MMA/MMAI 861
ANALYTICAL DECISION MAKING

Simulation Examples

Classes Three and Four
Yuri Levin

Static Simulation Models

Newsvendor Problem: Set-Up

Inputs:

- Each paper sells for p = \$.45
- Each paper costs c = \$.35
- Leftover papers sell for v = \$.05
- The firm is facing a random demand D – demand history is given

Objective:

- Find an optimal order quantity to maximize expected profit!

The "Two Much-Too Little Problem"

- Order too much and
  - Inventory is left over at the end of the season
  - High inventory cost

- Order too little and
  - Sales are lost
  - High opportunity cost

Let's try in Excel!

Three Approaches

- **Approach One: Trial-and-Error**
  - Try different order quantities and see what happens
  - Monitor several days
    - Too many units left, adjust upward
    - Too many units left, adjust downward
- **Approach Two: Mathematical Optimization**
  - Build mathematical cost model
  - Collect data or estimate demand variation
  - Calculate an 'optimal' order quantity with mathematical analysis (will do later)
  - Or…

Three Approaches (continued...)

- **Approach Three: A Compromise – Simulation**
  - Collect data or estimate demand variation
  - Build spreadsheet cost model
  - Fix a 'scenario': i.e. order quantity
    - Simulate thousands of days in the computer!
    - Collect and summarize the profit outcomes of all those days
  - Repeat for other scenarios and look for the 'best'

Demand History

Demand	Probability	Cumulative
20	1%	1%
21	2%	3%
22	3%	6%
23	4%	10%
24	5%	15%
25	6%	21%
26	6%	27%
27	7%	34%
28	7%	41%
29	7%	48%
30	7%	55%
31	7%	62%
32	7%	69%
33	6%	75%
34	5%	80%
35	4%	84%
36	4%	88%
37	3%	91%
38	3%	94%
39	3%	97%
40	3%	100%

29.99	Expected Value (average)
4.97	Standard Deviation (measure of variability)

Newsvendor Profit

Total Profit =

= Revenue - Cost + Return Credit

= .45 x Units Sold - .35 x Order Qnty + .05 x Units Unsold

Units Sold =

= min(Demand, Order Qnty)

Units Unsold =

= Order Qnty - Units Sold

Approach Two: Marginal Analysis

"Too Much" and "Too Little" Costs

- C_o = overage cost
  - The cost of ordering one more unit than what you would have ordered had you known demand
  - Suppose you had leftover inventory (i.e. you over ordered). C_o is the increase in profit you would have enjoyed had you ordered one fewer unit
  - C_o = *Cost - Salvage value = c - v* = .35 - .05 = .30
- C_u = underage cost
  - The cost of ordering one fewer unit than what you would have ordered had you known demand
  - Suppose you had lost sales (i.e. you under ordered). C_u is the increase in profit you would have enjoyed had you ordered one more unit
  - C_u = *Price - Cost = p - c* = .45 - .35 = .10

Balancing Risk and Benefit of Ordering a Unit

- Ordering one more unit increases the chance of overage…
  - Expected loss on the Q^{th} unit = C_o x $F(Q)$
  - $F(Q)$ = Distribution function of demand = $Prob\{Demand <= Q\}$
- …But the benefit/gain of ordering one more unit is the reduction in the chance of underage:
  - Expected gain on the Q^{th} unit = C_u x $(1-F(Q))$

- As more units are ordered, the expected benefit from ordering one unit decreases while the expected loss of ordering one more unit increases

Newsvendor Expected Profit Maximizing Order Quantity

- To maximize expected profit order *Q* units so that the expected loss on the *Q*th unit equals the expected gain on the *Q*th unit:

$$C_o \times F(Q) = C_u \times \left(1 - F(Q)\right)$$

- Rearrange terms in the above equation ->

$$F(Q) = \frac{C_u}{C_o + C_u}$$

- The ratio C_u / (C_o + C_u) is called the ***critical ratio***
- Hence, to maximize profit, choose *Q* such that the probability that we do not lose any sales (demand is *Q* or lower) equals the critical ratio

Expected Profit Maximizing Order Quantity Using Marginal Analysis

- Inputs: p = .45; c = .35; v = .05;
- C_u = .45-.35 = .10;
- C_o = .35-.05 = .30;
- Critical ratio = 0.25;
- Optimal order quantity Q^* is such that

$$F(Q^*) = .25$$

$$Q^* = 26$$

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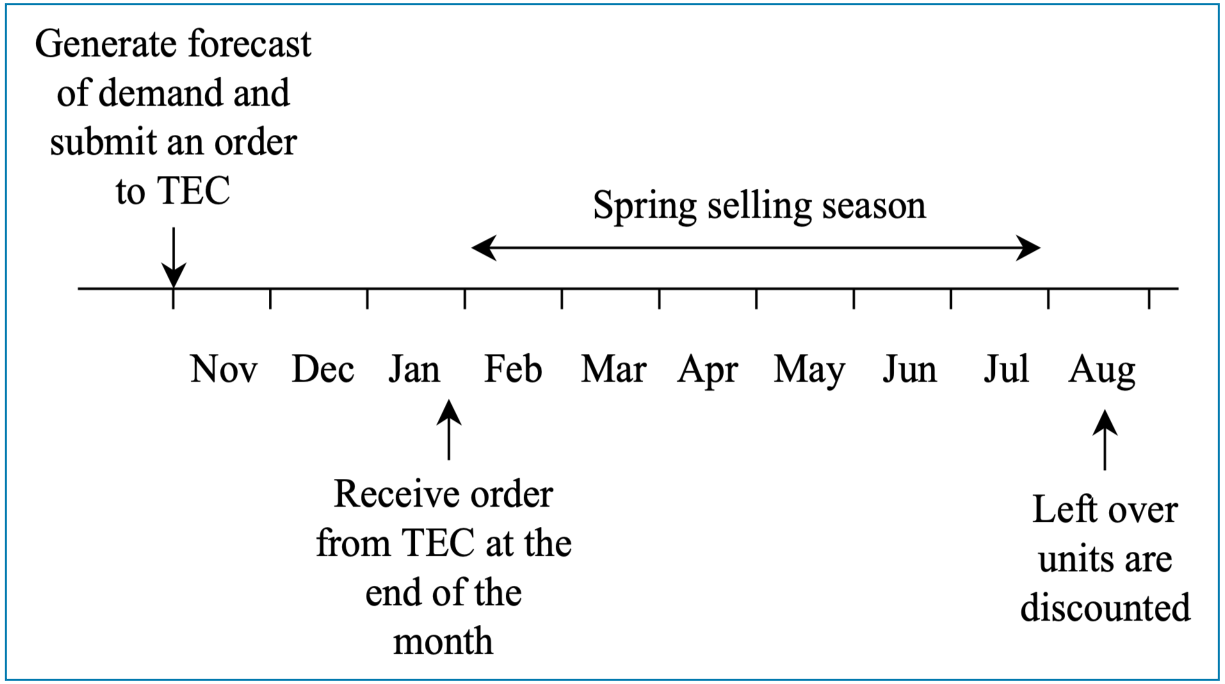
Another Example: O'Neill's Hammer 3/2 Wetsuit

Hammer 3/2 Timeline and Economics



Marketing's forecast for sales is *3,200* units.

Economics:

Each suit sells for *p* = *$180*

Each suit costs *c* = *$110*

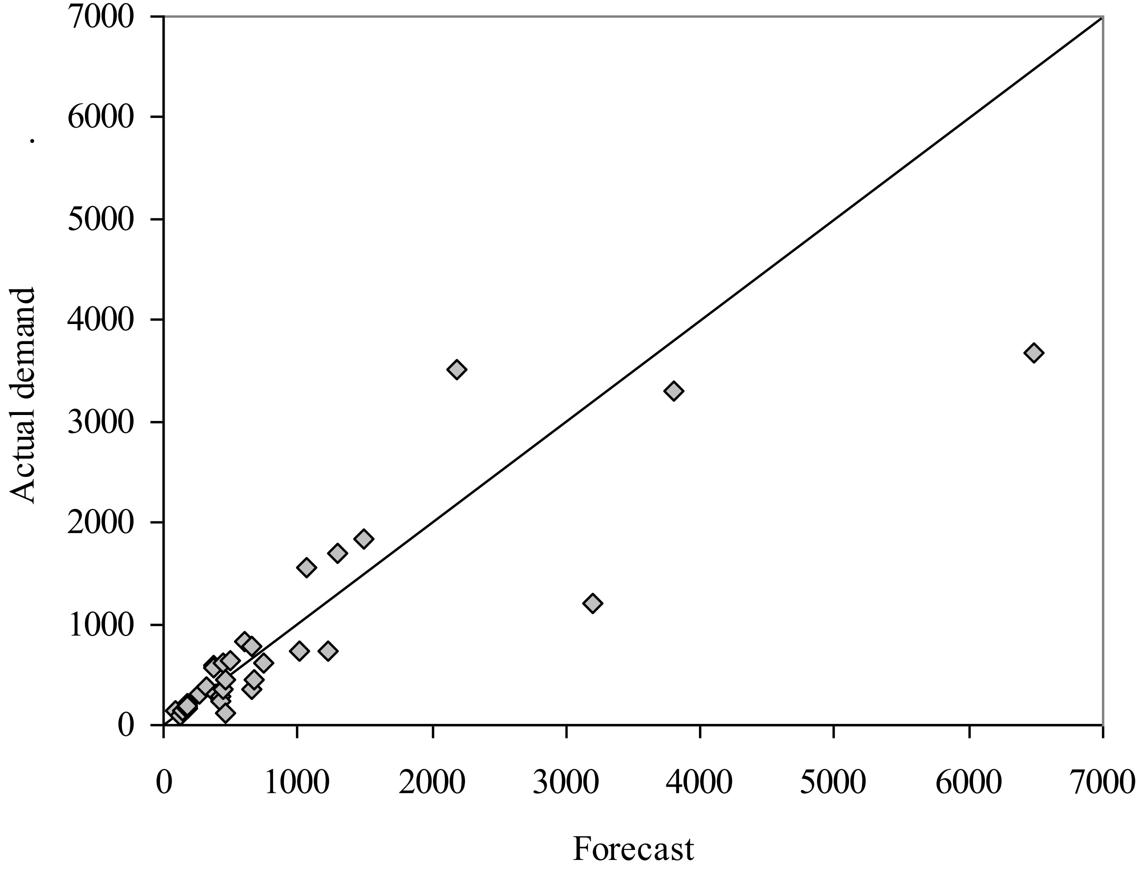
Leftover suits sell for *v* = *$90*

Model Implementation Steps

- Gather economic inputs
  - Selling price, production/procurement cost, salvage value of inventory
- Generate a demand model
  - Use empirical demand distribution or choose a standard distribution function to represent demand, e.g. the normal distribution
- Choose an objective
  - E.g. maximize expected profit or satisfy a fill rate constraint
- Choose a quantity to order

How to develop a demand forecast for O'Neill's Hammer 3/2 wetsuit?

Historical Forecast Performance at O'Neill



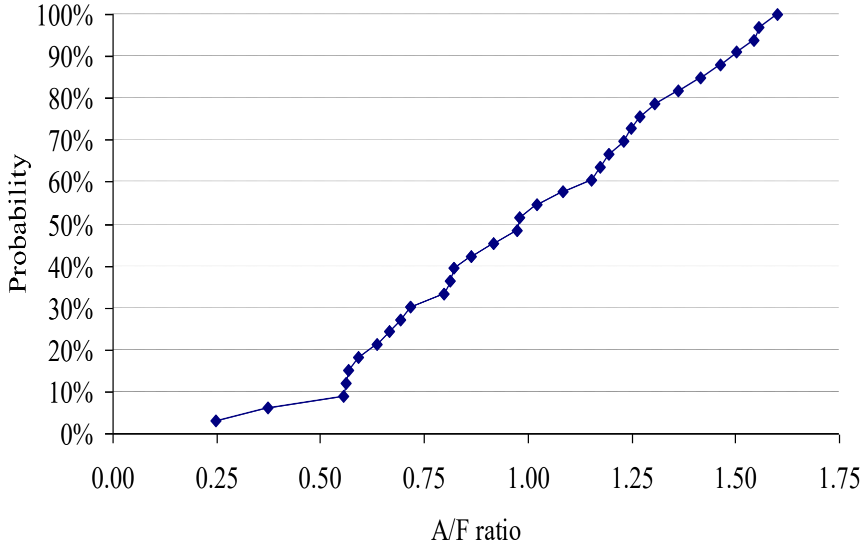
Forecasts and actual demand for surf wetsuits from the previous season

Empirical Distribution of Forecast Accuracy

Product description	Forecast	Actual demand	Error*	A/F Ratio**
JR ZEN FL 3/2	90	140	-50	1.56
EPIC 5/3 W/HD	120	83	37	0.69
JR ZEN 3/2	140	143	-3	1.02
WMS ZEN-ZIP 4/3	170	163	7	0.96
HEATWAVE 3/2	170	212	-42	1.25
JR EPIC 3/2	180	175	5	0.97
WMS ZEN 3/2	180	195	-15	1.08
ZEN-ZIP 5/4/3 W/HOOD	270	317	-47	1.17
WMS EPIC 5/3 W/HD	320	369	-49	1.15
EVO 3/2	380	587	-207	1.54
JR EPIC 4/3	380	571	-191	1.50
WMS EPIC 2MM FULL	390	311	79	0.80
HEATWAVE 4/3	430	274	156	0.64
ZEN 4/3	430	239	191	0.56
EVO 4/3	440	623	-183	1.42
ZEN FL 3/2	450	365	85	0.81
HEAT 4/3	460	450	10	0.98
ZEN-ZIP 2MM FULL	470	116	354	0.25
HEAT 3/2	500	635	-135	1.27
WMS EPIC 3/2	610	830	-220	1.36
WMS ELITE 3/2	650	364	286	0.56
ZEN-ZIP 3/2	660	788	-128	1.19
ZEN 2MM S/S FULL	680	453	227	0.67
EPIC 2MM S/S FULL	740	607	133	0.82
EPIC 4/3	1020	732	288	0.72
WMS EPIC 4/3	1060	1552	-492	1.46
JR HAMMER 3/2	1220	721	499	0.59
HAMMER 3/2	1300	1696	-396	1.30
HAMMER S/S FULL	1490	1832	-342	1.23
EPIC 3/2	2190	3504	-1314	1.60
ZEN 3/2	3190	1195	1995	0.37
ZEN-ZIP 4/3	3810	3289	521	0.86
WMS HAMMER 3/2 FULL	6490	3673	2817	0.57

* Error = Forecast - Actual demand

** A/F Ratio = Actual demand divided by Forecast



Empirical distribution function for the historical A/F ratios.

Using Historical A/F Ratios to Choose a Normal Distribution for the Demand Forecast

- Start with an initial forecast generated from hunches, guesses, etc.
  - O'Neill's initial forecast for the Hammer 3/2 = 3,200 units
- Evaluate the A/F ratios of the historical data

$$A/F\ ratio = \frac{Actual\ demand}{Forecast}$$

- Set the mean of the normal distribution to:

$$Expected\ actual\ demand = Expected\ A/F\ ratio \times Forecast$$

- Set the standard deviation of the normal distribution to:

$$Standard\ deviation\ of\ actual\ demand = Standard\ deviation\ of\ A/F\ ratios \times Forecast$$

O'Neill's Hammer 3/2 Normal Distribution Forecast

Product description	Forecast	Actual demand	Error	A/F Ratio
JR ZEN FL 3/2	90	140	-50	1.5556
EPIC 5/3 W/HD	120	83	37	0.6917
JR ZEN 3/2	140	143	-3	1.0214
WMS ZEN-ZIP 4/3	170	156	14	0.9176
•••	•••	•••	•••	•••
ZEN 3/2	3190	1195	1995	0.3746
ZEN-ZIP 4/3	3810	3289	521	0.8633
WMS HAMMER 3/2 FULL	6490	3673	2817	0.5659
Average				0.9975
Standard deviation				0.3690

$$Expected\ actual\ demand = 0.9975 \times 3200 = 3192$$

$$Standard\ deviation\ of\ actual\ demand = 0.369 \times 3200 = 1181$$

O'Neill should choose a *normal distribution* with mean *3192* and standard deviation *1181* to represent demand for the Hammer 3/2 during the spring season.

Expected Profit Maximizing Order Quantity Using the Normal Distribution

- Inputs: p = 180; c = 110; v = 90;
- C_u = 180-110 = 70;
- C_o = 110-90 =20;
- Critical ratio = 0.7778;
- μ = 3192; σ = 1181
- Optimal order quantity:

$$\begin{aligned} Q^* &= NORMINV(\text{critical ratio}, 3192, 1181) \\ &= NORMINV(.7778, 3192, 1181) \\ &= 4095 \end{aligned}$$

Newsvendor Model Performance Measures

For any order quantity we can evaluate the following performance measures:

- Expected lost sales
  - Average number of units demand exceeds the order quantity
- Expected sales
  - The average number of units sold
- Expected left over inventory
  - The average left over at the end of the season
- Expected profit
- Expected fill rate
  - The fraction of demand that is satisfied immediately
- In-stock probability (Probability all demand is satisfied)
- Stockout probability (Probability some demand is lost)

Newsvendor Model Summary

- The model can be applied to settings in which…
  - There is a single order/production/replenishment opportunity
  - Demand is uncertain
  - There is a "too much-too little" challenge:
    - If demand exceeds the order quantity, sales are lost
    - If demand is less than the order quantity, there is left over
- Firm must have a demand model that includes an expected demand and uncertainty in that demand
  - With the normal distribution, uncertainty in demand is captured with the standard deviation parameter
- At the optimal order quantity Q the probability that demand is less than Q equals the critical ratio:
  - The expected profit maximizing order quantity balances the "too much-too little" costs

Airline Overbooking

You are taking reservations for an airline flight. This particular flight uses an aircraft with 50 first-class seats and 190 economy-class seats.

First-class tickets on the flight cost $600, with demand to purchase them distributed like a Poisson random variable with mean 50. Each passenger who buys a first-class ticket has a 93% chance of showing up for the flight. If a first-class passenger does not show up, he or she can return their unused ticket for a full refund. Any first class passengers who show up for the flight with tickets but are denied boarding are entitled to a full refund plus a $500 inconvenience penalty.

Economy tickets cost $300. Demand for them is Poisson distributed with a mean of 200, and is independent of the demand for first-class tickets. Each ticket holder has a 96% chance of showing up for the flight, and "no shows" are not entitled to any refund. If an economy ticket holder shows up and is denied a seat, however, they get a full refund plus a $200 penalty. If there are free seats in first class and economy is full, economy ticket holders can be seated in first class.

Airline Overbooking (continued…)

The airline allows itself to sell somewhat more tickets than it has seats. This is a common practice called "overbooking". The firm is considering the 18 possible policies obtained through all possible combinations of:

Allowing overbooking of up to 0, 5, or 10 first-class seats

Allowing overbooking of up to 0, 5, 10, 15, 20, or 25 economy seats

Which option gives the highest average profit? What are the average numbers of first-class and economy passengers denied seating under this policy. If no overbooking of first class is allowed, what is the best policy?

Hiring Translators

Lingua Translations Company wants to determine how many part-time Hungarian translators it should hire. Examining its records for the last few years, the firm discovers that the number of translation orders per day has followed the following pattern: Level (frequency)

0-1%, 1-2%, 2-5%, 3-5%, 4-7%, 5-8%, 6-9%, 7-11%, 8-13%, 9-13%, 10-9%, 11-9%, 12-5%, 13-1%, 14-1%, 15-1%.

Each translator can process one order a day. Lingua pays a fixed amount of $10 per day to each translator, whether or not the translator has to fill an order.

Hiring Translators (continued…)

Furthermore, a translator gets $50 for translating an order. If the company receives more orders than it has translators, it asks the translators whether they are available for overtime. For each translator, the chance of being available for overtime is 35%, independent of all other translators and the number of translation orders received. A translator available for overtime can process one additional order, but at a cost of $75. The company charges its customers $95 per translation order.

Assume that the historical demand pattern will continue. In order to maximize its average daily profits, how many translators should Lingua keep on staff? At this staffing level, what is the average number of translation orders per day that Lingua must refuse for lack of staff?

Experiment with 5, 6, 7, … , 12 translators with a sample size of 1,000.

Insurance

The automobile insurance division of the Great Benefit insurance company expects an average of 1,000 claims in the forthcoming year, with the actual number of claims being random and well described by a Poisson distribution. The value of each claim is a random variable, independent of all other claims, with a mean of $5,000 and a standard deviation of $1,500. The division has $6 million of capital, which is split into two parts. The first part is the reserve capital needed to pay claims over the next year. The remainder is invested in short-term bonds, which provide a random return, equally likely to be any value between 5% and 8%. If the reserve capital turns out to be less than the total value of claims for the year, the division has to borrow enough money, at a cost of 10% of the amount borrowed, to make up the difference.

The firm would like to find an capital allocation that maximizes the expected amount of cash they have left at the end of the year. Suppose they have narrowed down their choice to the following possible amounts of reserve capital: $4.7 million, $4.8 million, $4.9 million, $5.0 million, and $5.1 million. Based on 1,000 simulation trials each, which option is the best?

Dynamic Simulation Models

Inventory Model

You sell a product for which monthly demand is Poisson with a mean of 400. The units cost you $1,500 each, and you sell them for $2,800. You can carry inventory from month-to-month, and estimate your inventory holding cost as $10 per unit left in inventory at the end of a month.

Every time you order, there is a fixed cost of $600, plus the $1,500 per unit cost of the products ordered.

You want to simulate a 24-month period, at the outset of which you have 700 units in stock. For every unit in stock at the end of this period, you assess a "salvage" credit of $1,500.

You are considering ordering policies of the following form: if the ending inventory for a given month is less than or equal to some "threshold" value R, immediately order another Q units. For simplicity, assume that these units become available immediately at the beginning of the next month.

Inventory Model (continued...)

Your boss asks you to evaluate the following possible combinations of *R* and *Q*. Which one seems to yield the highest expected profit over the 24-month period?

Policy	*R*	*Q*
1	400	800
2	400	1000
3	400	1200
4	500	1000
5	500	1200
6	600	1000
7	600	1200

For each policy, you also wish to estimate the probability of having a "stockout" at some time during the 24-month period. A "stockout" means that there is insufficient stock to meet customer demand.

Repair Shop

The Gotham Taxi Company has a fleet of 500 taxicabs. On any given day of use, a taxi has an 0.4% chance of breaking down. Broken-down taxis are towed overnight to the company repair shop. They return to service the day after they are fixed. Each day a taxi spends in the shop costs the company $350 in lost profits.

There are three mechanics Gotham is considering hiring to work in the repair shop: Larry, Moe and Curly. Each can fix one to three taxis per day.

Larry would cost the company $300 per day. On any given day, there is 20% probability he can only fix one taxi, and a 40% probability he will be able to fix either two or three.

Moe costs $250 per day. He has an equal probability of being able to fix either one, two, or three taxis on any given day.

Curly costs $200 per day. On any given day, there is a 50% chance he can fix only one cab, a 30% chance he will be able fix two, and a 20% chance that he will be able to fix three.

Repair Shop (continued…)

The company may hire any combination of the three mechanics: any one, any two, or all three. Explain why you can tell, prior to performing any simulation, that the option of hiring just Curly will not be workable. Simulate each possibility by 200 trials of 100 days each. Which possibility gives them the lowest average cost? What is the average number of taxis in the shop when you adopt this policy?

Horses

As part of its program to restore the area's natural ecosystem, the federal government is trying to eliminate wild horses from the Grand River National Wilderness. Your company, Westland Wranglers Inc. (WWI), has a contract with the government to capture the horses. Fortunately, ranchers and hobbyists from neighbouring areas are willing to purchase the horses, so they do not have to be destroyed.

The number of horses your cowboys capture each day is well-modeled by a Poisson random variable with a mean value of 4. You charge your customers $150 each for the horses, and at that price the daily demand for them is well-modeled by a Poisson random variable with mean 4.1. If demand exceeds the number of horses you have available, you lose the additional sales.

Horses (continued…)

Any horses that are not sold at the end of the day are placed in a corral. The cost of keeping a horse in the corral for one night is $8.00, which covers feed, water, and sanitation. These horses are available for sale the following day, along with newly captured horses.

However, only a limited number of horses can fit in a corral. If there are too many horses to fit, the extra ones must be released.

Horses (continued...)

WWI is considering four different options for the corral:

- Rent a small corral, holding up to 5 horses, for $35 per day.
- Rent a medium-sized corral, holding up to 10 horses, for $50 per day.
- Rent both the small and medium-sized corrals. This option would in effect yield a combined corral holding 5 + 10 = 15 horses at a rent of $35 + $50 = $85 per day.
- Do not rent a corral. This option may be considered as equivalent to a corral holding zero horses at a cost of $0 per day. Which option will earn the greatest expected profit? Evaluate each option based on 500 trials, each trial simulating 100 consecutive days of operation. Horses left in the corral at the end of 100 days should be assigned a "salvage" value of $130. You are also interested in how many horses, on average, are left in the corral at the end of day, and the probability that your total profit, without the salvage adjustment, will be less than $45,000.

Belt Replacement Model

Your factory's production equipment contains a belt that must operate under extreme environmental conditions. The belts fail frequently, and the exact probability of failure depends on a belt's age, as follows:

Belt Age (Days)	Chance of Belt Failure
1 ("fresh")	3%
2	7%
3	12%
4	20%
5	34%
6 or more	40%

If a belt fails while in use, it must be replaced on an emergency basis. This causes you to lose the remainder of the day's production on the equipment, with a cost uniformly distributed between $1,000 and $2,000. In this case, you start the next day with a fresh belt.

Belt Replacement Model (continued…)

A working belt can also be replaced just before the start of any day's production. This scheduled replacement is much cheaper than emergency replacement, costing only $450, and allows you to start that day with a fresh belt.

The firm's strategy is to replace each belt after *n* days of use, or as soon as it fails, whichever comes first. What is the best choice of *n* out of the possibilities 1, 2, 3, 4, and 5? Simulate each policy for 100 days with a sample size of 500. Assume that you start the 100-day period with a scheduled replacement.

For the best policy, what is the average number of scheduled and emergency replacements in the 100-day period?