Exercises for Inventory Management

Problem 1

Dan McClure owns a thriving independent bookstore in artsy New Hope, Pennsylvania. He must decide how many copies to order of a new book, power and Self-Destruction, an exposé on a famous politician's lurid affairs. Interest in the book will be intense at first and then fizzle quickly as attention turns to other celebrities. The book's retail price is $20 and the wholesale price is $12. The publisher will buy back the retailer's leftover copies at a full refund, but McClure Books incurs $4 in shipping and handling costs for each book returned to the publisher. Dan believes his demand forecast can be represented by a normal distribution with mean 200 and standard deviation 80.

a) Dan will consider this book to be a blockbuster for him if it sells more than 400 units. What is the probability Power and Self-Destruction will be a blockbuster?

b) Dan considers a book a "dog" if it sells less than 50 percent of his mean forecast. What is the probability this expose is a "dog"?

c) What is the probability demand for this book will be within 20 percent of the mean forecast?

d) What order quantity maximizes Dan's expected profit?

e) Dan prides himself on good customer service. In fact, his motto is "McClure's got what you want to read." How many books should Dan order if he wants to achieve a 95 percent in-stock probability?

f) If Dan orders the quantity chosen in part 3 to achieve a 95 percent in-stock probability, then what is the probability that "Dan wont' have what some customer wants to read" (i.e., what is the probability some customer won't be able to purchase a copy of the book)?

g) Suppose Dan orders 300 copies of the book. What would Dan's expected profit be in this case?

SOLUTIONS

a)

$$z = \frac{Q-\mu}{\sigma} = \frac{400-200}{80} = 2.5$$

Using Excel, "=norm.s.dist(2.5, TRUE)" yields 0.99379

Alternatively, a z-table may be used to find a similar result.

This corresponds to the probability of selling 400 or *fewer* units. By extension, the probability of selling *more* than 400 units is 1-0.99379 = 0.621%

b)

$$z = \frac{Q-\mu}{\sigma} = \frac{0.5(200)-200}{80} = -1.25$$

Again, using either Excel or a z-table, we find this corresponds to a probability of 10.56%

c)

$$z = \frac{Q-\mu}{\sigma} = \frac{1.2(200)-200}{80} = 0.5 \rightarrow p = 0.691462$$

$$z = \frac{Q-\mu}{\sigma} = \frac{0.8(200)-200}{80} = -0.5 \rightarrow p = 0.308588$$

$$\begin{aligned} p(160 \leq x \leq 240) &= 0.691462 - 0.308588 \\ &= 38.2925\% \end{aligned}$$

d)

$c_u = 20 - 12 = \$8$
$c_o = \$4$

$$F(Q) = \frac{c_u}{c_o + c_u} = \frac{8}{4+8} = 0.\overline{6}$$

This is a probability. Using Excel, "=norm.s.inv(2/3)" returns a corresponding z-value of 0.430727

Alternatively, we can seek the probability 0.6666 within a z-table and obtain a similar z-value.

$$\begin{aligned} Q^* &= \mu + z\sigma \\ &= 200 + (0.430727)(80) \\ &\cong 235 \text{ books} \end{aligned}$$

e)

For $p = 0.95$, the corresponding z-value is 1.645

$$\begin{aligned} Q^* &= \mu + z\sigma \\ &= 200 + (1.645)(80) \\ &\cong 332 \text{ books} \end{aligned}$$

f) The probability of stock-out is 100%-95% = 5%

g) Regardless of the purchase quantity, some probability of lost sales remains. The question becomes how many lost sales can be expected? Start by finding the z-value corresponding to your order size…

$$z = \frac{Q-\mu}{\sigma} = \frac{300-200}{80} = 1.25$$

We know require a *standard normal loss function*, $L(z)$. In Excel,

"=norm.s.dist(z, FALSE) - z*(1-norm.s.dist(z,TRUE))"

"=norm.s.dist(1.25, FALSE) - 1.25*(1-norm.s.dist(1.25,TRUE))"

This yields a value of 0.050587. Alternatively, using a standard normal loss function table, a value of 0.0506 is found corresponding to a z-value of 1.25.

This is used to determine the "expected loss sales"

$$\begin{aligned} Expected\ Lost\ Sales &= \sigma \times L(z) \\ &= (80)(0.050587) \\ &= 4.047 \end{aligned}$$

Naturally, the expected demand (μ) consists of the total of expected sales and expected lost sales…

$$\begin{aligned} Expected\ Demand &= Expected\ Sales + Expected\ Lost\ Sales \\ 200 &= Expected\ Sales + 4.047 \\ Expected\ Sales &= 195.95 \end{aligned}$$

And since our order size, Q, is 300 units, we can naturally expect to have $300 - 195.95 \approx 104$ books remaining in inventory at the end of the month.

The profit on books sold is: $\$8 \times 196 = \$1{,}568$

The losses on returned books total: $-\$4 \times 104 = -\$416$

Total profit is therefore: $\$1{,}568 - \$416 = \$1{,}151.40$

Problem 2

Flextrola, Inc., an electronics systems integrator, is planning to design a key component for their next-generation product with Solectrics. Flextrola will integrate the component with some software and then sell it to consumers. Given the short life cycles of such products and the long lead times quoted by Solectrics, Flextrola only has one opportunity to place an order with Solectrics prior to the beginning of its selling season. Flextrola's demand during the season is normally distributed with a mean of 1000 and a standard deviation of 600. Solectrics' production cost for the component is \$52 per unit and it plans to sell the component for \$72 per unit to Flextrola. Flextrola incurs essentially no cost associated with the software integration and handling of each unit. Flextrola sells these units to consumers for \$121 each. Flextrola can sell unsold inventory at the end of the season in a secondary electronics market for \$50 each. The existing contract specifies that once Flextrola places the order, no changes are allowed to it. Also, Solectrics does not accept any returns of unsold inventory, so Flextrola must dispose of excess inventory in the secondary market.

a) What is the probability that Flextrola's demand will be within 25 percent of its forecast?

b) What is the probability that Flextrola's demand will be more than 40 percent greater than its forecast?

c) Under this contract, how many units should Flextrola order to maximize its expected profit?

For parts d through i, assume Flextrola orders 1,200 units.

d) What are Flextrola's expected sales?

e) How many units of inventory can Flextrola expect to sell in the secondary electronics market?

f) What is Flextrola's expected gross margin percentage, which is (Revenue - Cost)/Revenue?

g) What is Flextrola's expected profit?

h) What is Solectrics' expected profit?

i) What is the probability that Flextrola has lost sales of 400 units or more?

j) A sharp manager at Flextrola noticed the demand forecast and became wary of assuming that demand is normally distributed. She plotted a histogram of demands from previous seasons for similar products and concluded that demand is better represented by the log normal distribution. Figure A below plots the density function for both the log normal and the normal distribution, each with mean of 1,000 and standard deviation of 600. Figure B plots the distribution function for both the log normal and the normal. Using the more accurate forecast (i.e., the log normal distribution), approximately how many units should Flextrola order to maximize its expected profit?

SOLUTIONS

a)

$$z = \frac{Q-\mu}{\sigma} = \frac{1.25(1000) - 1000}{600} = 0.41\bar{6} \rightarrow p = 0.661539$$

$$z = \frac{Q-\mu}{\sigma} = \frac{0.75(1000) - 1000}{600} = -0.41\bar{6} \rightarrow p = 0.338461$$

$$\begin{aligned} p(750 \leq x \leq 1{,}250) &= 0.661539 - 0.338461 \\ &= 32.30\% \end{aligned}$$

b)

$$z = \frac{Q-\mu}{\sigma} = \frac{1.4(1000) - 1000}{600} = 0.\bar{6} \rightarrow p = 0.7475$$

$$1 - 0.7475 = 25.2\%$$

c)

$c_u = 121 - 72 = \$49$
$c_o = \$72 - 50 = \$22$

$$F(Q) = \frac{c_u}{c_o + c_u} = \frac{49}{22+49} = 0.69 \rightarrow z = 0.5$$

$$\begin{aligned} Q^* &= \mu + z\sigma \\ &= 1000 + (0.5)(600) \\ &\cong 1{,}300 \end{aligned}$$

d)

$$z = \frac{Q-\mu}{\sigma} = \frac{1200-1000}{600} = 0.\overline{3} \rightarrow L(z) = 0.254236$$

$$\begin{aligned} \mathit{Expected\ Lost\ Sales} &= \sigma \times L(z) \\ &= (600)(0.254236) \\ &\cong 153 \end{aligned}$$

$$\begin{aligned} \mathit{Expected\ Demand} &= \mathit{Expected\ Sales} + \mathit{Expected\ Lost\ Sales} \\ 1{,}000 &= \mathit{Expected\ Sales} + 153 \\ \mathit{Expected\ Sales} &= 847 \end{aligned}$$

e)

And since our order size, Q, is 1,200 units, we can naturally expect to have $1{,}200 - 847 =$ 353 units remaining for sale on the secondary market.

f)

$\text{Revenue} = \$121(847) + \$50(353) = \$120{,}137$

$\text{Costs}{=}\$72(1200){=}\$86{,}400$

$$\frac{\text{Revenue} - \text{Costs}}{\text{Revenue}} = \frac{\$120{,}137 - \$86{,}400}{\$120{,}137} = 28\%$$

g)

$$\text{Profit} = \$120{,}137 - \$86{,}400 = \$33{,}737$$

h)

$$\text{Profit} = 1{,}200(\$72 - \$52) = \$24{,}000$$

i)

$$\begin{aligned} \mathit{Expected\ Lost\ Sales} &= 400 \text{ (or more)} \\ \mathit{Expected\ Lost\ Sales} &= \sigma \times L(z) \\ 400 &= 600 \times L(z) \\ L(z) &= 0.\overline{6} \\ &\rightarrow z \cong -.44 \\ &\rightarrow p \cong 33\% \end{aligned}$$

j)

From part c) of the problem, we found that $F(Q) = 0.69$ but instead of finding z (corresponding to a normal distribution) we will instead find the corresponding value from Figure B for the lognormal distribution. Here a reasonable approximation appears to be 1,100 units.

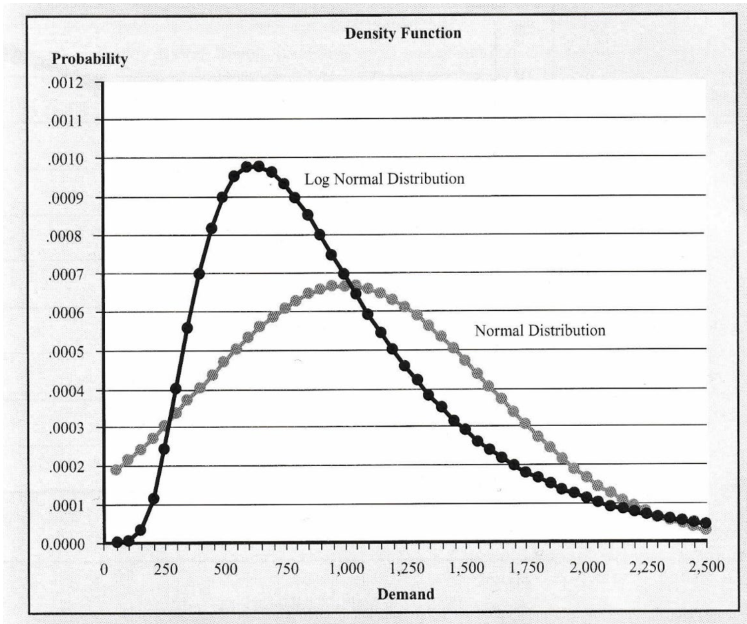

Figure A

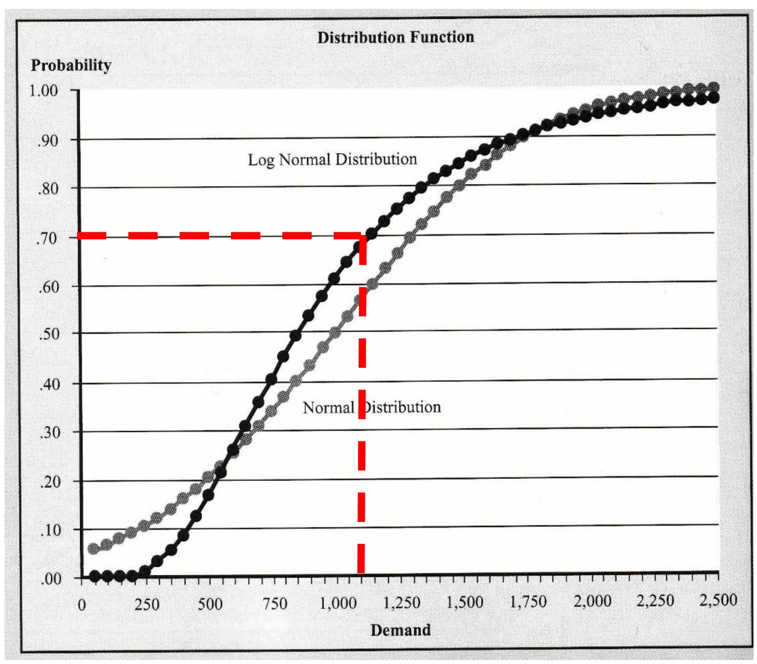

Figure B

Problem 3

Fashionables is a franchisee of The Limited, the well-known retailer of fashionable clothing. Prior to the winter season, The Limited offers Fashionables the choice of five different colors of a particular sweater design. The sweaters are knit overseas by hand, and because of the lead times involved, Fashionables will need to order its assortment in advance of the selling season. As per the contracting terms offered by The Limited, Fashionables also will not be able to cancel, modify, or reorder sweaters during the selling season. Demand for each color during the season is normally distributed with a mean of 500 and a standard deviation of 200. Further, you may assume that the demands for each sweater are independent of those for a different color. The Limited offers the sweaters to Fashionables at the wholesale price of $40 per sweater and Fashionables plans to sell each sweater at the retail price of $70 per unit. The Limited delivers orders placed by Fashionables in truckloads at a cost of $2,000 per truckload. The transportation cost of $2,000 is borne by Fashionables. Assume unless otherwise specified that all the sweaters ordered by Fashionables will fit into one truckload. Also assume that all other associated costs, such as unpacking and handling, are negligible. The Limited does not accept any returns of unsold inventory. However, Fashionables can sell all of the unsold sweaters at the end of the season at the fire-sale price of $20 each.

a) How many units of each sweater type should Fashionables order to maximize its expected profit?

b) If Fashionables wishes to ensure a 97.5 percent in-stock probability, what should its order quantity be for each type of sweater?

For parts c and d, assume Fashionables orders 725 of each sweater.

c) What is Fashionables' expected profit?

d) What is the stock out probability for each sweater?

e) Now suppose that The Limited announces that truckload capacity is 2,500 total units of sweaters. If Fashionables orders more than 2,500 units in total (from 2,501 to 5,000 units in total), it will have to pay for two truckloads. What now is Fashionables' optimal order quantity for each sweater?

SOLUTIONS

a)

$c_u = 70 - 40 = \$30$
$c_o = 40 - 20 = \$20$

$$F(Q) = \frac{c_u}{c_o + c_u} = \frac{30}{20 + 30} = 0.6 \rightarrow z = 0.253347$$

$$\begin{aligned} Q^* &= \mu + z\sigma \\ &= 500 + (0.253347)(200) \\ &\cong 552 \end{aligned}$$

b)

$$p = 0.975 \rightarrow z = 1.96$$

$$\begin{aligned} Q &= \mu + z\sigma \\ &= 500 + (1.96)(200) \\ &\cong 892 \end{aligned}$$

c)

$$z = \frac{Q - \mu}{\sigma} = \frac{725 - 500}{200} = 1.125 \rightarrow L(z) = 0.065295$$

$$\begin{aligned} Expected\ Lost\ Sales &= \sigma \times L(z) \\ &= (200)(0.065295) \\ &\cong 13 \end{aligned}$$

$$\begin{aligned} Expected\ Demand &= Expected\ Sales + Expected\ Lost\ Sales \\ 500 &= Expected\ Sales + 13 \\ Expected\ Sales &= 487 \end{aligned}$$

And since our order size, Q, is 725 units, we can naturally expect to have $725 - 487 = 238$ units remaining in inventory.

$\text{Revenue} = \$70(487) + \$20(238) = \$38{,}850$

$\text{Costs} = \$40(725) = \$29{,}000$

$\text{Profit} = \$38{,}850 - \$29{,}000 = \$9{,}850$

d)

$$z = 1.125 \rightarrow p = 0.869705$$

$$1 - 0.869705 \cong 13\%$$

e)

In part a) of the problem, we found an optimum order quantity of 552 units. This is for each colour of sweater, for a total order quantity of 2,760 units, exceeding the capacity of a single truck. Alternatives are either ordering 500 units of each sweater type of 1,000 units of each sweater type to fill either 1, or 2 trucks respectively.

<u>Let $Q = 500$ (or a single truckload)</u>

$$z = \frac{Q - \mu}{\sigma} = \frac{500 - 500}{200} = 0 \rightarrow L(z) = 0.398942$$

$$\begin{aligned} Expected\ Lost\ Sales &= \sigma \times L(z) \\ &= (200)(0.398942) \\ &\cong 80 \end{aligned}$$

$$\begin{aligned} Expected\ Demand &= Expected\ Sales + Expected\ Lost\ Sales \\ 500 &= Expected\ Sales + 80 \\ Expected\ Sales &= 420 \end{aligned}$$

And since our order size, Q, is 500 units, we can naturally expect to have $500 - 420 = 80$ units remaining in inventory.

$\text{Revenue} = \$70(2{,}100) + \$20(400) = \$155{,}000$

$\text{Costs} = \$40(2{,}500) + \$2{,}000 = \$102{,}000$ (the $2,000 is the cost of a single truck)

$\text{Profit} = \$155{,}000 - \$102{,}000 = \$53{,}000$

<u>Let $Q = 1{,}000$ (or two truckloads)</u>

$$z = \frac{Q - \mu}{\sigma} = \frac{1{,}000 - 500}{200} = 2.5 \rightarrow L(z) = 0.002004$$

$$\begin{aligned} Expected\ Lost\ Sales &= \sigma \times L(z) \\ &= (200)(0.002004) \\ &\cong 0 \end{aligned}$$

$$\begin{aligned} Expected\ Demand &= Expected\ Sales + Expected\ Lost\ Sales \\ 500 &= Expected\ Sales + 0 \\ Expected\ Sales &= 500 \end{aligned}$$

And since our order size, Q, is 1,000 units, we can naturally expect to have $1{,}000 - 500 = 500$ units remaining in inventory.

$\text{Revenue} = \$70(2{,}500) + \$20(2{,}500) = \$225{,}000$

$\text{Costs} = \$40(5{,}000) + \$4{,}000 = \$204{,}000$ (the $4,000 is the cost of two trucks)

$\text{Profit} = \$155{,}000 - \$102{,}000 = \$21{,}000$

<u>Conclusion: </u>Profit is significantly higher by simply ordering 500 of each sweater type.

Problem 4

Teddy Bower is an outdoor clothing and accessories chain that purchases a line of parkas at $10 each from its Asian supplier, TeddySports. Unfortunately, at the time of order placement, demand is still uncertain. Teddy Bower forecasts that its demand is normally distributed with mean of 2,100 and standard deviation of 1,200. Teddy Bower sells these parkas at $22 each. Unsold parkas have little salvage value; Teddy Bower simply gives them away to a charity.

a) What is the probability this parka turns out to be a "dog", defined as a product that sells less than half of the forecast?

b) How many parkas should Teddy Bower buy from TeddySports to maximize expected profit?

c) If Teddy Bower wishes to ensure a 98.5 percent in-stock probability, how many parkas should it order?

For parts d and e, assume Teddy Bower orders 3,000 parkas.

d) Evaluate Teddy Bower's expected profit.

e) Evaluate Teddy Bower's stockout probability

SOLUTIONS

a)

$$z = \frac{Q-\mu}{\sigma} = \frac{(0.5)(2{,}100) - 2{,}100}{1{,}200} = -0.875 \ \rightarrow p \cong 19\%$$

b)

$c_u = 22 - 10 = \$12$
$c_o = \$10$

$$F(Q) = \frac{c_u}{c_o + c_u} = \frac{12}{10+12} = 0.\overline{54} \rightarrow z = 0.1142$$

$$\begin{aligned} Q^* &= \mu + z\sigma \\ &= 2{,}100 + (0.1142)(1{,}200) \\ &\cong 2{,}237 \end{aligned}$$

c)

$$p = 0.985 \rightarrow z = 2.17$$

$$\begin{aligned} Q &= \mu + z\sigma \\ &= 2{,}100 + (2.17)(1{,}200) \\ &\cong 4{,}704 \end{aligned}$$

d)

$$z = \frac{Q - \mu}{\sigma} = \frac{3{,}000 - 2{,}100}{1{,}200} = 0.75 \rightarrow L(z) \cong 0.1312$$

$$\begin{aligned} Expected\ Lost\ Sales &= \sigma \times L(z) \\ &= (1{,}200)(0.1312) \\ &\cong 157 \end{aligned}$$

$$\begin{aligned} Expected\ Demand &= Expected\ Sales + Expected\ Lost\ Sales \\ 2{,}100 &= Expected\ Sales + 157 \\ Expected\ Sales &= 1{,}943 \end{aligned}$$

$$Leftover\ Inventory = 3{,}000 - 1{,}943 = 1{,}057$$

$\text{Revenue} = \$22(1{,}943) = \$42{,}746$

$\text{Costs} = \$10(3{,}000) = \$30{,}000$

$\text{Profit} = \$42{,}746 - \$30{,}000 = \$12{,}746$

e)

$$z = 0.75 \rightarrow p = 0.773$$

$$1 - 0.773 = 22.7\%$$

Problem 5

To ensure a full line of outdoor clothing and accessories, the marketing department at Teddy Bower insists that they also sell waterproof hunting boots. Unfortunately, neither Teddy Bower nor TeddySports has expertise in manufacturing those kinds of boots. Therefore, Teddy Bower contacted several Taiwanese suppliers to request quotes. Due to competition, Teddy Bower knows that it cannot sell these boots for more than $54. However, $40 per boot was the best quote from the suppliers. In addition, Teddy Bower anticipates excess inventory will need to be sold off at a 50 percent discount at the end of the season. Given the $54 price, Teddy Bower's demand forecast is for 400 boots, with a standard deviation of 300.

a) If Teddy Bower decides to include these boots in its assortment, how many boots should it order from its supplier?

b) Suppose Teddy Bower orders 380 boots. What would its expected profit be?

c) John Briggs, a buyer in the procurement department, overheard at lunch a discussion of the "boot problem." He suggested that Teddy Bower ask for a quantity discount from the supplier. After following up on his suggestion, the supplier responded that Teddy Bower could get a 10 percent discount if they were willing to order at least 800 boots. If the objective is to maximize expected profit, how many boots should he order given this new offer?

SOLUTIONS

a)

$c_u = 54 - 40 = \$14$
$c_o = 40 - 0.5(54) = \$13$

$$F(Q) = \frac{c_u}{c_o + c_u} = \frac{14}{13 + 14} = 0.5185 \rightarrow z = 0.0464$$

$$\begin{aligned} Q^* &= \mu + z\sigma \\ &= 400 + (0.0464)(300) \\ &\cong 414 \end{aligned}$$

b)

$$z = \frac{Q - \mu}{\sigma} = \frac{380 - 400}{300} = -0.0\overline{6} \rightarrow L(z) \cong 0.4332$$

$$\begin{aligned} \textit{Expected Lost Sales} &= \sigma \times L(z) \\ &= (300)(0.4332) \\ &\cong 130 \end{aligned}$$

$$Expected\ Demand = Expected\ Sales + Expected\ Lost\ Sales$$
$$400 = Expected\ Sales + 130$$
$$Expected\ Sales = 270$$

$$Leftover\ Inventory = 380 - 270 = 110$$

$\text{Revenue} = \$54(270) + \$27(110) = \$17{,}550$

$\text{Costs} = \$40(380) = \$15{,}200$

$\text{Profit} = \$17{,}550 - \$15{,}200 = \$2{,}350$

c)

$c_u = 54 - (0.9)40 = \$18$
$c_o = (0.9)40 - 0.5(54) = \$9$

$$F(Q) = \frac{c_u}{c_o + c_u} = \frac{18}{9 + 18} = 0.\bar{6} \rightarrow z = 0.4307$$

$$\begin{aligned} Q^* &= \mu + z\sigma \\ &= 400 + (0.4307)(300) \\ &\cong 529 \end{aligned}$$

Recall that the discount became available to us only if we ordered at least 800 units. The above order quantity of 529 is therefore insufficient to obtain the discount. We will therefore adjust the order size to the minimum of 800 units and compare that against the profit we calculated in part b) to identify the optimum order quantity in light of a potential discount…

$$z = \frac{Q - \mu}{\sigma} = \frac{800 - 400}{300} = 1.\bar{3} \ \rightarrow L(z) \cong 0.0424$$

$$\begin{aligned} Expected\ Lost\ Sales &= \sigma \times L(z) \\ &= (300)(0.0424) \\ &\cong 13 \end{aligned}$$

$$Expected\ Demand = Expected\ Sales + Expected\ Lost\ Sales$$
$$400 = Expected\ Sales + 13$$
$$Expected\ Sales = 387$$

$$Leftover\ Inventory = 800 - 387 = 413$$

$\text{Revenue} = \$54(387) + \$27(413) = \$32{,}049$

$\text{Costs} = \$36(800) = \$28{,}800$

$\text{Profit} = \$32{,}049 - \$28{,}800 = \$3{,}249$

Since this is greater than the profit we calculated in part b), we should proceed with ordering 800 units.

Problem 6

Geoff Gullo owns a small firm that manufactures "Gullo Sunglasses", He has the opportunity to sell a particular seasonal model to Land's End. Geoff offers Land's End two purchasing options:

Option 1. Geoff offers to set his price at $65 and agrees to credit Land's End $53 for each unit Land's End returns to Geoff at the end of the season (because those units did not sell). Since styles change each year, there is essentially no value in the returned merchandise.

Option 2. Geoff offers a price of $55 for each unit, but returns are no longer accepted. In this case, Land's End throws out unsold units at the end of the season.

This season's demand for this model will be normally distributed with mean of 200 and standard deviation of 125. Land's End will sell those sunglasses for $100 each. Geoff's production cost is $25.

a. How much would Land's End buy if they chose option 1?

b. How much would Land's End buy if they chose option 2?

c. Which option will Land's End choose?

d. Suppose Land's End chooses option 1 and orders 275 units. What is Geoff's expected profit?

SOLUTIONS

a)

$c_u = 100 - 65 = \$35$
$c_o = 65 - 53 = \$12$

$$F(Q) = \frac{c_u}{c_o + c_u} = \frac{35}{12 + 35} = 0.7447 \rightarrow z = 0.6579$$

$$\begin{aligned} Q^* &= \mu + z\sigma \\ &= 200 + 0.6579(125) \\ &\cong 282 \end{aligned}$$

b)

$c_u = 100 - 55 = \$45$
$c_o = \$55$

$$F(Q) = \frac{c_u}{c_o + c_u} = \frac{45}{55 + 45} = 0.45 \rightarrow z = -0.1257$$

$$\begin{aligned} Q^* &= \mu + z\sigma \\ &= 200 - 0.1257(125) \\ &\cong 184 \end{aligned}$$

c) In order to answer this, we must look at the profit resulting from each of the two Options

Option 1:

$$z = 0.6579 \rightarrow L(z) \cong 0.1534$$

$$\begin{aligned} Expected\ Lost\ Sales &= \sigma \times L(z) \\ &= (125)(0.1534) \\ &\cong 19 \end{aligned}$$

$$\begin{aligned} Expected\ Demand &= Expected\ Sales + Expected\ Lost\ Sales \\ 200 &= Expected\ Sales + 19 \\ Expected\ Sales &= 181 \end{aligned}$$

$$Leftover\ Inventory = 282 - 181 = 101$$

$\text{Revenue} = \$100(181) + \$53(101) = \$23{,}453$

$\text{Costs} = \$65(282) = \$18{,}330$

$\text{Profit} = \$23{,}453 - \$18{,}330 = \$5{,}123$

Option 2:

$$z = -0.1257 \rightarrow L(z) \cong 0.4649$$

$$\begin{aligned} Expected\ Lost\ Sales &= \sigma \times L(z) \\ &= (125)(0.4649) \\ &\cong 58 \end{aligned}$$

$$\begin{aligned} Expected\ Demand &= Expected\ Sales + Expected\ Lost\ Sales \\ 200 &= Expected\ Sales + 58 \\ Expected\ Sales &= 142 \end{aligned}$$

$$Leftover\ Inventory = 184 - 142 = 42$$

$\text{Revenue} = \$100(142) = \$14{,}200$

$\text{Costs} = \$55(184) = \$10{,}120$

$\text{Profit} = \$14{,}200 - \$10{,}120 = \$4{,}080$

The largest profit is associated with Option 1 and it is therefore the preferred choice.

d) Before determining Geoff's expected profit, we need to determine how many units will be returned to him…

$$z = \frac{Q-\mu}{\sigma} = \frac{275-200}{125} = 0.6 \ \rightarrow L(z) \cong 0.1687$$

$$\begin{aligned} Expected\ Lost\ Sales &= \ \sigma \times L(z) \\ &= (125)(0.1687) \\ &\cong 21 \end{aligned}$$

$$\begin{aligned} Expected\ Demand &= Expected\ Sales + Expected\ Lost\ Sales \\ 200 &= Expected\ Sales + 21 \\ Expected\ Sales &= 179 \end{aligned}$$

$$Leftover\ Inventory = 275 - 179 = 96$$

Therefore, Geoff can expect 96 units to be returned to him. His profit is therefore…

$\text{Revenue} = \$65(275) = \$17{,}875$

$\text{Costs} = \$25(275) + \$53(96) = \$11{,}963$

$\text{Profit} = \$17{,}875 - \$11{,}963 =$ $\$5{,}912$

Problem 7

CPG Bagels starts the day with a large production run of bagels. Throughout the morning, additional bagels are produced as needed. The last bake is completed at 3pm and the store closes at 8 pm. It costs approximately \$0.20 in materials and labor to make a bagel. The price of a fresh bagel is \$0.60. Bagels not sold by the end of the day are sold the next day as "day old" bagels in bags of six, for \$0.99 a bag. About two-thirds of the day-old bagels are sold; the remainder are just thrown away. There are many bagel flavors, but for simplicity, concentrate just on the plain bagels. The store manager predicts that demand for plain bagels from 3 pm until closing is normally distributed with mean 54 and standard deviation of 21.

a. How many bagels should the store have at 3 pm to maximize the store's expected profit (from sales between 3 pm until closing)? (Hint. Assume day-old bagels are sold for \$0.99/6=\$0.165 each; i.e., don't worry about the fact that day-old bagels are sold in bags of six.)

b. Suppose that the store manager is concerned that stockouts might cause a loss of future business. To explore this idea, the store manager feels that it is appropriate to assign a stockout cost of \$5 per bagel that is demanded but not filled. (Customers frequently purchase more than one bagel at a time. This cost is per bagel demanded that is not satisfied rather than per customer that does not receive a complete order.) Given additional stockout cost, how many bagels should the store have at 3 p.m. to maximize the store's expected profit?

c. Suppose the store manager has 101 bagels at 3 p.m. How many bagels should the store manager expect to have at the end of the day?

SOLUTIONS

a)

$$c_u = 0.60 - 0.20 = \$0.40$$
$$c_o = 0.20 - (2/3)0.165 = \$0.09$$

$$F(Q) = \frac{c_u}{c_o + c_u} = \frac{40}{9 + 40} = 0.8163 \rightarrow z = 0.9014$$

$$\begin{aligned} Q^* &= \mu + z\sigma \\ &= 54 + 0.9014(21) \\ &\cong 73 \end{aligned}$$

b)

$$c_u = 0.60 - 0.20 + 5 = \$5.40$$
$$c_o = 0.20 - (2/3)0.165 = \$0.09$$

$$F(Q) = \frac{c_u}{c_o + c_u} = \frac{540}{9 + 540} = 0.9836 \rightarrow z = 2.1345$$

$$\begin{aligned} Q^* &= \mu + z\sigma \\ &= 54 + 2.1345(21) \\ &\cong 99 \end{aligned}$$

c)

$$z = \frac{Q - \mu}{\sigma} = \frac{101 - 54}{21} = 2.2381 \rightarrow L(z) \cong 0.004382$$

$$\begin{aligned} Expected\ Lost\ Sales &= \sigma \times L(z) \\ &= (21)(0.004382) \\ &\cong 0 \end{aligned}$$

$$\begin{aligned} Expected\ Demand &= Expected\ Sales + Expected\ Lost\ Sales \\ 54 &= Expected\ Sales + 0 \\ Expected\ Sales &= 54 \end{aligned}$$

$$Leftover\ Inventory = 101 - 54 = 47$$

Problem 8

Johnson Electronics sells electrical and electronic components through catalogs. Catalogs are printed once every two years. Each printing run incurs a fixed cost of \$25,000, with a variable production cost of \$5 per catalog. Annual demand for catalogs is estimated to be normally distributed with a mean of 16,000 and standard deviation of 4,000. Data indicates that, on average, each customer ordering a catalog generates a profit of \$35 from sales. Assuming that Johnson wants only one printing run in each two-year cycle, how many catalogs should be printed in each run?

SOLUTIONS

$c_u = 35 - 5 = \$30$
$c_o = \$5$

$\mu = 16{,}000 + 16{,}000 = 32{,}000$
$\sigma = \sqrt{4{,}000^2 + 4{,}000^2} = 5{,}657$

$$F(Q) = \frac{c_u}{c_o + c_u} = \frac{30}{5+30} = 0.8571 \rightarrow z = 1.067 \rightarrow L(z) = 0.0728$$

$$\begin{aligned} Q^* &= \mu + z\sigma \\ &= 32{,}000 + (1.067)(5{,}657) \\ &\cong 38{,}036 \end{aligned}$$

$$\begin{aligned} Expected\ Lost\ Sales &= \sigma \times L(z) \\ &= (5{,}657)(0.0728) \\ &\cong 412 \end{aligned}$$

$$\begin{aligned} Expected\ Demand &= Expected\ Sales + Expected\ Lost\ Sales \\ 32{,}000 &= Expected\ Sales + 412 \\ Expected\ Sales &= 31{,}588 \end{aligned}$$

$$Leftover\ Inventory = 38{,}036 - 31{,}588 = 6{,}448$$

$\text{Profit} = \$30(31{,}588) - \$5(6{,}448) - \$25{,}000 = \$890{,}400$

Problem 9

As owner of Catch-of-the-Day Fish Shop, you can purchase fresh fish at $18 per crate each morning from the Walton Fish Market. During the day, you sell crates of fish to local restaurants for $120 each. Coupled with the perishable nature of your product, your integrity as a quality supplier requires you to dispose of each unsold crate at the end of the day. Your cost of disposal is $2 per crate. You have a problem, however, because you do not know how many crates our customers will order each day. The demand is normally distributed with a mean 10 and standard deviation of 3.

SOLUTIONS

$c_u = 120 - 18 = \$102$
$c_o = 18 + 2 = \$20$

$$F(Q) = \frac{c_u}{c_o + c_u} = \frac{102}{20 + 102} = 0.836 \rightarrow z = 0.978$$

$$\begin{aligned} Q^* &= \mu + z\sigma \\ &= 10 + (0.978)(3) \\ &\cong 13 \end{aligned}$$

Problem 10

The residents of Bucktown, Illinois, place their trash at the curb each Wednesday morning to be picked up by municipal crews. Experience shows that the total amount of trash put out has normal distribution with a mean of 35 tons and a standard deviation of 9 tons. Crews of full-time city employees assigned to trash collection collect trash. Each crew can collect 5 tons of trash per working day. The city has plenty of trucks of the kind used for trash collection. The marginal cost of operating one trash collection crew for one working day, including both personnel-related costs and truck-related costs, is reckoned at $625. Whatever trash remains at the end of the work day must be collected that evening by an outside contractor who charges $650 per ton. How many crews should the city assign to trash collection? For simplicity, treat the number of crews as a continuous variable.

SOLUTIONS

Cost per ton: $625 ÷ 5 = $125/ton

$c_u = 650 - 125 = \$525$
$c_o = \$125$

$$F(Q) = \frac{c_u}{c_o + c_u} = \frac{525}{125 + 525} = 0.8077 \rightarrow z = 0.8695$$

$$\begin{aligned} Q^* &= \mu + z\sigma \\ &= 35 + (0.8695)(9) \\ &\cong 42.825 \text{ tons} \end{aligned}$$

$$\text{Number of crews} = \frac{42.825 \text{ tons}}{5 \text{ tons /crew}} = 8.565 \text{ crews}$$

Problem 11

Northwest Airlines runs daily flights from Detroit to Amsterdam. They face a fixed cost of $70,000 for each flight independent of the actual number of passengers on the plane. There are 310 seats available on a plane. One-way tickets generate revenues of $600 apiece when used but are fully refundable if not used. On a typical weekday, the airline estimates that the number of no-shows will range between 0 and 20; all intermediate values are equally likely. By law, an airline is allowed to overbook flights, but must give compensation of $250 to all ticketed passengers not allowed to board. In addition, it must provide those passengers with alternative transportation on another carrier (the cost of providing the alternative transportation just wipes out the $600 revenue). How many tickets should Northwest book on its flight from Detroit to Amsterdam?

SOLUTIONS

$c_u = \$600$
$c_o = \$250$

$$F(Q) = \frac{c_u}{c_o + c_u} = \frac{600}{250 + 600} = 0.7$$

In the past, we've used used $F(Q)$ to obtain a z-value from a z-table. Doing so naturally assumed that the demand is normally distributed. However, in this specific problem, we're told that the number of no-shows is **uniformly distributed** from 0 to 20.

Recalling that $F(Q)$ is a cumulative probability, we are seeking a value such that 70% of the probability distribution falls to the left (equal to or less than) of that quantity. For a uniform distribution, we need only multiply $F(Q)$ by the maximum quantity bounding the distribution (in this case, 20). In other words, for a uniform distribution from 0 to 20, 70% of all values are equal to or less than 14.

The airline should therefore book 310 + 14 = 324 tickets.