MMA/MMAI 861
ANALYTICAL DECISION MAKING

Optimization Models (Two): Integer Programming

Class Two

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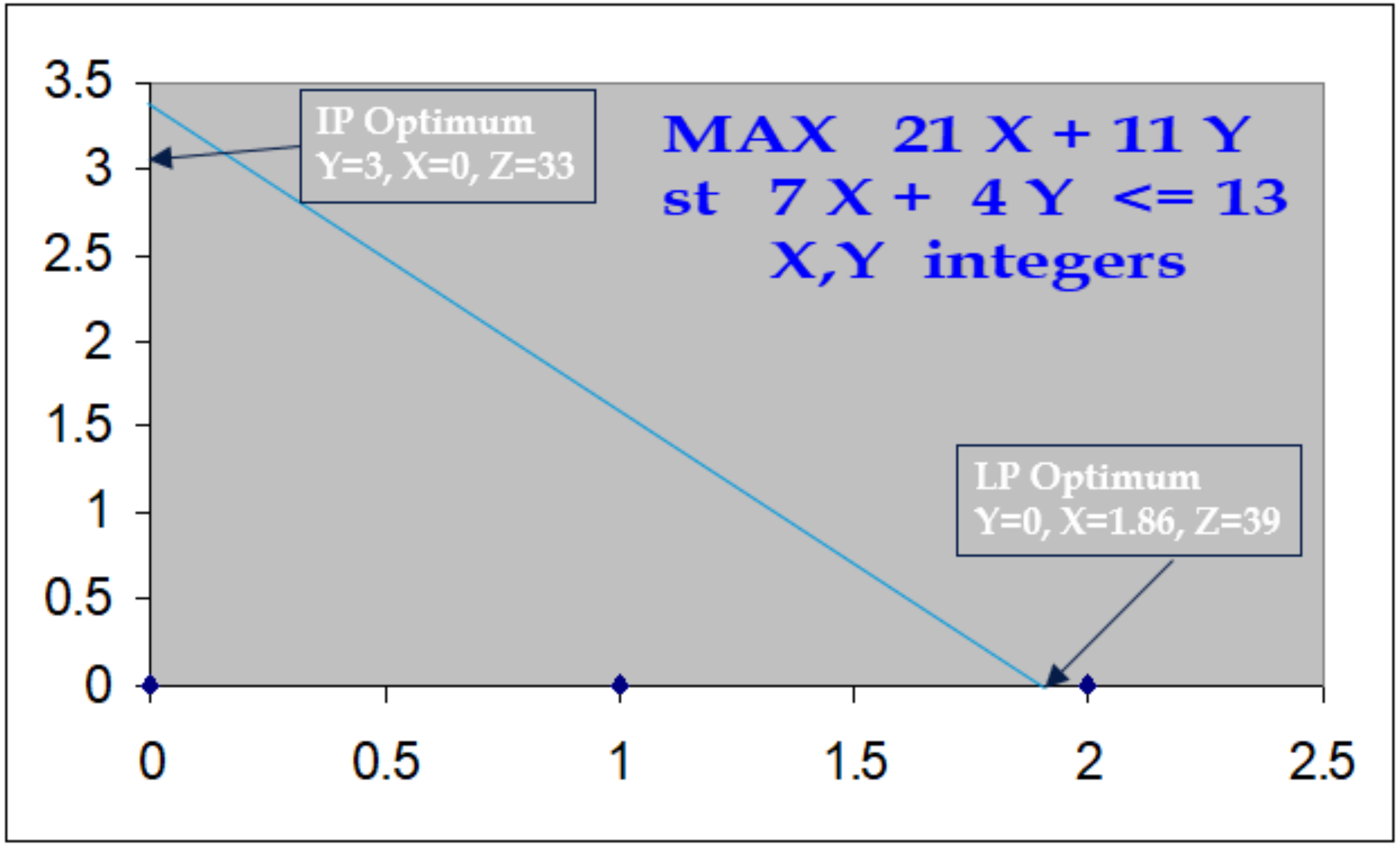
Today's Objectives

- Review problems
- Integer programming
  - Rationale
  - Solution principles
  - Use in formulating common problems

Integer Programming

- In some cases, only integral values possible, either by definition or for special modeling problems… integer programming
- Particularly important when variable values are "small"
- Why not simply round off LP results?

Integer Program: Illustration



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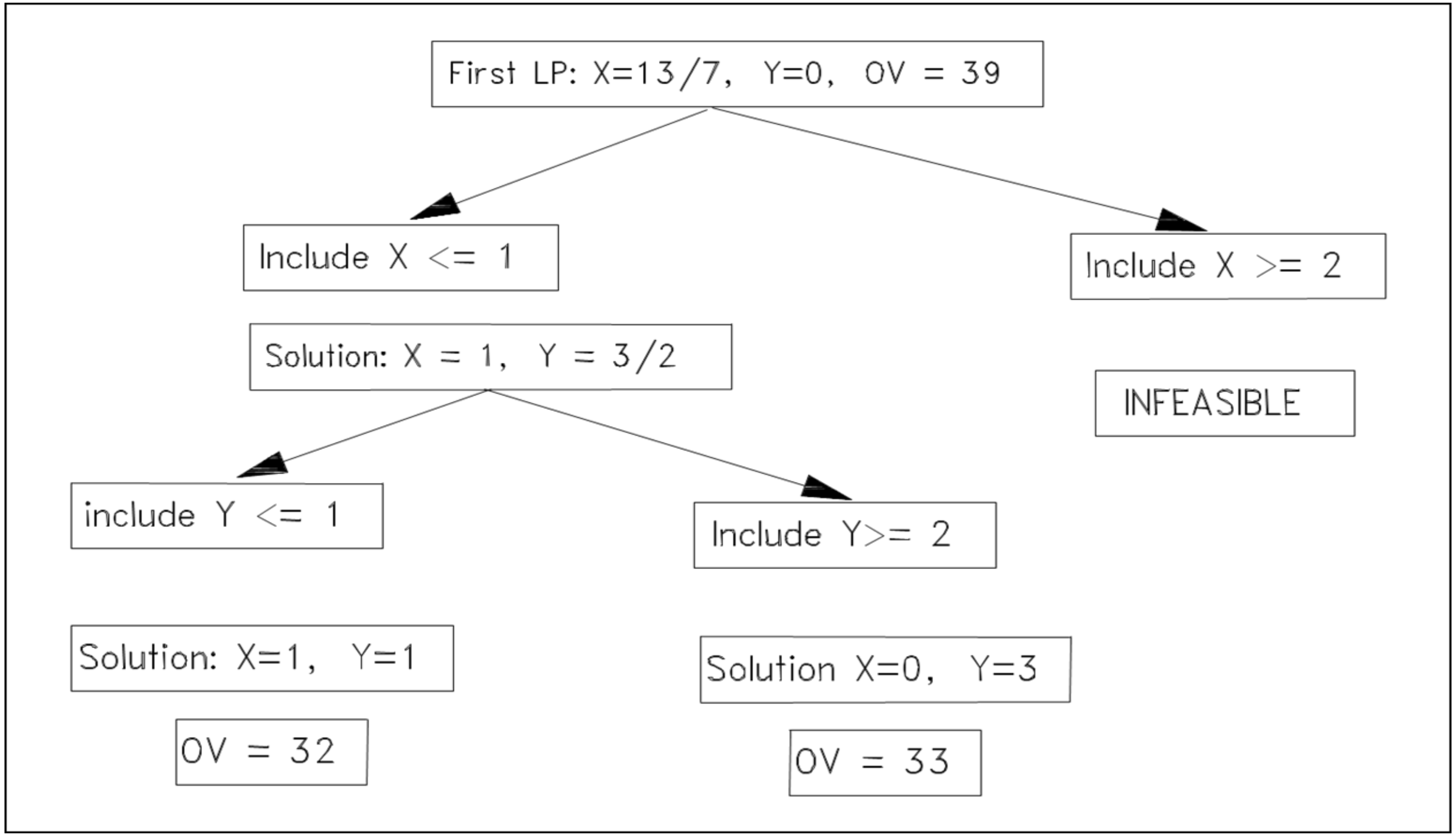
Solving IP Problems

- Cutting plane techniques
- Branch and bound techniques
  - Start with LP solution: choose one current non-integral value (say, X=13/7) and create two new LP problems from the first, one with the constraint X <= 1 and the other with the constraint X >= 2
  - Continue branching, following most likely prospects, until integer solution found

Branch and Bound Method



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Integer Programming and Formulation

0-1 (binary) variables permit modeling some non-linear problem types:

- Yes/no decisions
- Set-up costs and economies of scale
- Batch size restrictions
- Scheduling and routing problems
- Logical relations, etc.

A Simple "Knapsack" Integer Programming Problem with Binary Variables: StockCo

StockCo is considering investing up to $14,000 cash in some subset of the following four investments:

	Cash Required	NPV Added
Investment one	$5,000	$16,000
Investment two	$7,000	$22,000
Investment three	$4,000	$12,000
Investment four	$3,000	$8,000

Each investment is "all or nothing" – it is not possible to purchase partial shares.
How can the firm maximize its NPV?

StockCo: Mathematical Model

Decision variables?

Objective?

Constraints?

Excel implementation

Milkem Asset Sales Problem: Binary Variable Grids

Boris Milkem owns six assets that he must sell over the next three years. He expects that the sales will generate the following revenue, in millions of $.

	Year One	Year Two	Year Three
Asset One	15	20	24
Asset Two	16	18	21
Asset Three	22	30	26
Asset Four	10	20	30
Asset Five	17	19	22
Asset Six	19	25	29

For example, if he sells asset three in year two, he receives $30 million.
Each asset can be sold at most once, of course. He must sell at least $20 million of assets in year one, $30 million in year two, and $35 million in year three.

How can he maximize his total revenue from the sales?

Models with Logical Relations: Pitchers

The Cubs are trying to determine which of the following free-agent pitchers should be signed: Rick Sutcliffe (RS), Bruce Sutter (BS), Dennis Eckersley (DE), Steve Trout (ST) or Tim Stoddard (TS). The cost of signing each pitcher and the predicted number of victories each pitcher will add to the Cubs are:

Pitcher	Cost of Signing (millions)	Victories Added to Cubs
RS (righty)	$6	6
BS (righty)	$4	5
DE (righty)	$3	3
ST (lefty)	$2	3
TS (righty)	$2	2

Subject to the following restrictions, the Cubs want to sign the pitchers who will add the most victories to the team.

- At most $12 million can be spent
- At most two-right-handed pitchers can be signed
- The Cubs cannot sign both BS and RS

Models with Logical Relations: Pitchers (continued…)

Decisions?

Objective?

Constraints?

Set-Covering Models: Airhubs

Western Airlines has decided that it wants to design a "hub" system in the United States. Each hub is used for connecting flights among the following cities within 1,000 miles of the hub. Western runs flights among the following cities: Atlanta, Boston, Chicago, Denver, Houston, Los Angeles New Orleans, New York, Pittsburgh, Salt Lake City, San Francisco and Seattle. The company wants to determine the smallest number of hubs it will need to cover all of these cities, where a city is covered if it is within 1,000 miles of at least one hub. The following table lists which cities are within 1,000 miles of other cities.

Set-Covering Models: Airhubs (continued…)

	Cities within 1,000 Miles
Atlanta (AT)	AT, CH, HO, NO, NY, PI
Boston (BO)	BO, NY, PI
Chicago (CH)	AT, CH, NY, NO, PI
Denver (DE)	DE, SL
Houston (HO)	AT, HO, NO
Los Angeles (LA)	LA, SL, SF
New Orleans (NO)	AT, CH, HO, NO
New York (NY)	AT, BO, CH, NY, PI
Pittsburgh (PI)	AT, BO, CH, NY, PI
Salt Lake City (SL)	DE, LA, SL, SF, SE
San Francisco (SF)	LA, SL, SF, SE
Seattle (SE)	SL, SF, SE

Set-Covering Models: Airhubs (continued…)

Decisions?

Objective?

Constraints?

A Product Mix Problem with "Fixed Charge" Variables: Gandy

Gandy cloth company can manufacture three kinds of clothing: shirts, shorts and pants. Making each kind of clothing requires renting a particular kind of machine, which has effectively unlimited capacity, but cannot be used for any other kind of clothing. The following table describes the production process:

Gandy can obtain up to 150 hours of labour and 160 yards of cloth per week.

How can the firm maximize its profits?

Gandy Problem

	Cost of machine ($/week)	Labour hours per unit	Yards of cloth per unit	Revenue per unit	Variable cost per unit
Shirts	$200	3	4	$12	$6
Shorts	$150	2	3	$8	$4
Pants	$100	6	4	$15	$8

Combination of Models: Hiring

This model contains two sub-models:

- Fixed charge
- Set-covering

Hiring Example

The campus computer store has decided to hire some students to work part-time helping customers and performing various functions that used to be contracted out to outside vendors.

There are two work shifts per day (AM and PM), five days per week. Exactly one student must be assigned to work each shift. To become sufficiently familiar with the store's operation, each student hired must work at least two shifts per week. The store has identified a pool of eight qualified students from which to hire.

For each student, the store has estimated how much he or she would add to the store's profit, per shift worked. This information is shown in the table below. The table also marks each shift for which a particular student is available with a "1". Finally, it also gives the maximum number of shifts per week that each student can work.

To maximize its profit, which students should the store hire, and which shifts should each student work?

Hiring Example (continued…)

Initial	Profit /Shift	AM	P M	AM	P M	AM	PM	AM	PM	AM	PM	Max shifts
CM	$100	1	--	--	--	1	1	1	--	--	--	3
FI	$50	--	1	--	1	--	1	--	1	--	1	4
GR	$95	--	1	--	--	--	1	--	1	1	--	2
HS	$75	--	--	1	--	--	--	--	--	1	1	2
JD	$70	--	--	--	--	1	--	--	--	1	--	2
JE	$90	--	--	--	--	--	--	1	1	1	--	2
SR	$80	1	--	--	1	1	--	--	1	1	1	3
TR	$85	--	1	--	--	--	1	--	--	--	1	2

End of Class Two