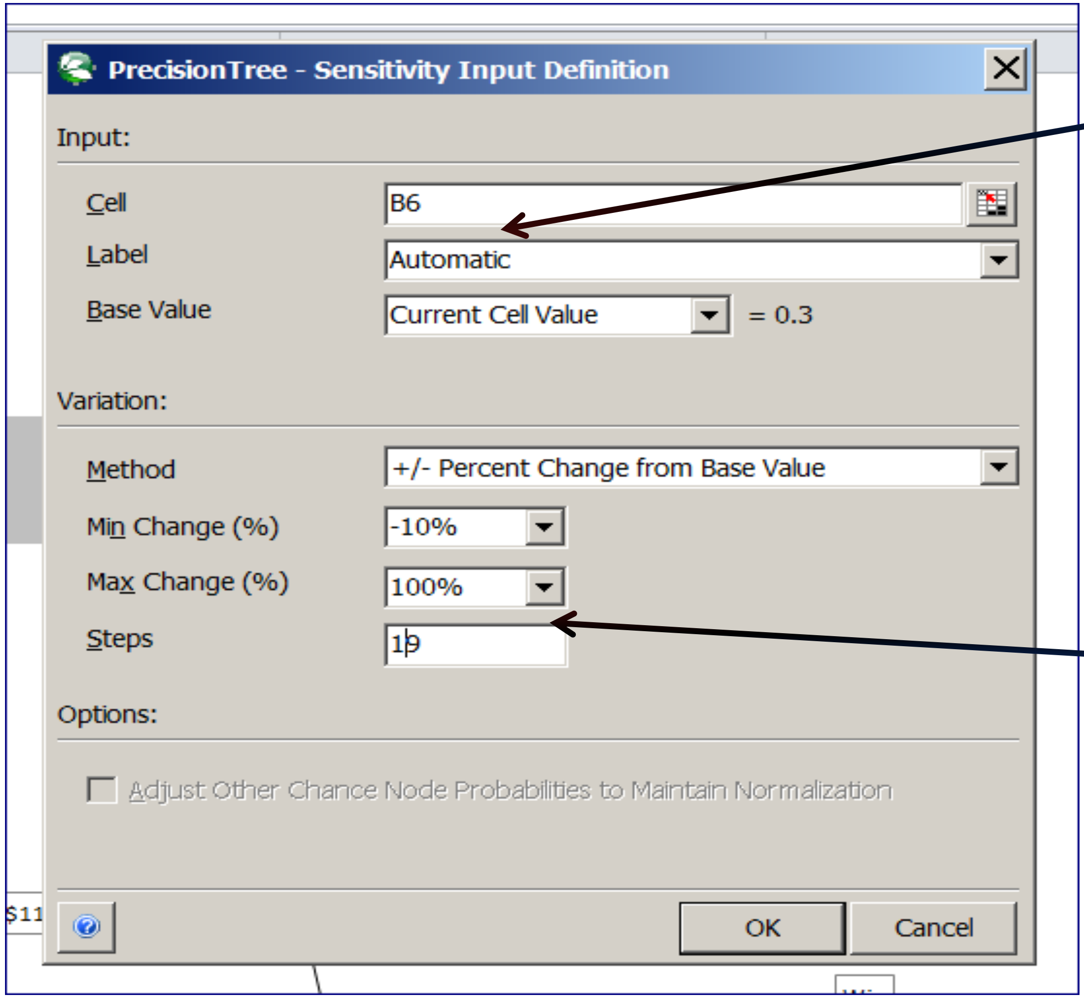


Optimal decision is to bid $115K with 'expected' payoff of $12,200

Sensitivity Analysis

- We really just guessed at the input data
  - E.g. 30% chance of no competing bid
- How much faith can we put in a decision to bid $115K?
  - How 'stable' is our decision
- Can try different values for important assumptions and see how things change…

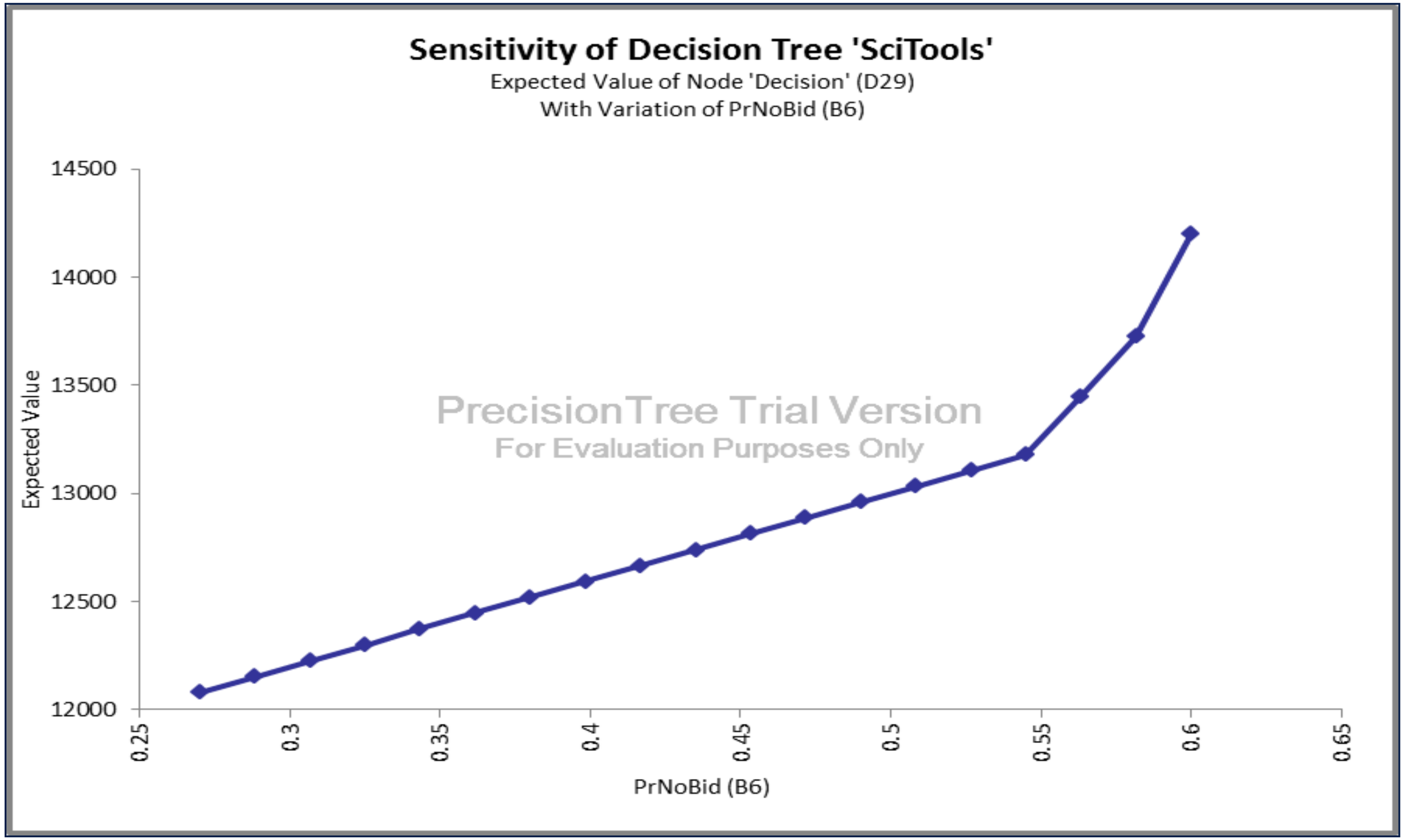
Sensitivity Analysis: No Competition – Probability



Cell to change

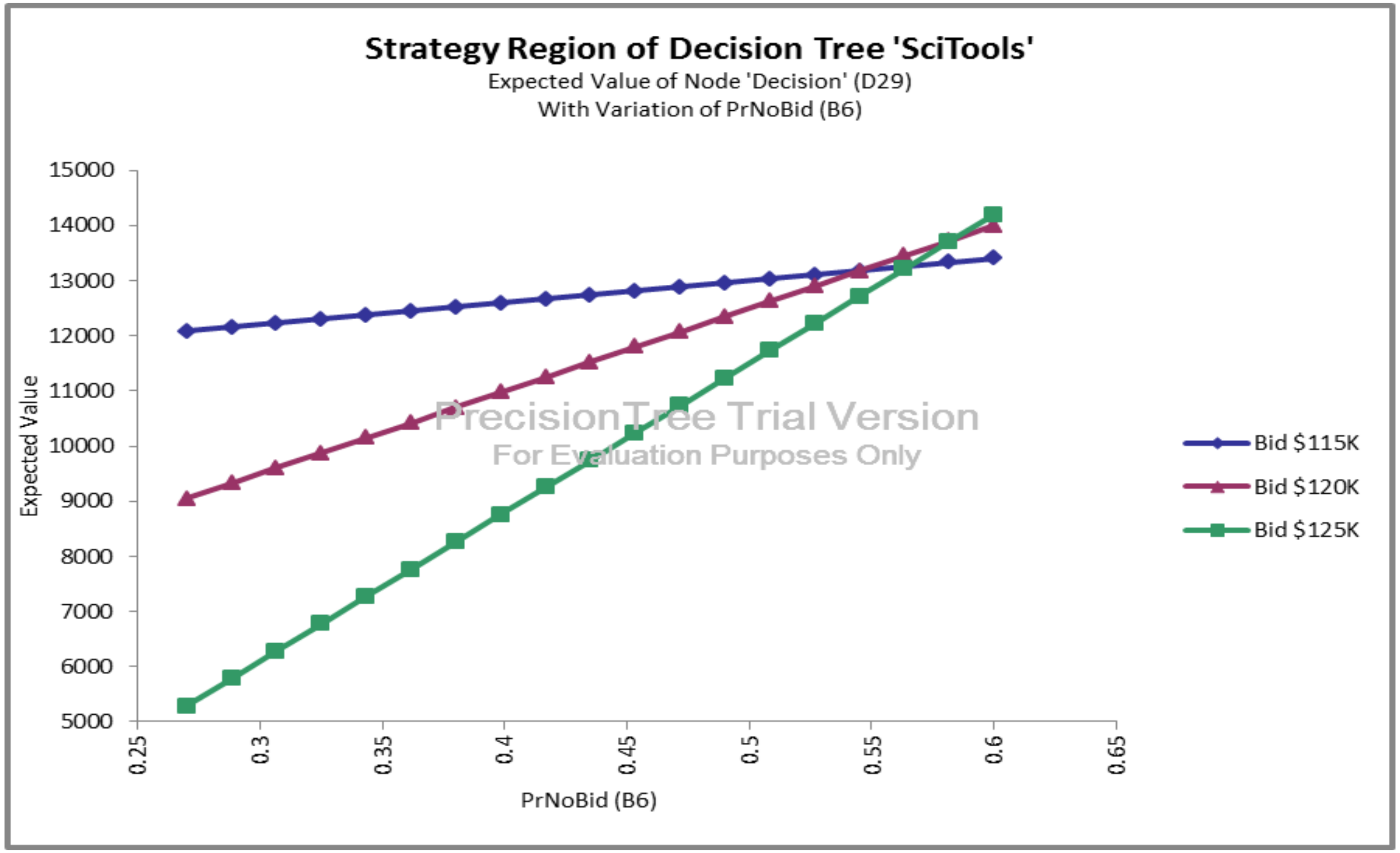
Range of values to try

Sensitivity Analysis: Change in EV with No Comp Prob



It looks like something changes at around 55%

Sensitivity Analysis: EV with Different Strategies



$115K remains optimal all the way up to near 55%, then $120K is briefly optimal, then $125K

Sensitivity Analysis: EV with Different Strategies (continued…)

Strategy Region Data

	Input		Bid $115K		Bid $120K		Bid $125K	
	Value	Change (%)	Value	Change (%)	Value	Change (%)	Value	Change (%)
#1	0.27	-10.00%	12080	-0.98%	9050	-25.82%	5290	-56.64%
#2	0.288333333	-3.89%	12153.33333	-0.38%	9325	-23.57%	5785	-52.58%
#3	0.306666667	2.22%	12226.66667	0.22%	9600	-21.31%	6280	-48.52%
#4	0.325	8.33%	12300	0.82%	9875	-19.06%	6775	-44.47%
#5	0.343333333	14.44%	12373.33333	1.42%	10150	-16.80%	7270	-40.41%
#6	0.361666667	20.56%	12446.66667	2.02%	10425	-14.55%	7765	-36.35%
#7	0.38	26.67%	12520	2.62%	10700	-12.30%	8260	-32.30%
#8	0.398333333	32.78%	12593.33333	3.22%	10975	-10.04%	8755	-28.24%
#9	0.416666667	38.89%	12666.66667	3.83%	11250	-7.79%	9250	-24.18%
#10	0.435	45.00%	12740	4.43%	11525	-5.53%	9745	-20.12%
#11	0.453333333	51.11%	12813.33333	5.03%	11800	-3.28%	10240	-16.07%
#12	0.471666667	57.22%	12886.66667	5.63%	12075	-1.02%	10735	-12.01%
#13	0.49	63.33%	12960	6.23%	12350	1.23%	11230	-7.95%
#14	0.508333333	69.44%	13033.33333	6.83%	12625	3.48%	11725	-3.89%
#15	0.526666667	75.56%	13106.66667	7.43%	12900	5.74%	12220	0.16%
#16	0.545	81.67%	13180	8.03%	13175	7.99%	12715	4.22%
#17	0.563333333	87.78%	13253.33333	8.63%	13450	10.25%	13210	8.28%
#18	0.581666667	93.89%	13326.66667	9.23%	13725	12.50%	13705	12.34%
#19	0.6	100.00%	13400	9.84%	14000	14.75%	14200	16.39%

$120K better at 56%

$125K at 60% and above

Summary

- Can lay out general structure of decision problem
- Allows common framework for discussion
- Can be made quite complex, but this can destroy 'overview'
- Should always be subjected to sensitivity analysis
- People make decisions, not computers!

Book Sales

- I am a new author making a deal for the income I'll get from my new book. Two options are:
  - A fixed fee plus a small commission
  - Straight commission on sales
- I figure sales will be high, medium or low, and my payoffs under each alternative are:

Payoff Table		
SALES	FEE	COMMISSION
High	$40K	$80K
Medium	$35K	$40K
Low	$30K	$10K

- Which deal should I choose?

Book Sales (continued…)

To evaluate the decision, we need some measure of the relative likelihood of each level of book sales.

Suppose you examine all of the available evidence, and your own beliefs, to estimate:

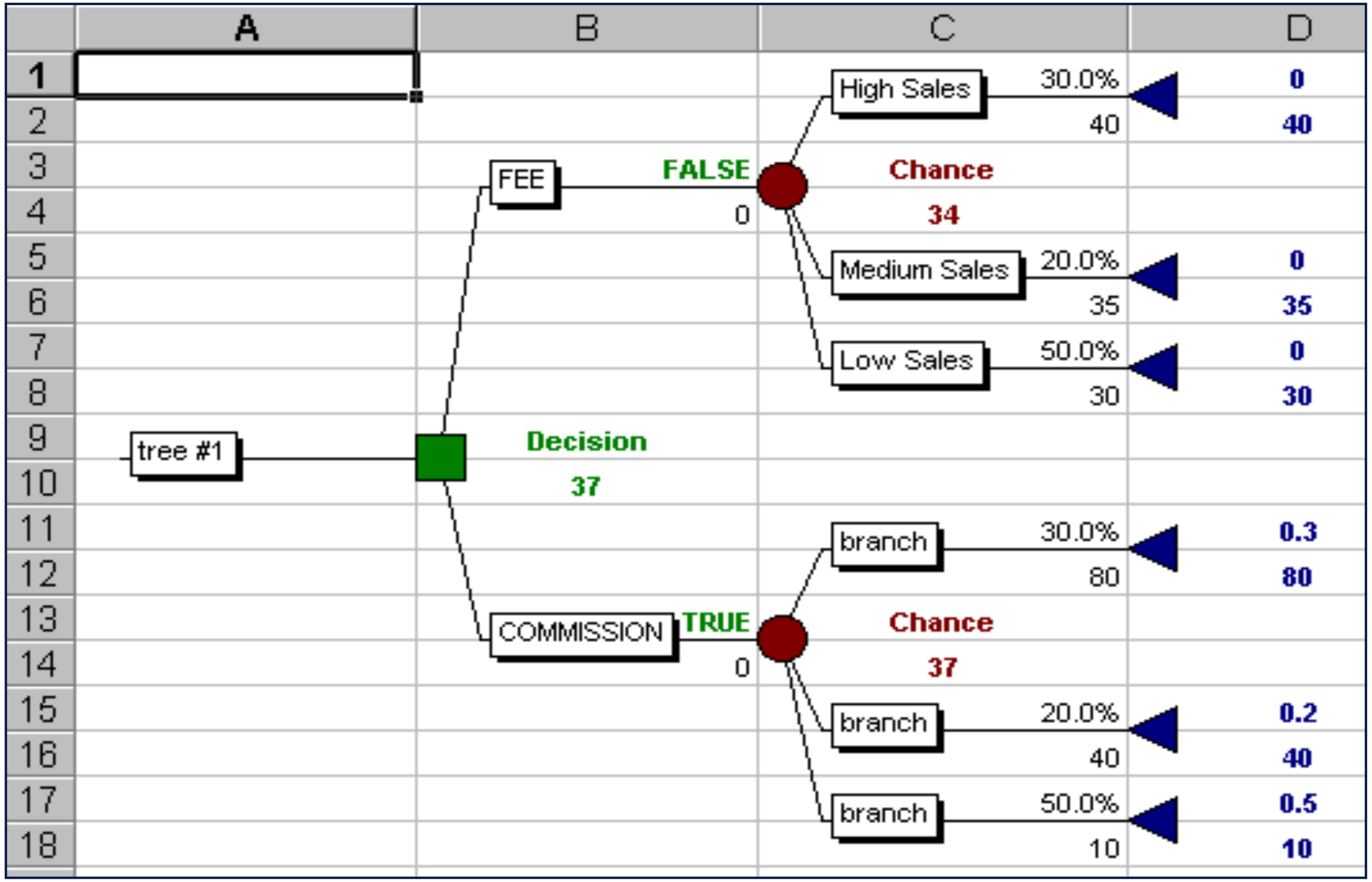
Probability (high sales) = 0.3

Probability (medium sales) = 0.2

Probability (low sales) = 0.5

Then the tree will appear:

Book Sales Decision Tree



Value of Information

- How much is information about your book sales worth to you?
- To start, how much would you pay for perfect information on sales?
  - Imagine a clairvoyant, who'll tell all for $$$. Would you pay $1,000? $20,000?
- To answer, what is your best estimate that she'll say, "high sales"? "medium"? "low"?

- Suppose, as before, your beliefs are:
  - P("high") = .3, P("med") = .2, P("low") = .5

Value of Information (continued…)

- If she says "high", you'll choose commission; if "medium", you'll choose commission; but if she says "low", you'll choose the fee option
- Your expected payoff with her information is thus equal to $80K (.3) + $40K (.2) + $30K (.5) = $47K
- Since before getting the information, with your best choice (commission) your expected payoff was $37K, her data is worth the difference, ($47K - $37K =) $10K
- Called the Expected Value of Perfect Information, or EVPI

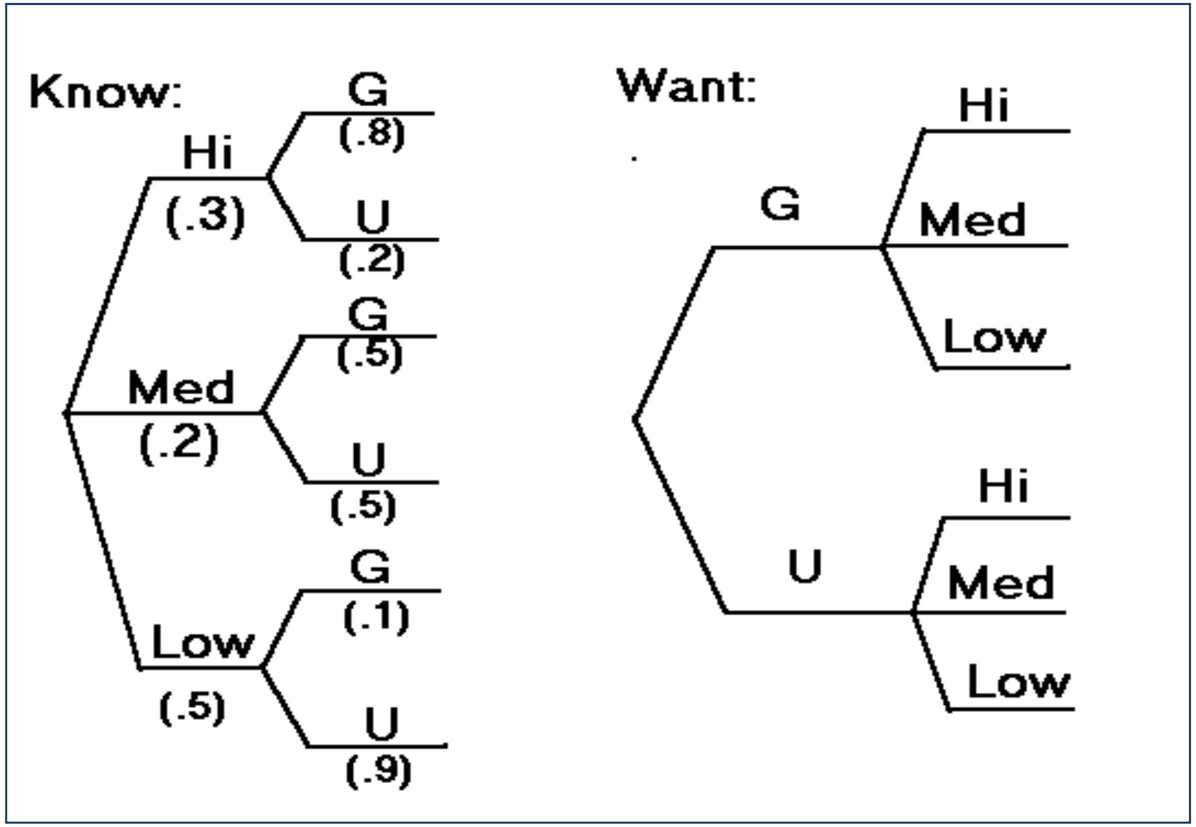
Value of Information (continued...)

- Perfect information doesn't generally exist, but this (easy) computation places an upper bound on the value of getting any information at all.
- How to deal with imperfect information?
- Continue with example, but suppose just before signing, a survey company offers to run a survey which will conclude one of two things: "Looks Good (G)" or "Unpromising (U)".
- You say, "How do I make sense of this information?"

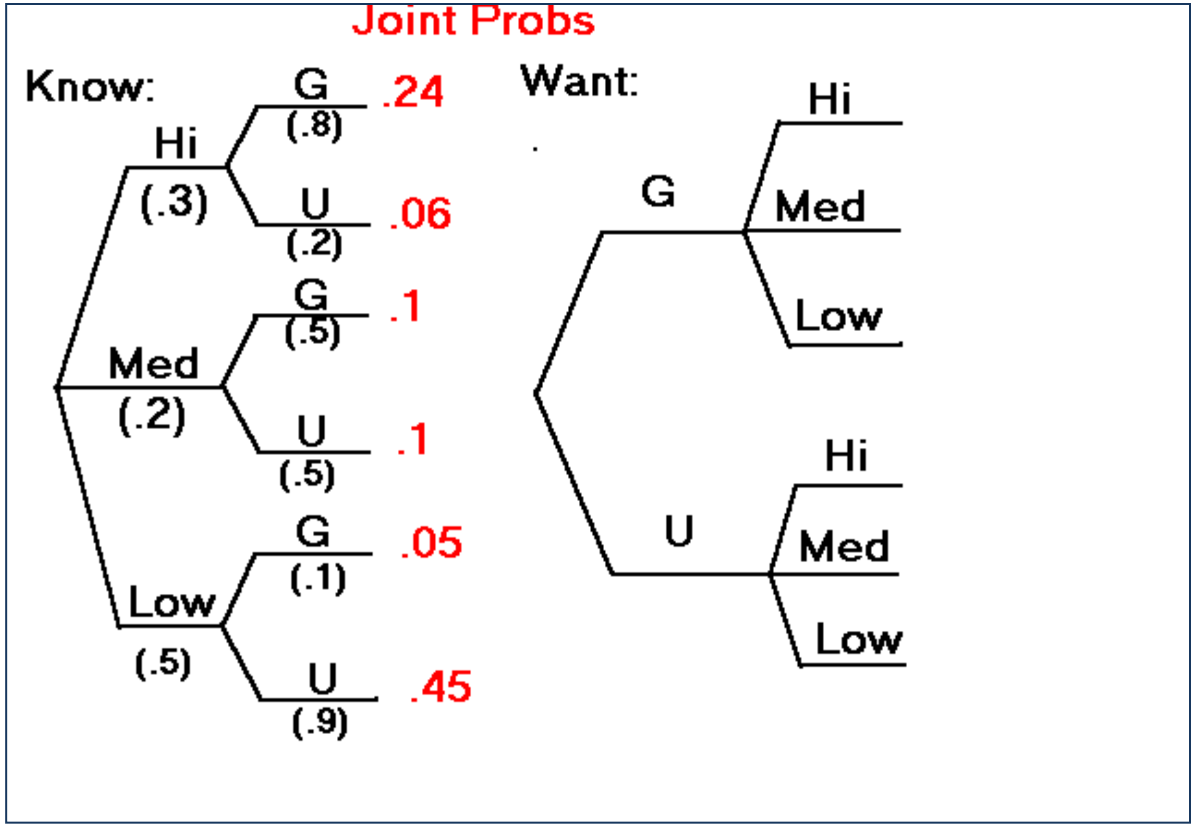
Value of Information (continued…)

- They say, "Well, it's your book. But in the past, for books which turned out to be "high", we had said "G" 80% of the time. In "medium" books, we had said "G" 50% of the time; and for "low" sellers, we said "G" 10% of the time."
- What is their imperfect survey worth to you?
  Note: it can't be worth more than $10K, the EVPI.
- Observe that the information you get depends on how good the book is – your prior beliefs describe the likelihood of each 'state'.
- How will the sample info change these beliefs?

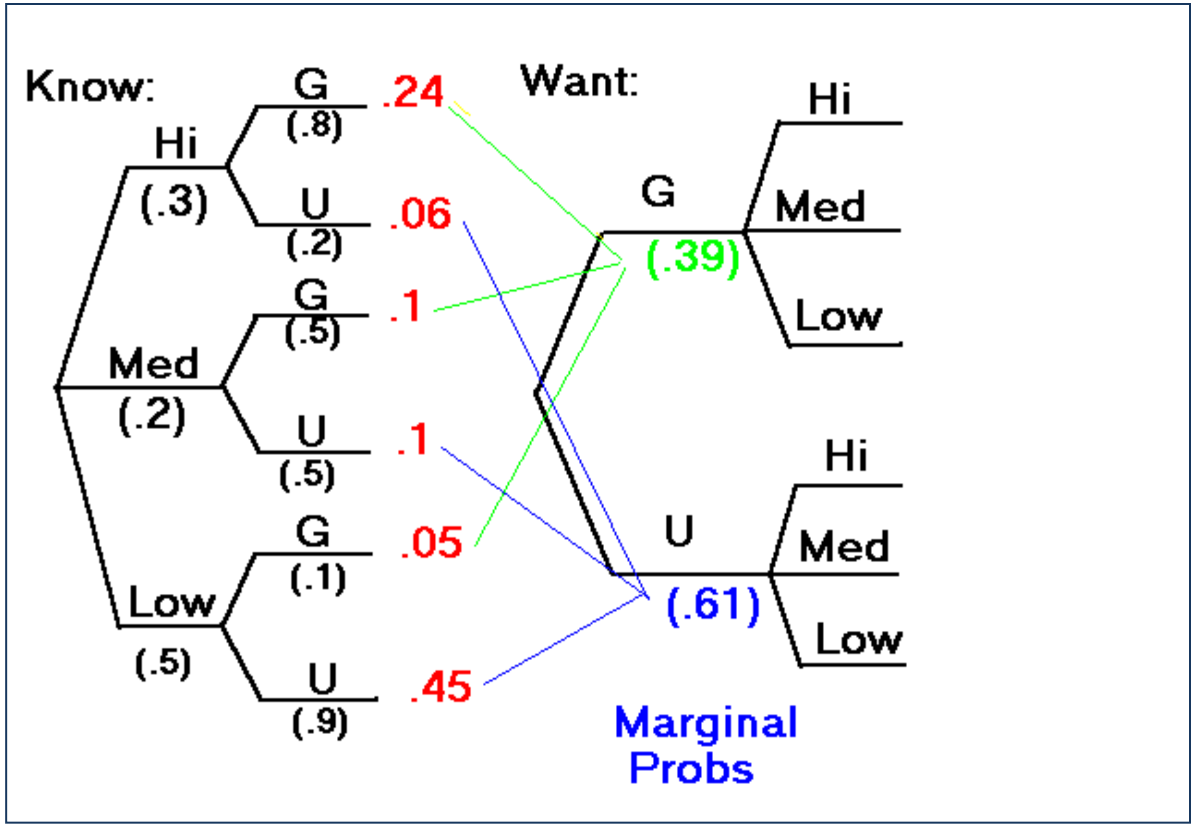
Value of Information (continued…)



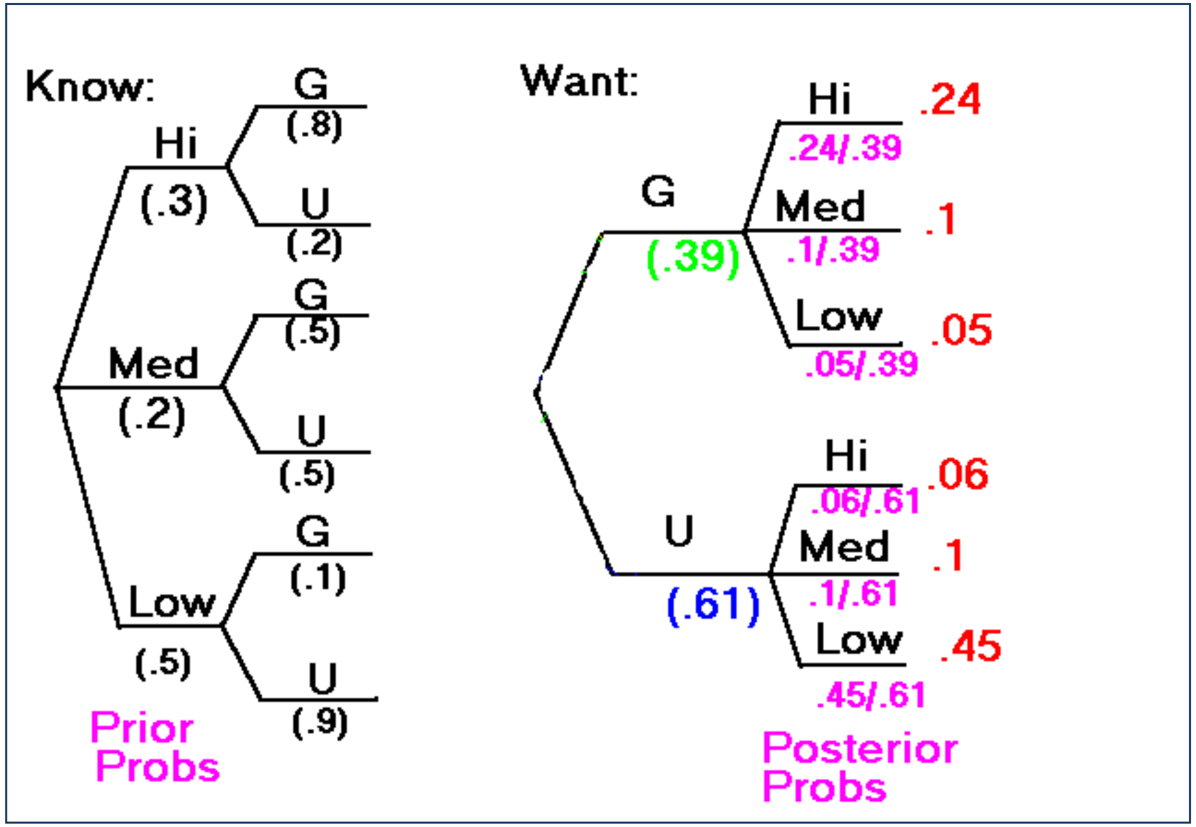
Value of Information (continued…)



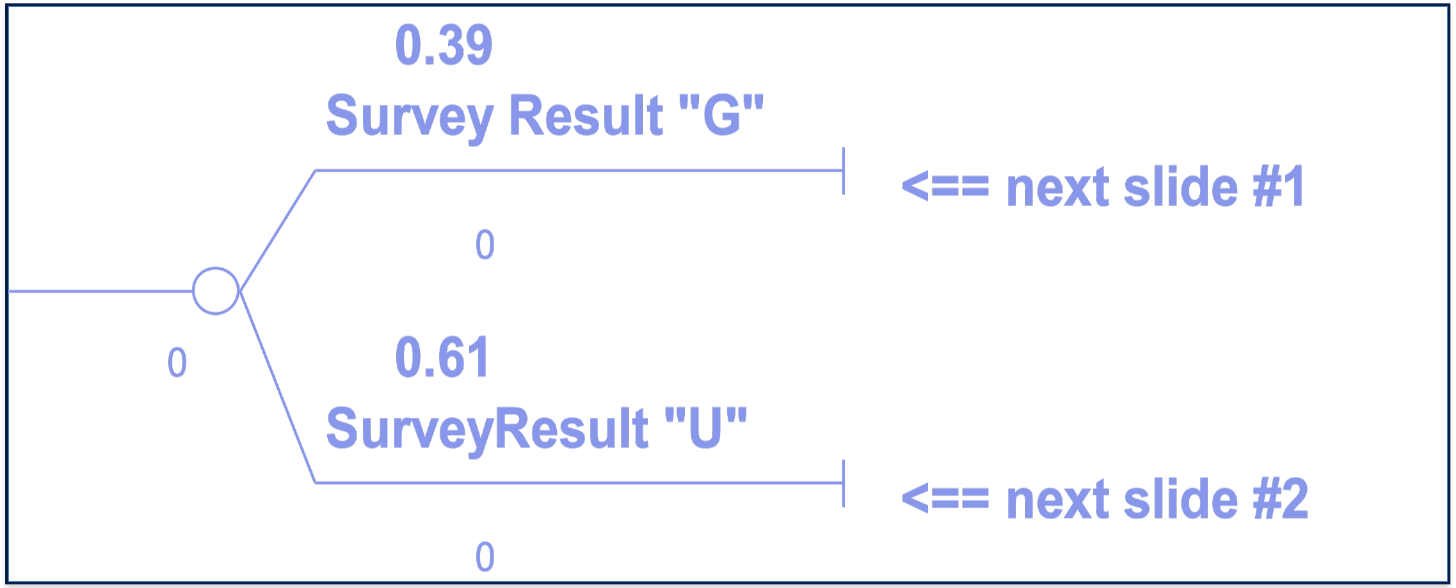
Value of Information (continued...)



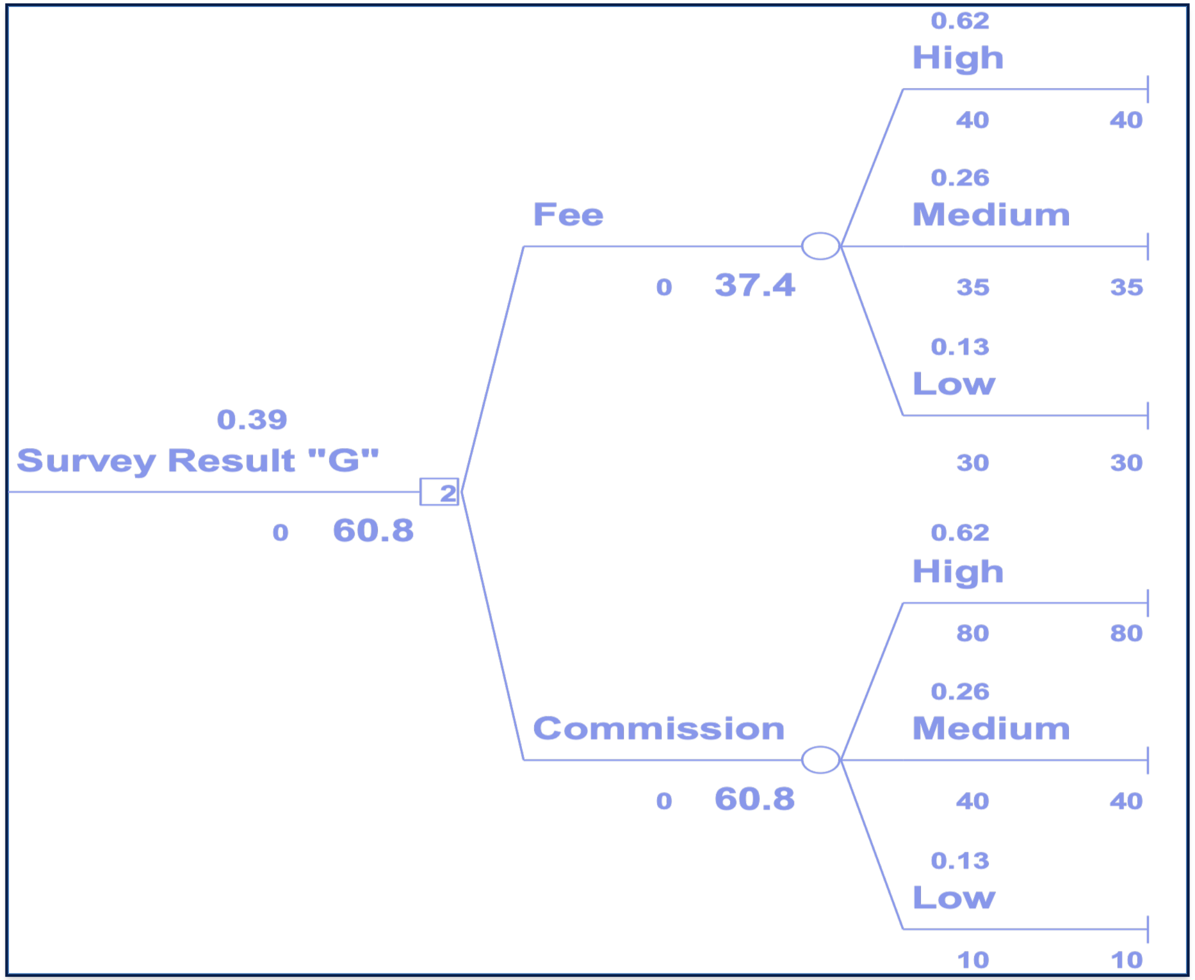
Value of Information (continued…)



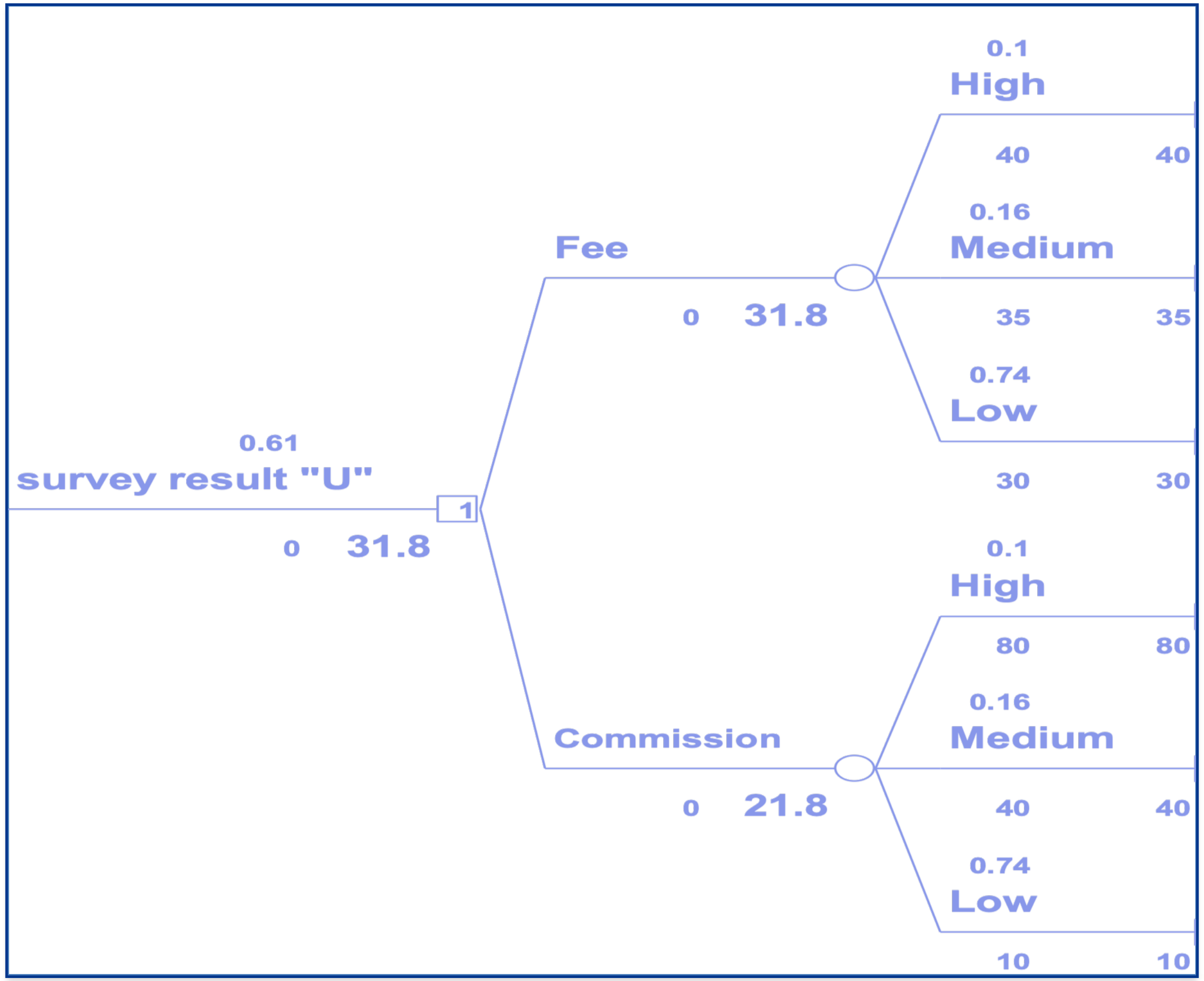
Value of Information (continued…)



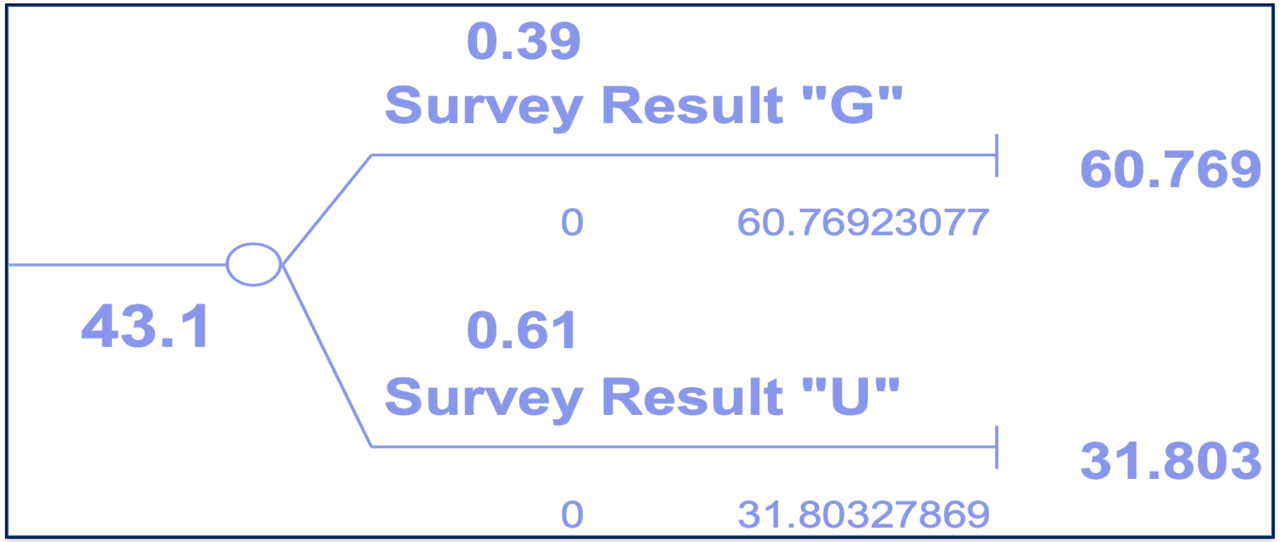
Slide One: Decision with Info "G"



Slide Two: Decision with Info "U"



Value of Sample Information



Expected value with the information = $43,100

Expected value without information = $37,000

Thus, expected value of sample information = $6,100

Summary

- With no information, the EV was $37K
- The survey information yielded an EV of $43.149K
- The value of the sample information, EVSI, is (43.149 - 37) = $6,149
- N.B. The information was included thru its effect on changing your prior beliefs about the true state of nature to the posterior beliefs, and thus affecting your decision in some cases.
- Information which doesn't change a decision has no value!

Practice Problem: Texaco *versus* Pennzoil

In 1984, Pennzoil and Getty Oil agreed to merge. But before they signed, Texaco offered Getty a better price, and Getty sold to Texaco. Pennzoil cried "foul" and sued Texaco for illegal interference. Pennzoil won, for $10.3 billion. James Kinnear, Texaco CEO, had said Texaco would file for bankruptcy if Pennzoil was allowed to file liens against Texaco's assets, and further promised to fight the case up to the Supreme Court (partly because Pennzoil had not followed SEC regulations in dealing with Getty).

Practice Problem: Texaco *versus* Pennzoil (continued…)

In April 1987, just before Pennzoil began to file liens, Texaco offered to pay Pennzoil $2 billion to settle the entire case.

Hugh Liedtke, chairman of Pennzoil, indicated that he had been advised that a settlement of between $3 billion and $5 billion would be fair. Also, in considering what would happen with the Supreme Court, he felt that there was some chance that the original $10.3 billion would stand, a more likely chance that a mid-point would be suggested by the court, and quite a high chance that the court would overthrow the judgment.