MMA/MMAI 861
ANALYTICAL DECISION MAKING

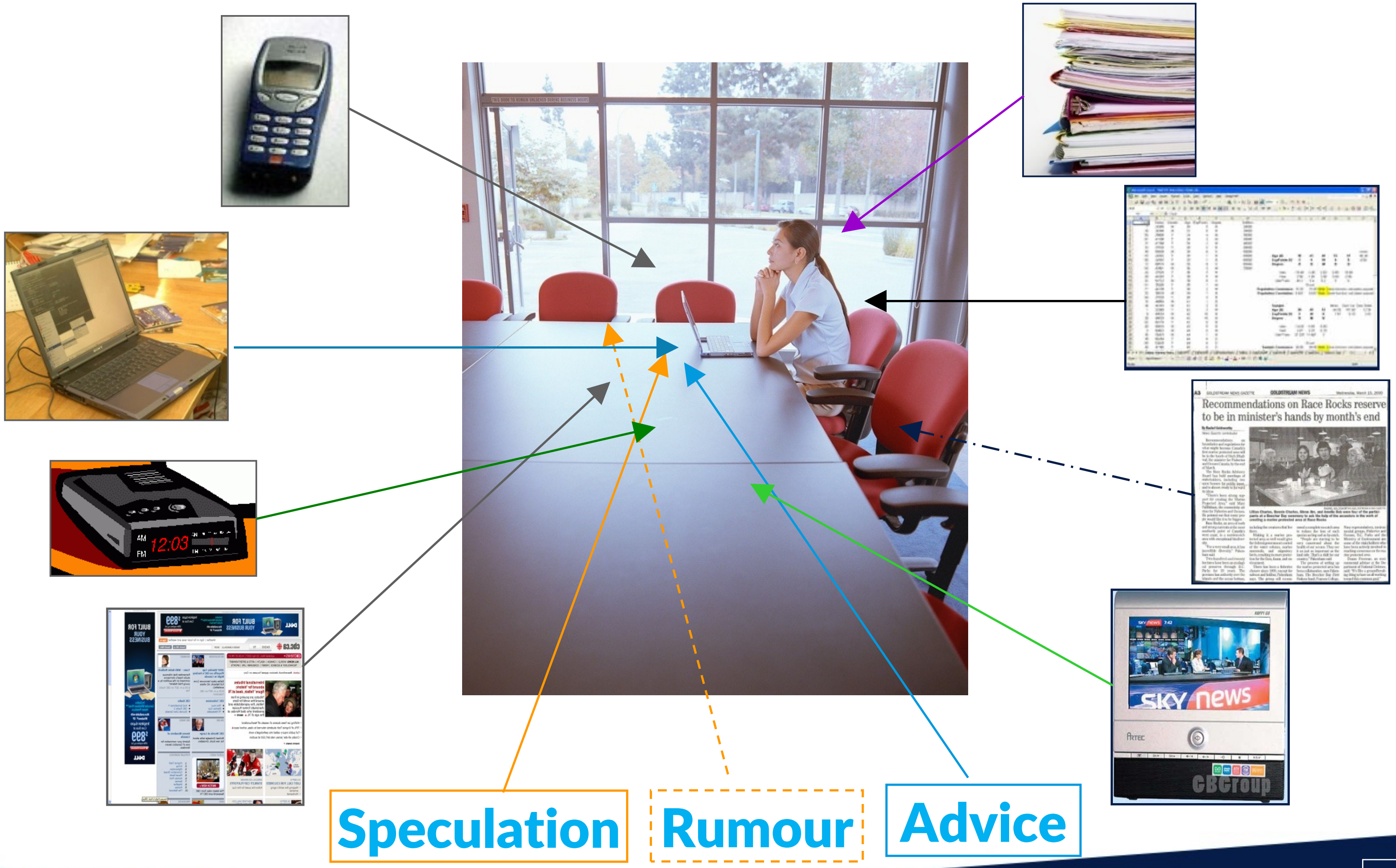
Introduction to Modeling
Optimization Models

Class One
Yuri Levin

Course Administration

- Welcome and introduction
- Course overview
- Support material
  - Winston, Albright
  - Notes and slides
- Assignments (team and individual)
- Term project
- Class presentations

Data Complexity



A Simple Game

1. I will auction a $20 bill
2. You can bid in increments of $1
   - That is, if you want to raise the current bid, you can only do so by $1
3. The bidders with the highest and second highest bids pay their bids
4. The highest bidder gets the bill

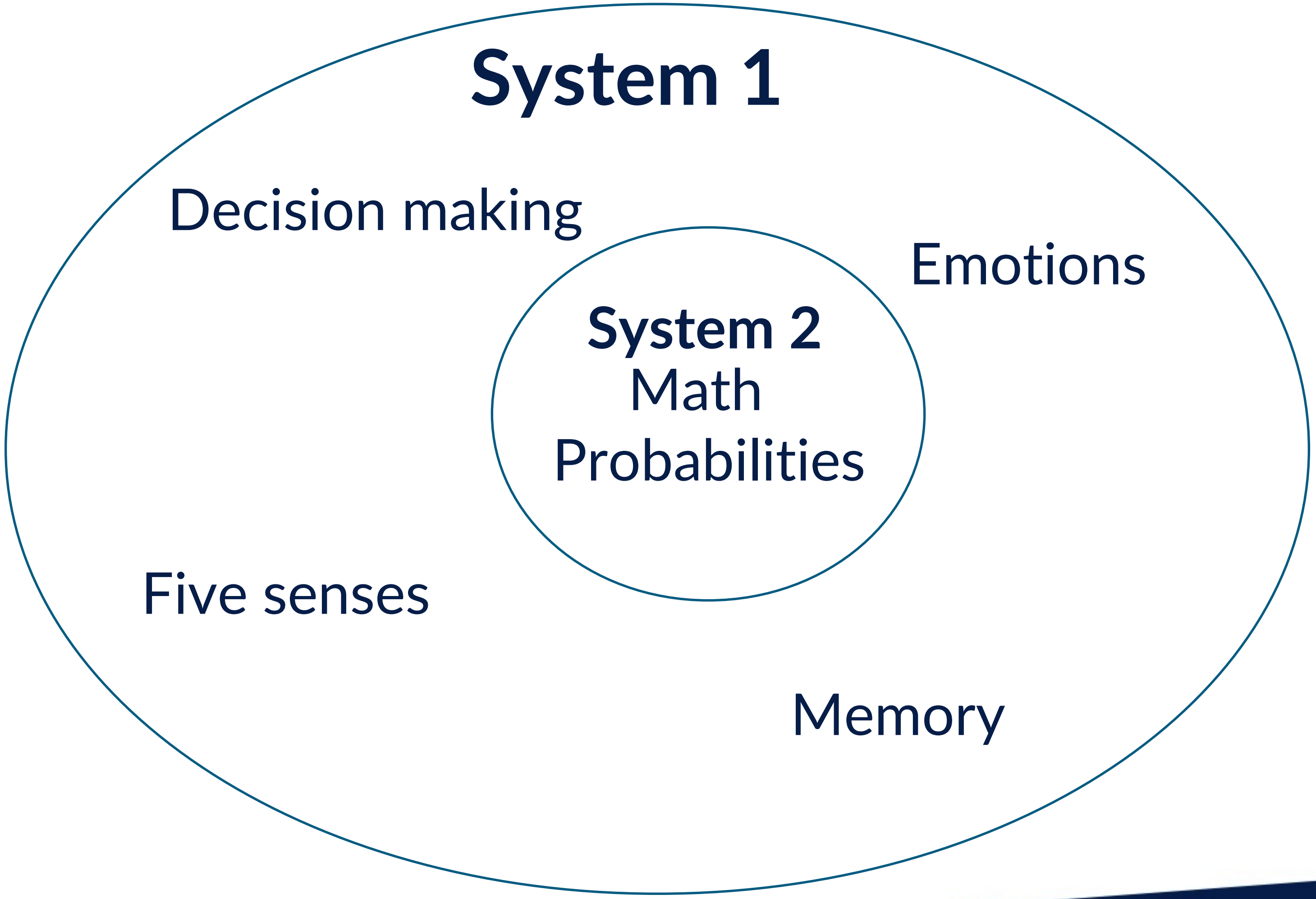
What is your Basic Philosophical Tendency when Faced with Important Decisions?

- Statement 1: Too much analysis can cause you to get "lost in the weeds" and miss the more important consequences of decisions

Intuition, gut, emotion… SYSTEM 1

- Statement 2: Thorough, rigorous analysis, with numbers, is essential for making good decisions by helping you avoid the pitfalls of faulty intuition

Rational thinking, analysis… SYSTEM 2


System 1
Decision making
Emotions
System 2
Math
Probabilities
Five senses
Memory

… and the winner is?

SYSTEM 1!!!

(if the situation is sufficiently *typical* it makes decisions automatically – you are not even aware)

But it is not the panacea that many of you think (hope?) it is…

Question 1: Tough Choices

You are the CEO of a company faced with a difficult choice. Due to structural changes in the economy, one of the divisions, which currently employs 6000 workers, is threatened with shutdown. Your staff has identified two options to address this crisis.

- **If Plan A is adopted, 4000 jobs will be lost.**
- **If Plan B is adopted, there is a one-third probability that no jobs will be lost, and a two-thirds probability that 6000 jobs will be lost.**

- **If Plan A is adopted, 2000 jobs will be saved.**
- **If Plan B is adopted, there is a one-third probability that 6000 jobs will be saved, and a two-thirds probability that no jobs will be saved.**

FRAMING BIAS

Framing

- Why is this a problem?
  - The way the problem is stated is an anchor
  - Frames are hard to see
  - Drastic differences in decision about the same problem framed differently
- Curious examples:
  - "Can I smoke while praying, vs. can I pray while smoking"
  - Cash-match pension contributions, DMV organ donations
- What can be done?
  - Don't automatically accept the initial frame (your own or from others)
  - Reframe the problem in a new way
  - Try posing the problem in a neutral way, combine gains and losses
  - Ask yourself how your thinking will change if the frame changes
  - If others recommend decisions, examine how they framed the problem

Question 2: Potential Market Size

- What is your best estimate of the population of Fiji?

- Do you believe that the population of Fiji is more than 1.5 million people?

- Do you believe that the population of Fiji is less than 500 thousand people?

ANCHORING BIAS

Anchoring

- Why is this a problem?
  - Your judgment is severely affected by the first information you receive
  - This info could be totally random
  - Or, worse, specifically engineered to anchor you to a disadvantageous point
- What can be done?
  - View problem from different perspectives
  - Think through the problem on your own before talking to others
  - Be open-minded (seek information, widen your frame)
  - Be wary of anchors in negotiations, consultations
  - Try to use anchors to your own advantage

Question 3: Causes of Death

What causes more deaths in the U.S.?

I usually see

- Stomach cancer (95,000) **25%**

or

- Motor vehicle accidents (46,000) **75%**

- Tuberculosis (4,000)

or

- Fire and flames (5,000)

VISIBILITY/BASE RATE/RECALLABILITY BIAS

Base Rate/Rare Events, Recallability/Confirmation

- Why is this a problem?
  - Contaminates the way you process information
  - Tendency to disregard information that conflicts your existing belief
  - Or focus on the information most widely available
- Curious examples:
  - What is more likely: to die from stomach cancer or car accident?
- What can be done?
  - Get someone to play devil's advocate
  - Build counter arguments yourself
  - Expose yourself with conflicting information
  - Seek others opinion but don't ask leading questions that invite them to confirm your thinking
  - Get statistics!

Question 4: Uncertain Quantities

1. Books sold in the U.S. in year 2000
2. U.S. workforce in year 2000
3. Gold medals won by team Korea in 2004 Olympics
4. Population of Lichtenstein as of Dec 31, 2004
5. Number of pirate attacks in 2004
6. Num. cigarettes smoked per capita in Greece in 2004
7. Num. of cars per capita in Spain in 2000
8. Length of Mekong river in km
9. Year when General Motors' sales first reached $1bln
10. Market value of Heinz as of March 14, 2003

1. 2.36 bln
2. 140 mln
3. 9
4. 33436
5. 325
6. 3100
7. 0.408
8. 4425
9. 1926
10. 10.6 bln

OVERCONFIDENCE BIAS

Overconfidence

- Why is this a problem?
  - Severe underestimation of risk or opportunity
  - Propagates through organization (your direct reports are overconfident -> you don't receive correct info to make decisions)
- Curious examples:
  - "I think there is a world market for about five computers" T.J. Watson, IBM Chair, 1943
  - 82% of drivers say they drive safer than average, 68% of lawyers believe they will win
- Organizational pressures to be overconfident:
  - Experts report tighter estimates to demonstrate superior knowledge with respect to their peers
  - “You are paid to know, not to not know…"
- What can be done?
  - Concentrate on extremes, challenge your extremes
  - Anchor and adjust heuristic, but adjust for MUCH MORE
  - Systematically track the accuracy of your experts' (colleagues', subordinates') estimates
  - Specialized training to be decisive and calibrated, decomposition heuristic

Sunk-cost and Status Quo

- Why is this a problem?
  - Sense of ownership with respect to the alternatives one selected earlier
  - Failures (sunk-costs) are psychologically difficult to forgo
  - Lead to major errors in decision making
- Curious quote:
  - "When you find yourself in a hole, the best thing you can do is stop digging", Warren Buffet
- What can be done?
  - Seek views from the people who were not involved in the earlier decisions
  - Why admitting a mistake distresses you? Remember – a smart decision does not always mean a good outcome
  - Be aware of sunk-cost bias of your subordinates (colleagues, etc.)
  - Always remind yourself of the objectives
  - Never think your existing alternative is the only one
  - Would you have chosen your current alternative if you had to do that again?

Decision Biases

System 1 quickly "wins" relying on rules of thumb and intuition, but is easily biased (translation: wrong!)

- Framing
- Anchoring
- Visibility/base rate/recallability (rare events, confirmation)
- Overconfidence
- Sunk-cost
  - Bankers who originated "problem loans" are far more likely to advance additional funds to the "problem borrowers" (hoping that they will then recover), than the bankers who took over the "problem accounts"
- Status quo
  - The more choices you have, the more likely you are to decide to stay where you are now

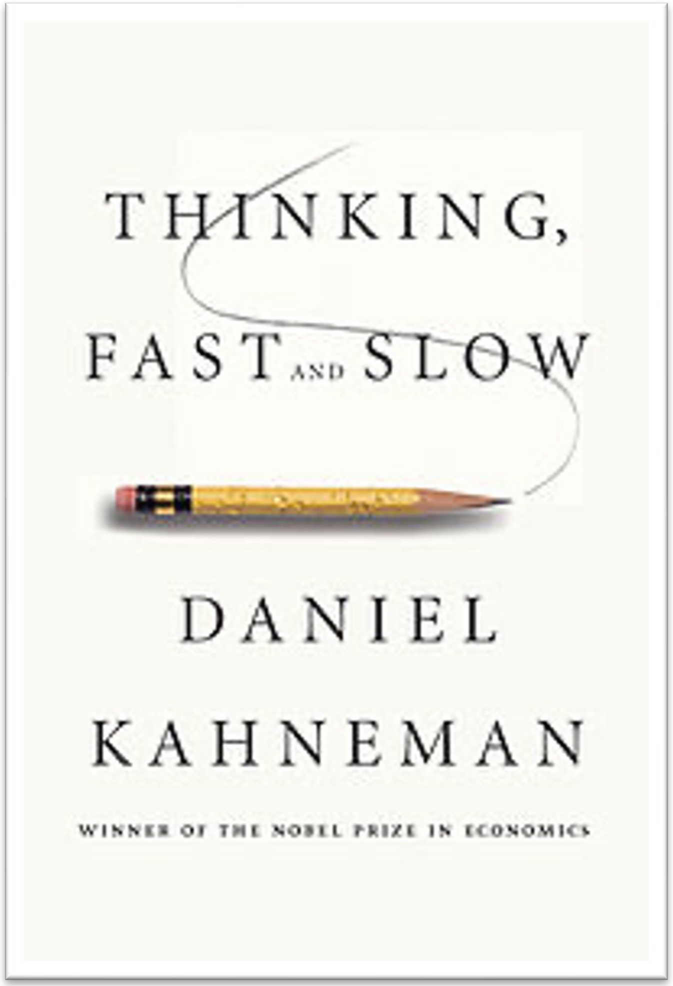
What Can We Do About It?

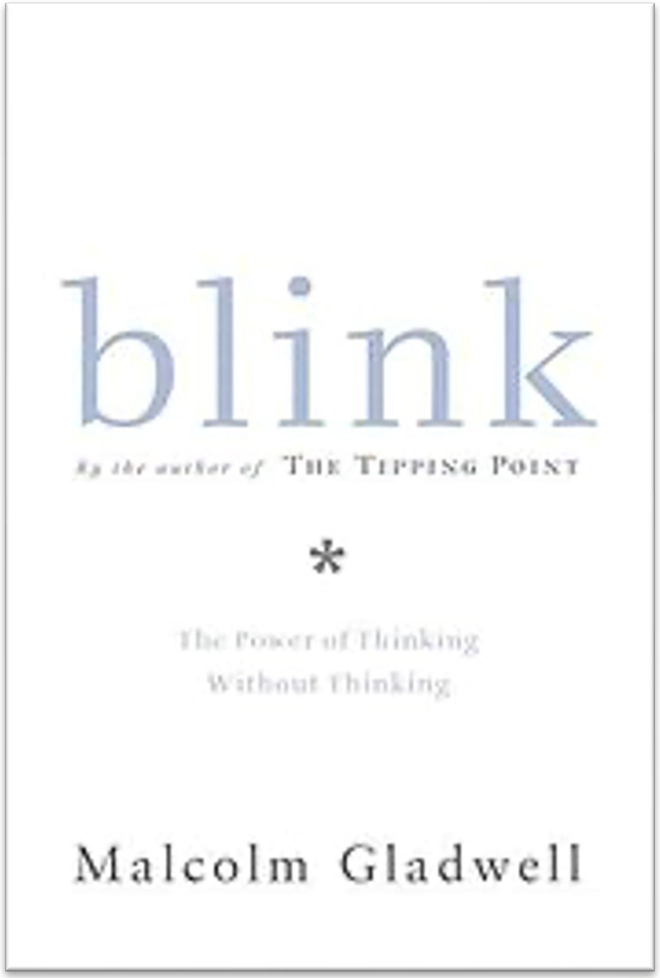
- Defer to SYSTEM 2
  - System 2 has its own set of issues/limitations
  - http://www.youtube.com/watch?v=IGQmdoK_ZfY
  - When System 2 is engaged, other functions (such as sensing) shut down, because System 2 is work and work requires resources
  - Multi-tasking vs "multi-failing"?

- For IMPORTANT decisions, you CAN NOT override System 1…
  - Thus must train your System 1 to recognize the need to utilize System 2 for some decisions

ANALYTICAL DECISION MAKING

Amazing Reads…





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Tons of Data - What about Information?

- Management decisions must be based on 'information'
- Traditionally such information is informal
  - Management experience, impressions, opinions...
- Increasing pressure to base decisions on 'hard' information
  - 'Due diligence', 'evidence based decision-making'
- Many organizations are drowning in data, not information
- Conversion of data to information
  - Statistical analysis
  - Probability analysis
  - Models...

Quick Question

- 100 units for sale each week
- Sales history for the last 3 weeks:

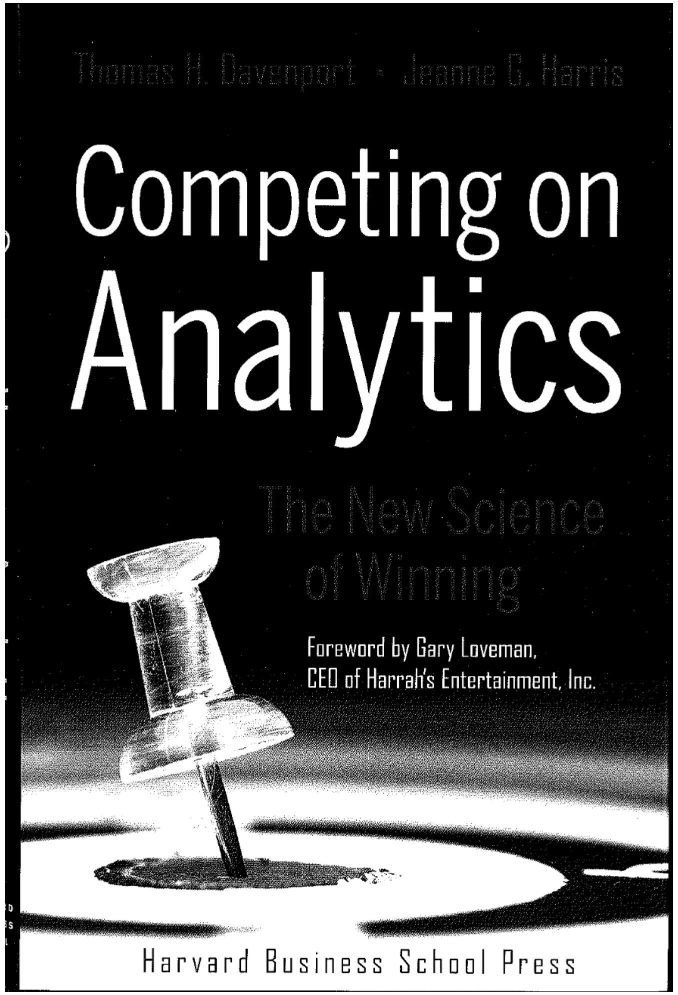
  week 1 – 100

  week 2 – 70

  week 3 – 100

What is the average demand? What do you think about the quality of the current pricing policy?

Competing on Analytics



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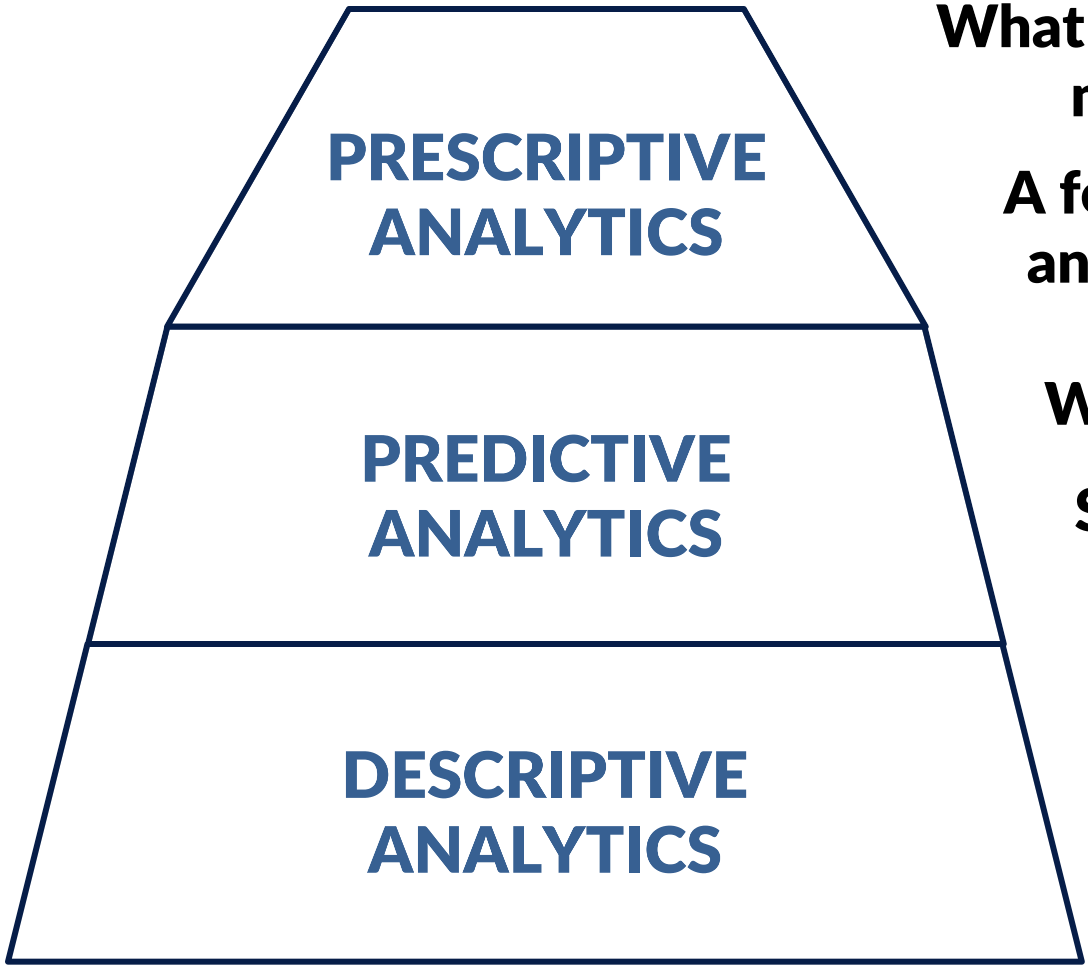
Analytics Evolution



What is best? How do we make it happen?

A few organizations use analytics: Optimization

What is going to happen?

Some organizations use analytics: Modeling

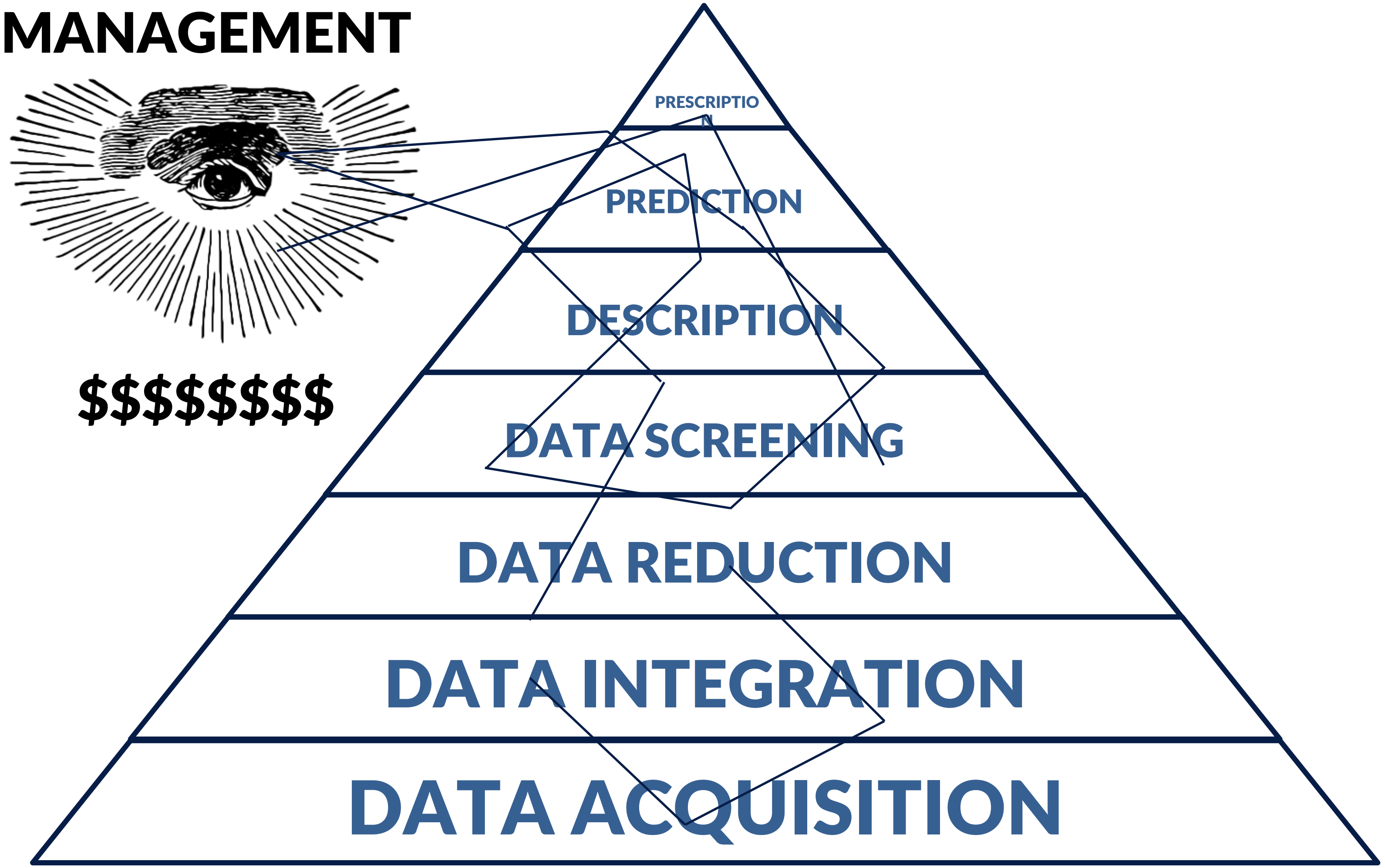
What is happening?

Many organizations use analytics: Statistics

The Full Big Data / Analytics Picture

MANAGEMENT

$$$$$$$$



Models

- All decisions are based on models
- *Implicit models*
  - Decision-maker's "view of the world"
- *Explicit models*
  - Conceptual (Theory X -- Theory Y)
  - Physical (model plane in wind tunnel)
  - Schematic (process flow chart)
  - Mathematical
- Example: (Revenue = price times quantity)

$$R = p \times q \quad \text{or} \quad R = p\,q$$

Mathematical Models

- Involve systems of relationships, for example:

$$R = p\,q$$

$$C = F + V\,q$$

total cost = fixed cost + variable cost × quantity

$$P = R - C$$

profit = revenue – total cost

$$P = (p - V)q - F$$

profit = contribution × quantity – fixed cost

Some Practical Issues

- Terrific, we have an equation that can predict profit!
- BUT
  - Costs can 'bounce around'
  - Quantity and price are related…
- How do we develop 'good' estimates of costs?
  - *Statistical sampling and estimation*
- How can we estimate the price/quantity relationship?
  - *Regression model*
- How do we find a price that may improve profit?
  - *Optimization*

More Challenges...

- If the world is such a variable place, how can we assess the variability of the profit that we have 'optimized'?
  - *Probability analysis*
  - *Simulation model*
- How might we assess a decision to enter a new market when the future is so uncertain?
  - *Decision analysis*
- Real decisions often have many impacts, not just one. How can we assist choices involving multiple factors?
  - *Multi-criteria decision analysis*

'Soft' Versus 'Hard' Data and Information

- 'Soft'
  - Intuition/guesswork/rumours/opinions…
  - Harder to explain decisions
  - Often all we have

- 'Hard'
  - Allows statistical and other analyses
  - Easier to explain decisions
  - Can be badly misused

- Many (most?) management decisions are influenced by both of these

A Key Challenge

- We rarely have the luxury of having complete data about <u>any</u> situation
- Usually must work with samples
- Samples subject to random fluctuations
- So-called 'hard' data is actually pretty shaky
- How can we assess patterns of 'shakiness'?

A Link Between 'Soft' and 'Hard' Data?

- Soft:
  Can assess degree of belief through probabilities
- Hard:
  Can assess patterns and reliability of estimates through probability analysis

Examples

- Soft:
  "This training program is going to cost us $400K, how sure are we that it will actually improve customer service?"
  "Well we are 80% sure..."
- Hard:
  "Among *100 employees with this profile, 23 quit within two years of hiring. The probability that a new employee with this profile will quit is...?"*
- We can perform structured decision-making combining these types of probabilities
- Playing the odds is not just for poker players

Course Topics

We will explore the most common and applicable mathematical models, including:

- Optimization models
  - Linear, integer
- Simulation models
- Decision analysis

Example: Beverage Planning

Want to maximize 'good time' within a budget of $30 and without exceeding a calorie intake of 1,500 calories

	Price ($/oz)	Alcohol (oz/oz)	Calories/oz
Beer	0.20	0.05	15
Wine	1.20	0.12	30
Limit	$30		1,500

Formulations Steps

- Define the objective in words
- Identify the decision variables and their units of measurement
  - Define meaningful symbols
- Write out the objective function
  - Check the units
- Describe each constraint in words
- Write out the constraints
  - Check the units

Problem Formulation

Objective?

– Maximize alcohol (ounces)

Decision variables?

– B: Beer consumption (ounces)

– W: Wine consumption (ounces)

Constraints?

– Budget: maximum $30

– Diet: maximum 1,500 calories

Mathematical Model

$$
\begin{array}{llclclcl}
\max & z & = & 0.05B & + & 0.12W & & \\
\text{s.t.} & & & 0.2B & + & 1.2W & \leq & 30 \\
 & & & 15B & & 30W & \leq & 1500 \\
 & & & B & & & \geq & 0 \\
 & & & & & W & \geq & 0
\end{array}
$$

Questions:

All elements present?

Units balance?

An Excel Spreadsheet Model: Setup

	A	B	C	D	E	F	G	H	I	J	K	L	M
1		RESOURCE ALLOCATION PROBLEM											
2		DATA:					MODEL:						
3			alcohol (oz/oz)	price/oz	calories/oz				Cost	Calories	Alcohol		
4		Beer	0.05	0.2	15		beer oz	1	0.2	15	0.05		
5		Wine	0.12	1.2	30		wine oz	1	1.2	30	0.12		
6		Limits		30	1500		TOTALS		1.4	45	0.17		
7													
8													
9													

=H4*D4

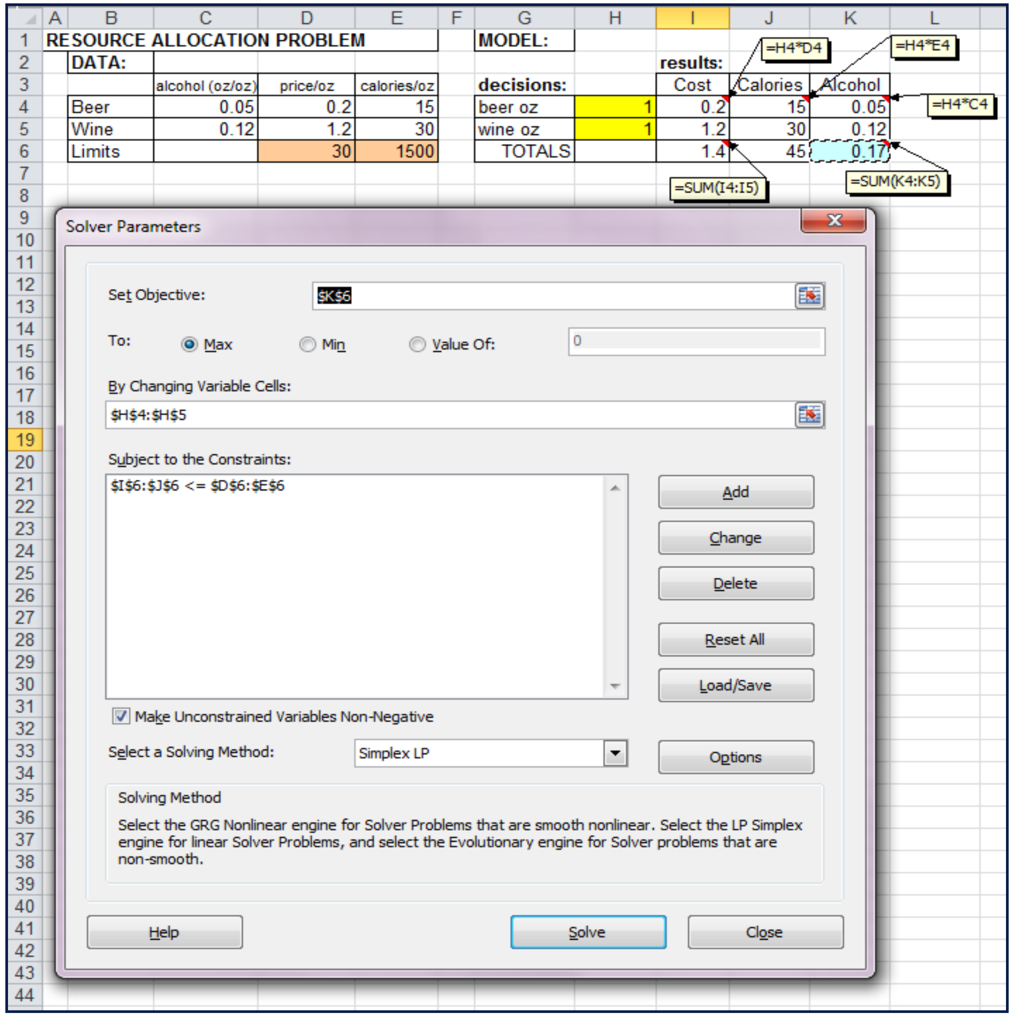
=H4*E4

=H4*C4

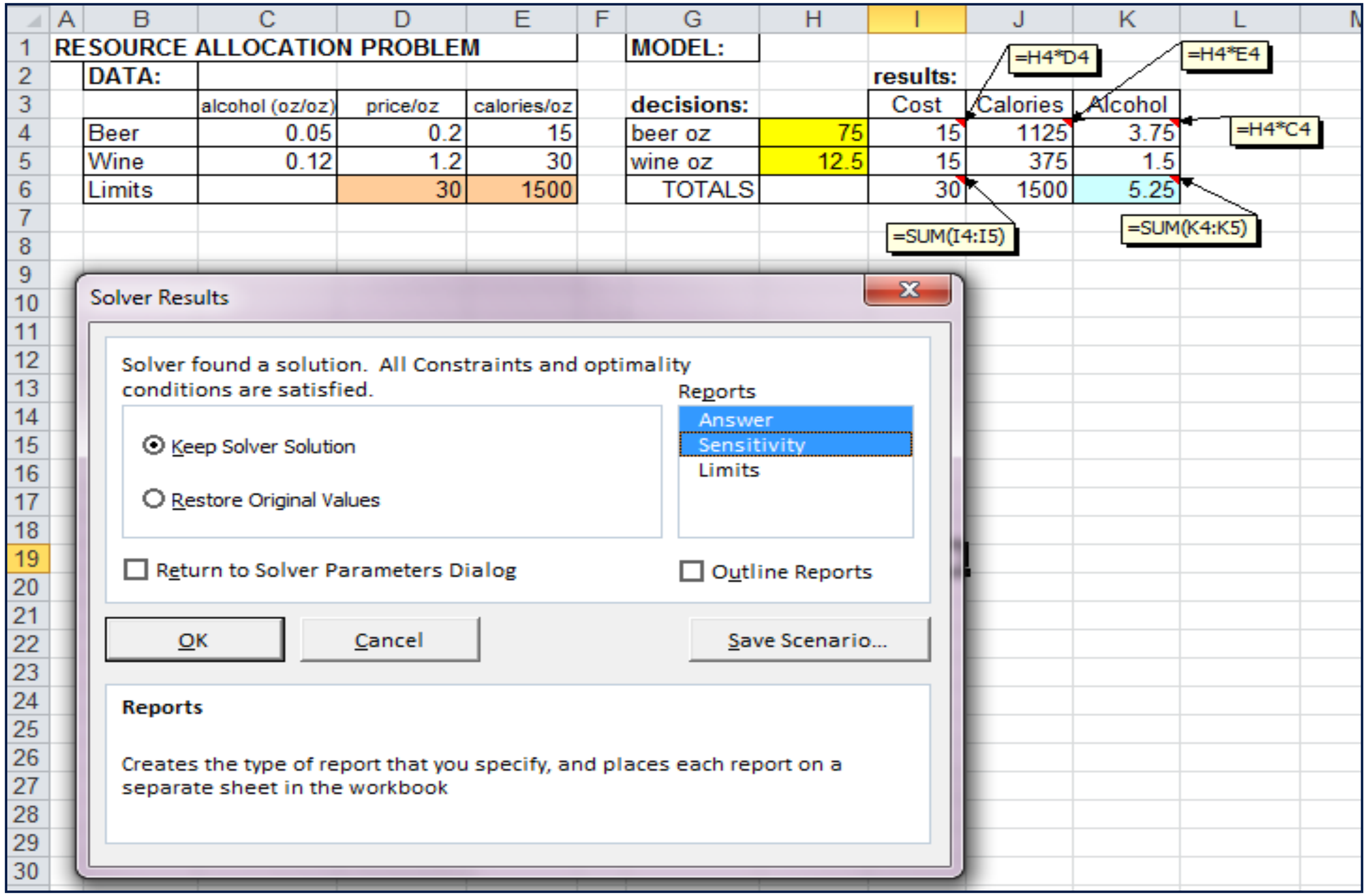
=SUM(I4:I5)

=SUM(K4:K5)

An Excel Model: Solver Dialog Box



An Excel Model: Solution



Solver Answer Report

Microsoft Excel 14.0 Answer Report
Worksheet: [beer_wine.xls]Good time layout #1
Report Created: 2/23/2012 2:17:59 PM
Result: Solver found a solution. All Constraints and optimality conditions are satisfied.
Solver Engine
Engine: Simplex LP
Solution Time: 0.016 Seconds.
Iterations: 2 Subproblems: 0
Solver Options
Max Time 100 sec, Iterations 100, Precision 0.000001
Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 5%, Solve Without Integer Constraints, Assume NonNegative

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$K$6	TOTALS Alcohol	0.17	5.25

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$H$4	beer oz	1	75	Contin
$H$5	wine oz	1	12.5	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$I$6	TOTALS Cost	30	$I$6<=$D$6	Binding	0
$J$6	TOTALS Calories	1500	$J$6<=$E$6	Binding	0

Solver Sensitivity Report

Microsoft Excel 14.0 Sensitivity Report
Worksheet: [beer_wine.xls]Good time layout #1
Report Created: 2/23/2012 2:17:59 PM

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$H$4	beer oz	75	0	0.05	0.01	0.03
$H$5	wine oz	12.5	0	0.12	0.18	0.02

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$I$6	TOTALS Cost	30	0.025	30	30	10
$J$6	TOTALS Calories	1500	0.003	1500	750	750

Optimization Models: Linear Programming Models

- Identification of LP problems
- Formulation of LP's
- Solution with EXCEL
- Sensitivity analysis
- Formulation examples:
  - Product-mix LP models
  - Multi-period production LP models
  - Blending models
  - Combination of models

Linear Programming: Background

- Kantorovich, Koopmans (1930's - 1940's)
  - Allocation of constrained resources
  - Linear relationships
- Dantzig: Simplex method (1948)
- Electronic computers
  - One of earliest applications
- Nobel prize (1975)
- Continuing research: e.g. Karmakar (1984)

Examples of Areas of Application

- Refinery production mix/scheduling
- Lumber/paper cutting
- Staff scheduling
- Blending (diets, pet foods)
- Logistics/transportation
- Airline crew scheduling
- …and many more

Formulations Steps

- Define the objective in words
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  - Define meaningful symbols
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  - Check the units
- Describe each constraint in words
- Write out the constraints
  - Check the units

Recognizing LP Problems

- Single, quantifiable objective
  - Otherwise *multiple criteria* problem
- Continuous decision variables
  - e.g. ounces of liquid, number of employees (if fractions make sense)
  - Otherwise: *integer programming problem*
- Linear constraints
  - Unit change in any decision variable changes resource consumption by a constant amount
- Linear objective function

Product-Mix LP Example: Bicycle Wheels

Free Wheelers, Inc. makes spokes and rims for mountain bike wheels. Recently, the firm has also started selling assembled wheels.

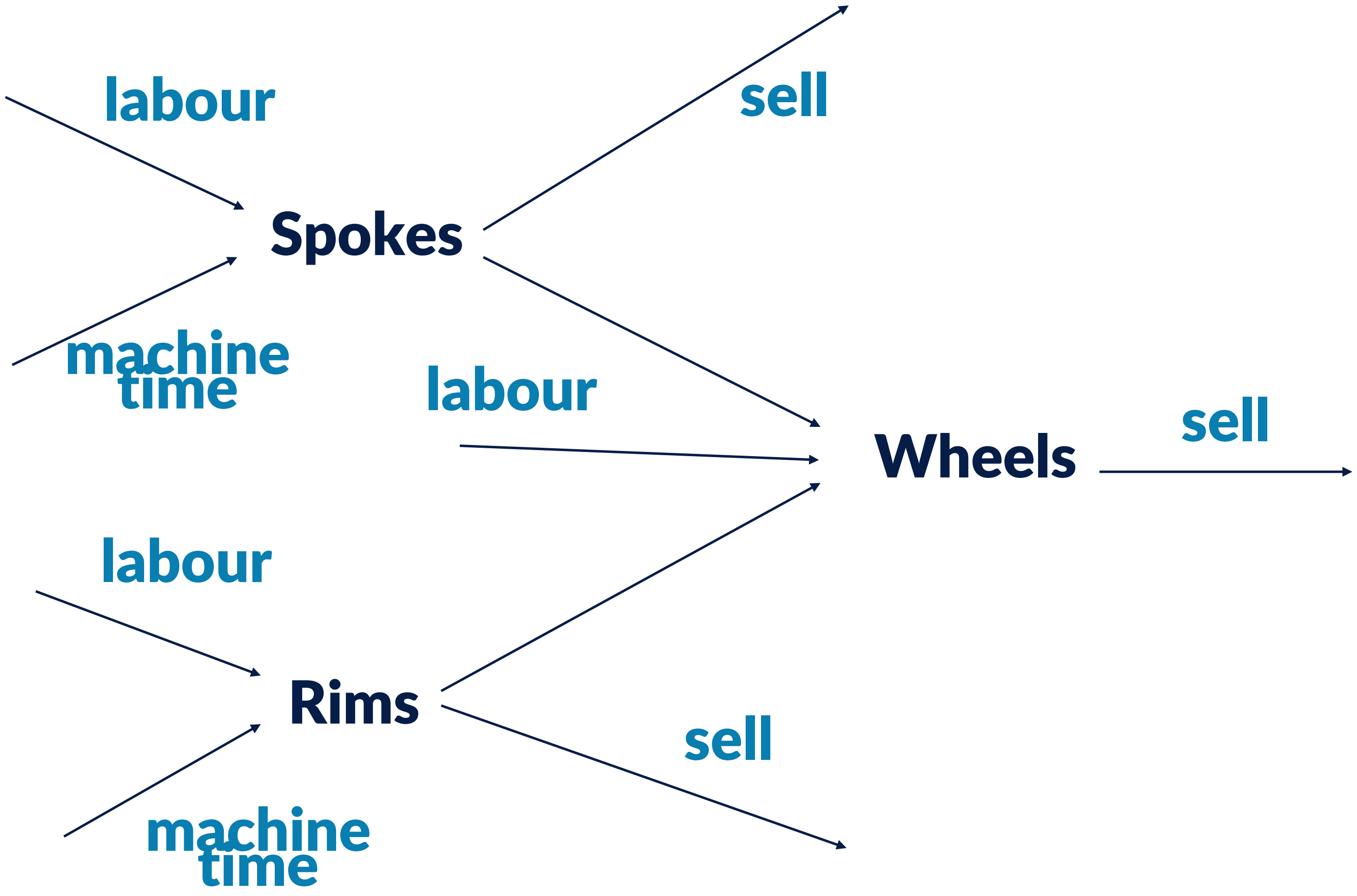
Each spoke requires 0.04 hours of machine time and 0.01 hours of labour, and costs $0.05 to make. Each rim requires 0.6 hours of machine time and 0.5 hours of labour, at a cost of $4.

An assembled wheel requires one rim and 72 spokes. Assembling a wheel takes 1.2 hours of labour, in addition to that needed to make its rim and spokes; no further machine time is needed. The assembly process also incurs additional direct costs of $11. Up to 378 hours of machine time and 267 hours of labour are available per week.

Assembled wheels sell for $50. Spokes not used to make assembled wheels can be sold for $0.10 each. Similarly, rims not used in assembled wheels can be sold for $10 each. Assume that, at these prices, the firm can sell all the products it can produce.

How to maximize Free Wheelers' weekly profits?

Bicycle Wheels (continued...)



Bicycle Wheels: Problem Formulation

Objective?

- Maximize weekly profits ($)

Decision variables?

- S: number of spokes to be produced
- R: number of rims to be produced
- W: number of wheels to be produced

Constraints?

- Machine time: maximum 378 hours
- Labour: maximum 267 hours
- What else?

Bicycle Wheels (mathematical model)

$$\text{Revenue} = 0.10\,(S-72W) + 10.0\,(R-W) + 50.0\,W$$
$$\text{Cost} = 0.05\,S + 4.0\,R + 11.0\,W$$
$$\text{Profit} = \text{Revenue} - \text{Cost}$$

$$\max\; Z = \text{Profit}$$

$$\begin{array}{llll}
\text{s.t.} & 0.04\,S + 0.6\,R & <= 378 & \text{(machine time)} \\
 & 0.01\,S + 0.5\,R + 1.2\,W & <= 267 & \text{(labour)} \\
 & S - 72\,W & >= 0 & \text{(sold spokes >=0)} \\
 & R - W & >= 0 & \text{(sold rims >=0)} \\
 & S,\; R,\; W & >= 0 & \text{(non-negativity)}
\end{array}$$

Multi-Period Production LP: Pigskin

The Pigskin company will produce footballs over the next six months. Forecasted demand and production costs over this time period are:

Month	1	2	3	4	5	6
Demand (Cases of 100)	100	150	300	350	250	100
Unit Production Cost $	12.50	12.55	12.70	12.80	12.85	12.95

Pigskin has a monthly production capacity of 300 cases. They currently have 50 cases of footballs in inventory, and have enough capacity to store up to 100 cases. The holding cost of keeping a football in inventory for a month is estimated to be 5% of the cost of producing in that month.

Pigskin has decided that they want to meet the entire demand for footballs over the six-month period. How can they do that at minimum cost?

Mathematical Model?

Sensitivity Analysis: Kingston Anchors

Ed makes two types of anchor, the danforth type (D) and the plow type (P). Each danforth contributes a net profit of $14, and each plow contributes $75. He has 20 hours of grinding time available, and each danforth requires an average of 12 minutes of grinding, and each plow 66 minutes. He has 12 hours of assembly capacity, with each danforth requiring 12 minutes and each plow 24 minutes. Nine hours of galvanizing tank capacity are available, with each danfoth using six minutes and each plow 24 minutes.

Kingston Anchors Formulation

$$
\begin{array}{lccccccl}
\max & z = & 14D & + & 75P & & & \\
\text{s.t.} & & 0.2D & + & 1.1P & \leq & 20 & \textit{grind} \\
 & & 0.2D & + & 0.4P & \leq & 12 & \textit{assemble} \\
 & & 0.1D & + & 0.4P & \leq & 9 & \textit{galvanize} \\
 & & D & , & P & \geq & 0 &
\end{array}
$$

Sensitivity Analysis

- "Solutions" are approximate because:
  - Statement is a model of reality
  - Data is uncertain
  - Can often 'relax' constraints (for a price)
- Key questions:
  - How sensitive is our solution to small changes in data?
  - Is a change in a constraint worth the price?
- We will consider:
  - Changes to RHS values
  - Changes to objective function coefficients

Computer Sensitivity Analysis

- Reduced cost
  - Cost, after rebalancing, of forcing a current inactive variable to a value of one
- Objective function coefficient ranging
  - Same active variables, with same values
  - Optimal objective value will change
- RHS ranging
  - Same set of active variables
  - Changed objective and decision values
- Dual price (shadow price)
  - Benefit of relaxing the constraint by one unit within allowable range

Computer Solution with Excel

Kingston Anchors					
Dec Vars:	D	P			
	37.14286	11.42857			
Profit	14	75	1377.143		
Grind	0.2	1.1	20	<=	20
Ass'y	0.2	0.4	12	<=	12
Galvanize	0.1	0.4	8.285714	<=	9

Computer Solution with Excel (continued…)

Target Cell (Max)

Cell	Name	Original Value	Final Value
$D$4	Profit	89	1377.142857

Adjustable Cells

Cell	Name	Original Value	Final Value
$B$3	D	1	37.14285714
$C$3	P	1	11.42857143

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$D$5	Grind	20	$D$5<=$F$5	Binding	0
$D$6	Ass'y	12	$D$6<=$F$6	Binding	0
$D$7	Galvanize	8.285714286	$D$7<=$F$7	Not Binding	0.714285714

Computer Solution with Excel (continued...)

Changing Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$3	D	37.14285714	0	14	23.5	0.363636364
$C$3	P	11.42857143	0	75	2	47

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$5	Grind	20	67.14285714	20	2.5	1E+30
$D$6	Ass'y	12	2.857142857	12	3.333333333	1E+30
$D$7	Galvanize	8.285714286	0	9	1E+30	0.714285714

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Sensitivity Analysis for Kingston Anchors

Profit for danforths increases to $20?

- Increase of $6 within allowable range
- Effect is $6 x 37.14 = $222.84

Profit for plows drops to $50?

- Drop of $25 within allowable range
- Profit will drop by $25 x 11.43 = $285.75

Profit for danforths drops to $10?

- Drop of $4 exceeds allowable decrease
- Effect is uncertain - re-solve problem
- (From graph: stop producing danforths)

Sensitivity Analysis for Kingston Anchors (continued…)

Galvanizing availability increases to 20 hours?

- Any increase is within allowable range
- No effect: dual price zero, inactive constraint

Grinder availability increases to 25 hours?

- Increase of five hours exceeds range
- Uncertain effect – re-solve problem

Assembly availability drops to 10 hours?

- Drop of two hours within allowable decrease
- Effect is profit drop: \$2.86 x 2 = \$5.72

Sensitivity Analysis for Kingston Anchors (continued…)

Can get two more assembly hours for $8 total?

- Two hour increase is within allowable range
- Not worth it since $2.86 x 2 = $5.72

Can get three more grinder hours for $150 total?

- Increase of three hours exceeds range
- Profit will increase by at least $67.14 x 2.5 = $167.85
- Take the deal

Blending LP Model: Chandler

Chandler Oil has 5,000 barrels of crude oil one and 10,000 barrels of crude oil two available. Chandler sells gasoline and heating oil. These products are produced by blending together the two crude oils. Each barrel of crude one has a "quality level" of ten and each barrel of crude two has a quality level of five. Gasoline must have an average quality level of at least eight, whereas heating oil must have an average quality level of at least six. Gasoline sells for $25 per barrel and heating oil sells for $20 per barrel. The advertising cost to sell one barrel of gasoline is $0.20 and the advertising cost to sell one barrel of heating oil is $0.10. We assume that demand for heating oil and gasoline is unlimited, so that all of Chandler's production can be sold. Chandler wants to maximize its profit.

Chandler Model: Formulation

- Decisions?
- Objective?
- Constraints?

Transportation Model

	DESTINATION DEPOT				
Plant	**T**	**O**	**R**	**W**	**Supply**
P	$9.60	$8.70	$8.90	$12.60	10,000
E	$7.90	$9.20	$8.75	$7.60	7,000
Ro	$10.20	$6.50	$7.60	$11.80	3,500
Demand	6,000	2,500	5,000	1,500	

Transportation Model: Formulation

Objective?

- Minimize total shipping costs

Decision Variables?

- PT = number of tons Picton to Toronto
- RoT = number of tons Rome to Toronto
- …etc.

Constraints?

- Total from Picton cannot exceed 10,000 tons
- Total to Toronto 6,000 tons or more
- …etc.

Mathematical Model

$$
\begin{array}{lcccccccccc}
\min & z & = & 9.6PT & + & 8.7PO & + & 8.9PR & + & 12.6PW \\
 & & + & 7.9ET & + & 9.2EO & + & 8.75ER & + & 7.6EW \\
 & & + & 10.2RoT & + & 6.5RoO & + & 7.6RoR & + & 11.8RoW \\
\hline
\text{s.t.} & PT & + & PO & + & PR & + & PW & \leq & 10000 \\
 & ET & + & EO & + & ER & + & EW & \leq & 7000 \\
 & RoT & + & RoO & + & RoR & + & RoW & \leq & 3500 \\
\hdashline
 & & & PT & + & ET & + & RoT & \geq & 6000 \\
 & & & PO & + & EO & + & RoO & \geq & 2500 \\
 & & & PR & + & ER & + & RoR & \geq & 5000 \\
 & & & PW & + & EW & + & RoW & \geq & 1500 \\
\hdashline
 & & & & & & & \text{all} & \geq & 0
\end{array}
$$

Combination of Models: Conmine

Consolidated Mining Company mines zinc ore at two locations: Blue Mesa (New Mexico) and Dry Pass (Washington state). Once mined, each ton of ore must be moved to one of two processing plants, one near Boise (Idaho), and the other in West Texas.

The processed ore is then shipped to three customers, Galvanic Industries, MunchCo, and American Metals. These customers require 600, 400 and 700 tons per day of processed ore, respectively.

Blue Mesa can produce up to 800 tons of ore per day at a cost of $12/ton. Dry pass can produce up to 1,000 tons of ore per day at a cost of $10/ton.

The Boise plant can process up to 1,000 tons of ore per day at a cost of $17/ton. The West Texas plant can handle up to 700 tons per day at $15/ton.

Conmine (continued...)

Shipping costs per ton between the mines, plants, and customers are given in the following two tables:

	Shipping Cost To:	
From Mine	Boise	West TX
Blue Mesa	$4.50	$3.00
Dry Pass	$3.50	$6.00

	Shipping Cost From:	
To Customer	Boise	West TX
Galvanic	$2.25	$5.75
MunchCo	$3.35	$2.95
American Metals	$6.00	$7.10

What pattern of production, processing, and shipping will allow the firm to meet customer demands at the lowest possible cost?

Conmine Model: Formulation

Decisions?

Objective?

Constraints?

End of Class One