Chapter Six

Decision Analysis

A Look Ahead

How do we decide what issues need our attention? How are decision problems identified and defined? Sometimes the need to make a decision is imposed by circumstances, when a crisis arises or an opportunity must be accepted or rejected. Very often, however, the choice of what problems to consider is discretionary – you face a variety of outstanding issues, which are more or less pressing and from which the decision agenda is selected. In addition, the very way in which a decision problem is *defined* can influence whether or not it appears on the agenda. For instance, a certain problem could be specified as "Should we hire Ms. Magenta as a buyer?" Alternative approaches are, "Do we need to hire more buyers?" or "What should our hiring policy for new buyers be?" Simpler and more explicit problems, like the first statement, are more likely to be selected for early resolution. This matter of identifying and defining decision problems, which we call **problem formulation**, is a critical part of the decision process.

We begin by describing the steps in problem formulation, and continue with a description of some useful models for structuring decision problems. This discussion may make problem formulation appear to be a tidy, sequential procedure. This gives a misleadingly simple impression – formulation is in fact a very *un*tidy, messy business that involves many false starts and mistakes. It is necessary to become immersed in the problem to understand the issues involved. In fact, the formulation of decision problems is more of an art than a science. The suggestions in this chapter should thus be viewed as a collection of ideas to consider (they may or may not apply to a given situation) when formulating decision problems, rather than as a recipe for "how it should be done".

We should also stress that decision analysis does not happen instantaneously. The process takes time, and during this period the problem agenda may shift, different criteria may become relevant, and other problems may dominate the current one. However, the principles underlying a good analysis do not alter with changing circumstances. Although this volume in general treats decision problems as if they were independent of other problems, you should continually keep in mind that the broader personal or organizational context is important to the process of problem resolution.

At the end of the chapter you will find a list of further readings to give you more source material for issues in formulating decision problems.

Formulating a Problem

A formulation of a decision problem is made up of the following components:

1. **Understanding** the problem.
2. Stating the **goals** of the decision maker.
3. Identifying **criteria** that correspond to goals.
4. Describing the **actions** and their impact on the criteria.
5. Identifying sources of **uncertainty** in actions and outcomes.

Step 1: Understanding

The first issue is to determine which problem to work on and to understand what the problem really is. Individuals and managers have a whole range of concerns at any given time. These concerns vary in terms of how specific they are, what time frame is relevant, how pressing the need for a decision is, and how important the issue is to the decision maker(s). There are no rules for selecting a problem to resolve, but you should note that people generally prefer to work on problems that are short-term, simple in structure, and concrete rather than abstract. Managers often complain that they spend too much time "fighting fires" (i.e. solving small, immediate problems), thus neglecting longer-term, broader issues. This complaint is rooted partly in the manager's personal characteristics, and it is helpful to realize that fire-fighting is sometimes a deliberate choice.

Step 2: Goals

Once a problem is selected, the decision maker (DM) must clarify what is involved in the decision. Problems often appear as *symptoms*, which could arise from a variety of causes. Resolving a problem effectively depends on defining it appropriately.

> **Example 6.1 Warehouse Congestion**
>
> A large grocery chain operates a warehouse, which receives bulk shipments from suppliers and breaks these down into truckload lots to be shipped to individual stores. Last year the warehouse manager became concerned because the system was becoming clogged, and a consultant was hired to determine what expansion in the truck fleet was necessary to "get the groceries out the door faster".
>
> However, the congestion was only a symptom of the real problem, which was that in some cases the suppliers' lot sizes were too large. Although economical from the perspective of shipping and handling, the large shipments were causing chaos at the warehouse level. By selectively reducing suppliers' lot sizes, and thus smoothing the product flow (at a small increase in cost), the warehouse was able to function smoothly with the current truck fleet.

This example shows that the statement of a problem as an obvious conclusion from looking at the symptoms may not give the most useful or appropriate formulation of the problem.

Step 3: Criteria

Once the problem is defined, the **goals** of the agency with respect to the problem should be clarified. What is the agency trying to achieve, and how will resolution of this problem affect these ends? For example, management expert Peter Drucker has suggested that corporate goals may include the following:

improvements in market standing	innovation
physical and financial resources	productivity
managerial performance and development	profitability
worker performance and attitude	public responsibility

Any individual decision-maker will likewise have a range of personal goals that may apply to a decision problem. Some examples of personal goals are:

increased income	a good living environment
more leisure time	a good work environment
good career growth	opportunity for travel
long-term career potential	opportunity for social involvement

In general, decision problems will affect more than one goal, and the goals themselves are often in conflict with one another (increased income may mean less leisure time, for instance).

The goals described above are general statements of objectives. To allow alternatives to be compared, these goals must be converted to some measurable criteria. For instance, a goal of increased income might involve such criteria as next year's salary, or the present value of the future income stream, or the anticipated income stream at retirement, or others. The goals and the appropriate criteria to represent them will depend on the problem at hand. A decision to fire a vice-president will have different goals and criteria than one about introducing a new product.

The criteria for representing goals may have a natural numeric scale (e.g. next year's income level) or may have a fuzzier, descriptive scale (e.g. the state of corporate morale). In Chapter Seven, we will describe methods for designing scales for different criteria and for making trade-offs between different criteria, in order to come up with an overall assessment of the outcome.

Step 4: Actions

Once the criteria are identified, the next step is to identify the range of alternative actions to consider, and to specify their potential impacts (or range of impacts) on the criteria. During this procedure, new goals and criteria may become relevant, and the problem may be re-defined. Formulating a problem is not a sequential list of activities to be completed in turn, but is a messy process in which any step is open to revision at any time.

Step 5: Uncertainty

The last step in formulation is to recognize which elements in the problem are subject to uncertainty, and how that uncertainty affects the outcome of each action choice. Much of this book is concerned with how to measure and make intelligent decisions in the face of uncertainty, so we won't go into these issues here. At this formulation stage, however, it is critical to recognize which elements are subject to some uncertainty, and what is the nature of that uncertainty (e.g. could you search for more information to resolve it? What causes the uncertainty?)

Example 6.2 Peter Moss.

As a sales manager for a large paper products firm, Moore Products, Peter Moss is responsible for selling pre-printed forms. He says, "After five years in this position, I feel that I'm in a rut and really need a change."

> Peter has an undergraduate degree in business and spent his first three years in the data-processing department of Moore Products. It became clear to him that, given the firm's strong marketing orientation, the best route to the top was through the sales division. He consequently changed departments and has made steady progress since that time. Peter is married, with two children, aged three and five. His wife, Lynn, is employed as a speech therapist at a local hospital and finds her work challenging and rewarding.

The first issue is, what is Peter's problem? Is it unhappiness with Moore Products, or the paper products industry, or working in sales, or his career progress, or does his discontent have some other source?

After some discussion with Peter, you discover that he feels that, given the age and seniority of his superiors, he will have a long wait before promotion to senior management. He feels stalled at his current level, and although normal salary increments will occur and his current salary is adequate for a decent life for his family, he doesn't see the "big break" coming. Furthermore, the paper forms industry has been stagnating as personal computer software that allows individuals to create their own forms has made heavy inroads into the market. You both conclude that his problem is how to select a new career.

The first task is to identify Peter's major goals in his career change. Like most people, Peter does not have a well-defined list of personal goals, and this is not an easy exercise. However, we know that Peter is unhappy about his career prospects with Moore, and so his future career development will figure strongly in his list. After some discussion, suppose you have concluded that a starting list for Peter's most important goals looks like the following:

1) reasonable income,
2) opportunity for personal growth,
3) potential for promotion to more responsible positions,
4) a good living environment for his family.

This list will almost surely change as the analysis progresses, as other considerations become relevant and some elements on the current list become less important.

How can we measure how well any alternative satisfies these goals? We need a set of measurable *criteria* corresponding to each goal. Which measures to use will depend on Peter's own understanding of what each goal means to him. For "income", he might well say that the real measure is the level of personal disposable income after normal living expenses have been deducted. An additional measure might be the present value at retirement of the pension plan. (Any goal may have several criteria. Later, we will discuss how to combine these.)

The income criteria have natural numeric measurement scales. Criteria for the other goals will not be so easily quantified. The goal of "personal growth opportunity" is an abstract, fuzzy concept, which might be measured by a criterion with a loose verbal description as to how well any alternative meets this objective. For instance, an alternative could be scored as having growth potential of one of *{low, moderate, medium, good, excellent}*. Any number of descriptions could be included in this list. The other goals could have similar scales.

Now that we have some idea of what Peter is looking for and how we might measure it, we can examine options for his future. Usually, the initial list of goals and measurement scales will be inadequate and will need to be modified. In fact, it is this interplay among choices, goals, and measurement that many consider the real strength of the analysis. As Zeleny (1982) puts it, "Most decision problems are not resolved by agonising between alternatives A and B, but by discovering [a new] alternative C."

Peter will undoubtedly be uncomfortable with assessing alternatives in terms of the criteria we've defined. He will say that for some of the choices, he simply doesn't know what values of the criteria he would observe. At the formulation stage, we can note that some elements are uncertain. Later we will create probability distributions corresponding to Peter's *beliefs* about the possible criteria values for each choice, as described in the previous chapter.

Modeling a Decision Problem

Decision problems are composed of choices, outcomes, and preferences for different outcomes (goals). This section describes some different formats to describe the relationship between these components, to develop a *model* of the problem.

One simple form is the **payoff table**. It consists of a list of the choices along one axis, and the criteria of choice along the other. Entries in the table are the scores of each choice for each criterion.

Example 6. 2 Peter Moss (Continued)

We can illustrate with Peter Moss's career decision from the preceding section. Suppose Peter has identified four possible job prospects. Using his criteria, Figure 6.1 shows the payoff table for the problem.

	CRITERIA			
	Income	**Growth**	**Promotion**	**Location**
Job A	43,000	Low	Average	Great
Job B	29,000	Medium	Excellent	Great
Job C	35,000	High	Average	Moderate
Job D	40,000	Medium	Good	Poor

Figure 6.1 A Payoff Table for Peter Moss

This model is effective for simple problems with a single decision and little or no uncertainty. As you can see from the table, Peter's problem is how to think about the trade-offs among competing criteria. The table clearly shows the profile across the criteria for each choice, allowing ready comparison with the profiles of the other choices for making the trade-offs. In the next chapter, we'll explore a formal method called the AHP for this kind of multiple criterion problem. This pay-off table approach is useful for a problem like Peter's, with a single choice to be made from a set of possibilities, but it is not very effective if there is a sequence of decisions to be made, or if the DM has some uncertainty about the ratings.

The next section describes more comprehensive ways to structure decisions to make informed, rational trade-offs among alternatives.

Setting up the Decision Problem

Now that we have formulated a decision problem, and identified the available actions and the consequences of each action, how can we structure the problem to let us choose the best action?

In the rest of this chapter, we will assume that the consequences of each action can be summarized in a single value, which we can suppose is the monetary impact of that consequence. (Other kinds of consequences can be measured using money through the so-called “willingness to pay” principle: the amount you would be willing to pay to obtain, or avoid, any particular outcome is a proxy for its personal value to you.)

Let’s begin by describing a handy tool for talking about decision problems, called a **decision tree**. The idea underlying the tree is to represent the sequence of decisions, and resulting possible outcomes, in their logical order to make the problem analysis easier. Decision trees combine the following components:

- Decision nodes: have branches, where each branch corresponds to a different decision that could be taken at this point. If there is a direct monetary impact on choosing a particular decision, it is shown adjacent to the branch. In any problem, a decision maker must choose exactly one branch at each decision node.
- Chance nodes: have branches, where each branch corresponds to one of the possible consequences that could arise at this point. A monetary impact of the consequence is shown adjacent to the branch, and the probability that this consequence will happen is also shown adjacent to the branch. The decision maker cannot control what happens at a chance branch. The actual consequence is ruled by the probabilities attached to each possibility.

- Terminal nodes: indicate the end of a series of decision and chance branches, and show the net monetary impact of all the preceding branches.

Here is an illustration of a decision tree. Suppose you are offered the chance to play one of the following two games, one time only.

Game 1. Costs $0.50 to play. A fair, six-sided die is rolled once. If the number of spots is 1, 2, or 3, you win $0; 4, 5, or 6 wins you $4.

Game 2. Costs $1.00 to play. A fair, six-sided die is rolled once. If the number of spots is 1,2,3, or 4, you win $0; 5 or 6 wins you $6.

The decision tree corresponding to this choice problem is illustrated in Figure 6.2. By convention, small open squares (□) designate decision nodes and open circles (❍) show chance nodes, as in Figure 6.2. Terminal nodes are shown by a vertical dash (|).

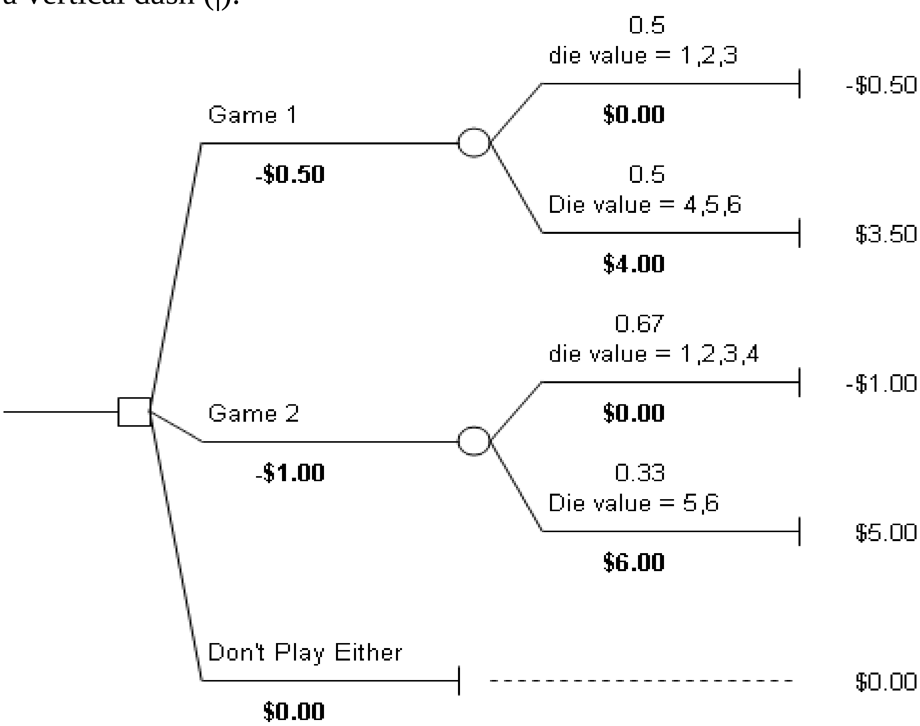
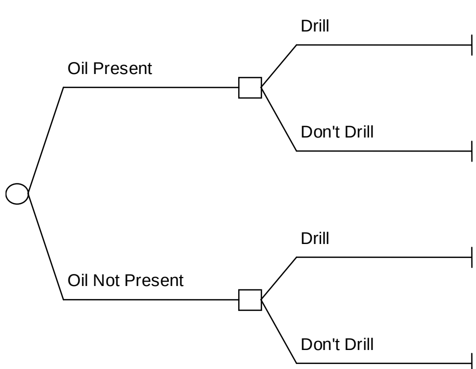


Figure 6.2 A Decision Tree

As you see, the money impact of any choice or chance event is shown adjacent to the corresponding branch (like the -$0.50 next to the choice of Game 1, or the $4.00 shown next to the event following the choice of Game 1 "a 1,2 or 3 is shown on the die". Probabilities are shown next to chance branches (e.g. following the choice of Game 1, the chance of seeing a die value of 1,2 or 3 is 0.5, so 0.5 is shown adjacent to this branch in the figure. Outcomes of each decision/chance pathway through the tree are placed at the terminal tip of each path. For example, if you choose Game 1 and the outcome (die = 4,5, or 6 occurs, the net monetary impact is $4.00 - $0.50 = $3.50, shown next to the terminal node corresponding to this sequence of decision and chance events.

The tree for any particular decision could have more decision or chance forks arranged in a logical sequence corresponding to the order in which information is known and decisions must be made. There is one important principle to bear in mind when constructing a tree: decisions must not follow chance events whose outcome would be unknown when the decision is made.

Suppose the decision here is whether or not to drill an oil well. The chance event is whether or not there is oil present. The tree pictured in Figure 6.3a is incorrect, since the decision must be made *before* it is known whether there is oil present or not. Figure 6.3b shows the proper representation of this problem because knowledge of the presence of oil *follows* the decision to drill the well.



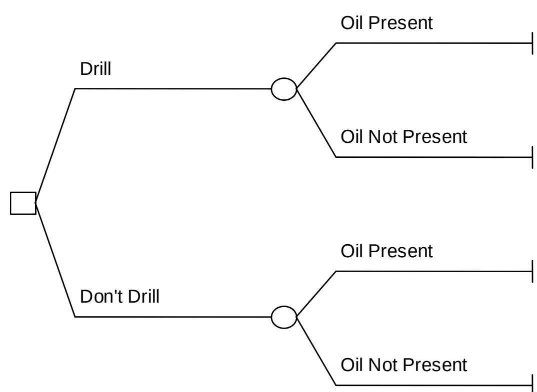


Figure 6.3a Improper Tree **Figure 6.3b Correct Tree**

Choosing the "Best" Action

One way to make a choice is to use criteria that do not require an explicit statement of the relative likelihood of alternative outcomes. These criteria generally focus on extreme outcomes. They can be thought of as "pessimistic", or conservative, versus "optimistic", or risk-taking, strategies.

The most elementary choice criterion is that of the dominated outcome. Examine the problem in Figure 6.4. For this problem, you can see that the decision to plant wheat yields a better outcome than planting corn regardless of the weather. Thus planting wheat dominates planting corn – wheat is always the better choice. Note that the likelihood of good or bad weather in the example is irrelevant: wheat is **always** a better choice than corn in this problem, no matter how likely each outcome is.

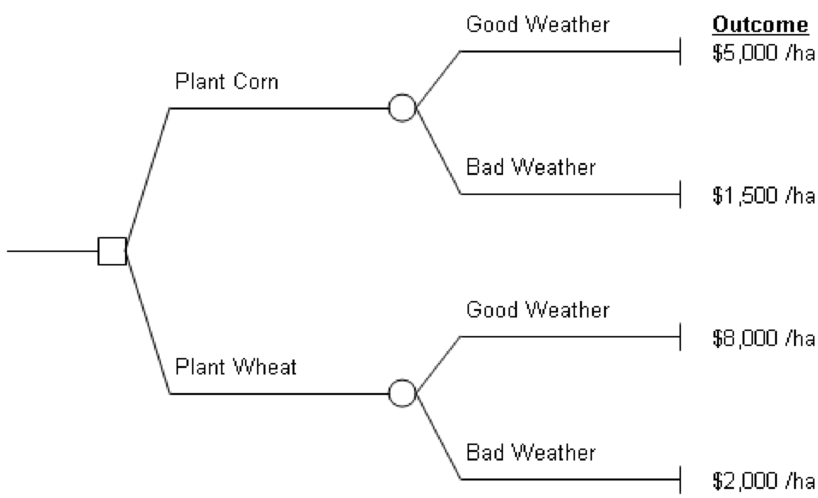


Figure 6.4 A Dominated Choice

If one outcome doesn't dominate the rest, a selection rule that doesn't require the decision-maker to make direct assessments about the relative chance of the various outcomes is called the **Maximin** rule, which can be stated as:

Maximin: Examine the worst outcome for each action, and choose the action that gives the best of these worst outcomes.

Examine Figure 6.5. You can see that if the decision-maker knew that the weather would be good, he would choose “Plant Wheat”; if he knew the weather would be bad, he would choose “Plant Corn”. Neither choice dominates the other. Let us apply the Maximin Rule to this problem.

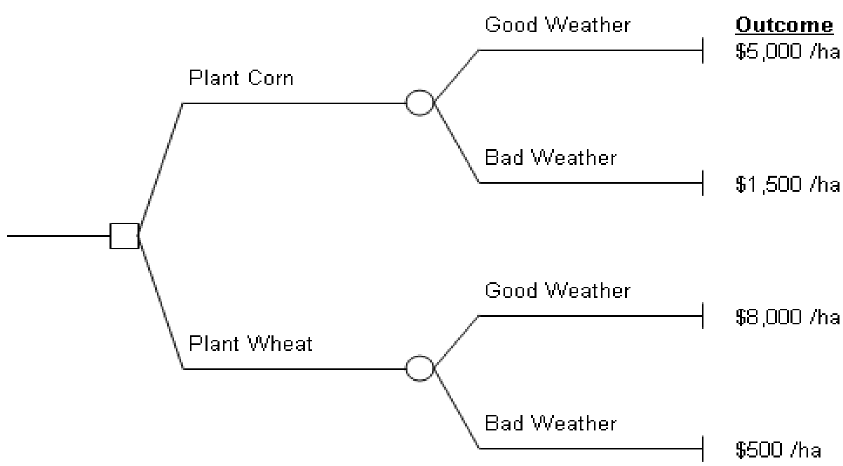


Figure 6.5 Illustrating the Maximin Rule

If the decision-maker chooses to plant corn, the worst outcome occurs with bad weather, and the result will be $1,500/ha. If, on the other hand, he chooses to plant wheat, the worst outcome again occurs with bad weather, with a yield of $500/ha. The “best” of these two worst cases is $1,500/ha, and so the Maximin rule would suggest he plant corn. In a sense, this rule is a pessimistic one, since only the worst possibility in each case is being considered.

A natural alternative to the pessimism of the Maximin rule is the **Maximax** rule: choose the option with the largest of the maximum payoffs. In the example, this rule would suggest Planting Wheat because its largest payoff ($8,000/ha) is larger than any other payoff. This is an optimistic rule, since it looks at only the best outcome for each decision.

Another rule for making a choice is called the **Minimax** criterion. Instead of using the actual value of each outcome to make the choice, this rule uses another quantity called the **regret** (or **opportunity loss**) for each outcome. For each decision and each chance outcome, the regret is the difference between two quantities: the payoff from *that* decision for that chance outcome, and the payoff from *the best* decision that could have been made for that particular chance outcome.

> **Minimax**: For each chance event, find the *regret* for each action should that event occur. Examine each action to find its maximum regret. Then choose the action with the smallest maximum regret.

We can illustrate this principle with the problem in Figure 6.6. Suppose the weather were to turn out to be Good. If we had chosen corn, we'd have earned \$5,000/acre, whereas wheat would have yielded \$8,000/acre. If the weather is Good, the regret in planting corn is the difference of \$3,000/acre. The regret in planting wheat in this case is zero. On the other hand, if the weather is Bad, corn yields \$1,500/acre and wheat yields \$500/acre, so the regret in planting wheat is the difference of \$1,000/acre. In this case, the regret from planting corn would be zero.

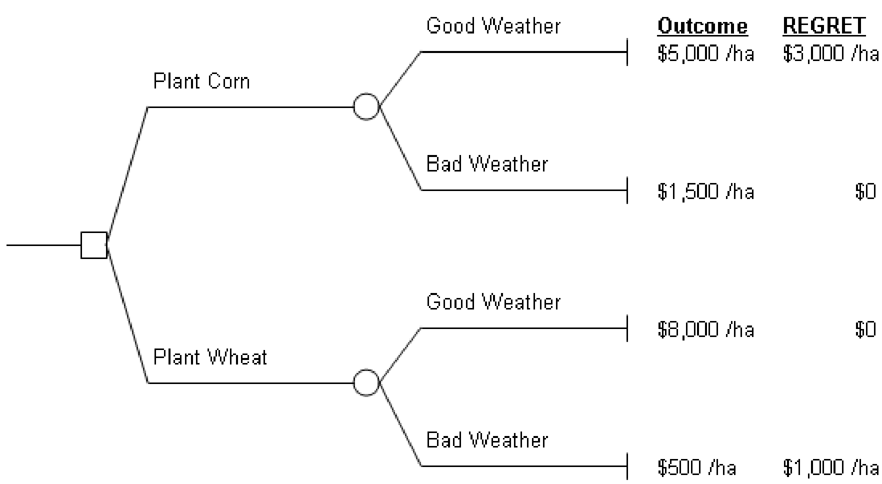


Figure 6.6 Illustrating the Minimax Rule

The decision tree shows that the maximum regret from planting corn is \$3,000/acre and from wheat is \$1,000/acre. The Minimax rule indicates that the DM should therefore plant wheat.

Regret in this sense is sometimes called **opportunity loss**, since it represents a lost opportunity to improve the outcome (if only the DM had known ahead of time that that event was to happen.)

In the three preceding methods, a choice was indicated without reference to the probability that each type of weather might occur. However, different choices were selected by each rule. Which is best? Note that each method focuses exclusively on *extreme* values, and the selection of the appropriate method depends on whether we are interested in worst cases (Maximin), best cases (Maximax), or in greatest lost opportunity (Minimax).

In most decision problems, however, we are not merely concerned with extreme outcomes, but with the whole *range* of possible outcomes. This requires that we consider the relative likelihood, or probability, of each chance outcome.

Choices Using Probability

Let’s look again at the decision problem from Example 6.4, as shown in Figure 6.7.

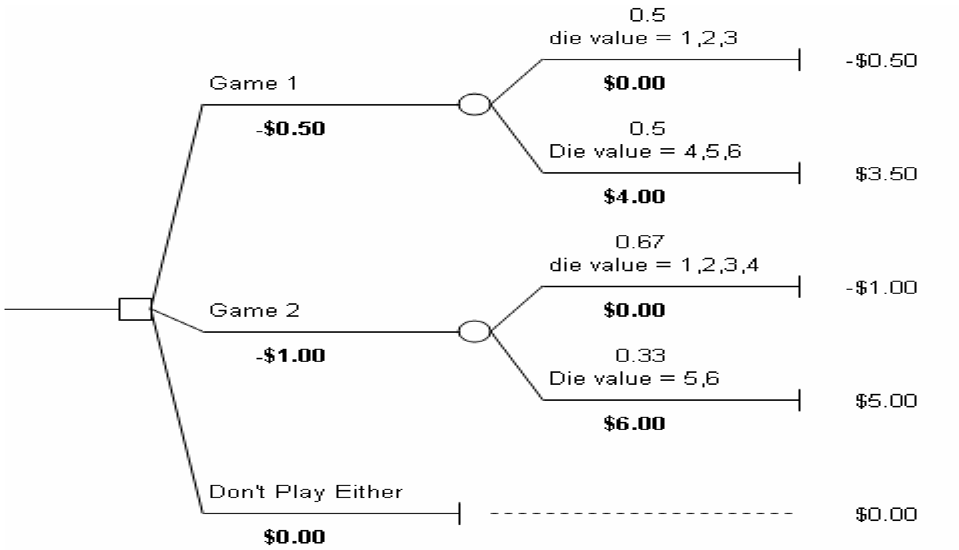


Figure 6.7 A Simple Decision Problem

The problem is to select among playing Game 1 once, playing Game 2 once, or not playing, with the costs, rewards, and outcomes for each choice as shown in the figure. With respect to the rules for extreme value choice, you can confirm that the Minimax rule would choose Game 2, and the Maximin would choose Don't Play Either.

However, we can use the probability assessments for each outcome to include *all* of the outcomes in the choice procedure, instead of just the extreme values. In Chapter 3, we observed that a rule of choice among alternatives with uncertain payoffs is to choose the action that has the payoff with the highest expected value.

The expected value of a variable was defined as the product of each possible value of the quantity times the probability that value occurs, summed over all the possible values. In symbols, if X is the variable quantity, which can have values $\{x_1, x_2, \ldots x_n\}$, with probabilities $p(x_1), p(x_2), \ldots p(x_n)$, then the expected value of X is:

$$E[\mathrm{X}] = \sum_{i=1}^{n} x_i \, p(x_i)$$

Thus, if we represent the value of Game 1 as X, with possible outcomes x_1 = -$0.50 and x_2 = $3.50 and probabilities $p(x_1) = 0.5$, $p(x_2) = 0.5$, then the above formula gives

$$E[\mathrm{X}] = x_1\, p(x_1) + x_2\, p(x_2) = (–\$0.50)(0.5) + (\$3.50)(0.5) = \$1.50.$$

Similarly, if we represent the outcome of Game 2 by Y, with values

$$y_1 = – \$1.00 \text{ and } y_2 = \$5.00$$

and probabilities $p(y_1) = 2/3$ and $p(y_2) = 1/3$, then

$$E[\mathrm{Y}] = y_1\, p(y_1) + y_2\, p(y_2) = (– \$1.00)(2/3) + (\$5.00)(1/3) = \$1.00.$$

Finally, if we represent “Don’t Play” by Z, which has only one possible value $z_1 = 0$ (which means that $p(z_1) = 1$), then

$$E[\mathrm{Z}] = z_1\, p(z_1) = (\$0)(1) = 0.$$

Game 1 yields the highest expected value. If we could play many times, we would probably be better off playing Game 1 all the time. It is thus a reasonable choice for a single play as well.

Now the expected value itself is not usually one of the outcomes. A single play of Game 1 will not give a win of $1.50, but will either be – $0.50 or +$3.50.

Why should expected value be a reasonable criterion for choice? If life consists of many choices, following a rule that makes these choices on the basis of the largest expected value will probably leave you better off in the long term than you would be if you followed a different rule.

Subjective Probabilities

Most people have little difficulty with using expected value for choosing in simple situations like the preceding example, for which the uncertainty arises from a physical process like flipping a coin or tossing a die. However, we are interested in much more than this: we want to resolve problems for which the uncertainty can only be represented by the DM's *degree of belief* in the likelihood of various outcomes.

The idea of using expected monetary value as a basis for choice in such cases is difficult for some people to accept because of its reliance on probability assessments that cannot be verified though experimentation and observation. They feel that the statement, "The probability of a 3 showing in one roll of a standard six-sided die is 1/6", is a different sort of statement than, "The probability that next year's sales will be high is 1/6". The first statement is treated as somehow more legitimate or valid than the second. However, both are really nothing more than statements of belief about the possibility of a particular value occurring in an observation.

For example, imagine we show someone a six-sided die and offer the choice between Game 1, Game 2, or Don't Play as in Figure 6.10. Most people indeed select Game 1, assigning a probability of 0.5 to the probability of seeing one of 1,2 or 3 in a single roll of a die. But now suppose that the die had been weighted in such a way that the numbers 1,2, or 3 show up ten times as often as the numbers 4,5, or 6, and we let the DM experiment with the die before making his final choice. He would likely change his choice to "Don't Play" with this new information.

What is the difference in these two situations? Before the DM learned that the die was weighted, the physical symmetry of the die gave rise to the assignment of probabilities as if each face was equally likely, and thus the belief that the probability of 1,2, or 3 on a single roll was 0.5. The initial choice was made on the basis that these are "true" probabilities. As shown by the later evidence, these beliefs were not verified, and the revised beliefs caused a change in the decision. But it is the same die in each case. What has changed is what the player believes about the die. Was the first decision correct? Yes, given his beliefs at that time!

Many people are reluctant to use degree of belief expressed through a probability distribution as the basis for decision making. The apparent root of this reluctance seems to be that the resulting probabilities are treated as "hard" numbers or, in other words, as "sure" assessments.

As the above example shows, however, probability is not an attribute of *things*, but only how we think about those things. Any statement of probability is nothing more than a statement of belief. Seen in this light, there is no essential difference between assessing probabilities for the die-roll and, say, assessing probabilities for next year's sales.

Decision analysis deals with uncertainty in the real world, for which there is no "true" probability; an event either occurs or it doesn't, and we are not usually in a position to observe repeated trials to find a long-run average occurrence rate. Thus, the assignment of probabilities in this case has little to do with the relative frequency interpretation of the meaning of probability: we cannot make a series of observations of next year's sales to derive a probability distribution. Rather, probability is an expression of belief in the likelihood that the event will happen.

Insofar as a DM's probabilities reflect his beliefs, computations of expected value using these probabilities are wholly justified. The whole point of the analysis is to assist a DM with problems that involve events whose consequences are uncertain. Probability allows us to express this uncertainty in a useful way.

The DM's beliefs are the culmination of his processing of all he knows about the variable in question and related phenomena in the world. It is entirely possible that someone else may have a different set of beliefs and so would come up with correspondingly different probability assessments. The argument above would suggest that if both people act according to their beliefs and probabilities, they are both right.

Isn't the existence of two contradictory probabilities for the same event inconsistent? The answer is that this is not inconsistent. From the perspective of decision analysis, probabilities are nothing more than statements of beliefs, and the beliefs of the two individuals differ. The statements can be inconsistent only if we assume that there is some "true" probability distribution for the events in question and that both people are trying to estimate this true distribution. But we have described probability not as an independent attribute of things, but only an expression of a state of mind. Hence, it *is* possible for two individuals to have differing probabilities about the occurrence of some event, and for both to be “right”.

This does not mean that one cannot become better at assessing distributions. Practice at making assessments can improve one’s abilities to summarize beliefs in the form of probability distributions. Also, seeking more information about the quantity at issue (e.g. through market surveys, or expert opinion, or evaluation of related variables) can “improve” the assessment by providing a more substantial basis for beliefs. The improvement, however, is not better in the sense of being closer to some ideal, true value. It simply reflects a revision of beliefs through better knowledge of the issue and circumstances.

Probabilistic Choice Criteria

Suppose that a decision maker has summarized his beliefs about the likelihood of various occurrences in the form of probability distributions. How should he use these data to make decisions?

A solution method based on the idea of expected value is called the maximum **Expected Monetary Value**, or *EMV*, approach. It proceeds as follows. The method begins at the tips of the decision tree and computes for each chance fork the expected value at that fork on the basis of the probabilities and values of each outcome. This expected value is then attached to that fork.

Now, this fork may be at the end of a branch of some predecessor fork, and the expected value just computed becomes the outcome for this branch of the earlier fork. If the earlier fork is another chance fork, the expected values for each tip are used in the same way as the outcomes were before, in computing the expected value of that earlier fork. If the earlier fork is a decision fork, then the best expected value from its branches is selected, and the other branches cut. That is, the expected value attached to a decision fork is the best of the expected values of any of its branches. Repeated application of this procedure allows the outcomes from the tree tips to be “rolled back” to the tree root, and an expected value for the tree as a whole results.

Example. Your company can sell 10,000 widgets, at a sale price of $5. You can either make these widgets, or buy them on the market. If you buy, your cost will be either $3 or $2.50, with equal probability. If you make, you must test the process first, which will cost $10,000. There is a 45% chance the test will succeed, and a 55% chance it will fail. If the test fails, you'll have to buy the widgets at the above prices, and probabilities. If the test passes, you have a second decision, to adopt an extended process for $5,000, which will result in widget costs of $1.20 with a 30% probability, or $.80 with a 70% probability. Otherwise you can proceed with the existing process, resulting in widget costs of either $2 each with probability 40%, or $1 cost with probability 60%.

The tree in Figure 6.8 describes the complicated series of decisions and chance nodes. (Computations are done in thousands of dollars, for clarity.)

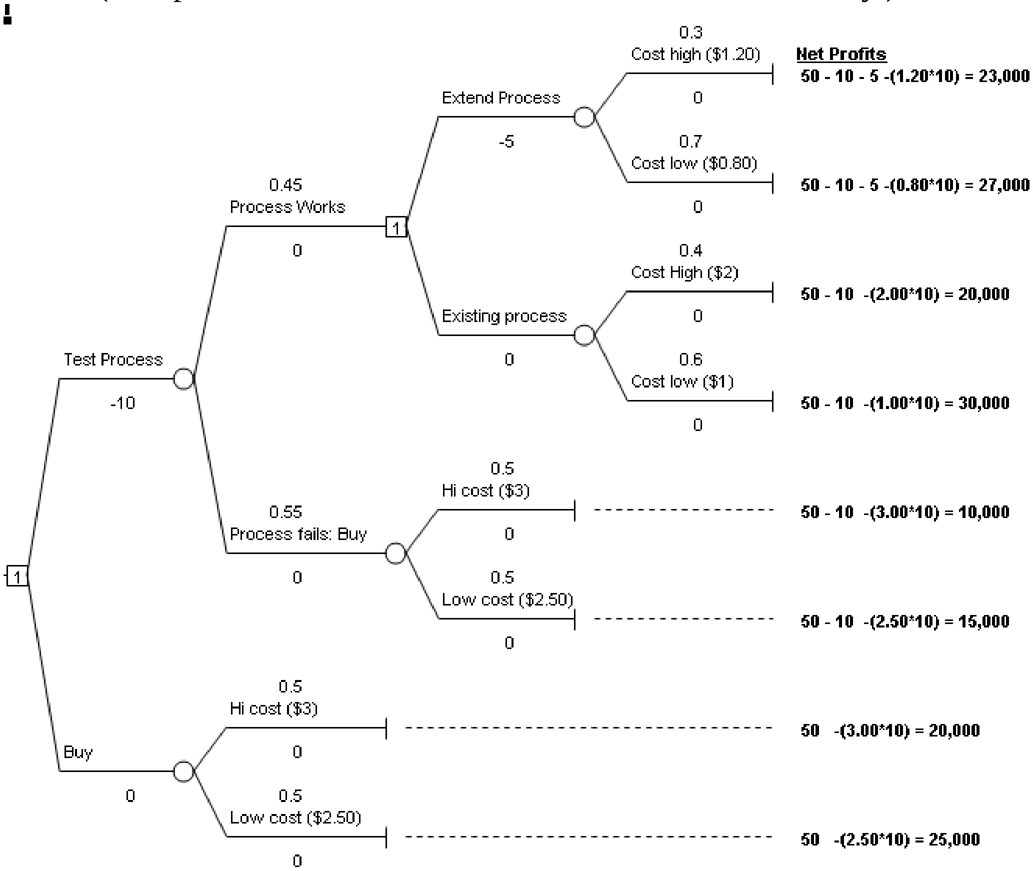


Figure 6.8 A Make-or-Buy Decision

The figure shows the net profit at each terminal node in the tree, giving the cumulative net effect of each predecessor decision and chance branch, as shown. We can use these values to find the expected value at each fork in the tree just prior to the terminal nodes. That is, for example, the expected value of the node at the end of the "Extend Process" decision, with the two possibilities "Cost high ($1.20)" or "Cost low ($.080), will be
0.3*23000 + 0.7*27000 = $25,800.
The expected value of the node at the end of the "Existing Process" decision will be 0.4*20000 + 0.6*30000 = $26,000.
We can insert these values, next to the respective nodes in the tree, giving Figure 6.9.

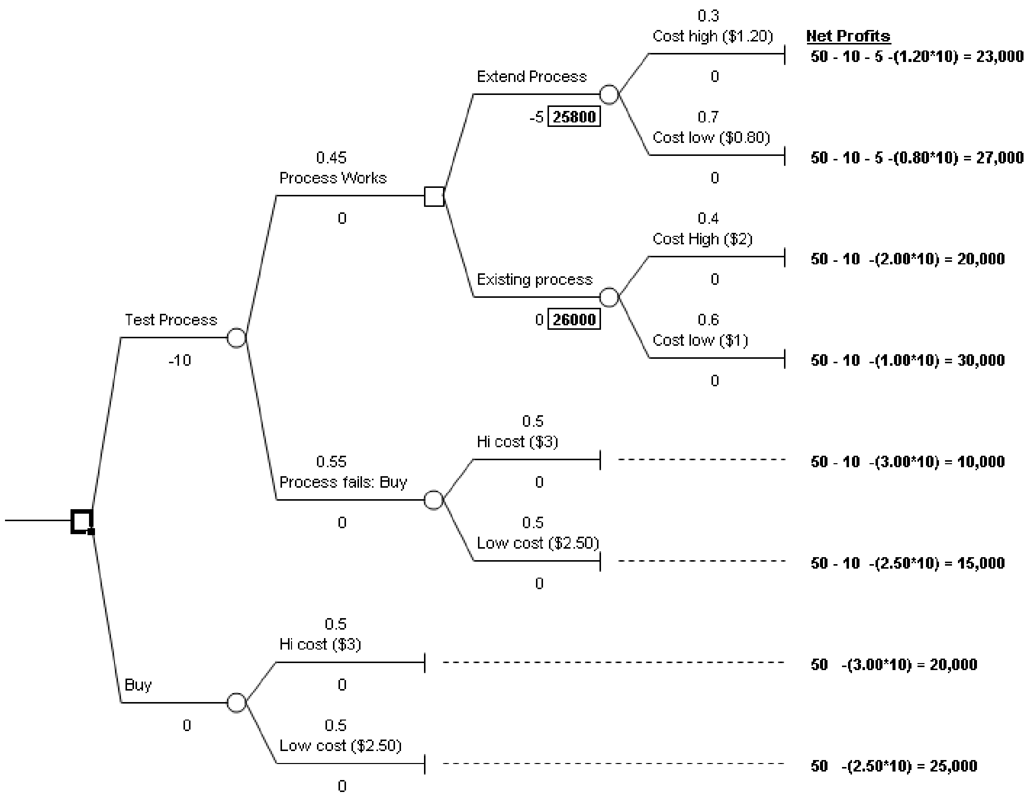


Figure 6.9 Make-or-Buy (Continued)

Now look at the decision following the “Test Process” “Process Works” path at the top of the tree. When we get to this decision, we can choose to Extend Process, with an expected value of $25,800, or we can choose Existing Process with an expected value of $26,000. As rational decision makers, we’d choose the second of these, which means the expected value at the decision point following Process Works is $26,000. We can insert this next to this decision node. We can also compute the expected value of the Process Fails: Buy node below this as 0.5*10000 + 0.5*15000 = $12,500, and put this in the tree, giving Figure 6.10.

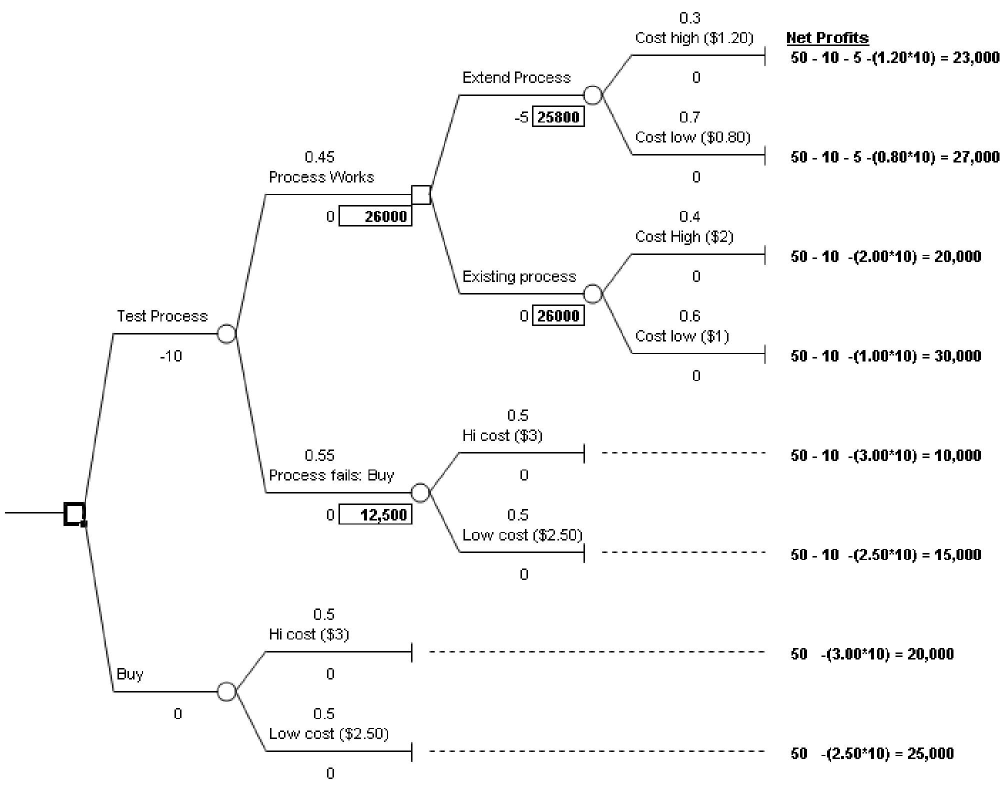


Figure 6.10 Make-or-Buy (Continued)

Now we can find the expected value of the Test Process decision by calculating the expected value of this choice, as .45*26000 + .55*12,500 =

$18,575. The expected value resulting from the Buy decision is 0.5*20,000 + 0.5*25,000 = $22,500. These are shown in Figure 6.11.

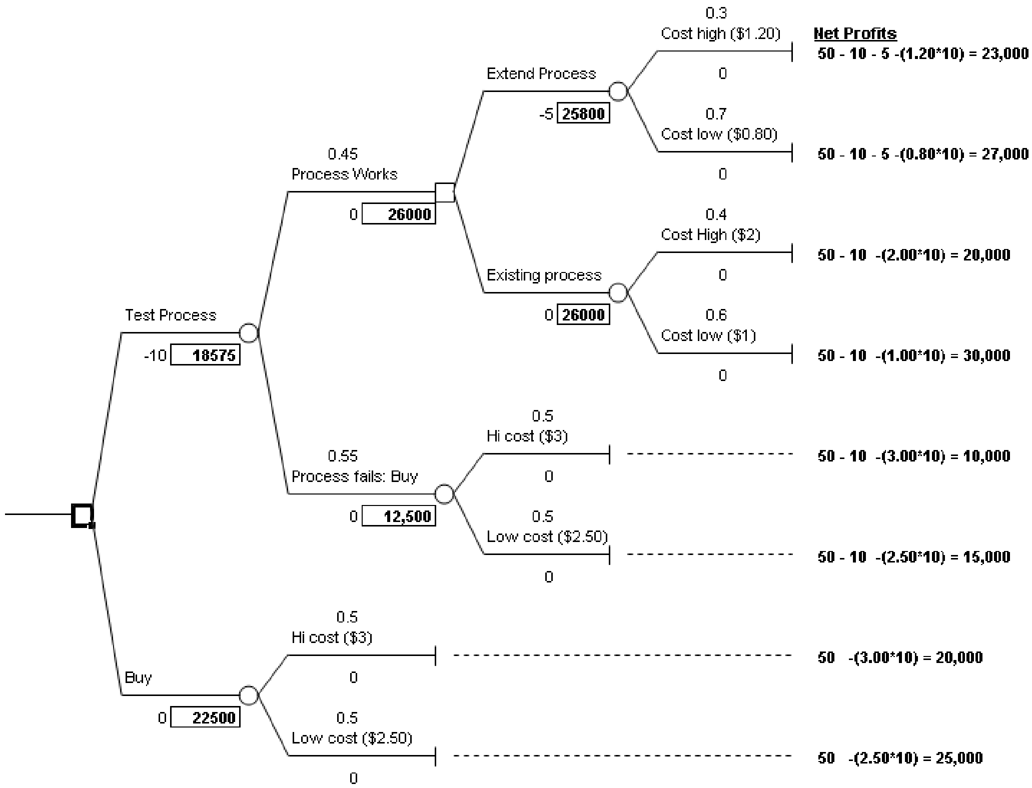


Figure 6.11 Make-or-Buy (Continued)

So our original decision problem is to pick Test Process, with an expected value of $18,575, or Buy with an Expected value of $22,500. Rational decision makers would choose to Buy, and the overall EMV of the decision problem is $22,500.

The example shows that this procedure not only identifies which actions to choose at every decision point, should it arise, but also computes the expected value resulting from the future possibilities for each fork in the tree. For example, if we selected “Test Process” and the event “Process Works” occurred, we can say that the remaining decisions and chance forks have an expected value of $26,000 from that point on.

This method gives a solution no matter how complex the tree becomes, so long as all the probabilities and outcomes are specified. This EMV decision criterion has many benefits:

1) EMV can be understood by people who have not had formal training, and its conclusions are therefore credible.

2) In contrast to the earlier methods we described, EMV takes all of the outcomes into account, and not just extreme values.

3) EMV automatically measures relative preference among the different actions, by comparing their different expected values.

4) You can do sensitivity analysis to see the effect on the eventual decision of changes in any of the numeric values in the problem (outcomes or probabilities).

However, there is one big drawback to EMV: it takes no account of how the decision maker feels about risk. Think about what you would do if you had the decision problem in Figure 6.12. Choice 1 involves flipping a fair coin, which we can assume has a 50% chance of landing either Heads or Tails

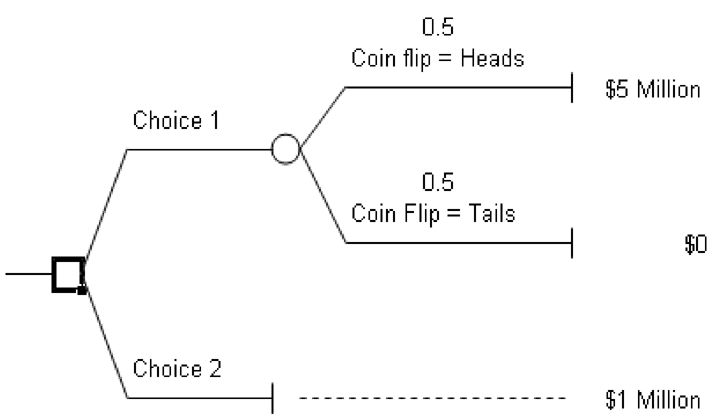


Figure 6.12 Risk Attitude Illustration

Would you choose Choice 1 or Choice 2?

You can calculate the EMV of Choice 1 as follows:
EMV of Choice 1 = 0.5*($5 Million) + 0.5*($0) = $2.5 Million.
Since Choice 2 gives you $1 Million for sure, EMV of Choice 2 = $1 Million.

The EMV criterion says you should pick Choice 1, and yet most of us will rationally choose Choice 2 for the $1 Million sure thing. The problem is that EMV ignores the difference in riskiness of the two choices, but we human beings take risk into account in comparing alternatives. How can we alter the criteria to include our risk attitudes?

Utility Theory

The problem with EMV is that it gives every dollar an equal weight in computing relative preference, and yet most of us put more value on the first million dollars than the second million, and more on the second than the third, and so on. (After all, its that first million that really matters to most of us!) This differential importance means is that $5 Million is not five times as valuable to us as $1 Million. Rather, money has a declining value to us as the amounts get larger.

What we need is some way to transform monetary outcomes into a suitable scale that corresponds to the intrinsic worth of these outcomes. This transformation of a monetary (or other measurement) scale into a scale corresponding to “preference for outcomes” is called a **utility function**. Figure 6.13 shows a typical utility function. In the graph, the function has been scaled so that $0 has a utility value of 0, and $5 Million has a utility value of 100. The other dollar values have utilities adjusted to this range.

From the figure, the utility score for $1 Million is 55. That is, if you select Choice 2, you expect to gain a utility of 55. What is your expected utility gain with Choice 1? We can use the same probability computation as we did with expected value to find the **expected utility** at a node in the tree. That is, the expected utility that you would derive from the chance event from Choice 1 is found as follows:
Expected Utility = Sum [utility for each chance branch * probability of that branch].

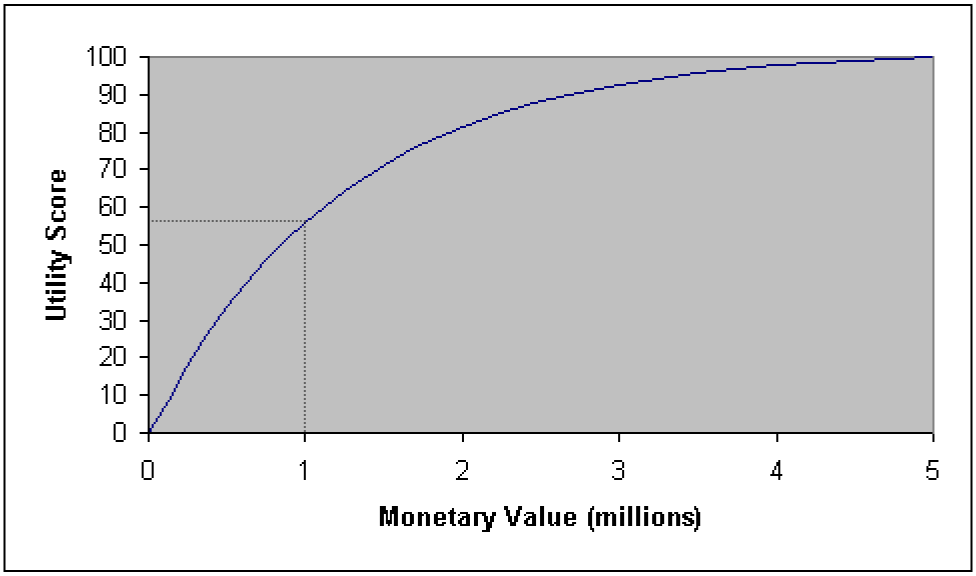


Figure 6.13 A Utility Function

By this logic, the expected utility of Choice 1, which we could denote as EU[Choice 1], will be

EU[Choice 1] = 0.5*Utility of ($0) + 0.5*Utility of ($5 Million)
= 0.5 * 0 + 0.5 * 100
= 50.

You choice is then between Choice 1 with an expected utility of 50, and Choice 2 with an expected utility of 55. Since we all want to make the choice with the highest worth, using the **maximization of expected utility** criterion, you'd choose Choice 2, the sure million dollars.

What we've seen is that the idea of expressing preference by a utility function rescues the use of a probability-weighted tool for deciding between possibilities that *does* take into account differing attitudes toward risk. Moreover, if we replace the net outcomes at the end of the monetary value tree with their corresponding utilities, the same "roll back" procedure can be used to evaluate any tree. That is, at each chance node we find the expected utility of that node, and at each decision node we choose the branch with the highest expected utility.

Note the shape of the utility function in Figure 6.13: it is increasing, but at a decreasing rate. This corresponds to the observation above that, as the total dollar amount gets larger and larger, the value that you derive from an additional dollar gets smaller and smaller. Such a function is called **concave**; this corresponds to **diminishing marginal utility**. We will return to some of the implications of this shape shortly.

Of course, the question is "How do I determine the utility of an outcome?" One way to answer this is to assume a relationship in the form of a mathematical expression. For example, we could assume that for a monetary value X (in, say, thousands of dollars), the DM's utility can be expressed as $U[X] = 1-e^{(-X/R)}$, where R is a numeric constant. This is called an *exponential* utility function. In order to use the function, we only have to assess one quantity, the value of R.

Example. Suppose you are faced with the decision shown in Figure 6.14a, with the indicated payoffs and probabilities corresponding to your beliefs that the events will happen under each choice.

First, how would an EMV criterion work here? The spreadsheet formulas show how to calculate the expected value of each choice, multiplying the value of each resulting chance branch by its probability and summing the results across the branches. The spreadsheet indicates that, if you use EMV, you should choose Action A to maximize your expected value, at $14,000.

Note the range of the outcomes from each choice here, though. Action A has a range of possible outcomes from $5,000 to $20,000, whereas Action B has a range from $10,000 to $15,000. The narrower range for B suggests that it is a less risky choice, and might be preferred by a risk averse decision maker.

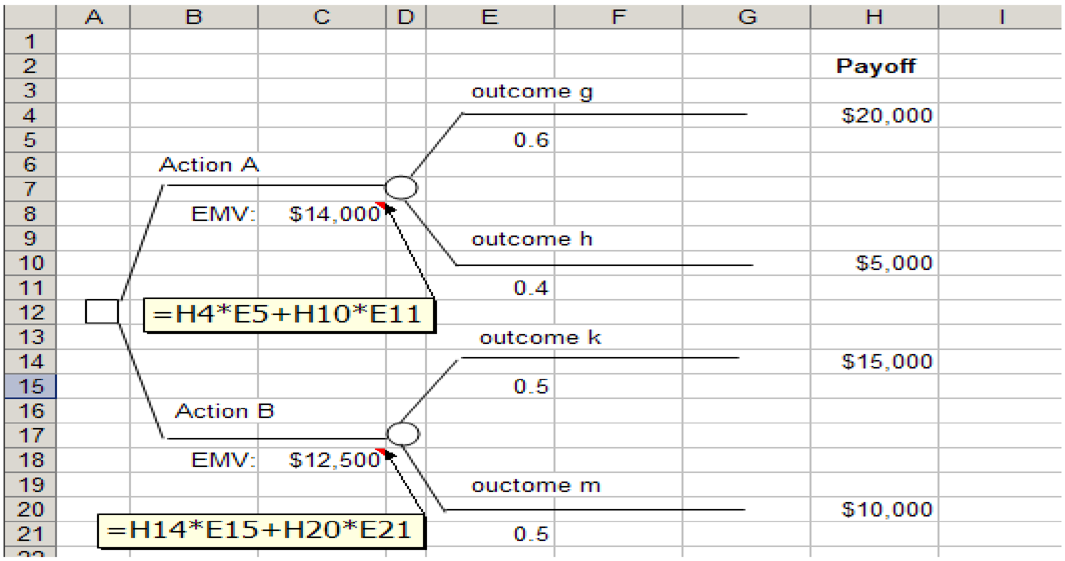


Figure 6.14a A Decision Problem, with Expected Values

Suppose you are willing to use the exponential utility form, and have assessed your constant as R = 8.4, for dollars expressed in thousands. (We'll talk about how to find R shortly.) We can insert the utility values for each payoff by determining the result of this expression into the decision tree, and calculate the expected utility of each choice as shown in Figure 6.14b.

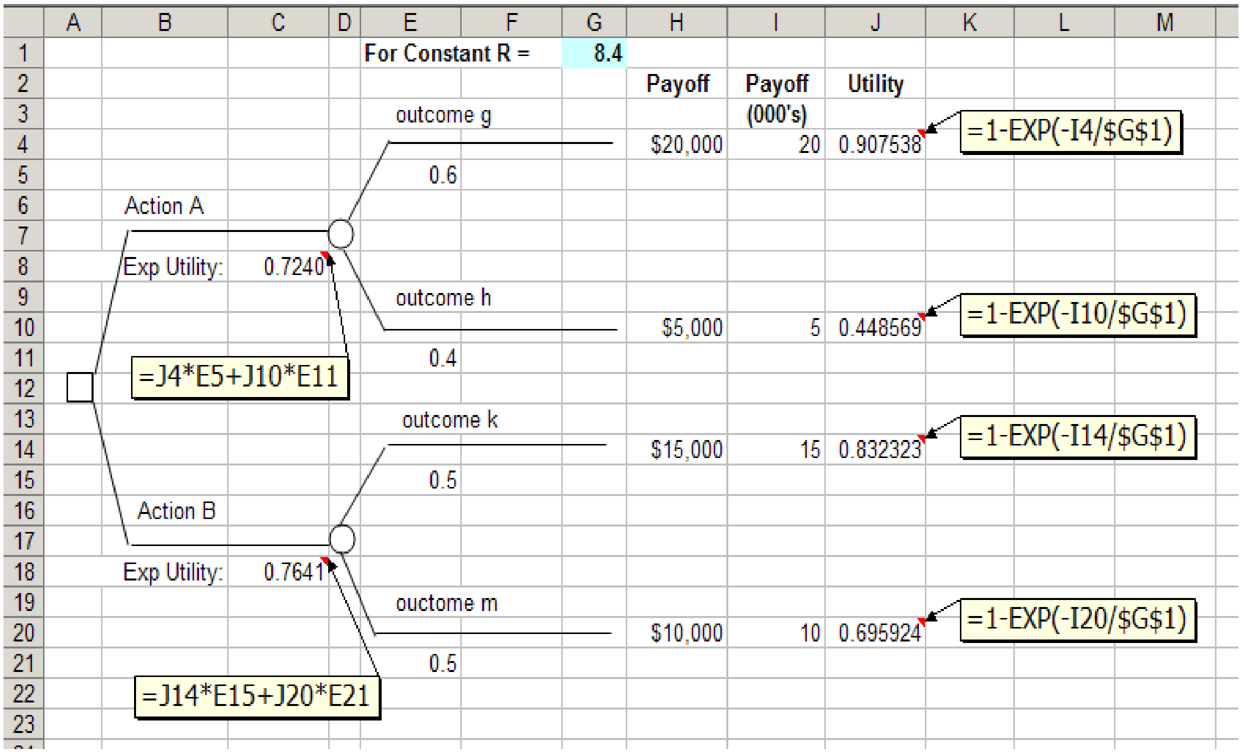


Figure 6.14b A Decision Problem, with Expected Utilities

Here we see that the introduction of your risk aversion by using the utility function shifts your decision to Action B, since it has the higher expected utility, of 0.7641.

An interesting question is, suppose you could sell the rights to make the choice above. What would you charge the buyer? Certainly you'd charge no less than $10,000, since you are guaranteed to make at least that much by choosing B. And you'd not certainly get more than $20,000, since that is the most a potential buyer could get by choosing A and having a lucky day. But what should you ask? We know that your expected utility, based on the above, was 0.7641. You should ask for that quantity of cash, as a sure thing, that gives you the same utility. That is, you should ask for the amount of cash M that solves this equation: $0.7641 = 1\text{-}e^{(-M/R)}$, or $0.7641 = 1\text{-}e^{(-M/8.4)}$. We can use the spreadsheet to find M = $12,133. (You can do this using trial and error, or Excel's GoalSeek or Solver tools.) This is called the Certainty Equivalent of the choice problem, defined as that quantity that you should be indifferent between receiving for sure versus making your best choice in the decision. The difference between the Expected Monetary Value (which assumes risk neutrality) and the Certainty Equivalent is called the Risk Premium, which here is equal to 14000 – 12,133 = $1,867. It represents the portion of the expected value that you'd be willing to forego in order to avoid the risk in the original decision.

A good way to keep these terms straight is to think of buying collision insurance for your car. For simplicity, suppose you think you'll either completely crash your $15,000 car, with a chance of .02 over the next year, or you'll have no accident. The expected cost of not having insurance is therefore $300. You might be willing to pay, say, up to $480 for complete collision insurance for a year. That is, it is worth an extra $180 to avoid the risk of having the big loss. Your certainty equivalent of the cost in this case is $480, and your risk premium would be $180.

The question remains, how can you possibly determine your own value for the constant R? (R is called the "risk tolerance": the higher R is, the more willing you are to accept risk.) One way to do this is as follows. Suppose you have the right to play the following game using a fair coin: Heads: you win $40,000; tails, you win $0. This is an attractive game, although the possibility exists that you'll get tails and wind up with nothing. Now suppose I offer to buy the game from you. That is, you'll get the sure amount I pay you, and I get to play the Heads/Tails game. What is the minimum amount you'd be willing to accept for the game? Would you take $15,000 for sure instead of playing? $10,000?

Suppose you settle on $12,500, meaning any less than this and you'd prefer to play, but if I offer you $12,500, you'd take my offer. It must be that you are assigning the same utility to my offer as you do to playing the game. Since the probabilities are 50/50, we can equate the two utilities (again, expressing the dollar values in thousands – you can use any scale you like, as long as you are consistent). That is, $1-e^{(-12.5/R)} = 0.5*(1-e^{(-40/R)}) + 0.5(1-e^{(-0/R)})$, so the utility of the sure thing is equal to the utility of the gamble. Now we could set up a spreadsheet with both sides of this equation and solve for the unknown value of R:

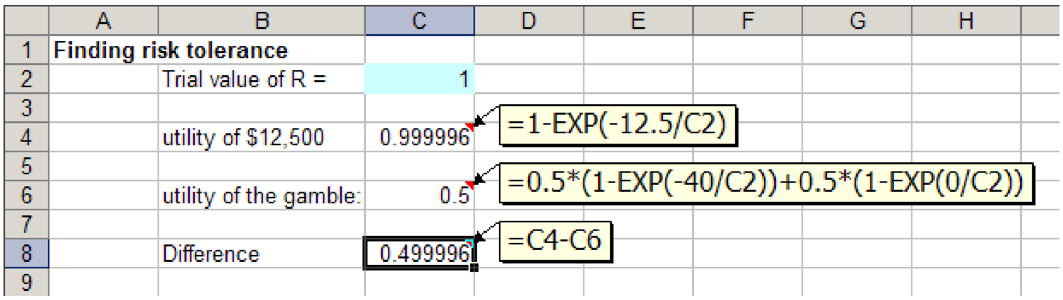


Figure 6.15a: Finding the Risk Tolerance, step 1

Now use GoalSeek, or Solver, to find the value for R in cell C2 that makes the difference in cell C8 = 0

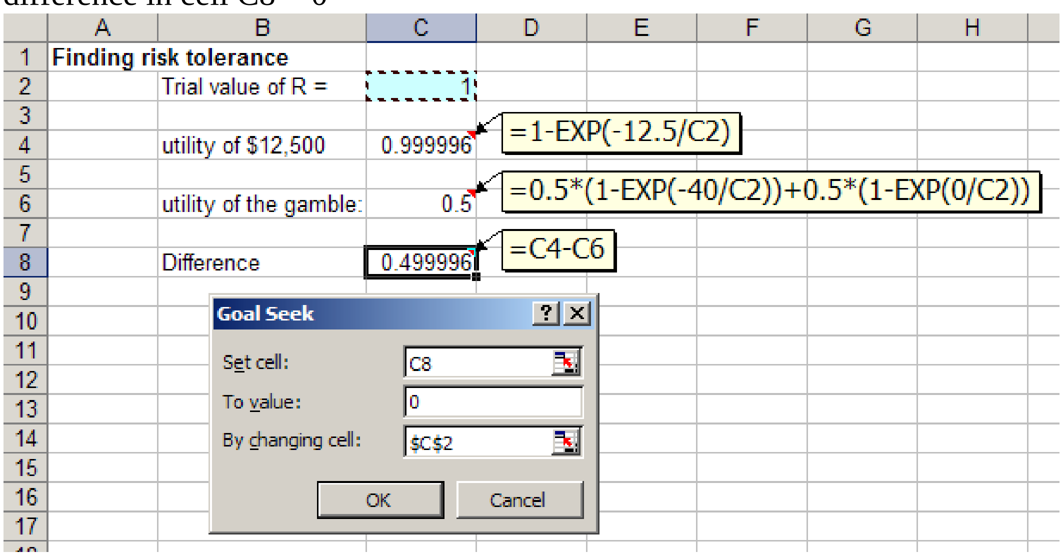


Figure 6.15b: Finding the Risk Tolerance, step 2

	A	B	C	D
1	**Finding risk tolerance**			
2		Trial value of R =	23.9403	
3				
4		utility of $12,500	0.406746	=1-EXP(-12.5/C2)
5				
6		utility of the gamble:	0.405954	=0.5*(1-EXP(-40/C2))+0.5*(1-EXP(0/C2))
7				
8		Difference	0.000792	=C4-C6
9				

Figure 6.15c: Finding the Risk Tolerance, step 3

Based on your value of $12,500 that you'd trade for the game, your risk tolerance R is 23.9, for X in thousands. That is, you could calculate the utility of any amount $X (thousand) as $U[X] = 1\text{-}e^{(-X/23.9)}$

TreePlan automatically calculates expected utility as well as expected value, using the exponential utility function. Of course, the use of the exponential utility is an approximation, but since you can control the risk aversion inherent in the function by increasing R to become more risk tolerant, or decreasing it to become more risk averse, you can explore the impact of your risk aversion in any decision problem. In particular, you could determine the value for R that would cause you to change your decision, and from this simply ask yourself "Am I more or less risk averse than this value of R suggests?" as a way to make the decision.

If this degree of abstraction is too extreme, you could also do some experiments to determine your own utility curve empirically.

Empirical Assessment of Utility Functions

Imagine we wish to assess the utility function for a particular decision-maker (or DM). That is, we suppose the DM has some internal conversion function *U(X)* with which he is implicitly converting monetary outcomes X into personal worth. Our task now is to measure *U(X)*.

First, we can start by establishing a feasible range of money values over which we will measure his utility. For example, for most of us there is no point in determining how we feel about outcomes in the millions of dollars, since we're unlikely to ever see such amounts, and it is unreasonable to expect clear preferences among uncertain outcomes with these magnitudes. Values similar in magnitude to those with which the DM is accustomed to working would be appropriate for the feasible range.

The principle behind the measurement system is the assumption that the utility function increases monotonically (that is, more money never has a smaller utility than less – you can always give some away!). A consequence of this is that decisions that result from combining utilities using any probability distribution will be **invariant under linear transformations**: decisions that would result from using a function $U(X)$ will be the same decisions which result from using $U'(X) = a + b\ U(X)$, where a and b are any real numbers.[1]

A temperature scale is a good example of a measure that is invariant under linear transformations: if any mixture of temperatures is higher than another on a Fahrenheit scale, it will be also higher on a Celsius scale (a linear transformation of Fahrenheit).

This invariance forms the basis for a utility measurement system. To understand it, first think about the Celsius temperature scale. This scale was created by arbitrarily fixing the freezing point of water at 0° and the boiling point of water at 100° and measuring any other temperature in relation to these two arbitrary assignments. We could have chosen any other values for the number of "degrees" to attach to the two reference points, the freezing and boiling points of water, as for example 32° and 212° on the Fahrenheit scale. We could likewise have selected any two other events to use as reference points (such as the freezing point of nitrogen and the melting point of lead). The result in any case is still a linear transformation of the Celsius scale.

Now, since we know that utility, like temperature, is also invariant under linear transformations, we can follow this same procedure to measure utility. That is, we can select any two monetary outcomes and attach an arbitrary

[1] Simple algebra will show that if any lottery A is preferred to another lottery B using any monotonic utility function $U(X)$, it will also be preferred using $U'(X)$.

utility to each one. Then we can measure the utility of the rest of the scale *relative* to these two arbitrary utilities, just as we did for temperature.

A specific example will clarify this procedure. Suppose that we are trying to find the DM's utility for money values between \$0 and \$20,000. We can choose any two monetary values and assign an arbitrary utility value to each. For example, we might select U(\$0) = 0 and U(\$20,000) = 100. The method proceeds by determining the DM's certainty equivalents for a sequence of lotteries, as follows.

We begin by finding the DM's certainty equivalent C for a 50/50 lottery between \$0 and \$20,000. To find this certainty equivalent, the simplest method is to offer the DM the choice between some sure amount \$$C$ and a coin flip, where tails wins \$0 and heads wins \$20,000. You can then vary C until the DM is just indifferent between accepting \$$C$ for sure and flipping the coin. This is the certainty equivalent of the lottery for that DM.

Suppose you have done so, and the DM settles on \$5,500 as his certainty equivalent for the lottery. Now, since the DM is willing to trade \$5,500 for sure in exchange for the lottery, he must be assigning the same worth to each one. Since we fixed the utilities of the coin flip outcomes at 0 and 100, the expected utility of the coin flip is:

$$\mathrm{E}[U] = 0.5\ U(\$0) + 0.5\ U(\$20{,}000) = 0.5\ (0) + 0.5\ (100) = 50.$$

Thus the implicit utility of the DM for a sure \$5,500 is 50. We can now repeat the process for outcomes of \$0 and \$5,500. Suppose the certainty equivalent for a 50/50 lottery over these outcomes is \$2,100 – then the utility of \$2,100 must satisfy:

$$U(\$2{,}100) = 0.5\ U(\$5{,}500) + 0.5\ U(\$0) = 0.5\ (50) + 0.5\ (0) = 25.$$

Now we might offer the 50/50 lottery over the outcomes \$5,500 and \$20,000. Suppose the DM gave a certainty equivalent of \$11,000 for this lottery. His utility for \$11,000 is then

$$U(\$11{,}000) = 0.5\ U(\$5{,}500) + 0.5\ U(\$20{,}000) = 0.5\ (50) + 0.5\ (100) = 75$$

We can continue making up new lotteries, using two payoffs for which the utility has been established to find the utility of a third payoff. In this way, we can establish enough utility values to fill in the rest of the utility function by fitting a smooth curve through the points, as in Figure 6.16. In the figure, the utilities corresponding to monetary values of $5,000, $10,000, and $15,000 are shown, for reference.

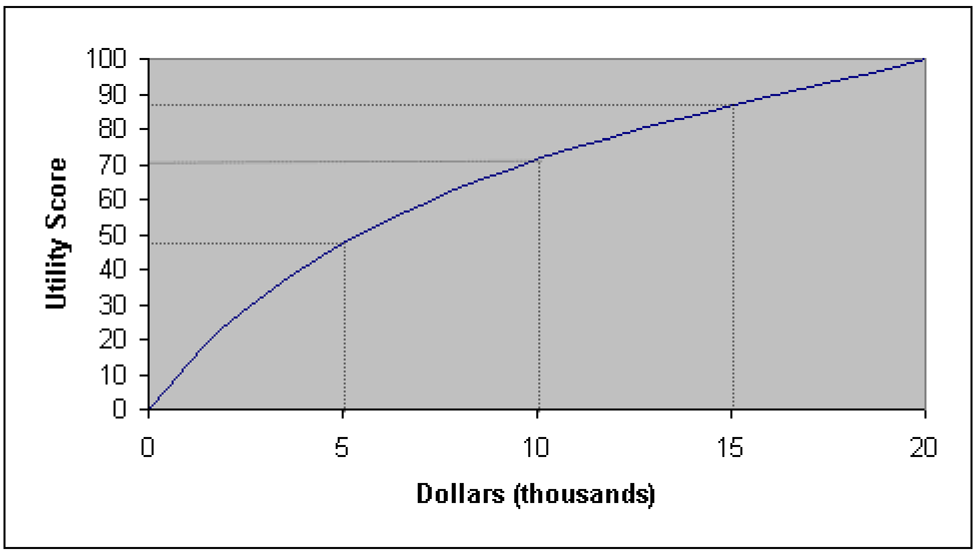


Figure 6.16 An Empirical Utility Function

In evaluating a utility function, it is important that the preferences be those *of the DM*. Questions should be probing (especially about inconsistent answers), but should not suggest that there is a "correct" answer. The point of the exercise is to capture the *DM's* perceptions about the relative values, and not your beliefs.

Example.
Review the decision problem we considered earlier in Figure 6.14. How would the empirical utility function shown in Figure 6.16 affect the decisions? We

can use the curve in Fig. 6.16 to determine the utility associated with each payoff in the problem. From the curve, the utility of $20,000 is 100, the utility of $5,000 is about 47, the utility of 15,000 is about 88, and the utility of 10,000 is about 71. These numbers are shown on the tree below in Figure 6.17, and the expected utility of each action computed as shown.

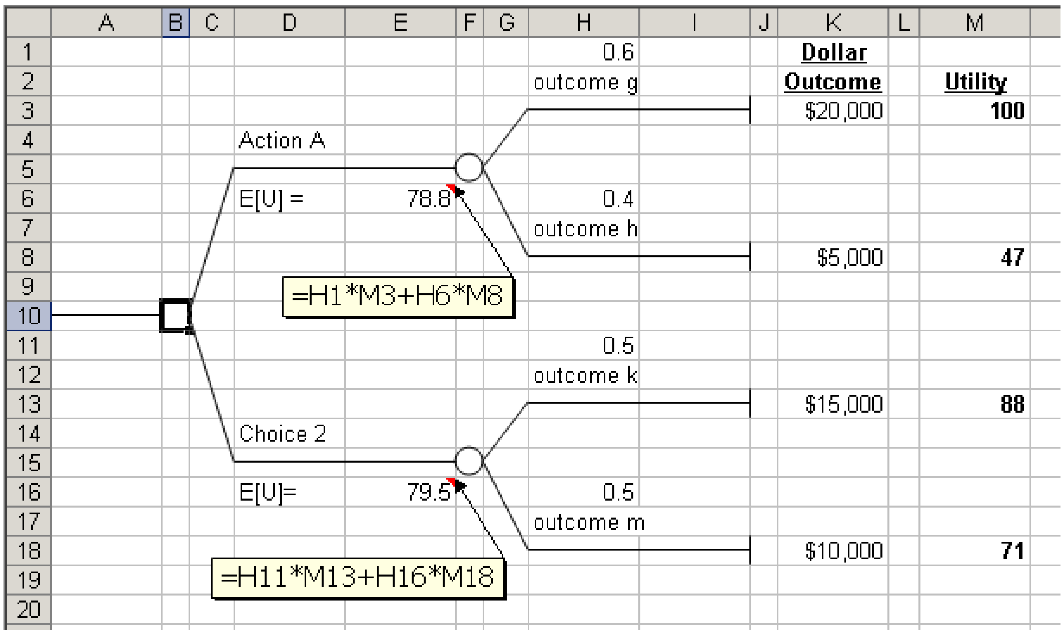


Figure 6.17 The Decision Problem, Again.

Using the empirical utility function, the expected utility of Action A is 78.8, and that of B is 79.5, so Action B is the preferred choice with the risk attitude as captured in the utility function. (Recall that the EMV of Action A is $14,000, and that of B is $12,500, so A would be preferred by a risk neutral, EMV based decision rule.)

We could also calculate the Certainty Equivalent of this decision problem. Since we know that the Expected Utility of the best choice is 79.5, we can find this value on the vertical axis of the utility function, and then find the corresponding horizontal axis value. The procedure is illustrated below in Figure 6.18. Carefully reading the graph, as shown, we can see that the certainty equivalent, which is the sure amount that yields an expected utility of 79.5, is about $12,300. The risk premium is the difference between the expected value of the best decision and the certainty equivalent of the best decision, which in this case is ($14,000-12,300 =) $1,700. Again, this can be interpreted as the reduction in expected value that this risk averse individual would be willing to accept to avoid the risk in the decision.

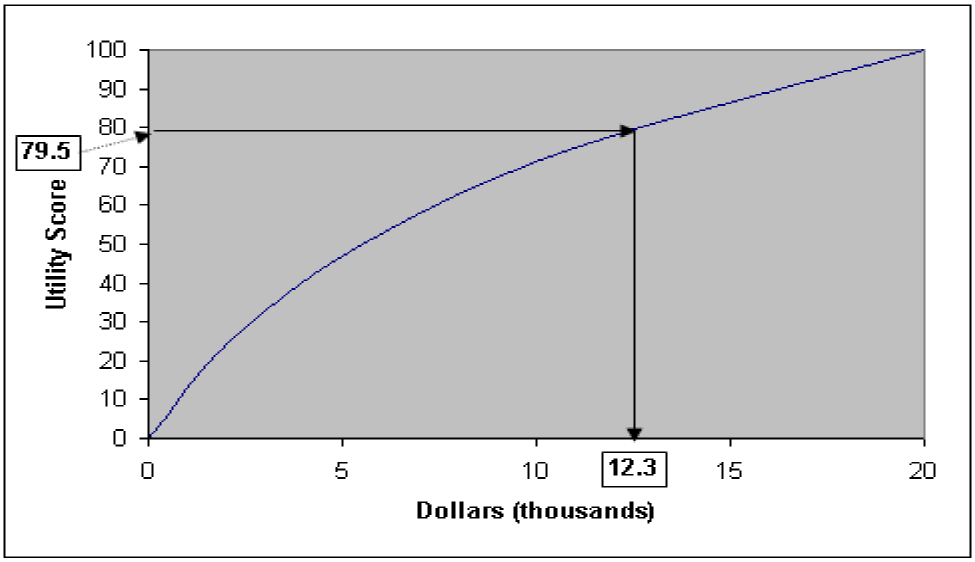


Figure 6.18 Finding the Certainty Equivalent

Comments on Empirical Utility Functions

The utility function illustrated in Figure 6.18 has a decreasing slope (it is **concave**), which corresponds to a diminishing marginal utility for money (i.e. the contribution to perceived total worth derived from each additional dollar is declining). We speak of people with this sort of utility function as **risk-averse**. This term arises from the fact that the certainty equivalent for lotteries for DMs with concave utility functions will always be less that the expected value of these same lotteries.

In the empirical example above, the risk-averse DM was indifferent between owning a sure \$5,500 and flipping a coin with outcomes \$0 or \$20,000 with 50/50 probabilities (which has an expected value of \$10,000). If the certainty equivalent is equal to the expected value, meaning that the DM would be indifferent between flipping the coin and owning \$10,000 for sure, he is called **risk-neutral**. In this case, his utility curve would be a straight line. If the DM's utility curve has an increasing slope (bending upward), it is called **convex**. His certainty equivalent will be higher than the expected value for any gamble, implying that he is a **risk-preferring** decision maker.

Most people are risk-averse for significant sums of money, as shown by the number of insurance contracts people buy for various kinds of loss. (As an aside, we observe that people *do* buy lottery tickets with expected values lower than the purchase price. Although this could be interpreted as risk-preferring behavior, we could also argue that buying a ticket lets the buyer dream about winning, which is worth something that is not included in the expected value.)

This chapter has focused on utility functions for individuals. Is there a utility function for a *group* of individuals, such as the board of directors in a business firm? While empirical evidence is slim, it appears difficult to draw any general conclusions about the merit of a corresponding notion for a group utility function. Indeed, *Arrow's Impossibility Theorem* (Arrow, 1963) indicates that there is no theoretical foundation for such a group utility function.

Does this mean we can't use the idea of risk aversion in a group decision setting? There are three responses to this question.

1) If the payoffs from the decision problem are relatively small compared to the whole agenda facing the group, the use of a risk-neutral (i.e. linear) utility function may be reasonable. Expected value is still the best criterion if there are many small uncertain situations to be dealt with.

2) If the payoffs are larger, a group could begin by determining the expected value for each alternative. The group could then collectively examine what an appropriate risk premium would be for each alternative, to measure the group's joint assessment of the riskiness of the alternative's outcomes.

Subtracting this risk premium from the expected value would give a certainty equivalent for each alternative, and the group can select the most attractive of these.

3) If there are many different alternatives or many outcomes to consider, the group could collectively think about selecting some *form* of utility function, such as the exponential utility function described in the text:

$$U(X) = 1-e^{-X/R}$$

Other functional forms can also be assumed, and sensitivity analysis can be used to identify ranges for the parameters of the function that result in different alternatives being selected. The group can debate which of these parameter ranges most closely approximates their collective risk attitude, to arrive at a decision.

Problems.

Question 6.1

A chain of supermarkets requires 24,000 fluorescent light bulbs for its stores. There are two suppliers of these bulbs. Supplier A offers them at $4.00 per bulb and will replace any defectives with guaranteed good ones for $4.00 each. Supplier B offers them at $4.15 each and guarantees to replace defectives for $1.00 each. The probability distributions of the percentage defective from the two suppliers is shown below:

	Probability of % Defective	
% Defective	**Supplier A**	**Supplier B**
3%	0.10	0.05
4%	0.20	0.10
5%	0.40	0.60
6%	0.30	0.25

These data have been estimated from historical records. For example, the probability is 0.4 that 5%, or 1,200, of 24,000 bulbs purchased from Supplier A will be defective. The supermarket plans to sell these bulbs for $4.40 each and charge nothing for replacing defectives. Which supplier should be chosen if the decision criterion is to maximize the expected monetary value (EMV)?

Question 6.2

A decision-maker has a utility function for monetary gains *x* given by

$$U(x) = \sqrt{x + 10{,}000}\,.$$

a) Show that the decision-maker will be indifferent between doing nothing (no loss, no gain), and a risky situation where she gains $80,000 with probability 1/3 or loses $10,000 with probability 2/3.

b) If there is a 10% chance that her painting valued at $10,000 will be stolen during the next year, what is the most that the decision maker would be willing to pay for insurance covering the theft?

Question 6.3

A decision-maker is going to invest his total wealth of $1,000 for a period of six months. Two potential investments are available: T-bills and gold. If he invests the sum in T-bills, he is certain to end the period with $1,296 (a gain of $296). If he invests in gold, there is a 0.75 probability that he will end the six months with $400 and a 0.25 probability that he will end with $10,000. His utility function of wealth *x* is $U(x) = \sqrt{x}$.

a) Should he invest in gold or T-bills?

b) Assume that he can invest some proportion *p* of the $1,000 in T-bills and the rest in gold. In this case, his gain or loss from each investment is reduced proportionally. For example, if he invests half his money in gold, his gold investment will be worth either $200 or $5,000. Assuming the same utility function, determine his best value for *p*. (You might want to consider a spreadsheet model with a data table to examine this.)

Question 6.4

Oilco, which follows a risk-neutral decision criterion, must decide whether or not to drill for oil in the South China Sea. It costs $100,000 to drill, and if oil is found, its estimated value will be $600,000. At present Oilco believes that there is a 20% chance that the field contains oil.

a) What is Oilco's best decision at this point?
b) Imagine that there is an "oil wizard" who knows for sure whether the proposed South China Sea field contains oil. Also assume that Oilco can pay this wizard for his information, and for the price the wizard will tell the truth; i.e. he will reveal whether there is oil or not. After receiving the wizard's advice, Oilco can decide to drill or not. (Obviously, Oilco will drill if the wizard indicates that there is oil, and will not drill otherwise.) How much should Oilco pay for the wizard's advice? (This quantity is sometimes called the **value of perfect information**.)
   Hint: Think about the probability that Oilco should assign to the wizard's saying that there is, in fact, oil there.
c) Now, assume that Oilco didn't hire the wizard, but, before drilling, it can hire a seismologist to obtain more information about the likelihood that the field will contain oil. The company believes that there is a 50% chance that the seismologist will issue a favourable report and a 50% chance of an unfavourable report. Given a favourable report, there is a 30% chance that the field contains oil. Given an unfavourable report, there is a 10% chance that the field contains oil. What is the expected value of the seismologist's advice to Oilco? Should the company pay $30,000 for this advice?

Question 6.5

You are responsible for the design of a new, sophisticated, remotely piloted vehicle that is used to survey remote electrical transmission lines. You must decide whether to provide zero, one, or two backup navigational computers. The first backup comes into service only in the event that the primary system fails, and the secondary backup comes into service only if the primary computer and first backup both fail. Each system (independent of the others) has a 0.01 probability of failure once it has been called into use. Once in flight, the vehicle will be lost only if the primary and all backups fail. Each backup system costs $70,000 and if the vehicle is lost, the power company must replace it at a cost of $8 million. How many backup systems would you recommend?

Question 6.6

You have just purchased a new car, and you believe your chances of having an accident in the next year are two in ten. In the event that you do have an accident, you know from insurance company data that the possible damage amounts are $100, $500, $2,000 and $15,000 with probabilities (conditional on having an accident) 0.6, 0.2, 0.1, and 0.1, respectively. Assume you can buy an insurance policy with a deductible clause (that is, a $150-deductible policy means you pay the first $150 of any claim and the insurance company pays the rest). Also suppose that claims do not affect your future insurance costs.

a) How much would you be willing to pay for a $150-deductible policy?

b) How much would you be willing to pay for a $250-deductible policy?

c) Intuitively, why isn't the price of a $150-deductible policy not $100 more than a $250-deductible policy?