Exercises for Waiting Lines

Problem 1

Entrepreneur John Doe has just founded Pizza-Ready, which will accept pizza orders for pickup over the phone. Pizza-Ready's strategy is to compete with established pizza restaurants by offering superior, fresh, made-to-order deep-dish pizza and excellent service. As part of his advertising campaign, Doe will publish an ad stating, "if your pizza is not ready in 20 minutes, that pizza plus your next order are on us." Doe has done extensive research on the pizza making process and knows that all fresh deep-dish pizzas require 15 minutes of oven time and 2 minutes of preparation. Moreover, as part of its excellent service, Pizza-Ready will accept orders whenever customers place them, and a marketing study estimates that Pizza-Ready can count on an average demand of 20 pizzas per hour. Doe, therefore, has ordered five pizza ovens, each of which is able to bake one pizza at a time. Doe is now looking for a silent partner to help carry the financial burden of his start-up company. Given the structure of this business, a potential partner has asked whether Pizza-Ready would be a profitable investment. What would you recommend, and why?

SOLUTION

Arrival Rate: $A = 20\text{ pizzas/hr}$

Service Rate: $S = 4\text{ pizzas/hr}$

This service rate assumes that the 2 minutes of preparation is irrelevant given the bottleneck created by the oven (i.e., the queue forms at the oven, not at the preparation station)

Utilization: $u = {}^{A}/_{MS} = {}^{20}/_{(5)(4)} = 100\%$

Notwithstanding the high utilization rate, the process risks consumers balking from the queue. Recall that the arrival rate is an *average* and is variable. At times when the arrival rate exceeds the service capacity of the firm, the queue will lengthen indefinitely. This is likely to irritate customers as they are almost certainly facing long wait times.

Problem 2

M.M Sprout, a catalog mail order retailer, has one customer service representative (CSR) to take orders at an 800 telephone number. If the CSR is busy, the next caller is put on hold. For simplicity, assume that any number of incoming calls can be put on hold and that nobody hangs up in frustration over a long wait. Suppose that, on average, one call comes in every 4 minutes and that it takes the CSR an average of 3 minutes to take an order. The coefficient of variation for both the inter-arrival and activity times are 1. The CSR is paid $20 per hour, and the telephone company charges $5 per hour for the 800 line. The company estimates that each minute a customer is kept on hold costs it $2 in customer dissatisfaction and loss of future business. Estimate the following:

a) The proportion of time that the CSR will be busy
b) The average time that a customer will be on hold. We are given that $CV_{IAT} = CV_{ST} = 1$
c) The average number of customers on line
d) The total hourly cost of service and waiting

SOLUTION

a)

Arrival Rate: $A = 15 \text{ calls/hr}$
Service Rate: $S = 20 \text{ calls/hr}$
Service Channels: $M = 1$

Utilization: $u = {}^{A}/_{MS} = {}^{15}/_{(1)(20)} = 75\%$

b)

$$L_q = \frac{u^{\sqrt{2(M+1)}}}{1-u} \times \frac{{CV_{IAT}}^2 + {CV_{ST}}^2}{2}$$

$$= \frac{0.75^{\sqrt{2(2)}}}{0.25} \times \frac{1^2 + 1^2}{2}$$

$$= \frac{0.75^2}{0.25}$$

$$= 2.25 \text{ calls}$$

$$W_q = \frac{L_q}{A}$$

$$= \frac{2.25}{15}$$

$$= 0.15 \text{ hours}$$
$$= 9 \text{ minutes}$$

c) $L = L_q + {}^{A}/_{S} = 2.25 + {}^{15}/_{20} = 3 \text{ calls}$

d)

Cost of CSR: $20/hr
Cost of Phone Utility: $5/hr
Cost of One Waiting Customer: $2/minute × 9 minutes = $18/customer
15 callers arrive every hour for a total waiting cost of $18/customer × 15 customers/hr = $270/hr

Total Cost = $20/hr + $5/hr + $270/hr = $295/hr

Problem 3

Heavenly Mercy Hospital wants to improve the efficiency of its radiology department and its responsiveness to doctors' needs. Administrators have observed that, every hour, doctors submit an average of 18 X-Ray films for examination by staff radiologists. For simplicity assume that the coefficient of variation for the inter-arrival times is 1. Each radiologist is equipped with a conventional piece of viewing equipment that reads one film at a time. Because of complications that vary from case to case, the actual time needed for report preparation is exponentially distributed with a mean of 30 minute. The coefficient of variation for the report preparation is 1. Together, the cost of leasing one piece of viewing equipment and each radiologist's salary is $100 per hour. Although it is difficult to put a dollar value on a doctor's waiting time, each doctor would like to get a radiologist's report within an average of 40 minutes from the time the film is submitted.

a) Determine the number of radiologists that the hospital should staff in order to meet doctors' requirements regarding job flow time. Compute the resulting hourly cost of operating the radiology department.

b) The hospital could also change its diagnostic procedure by leasing more sophisticated X-ray viewing devices. Administrators estimate that the new procedure would reduce a radiologist's average film processing time to 20 minutes, but would still have a coefficient of variation of 1. At the same time, however, higher equipment rental and salaries for additional support personnel would boost the hourly cost per radiologist to $150. Determine the number of radiologists that the hospital should staff under this new arrangement. Would the new arrangement be economically advantageous?

SOLUTION

a)

Arrival Rate: $A = 18 \text{ films/hr}$
Service Rate: $S = 2 \text{ films/hr}$
Service Channels, guess $M = 10$ (anything less would lead to 100% utilization or higher) and iteratively adjust until service standard is met.

Utilization: $u = {}^{A}/_{MS} = {}^{18}/_{(10)(2)} = 90\%$

$$L_q = \frac{u^{\sqrt{2(M+1)}}}{1-u} \times \frac{{CV_{IAT}}^2 + {CV_{ST}}^2}{2}$$

$$= \frac{0.9^{\sqrt{2(11)}}}{0.1} \times \frac{1^2 + 1^2}{2}$$

$$= 6.1007$$

$$W_q = \frac{L_q}{A}$$

$$= \frac{6.1007}{18}$$

$$= 0.3389 \text{ hours}$$
$$= 20.33 \text{ minutes}$$

Process flowtime is essentially the time spent waiting in the buffer (W_q) plus the time spent by the radiologists completing the request (30 minutes, on average).

$$W = W_q + \frac{1}{S} = 20.33 + 30 = 50.33 \text{ minutes}$$

Since this is higher than the service standard of 40 minutes, we'll have to add another server and try again.

Utilization: $u = {}^{A}/_{MS} = {}^{18}/_{(11)(2)} = 81.\overline{81}\%$

$$L_q = \frac{u^{\sqrt{2(M+1)}}}{1-u} \times \frac{CV_{IAT}{}^2 + CV_{ST}{}^2}{2}$$

$$= \frac{0.\overline{81}^{\sqrt{2(12)}}}{0.\overline{18}} \times \frac{1^2 + 1^2}{2}$$

$$= 2.05786$$

$$W_q = \frac{L_q}{A}$$

$$= \frac{2.05786}{18}$$

$$= 0.11433 \text{ hours}$$
$$= 6.86 \text{ minutes}$$

$$W = W_q + \frac{1}{S} = 6.86 + 30 = 36.86 \text{ minutes}$$

This meets our service standard of 40 minutes so we may conclude that 11 radiologists are required. Operating costs are 11×$100 = $1,100/hr b)

Arrival Rate: $A = 18$ films/hr
Service Rate: $S = 3$ films/hr
Service Channels, guess $M = 7$ (anything less would lead to 100% utilization or higher) and iteratively adjust until service standard is met.

Utilization: $u = {}^{A}/_{MS} = {}^{18}/_{(7)(3)} = 85.7\%$

$$L_q = \frac{u^{\sqrt{2(M+1)}}}{1-u} \times \frac{CV_{IAT}{}^2 + CV_{ST}{}^2}{2}$$

$$= \frac{0.857^{\sqrt{2(8)}}}{0.143} \times \frac{1^2 + 1^2}{2}$$

$$= 3.7784$$

$$W_q = \frac{L_q}{A}$$

$$= \frac{3.7784}{18}$$

$$= 0.2099 \text{ hours}$$
$$= 12.6 \text{ minutes}$$

$$W = W_q + \frac{1}{S} = 12.6 + 20 = 32.6 \text{ minutes}$$

This meets our service standard so we may conclude that 7 radiologists are sufficient.

The total cost is 7 × \$150 = \$1,050.

Problem 4

First Local Bank would like to improve customer service at its drive-in facility by reducing waiting and transaction times. On the basis of a pilot study, the bank's process manager estimates the average rate of customer arrivals at 30 per hour. All arriving cars line up in a single file and are served at one of four windows on a first-com/first-served basis. Each teller currently requires an average of 6 minutes to complete a transaction. The bank is considering the possibility of leasing high-speed information-retrieval and communication equipment that would cost $30 per hour. The new equipment would, however, serve the entire facility and reduce each teller's transaction-processing time to an average of 4 minutes per customer. Assume that the coefficient of variation for the inter-arrival and activity times are 1.

a) If our manager estimates the cost of a customer's waiting time in a queue (in terms of future business lost to the competition) to be $20 per customer per hour, can she justify leasing the new equipment on an economic basis?

b) Although the waiting-cost figure of $20 per customer per hour appears questionable, a casual study of the competition indicates that a customer should be in and out of a drive-in facility within an average of 8 minutes (including waiting). If First Local wants to meet this standard, should it lease the new high-speed equipment?

SOLUTIONS

Arrival Rate: $A = 30\text{ customers/hr}$
Service Rate: $S = 10\text{ customers/hr}$
Service Channels: $M = 4$
$CV_{IAT} = CV_{ST} = 1$

a)

Utilization: $u = {}^{A}/_{MS} = {}^{30}/_{(4)(10)} = 75\%$

$$L_q = \frac{u^{\sqrt{2(M+1)}}}{1-u} \times \frac{{CV_{IAT}}^2 + {CV_{ST}}^2}{2}$$

$$= \frac{0.75^{\sqrt{2(5)}}}{0.25} \times \frac{1^2 + 1^2}{2}$$

$$= 1.61\text{ customers}$$

$$W_q = \frac{L_q}{A}$$

$$= \frac{1.61}{30}$$

$$= 0.05368 \text{ hours}$$
$$= 3.22 \text{ minutes}$$

Hourly Waiting Costs: $\$20/\text{hr} \times 0.05368 \text{ hr/customer} \times 30 \text{ customers/hr} = \$32.21/\text{hr}$

With the new equipment…

$$u = {}^{A}\!/_{MS} = {}^{30}\!/_{(4)(15)} = 50\%$$

$$L_q = \frac{0.5^{\sqrt{2(5)}}}{0.5} = 0.223 \text{ customers}$$

$$W_q = \frac{0.223}{30} = 0.0074\overline{3} \text{ hr}$$

Hourly Waiting Costs: $\$20/\text{hr} \times 0.0074\overline{3} \text{ hr/customer} \times 30 \text{ customers/hr} = \$4.46/\text{hr}$

Total Cost: \$4.46/hr + \$30/hr (to lease new equipment) = \$34.46

By leasing the new equipment, our total cost has gone up and so the firm should ***not*** move forward leasing the new equipment.

b)

Utilization: $u = {}^{A}\!/_{MS} = {}^{30}\!/_{(4)(15)} = 50\%$

$$L_q = \frac{u^{\sqrt{2(M+1)}}}{1-u} \times \frac{{CV_{IAT}}^2 + {CV_{ST}}^2}{2}$$

$$= \frac{0.5^{\sqrt{2(5)}}}{0.5} \times \frac{1^2 + 1^2}{2}$$

$$= 0.2234 \text{ customers}$$

$$W_q = \frac{L_q}{A}$$

$$= \frac{0.2234}{30}$$

$$= 0.007447 \text{ hours}$$
$$= 0.45 \text{ minutes}$$

$$W = W_q + \frac{1}{S} = 0.45 + 4 = 4.45 \text{ minutes}$$

This meets our service standard of 8 minutes.

Problem 5

Since deregulation of the airline industry, increased traffic and fierce competition have forced Global Airlines to re-examine the efficiency and economy of its operations. As part of a campaign to improve customer service in a cost-effective manner, Global has focused on passenger check-in operations at its hub terminal. For best utilization of its check-in facilities, Global operates a common check-in system: passengers for all Global flights queue up in a single "snake line", and each can be served at any one of several counters as clerks become available. Arrival rate is estimated at an average of 52 passengers per hour. During the check-in process, an agent confirms the reservation, assigns a seat, issues a boarding pass, and weights, labels, and dispatches baggage. The entire process takes an average of 3 minutes. Agents are paid $20 an hour, and Global's customer relations department estimates that for every minute that a customer spends waiting in line, Global loses $1 in missed flights, customer dissatisfaction, and future business. For simplicity, assume that the coefficient of variation for the inter-arrival times and services times are equal to 1.

a) How many agents should Global airlines staff at its hub terminal?

b) Global has surveyed both its customers and its competition and discovered that 3 minutes is an acceptable average waiting time. If Global wants to meet this industry norm, how many agents should it hire?

SOLUTIONS

Arrival Rate: $A = 52\text{ passengers/hr}$
Service Rate: $S = 20\text{ passengers/hr}$
Service Channels: $M = 3$ (at least, in order to have utilization below 100%)
$CV_{IAT} = CV_{ST} = 1$

In the table below, I've used the same formulas for u, L_q, and W_q as used in previous problems. The hourly service cost is simply the number of agents (give in the first column) multiplied by the $20/hr wage. The waiting cost of a single customer would be the average wait in the queue in minutes (the fourth column in the table below, multiplied by 60 minutes per hour) multiplied by $1 per minute. To get the hourly waiting cost, we take this result and multiply it by the hourly flowrate of passengers through the system (i.e., the arrival rate of 52 passengers per hour).

Servers	Utilization	L_q	W_q	Hourly Service Cost	Hourly Waiting Cost	Total Cost
3	0.866666667	5.003575	0.096223	60	300.2145168	$360.21
4	0.65	0.731665	0.01407	80	43.89987597	$123.90
5	0.52	0.216255	0.004159	100	12.97527996	$112.98
6	0.433333333	0.07723	0.001485	120	4.633789514	$124.63

We can see the inflection point in the curve at 5 servers (the total cost increases at either 4, or 6 servers). And so the least expensive solution is to hire 5 agents.

b) A wait time of 3 minutes (or 0.05 hours) or less is obtained by having 4 agents or more.

Problem 6

When customers of Henniker Bank believe a mistake has been made on their account statements, their claims are forwarded to the bank's research department, whose trained clerks carefully research and document the transactions in question. On completing her investigation, a clerk phones the customer with her findings. The research department has three clerks. Each handles claims from a separate geographic district and never works on claims from outside her own district. The average number of complaints arising from each district is the same, 3.5 per week. The clerks are equally experienced and completely process the average claim in 1.2 days. Assume a five-day week. Assume, for simplicity, that the coefficient of variation for the inter-arrival times and services times are equal to 1.

a) Across all districts, how many claims are waiting to be processed on average?

b) The bank is considering introducing a new information system that would reduce the standard deviation of the service distribution by 50%, although the mean would remain unchanged. How would your answers to part a change?

SOLUTIONS

Arrival Rate: $A = 3.5 \text{ complaints/wk}$
Service Rate: $S = 4.1\bar{6} \text{ complaints/wk}$
Service Channels: $M = 1$
$CV_{IAT} = CV_{ST} = 1$

a) $L_q = 4.41$ complaints/district, or 13.23 complaints across all three districts.

b) $CV_{ST} = \sigma/\mu$. Our initial value for this was 1 but by reducing σ by 50%, this fraction becomes 0.5. The effect is reducing the average queue length by 37.5%

Problem 7

Burrito King, a new fast-food franchise, has had problems with its drive-through window operations. Customers arrive at an average rate of one every 30 seconds. The coefficient of variation for the interarrival times is equal to 1. Current service time has averaged 25 seconds with a standard deviation of 20 seconds. A suggested process change, when tested, results in an average service time of 25 seconds with a standard deviation of 10 seconds. Assume that no customers are blocked or abandon the system.

a) As a result of implementing this change, will the average waiting time in queue increase, decrease, or remain unchanged?

b) As a result of implementing this change, will the average server utilization increase, decrease, or remain the same?

SOLUTIONS

Arrival Rate: $A = 2 \text{ customers/min}$
Service Rate: $S = 2.4 \text{ customers/min}$
Service Channels: $M = 1$
$CV_{IAT} = 1$
Old System: $CV_{ST} = \frac{\sigma}{\mu} = \frac{20}{25} = 0.8$
New System: $CV_{ST} = \frac{\sigma}{\mu} = \frac{10}{25} = 0.4$

a)

$$L_q = \frac{u^{\sqrt{2(M+1)}}}{1-u} \times \frac{{CV_{IAT}}^2 + {CV_{ST}}^2}{2}$$

By simply observation, we can see that L_q decreases as CV_{ST} decreases (as is the case in the new system). And this can be extended to W_q using Little's Law. We may therefore conclude that based on the change to the new system, the waiting time will decrease.

b)

$$u = {}^{A}\!/_{MS}$$

Again, by simply observation of the formula for server utilization, we can see that the coefficient of variation doesn't appear, i.e., it has no influence on utilization and therefore utilization remains unchanged.

Problem 8

V.V. Ranger is a seller of industrial products. All purchases by customers are made through call centers where Ranger representatives take orders. Currently, Ranger has over 350 warehouses in the United States, each with its own call center. Customers call one of the call centers and wait on hold until a representative at that call center becomes available. Ranger is evaluating a switching system where customers will call one 800 number from where they will be routed to the first available representative in any of the call centers. If Ranger installs the switching system, will the average waiting time of customers increase, decrease, or remain the same? Explain.

SOLUTIONS

We're essentially consolidating 350 queues into a single queue having 350 servers. The effect will be decreased variability in the inter-arrival time (CV_{IAT}), more consistent server utilization and reduced wait times by extension.

Problem 9

Customers send e-mails to help desk of an online retailer every 2 minutes, on average, and the standard deviation of the interarrival time is also 2 minutes. The online retailer has three employees answering emails. It takes on average 4 minutes to write a response e-mail. The standard deviation of the processing times is 2 minutes.

a) Estimate the average customer wait before being served.

b) How many e-mails would there be, on average, that have been submitted to the online retailer but not yet answered?

SOLUTIONS

Arrival Rate: $A = 30 \text{ emails/hr}$
Service Rate: $S = 15 \text{ emails/hr}$
Service Channels: $M = 3$
$CV_{IAT} = \frac{\sigma}{\mu} = \frac{2}{2} = 1$
$CV_{ST} = \frac{\sigma}{\mu} = \frac{2}{4} = 0.5$

$u = 0.\bar{6}$

$L_q = 0.60$ customers

$W_q = 0.01985 \text{ hours} = 1.19 \text{ minutes}$

$L = 2.6$ customers

Problem 10

Mylaw.com is a recent start-up trying to cater to customers in search of legal services who are intimidated by the idea of talking to a lawyer or simply too lazy to enter a law office. Unlike traditional law firms, Mylaw.com allows for extensive interaction between lawyers and their customers via telephone and the internet. This process is used in the upfront part of the customer interaction, largely consisting of answering some basic customer questions prior to entering a formal relationship. In order to allow customers to interact with the firm's lawyers, customers are encouraged to send e-mails to mylawer@mylaw.com. From there, the incoming emails are distributed to the lawyer who is currently "on call". Given the broad skills of the lawyers, each lawyer can respond to each incoming request. Emails arrive from 8am to 6pm at a rate of 10 emails per hour (coefficient of variation for the arrival is 1). At each moment in time, there is exactly one lawyer "on call", that is, sitting at his or her desk waiting for incoming emails. It takes the lawyer, on average, 5 minutes to write the response email. The standard deviation of t is 4 minutes.

a) What is the average time a customer has to wait for the response to his/her email, ignoring any transmission times? Note: This includes the time it takes the lawyer to start writing he email and the actual writing time.

b) How many emails will a lawyer have received at the end of a 10-hour day?

c) When not responding to emails, the lawyer on call is encouraged to actively pursue cases that potentially could lead to large settlements. How much time in a 10-hour day can a Mylaw.com lawyer dedicate to this activity (assume the lawyer can instantly switch between emails and work on a settlement)?

To increase the responsiveness of the firm, the board of Mylaw.com proposes a new operating policy. Under the new policy, the response would be highly standardized, reducing the standard deviation for writing the response email to 0.5 minute. The average writing time would remain unchanged.

d) How would the amount of time a lawyer can dedicate to the search for large settlement cases change with this new operating policy?

e) How would the average time a customer has to wait for the response to his/her email change? Note: This includes the time until the lawyer starts writing the email and the actual writing time.

SOLUTIONS

Arrival Rate: $A = 10 \text{ customers/hr}$
Service Rate: $S = 12 \text{ customers/hr}$
Service Channels: $M = 1$
$CV_{IAT} = 1$
$CV_{ST} = \frac{\sigma}{\mu} = \frac{4}{5} = 0.8$

$u = 0.8\overline{3}$

$L_q = 3.41\overline{6}$ customers

$W_q = 0.341\overline{6}$ hours $= 20.5$ minutes

a) $W = 25.5$ minutes

b) 100 customers

c) $1.\overline{6}$ hours

d) This has no impact on utilization.

e)

$CV_{ST} = \frac{\sigma}{\mu} = \frac{0.5}{5} = 0.1$,

$L_q = 2.1$ customers

$W_q = 0.210$ hours $= 12.6$ minutes

$W = 17.6$ minutes

Reduced variability in service time yields lower flowtimes for customers in the process.

Problem 11

The airport branch of a car rental company maintains a fleet of 50 SUVs. The interarrival time between requests for an SUV is 2.4 hours, on average, with a standard deviation of 2.4 hours. There is no indication of a systematic arrival pattern over the course of a day. Assume that, if all SUVs are rented, customers are willing to wait until there is an SUV available. A SUV is rented, on average, for 3 days, with a standard deviation of 1 day.

a) What is the average number of SUVs parked in the company's lot?

b) Through a marketing survey, the company has discovered that if it reduces its daily rental price of $80 by $25, the average demand would increase to 12 rental requests per day and the average rental duration will become 4 days. Is this price decrease warranted? Provide an analysis!

c) What is the average time a customer has to wait to rent an SUV? Please use the initial parameters rather than the information in part (b).

d) How would the waiting time change if the company decides to limit all SUV rentals to exactly 4 days?

SOLUTIONS

Arrival Rate: $A = {}^{1}/_{2.4} = 0.417 \text{ requests/hr}$

Service Rate: $S = {}^{1}/_{3 \times 24} = 0.0139 \text{ requests/hr}$

$M = 50$

a)

$u = 0.6 \rightarrow 40\%$ of SUVs are sitting idle

40% of 50 SUVs = 20 vehicles

b)

In the original model, 20 vehicles were idle and 30 were rented. Revenue is therefore $30 \times \$80 = \$2{,}400$ per day.

Arrival Rate: $A_{new} = {}^{12}/_{24} = 0.5 \text{ requests/hr}$

Service Rate: $S = {}^{1}/_{4 \times 24} = 0.01041\bar{6} \text{ requests/hr}$

$u_{new} = 0.96 \rightarrow 4\%$ of SUVs are sitting idle

New revenue is $48 \times \$55 = \$2{,}640$. Since this is greater than the original model, we should indeed lower the price.

c)

$CV_{IAT} = 1$

$CV_{ST} = \frac{\sigma}{\mu} = \frac{1}{3} = 0.\overline{3}$

$u = 0.6$

$M = 50$

$L_q = 0.00798$ customers

$W_q = 0.019 \text{ hours} = 1.15 \text{ minutes}$

d)

If the service time is *exactly* 4 days then CV_{ST} would decrease, implying L_q decreases, implying that W_q decreases.

Problem 12

The following situation refers to Tom Opim, a first-year MBA student. In order to pay the rent, Tom decides to take a job in the computer department of a local department store. His only responsibility is to answer telephone calls to the department, most of which are inquiries about store hours and product availability. As Tom is the only person answering calls, the manager of the store is concerned about queuing problems. Currently, the computer department receives an average of one call every 3 minutes, with a standard deviation in this interarrival time of 3 minutes. Tom requires an average of 2 minutes to handle a call. The standard deviation in this processing time is 1minute. The telephone company charges $5 per hour for the telephone lines whenever they are in use either while a customer is in conversation with Tom or while waiting to be helped. Assume there are no limits on the number of customers that can be on hold and that customers do not hang up even if forced to wait a long time.

a) For one of his courses, Tom has to read a book (The Pole, by E. Silvermouse). He can read 1 page per minute. Tom's boss has agreed that Tom could use his idle time for studying, as long as he drops the book as soon as a call comes in. How many pages can Tom read during an 8-hour shift?

b) How long does a customer have to wait, on average, before talking to Tom?

c) What is the average total cost of telephone lines over and 8-hour shift? Note that the department store is billed whenever a line is in use, including when a line is used to put customers on hold.

SOLUTIONS

Arrival Rate: $A = {}^{1}/_{3} = 0.\overline{3}$ calls/min

Service Rate: $S = {}^{1}/_{2} = 0.5$ calls/min

$M = 1$

$u = 66.\overline{6}\% \rightarrow 33.\overline{3}\%$ idleness

a) 8 hours = 480 minutes, of which 160 are idle. At 1-page per minute, Tom can read 160 pages.

b)

$CV_{IAT} = 1$

$CV_{ST} = \frac{\sigma}{\mu} = \frac{1}{2} = 0.5$

$u = 0.\overline{6}$

$M = 1$

$L_q = 0.8\overline{3}$ customers

$W_q = 2.5$ minutes

c)

$L = 1.5$ customers (including both those waiting on-hold and those currently being served)

Total cost is 1.5 × \$5/hour = \$7.5/hour =\$60 per 8-hour shift.

Problem 13

RentAPhone is a new service company that provides European mobile phones to American visitors to Europe. The company currently has 80 phones available at Charles de Gaulle Airport in Paris. There are, on average, 25 customers per day requesting a phone. There's requests arrive uniformly throughout the 24 hours the store is open. (Note: This means customers arrive at a faster rate than 1 customer per hour.) The corresponding coefficient of variation is 1. Customers keep their phones on average 72 hours. The standard deviation of this time is 100 hours. Given that RentAPhone currently does not have a competitor in France providing equally good service, customers are willing to wait for the telephones. Yet, during the waiting period, customers are provided a free calling card. Based on prior experience, RentAPhone found that the company incurred a cost of $1 per hour per waiting customer, independent of day or night.

a) What is the average number of telephones the company has in its store?

b) How long does a customer, on average, have to wait for the phone?

c) What are the total monthly (30 days) expenses for telephone cards?

d) Assume RentAPhone could buy additional phones at $1000 per unit. Is it worth it to buy one additional phone? Why?

e) How would waiting time change if the company decides to limit all rentals to exactly 72 hours? Assume that if such a restriction is imposed, the number of customers requesting a phone would be reduced to 20 customers per day.

SOLUTIONS

a)

Arrival Rate: $A = 25 \text{ customers/day}$

Service Rate: $S = {}^{1}/_{(72 \div 24)} = 0.\overline{3} \text{ customers/day}$

$M = 80$

$u = 93.75\%$ → 6.25% idleness → 5 of 80 phones remain in the store

b)

$CV_{IAT} = 1$

$CV_{ST} = \frac{\sigma}{\mu} = \frac{100}{72} = 1.3\overline{8}$

$u = 0.9375\%$

$M = 80$

$L_q = 10.31$ customers

$W_q = 0.4122$ days

c) $10.31\ (\$1/\text{hr}) \times 24\ \text{hr}\ \times 30\ \text{days} = \$7{,}423.20$

d)

$M = 81$

$u = 0.9259\%$

$L_q = 7.3737$ customers

$W_q = 0.2949$ days

$7.3737\ \ (\$1/\text{hr}) \times 24\ \text{hr}\ \times 30\ \text{days} = \$5{,}309.06$

Definitely by the new phone. The monthly savings are a little over two thousand dollars. The capital acquisition cost is paid for in just half a month.

Problem 14

Webflux is an Internet-based DVD rental business specializing in hard-to-find, obscure films. Its operating model is as follows. When a customer finds a film on the Webflux Web site and decides to watch it, she puts it in the virtual shopping cart. If a DVD is available, it is shipped immediately (assume it can be shipped during weekends and holidays, too). If not available, the film remains in the customer's shopping cart until a rented DVD is returned to Webflux, at which point it is shipped to the customer if she is next in line to receive it. Webflux maintains an internal queue for each film and a returned DVD is shipped to the first customer in the queue (first-in, first-out). Webflux has one copy of the 1990 film Sundown, the Vampire in Retreat, starring David Carradine and Bruce Campbell. The average time between requests for the DVD is 10 days, with a coefficient of variation of 1. On average, a customer keeps the DVD for 5 days before returning it. It also takes 1 day to ship the DVD to the customer and 1 day to ship it from the customer back to Webflux. The standard deviation of the time between shipping the DVD out from Webflux and receiving it back is 7 days (i.e., it takes on average 7 days to (a) ship it, (b) have it with the customer, and (c) ship it back); hence, the coefficient of variation of this time is 1.

a) What is the average time that a customer has to wait to receive Sundown, the Vampire in Retreat DVD after the request? Recall it takes 1 day for a shipped DVD to arrive at a customer address (i.e., in your answer, you have to include the 1-day shipping time).

b) On average, how many customers are in Webflux's internal queue for Sundown/ Assume customers do not cancel their items in their shopping carts.

SOLUTIONS

a)

Arrival Rate: $A = 0.1 \text{ requests/day}$

Service Rate: $S = 0.1429 \text{ requests/day}$

$M = 1$

$u = 70\%$

$CV_{IAT} = 1$

$CV_{ST} = 1$

$L_q = 1.6\overline{3} \text{ requests}$

$W_q = 16.\overline{3} \text{ days}$ - we must add one more day to this to ship to the customer. So a customer can expect to wait $17.\overline{3}$ days after making the request before receiving the video.

b) On average, there are $1.6\overline{3}$ customers waiting in the queue.

Problem 15

A university offers a walking escort service to increase security around campus. The system consists of specifically trained uniformed professional security officers that accompany students from one campus location to another. The service is operated 24 hours a day, 7 days a week. Students request a walking escort by phone. Requests for escorts are received, on average, every 5 minutes with a coefficient of variation of 1. After receiving a request, the dispatcher contacts an available escort (via a mobile phone), who immediately proceeds to pick up the student and walk her/him to her/his destination. If there are no escorts available (that is, they are all either walking on student to her/his destination or walking to pick up a student), the dispatcher puts the request in a queue until an escort becomes available. An escort takes, on average, 25 minutes for picking up a student and taking her/him to her/his desired location (the coefficient of variation of this time is also 1). Currently, the university has 8 security officers who work as walking escorts.

a) How many security officers are, on average, available to satisfy a new request?

b) How much time does it take – on average – from the moment a student calls for an escort to the moment the student arrives at her/his destination?

SOLUTIONS

Arrival Rate: $A = 12 \text{ requests/hour}$

Service Rate: $S = 2.4 \text{ requests/hour}$

$CV_{IAT} = 1$

$CV_{ST} = 1$

$M = 8$

a) $u = 62.5\% \rightarrow$ 3 security officers are typically available.

b) $L_q = 0.363 \text{ requests}$ $W_q = 1.81 \text{ minutes}$ $W = 26.81 \text{ minutes}$

Problem 16

Mango Electronics Inc is a Fortune 500 company that develops and markets innovative consumer electronics products. The development process proceeds as follows. Mango researches new technologies to address unmet market needs. Patents are filed for products that have the requisite market potential. Patents are granted for a period of 20 years starting from the date of issue. After receiving a patent, the patented technologies are then developed into marketable products at 5 independent development centers. Each product is only developed at one center. Each center has all the requisite skills to bring any of the products to market (a center works on one product at a time). On average, Mango files a patent every 7 months (with standard deviation of 7 months). The average development process lasts 28 months (with standard deviation of 56 months).

a) What is the utilization of Mango's development facilities?

b) How long does it take an average technology to go from filing a patent to being launched in the market as a commercial product?

c) How many years of patent life are left for an average product launched by Mango Electronics?

SOLUTIONS

Arrival Rate: $A = 0.1429 \text{ patent/month}$

Service Rate: $S = 0.03571 \text{ patent/month}$

$CV_{IAT} = 1$

$CV_{ST} = \frac{56}{28} = 2$

$M = 5$

a) $u = 0.8$

b) $L_q = 5.77 \text{ patents} \quad W_q = 40.39 \text{ months.} \;\; W = 68.39 \text{ months}$

c) $20 - \frac{68.39}{12} = 14.3 \text{ years}$