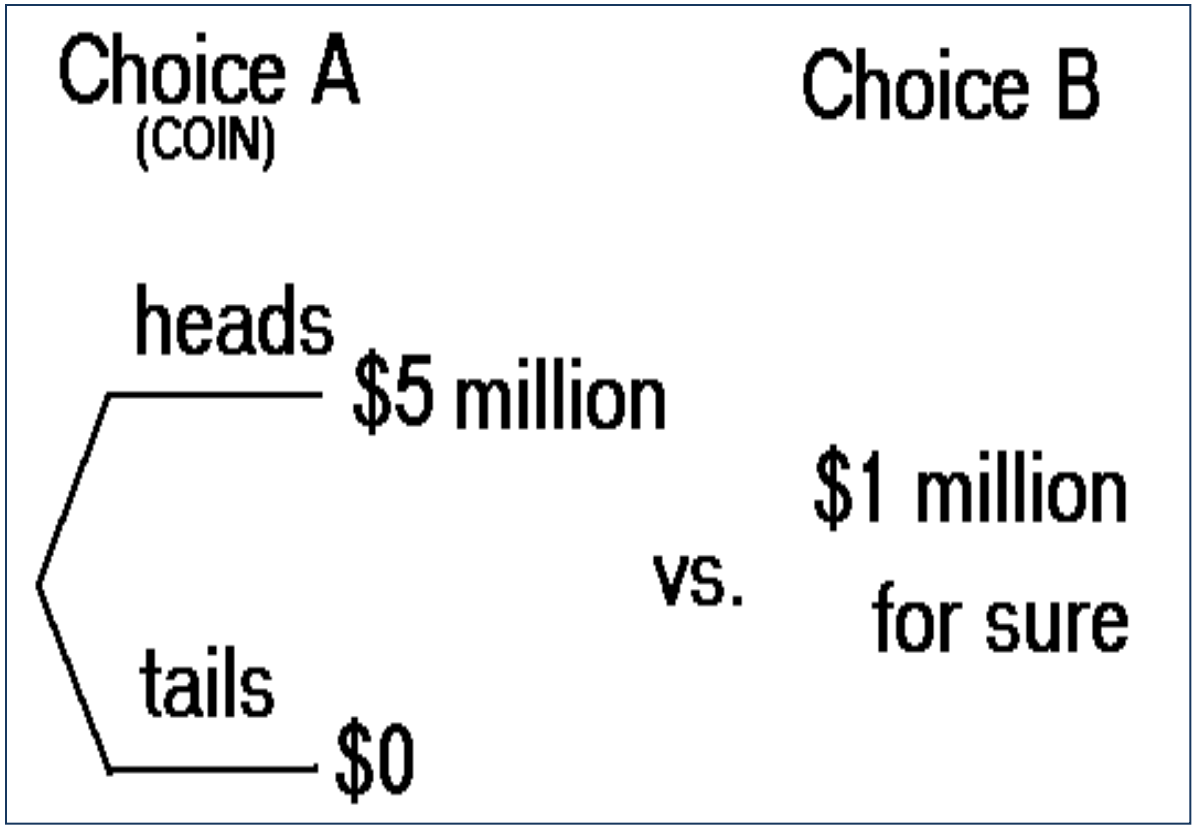
MMA/MMAI 861
ANALYTICAL DECISION MAKING

Decision Analysis (Two)

Class Six
Yuri Levin

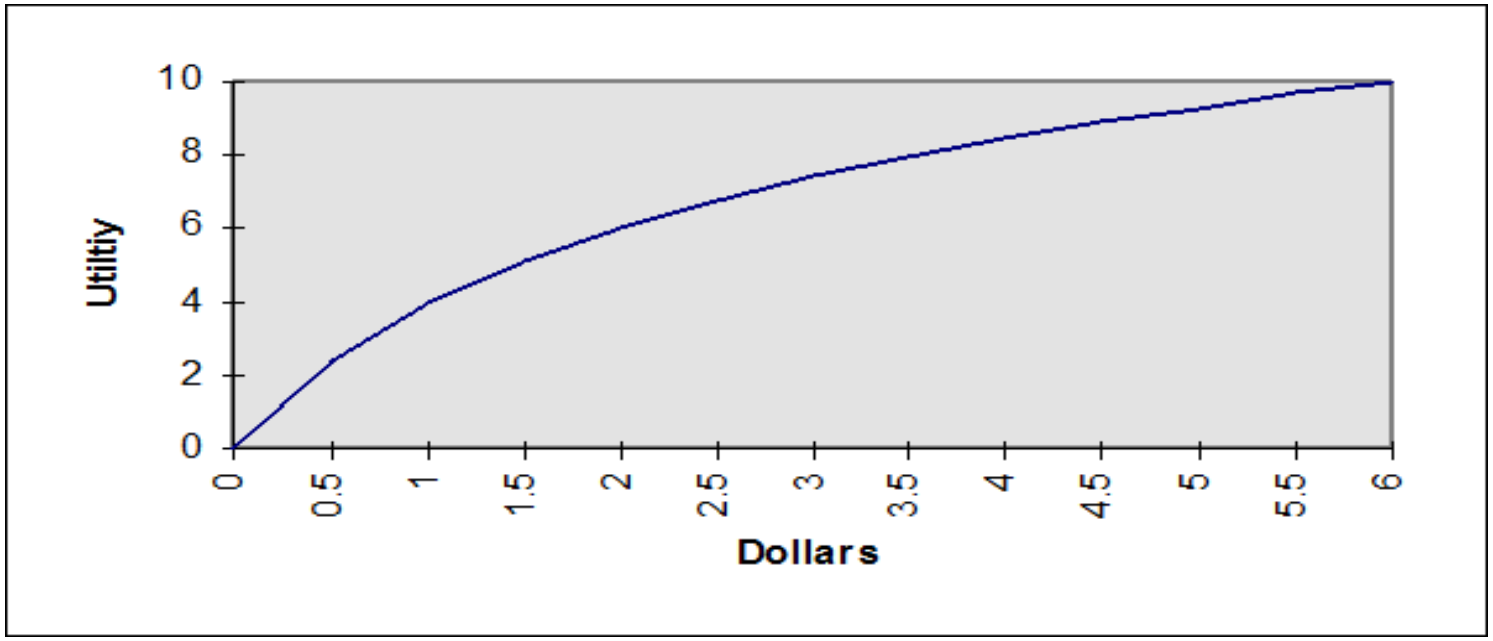
Coping with Risk

- Why not use Expected Value?
- What choice would you make here:



Coping with Risk (continued...)

- EV of A = $2.5M, but most choose B for $1M. How come?
- One way to explain is to suppose the existence of a transformation from $$ to personal worth: a "utility function".



Coping with Risk (continued…)

- Supposition: people act to maximize expected utility

- Two approaches:
  - Assume existence and measure utility function empirically
  - Theoretical: identify reasonable axioms and prove existence

Empirical Utility

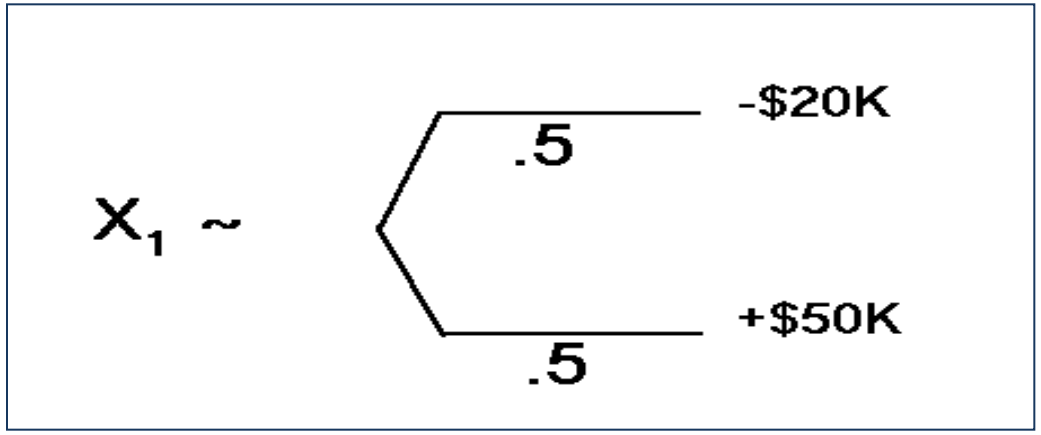
- Easy to prove that if utility is monotonically increasing, then a decision based on expected utility using probabilities will be the same decision as that made with a linear transformation of the original utility function.

- Impact: utility is a cardinal measure – like a temperature scale, we can arbitrarily fix a utility value for two dollar amounts, and measure the utility of other amounts based on these values.

Empirical Utility (continued…)

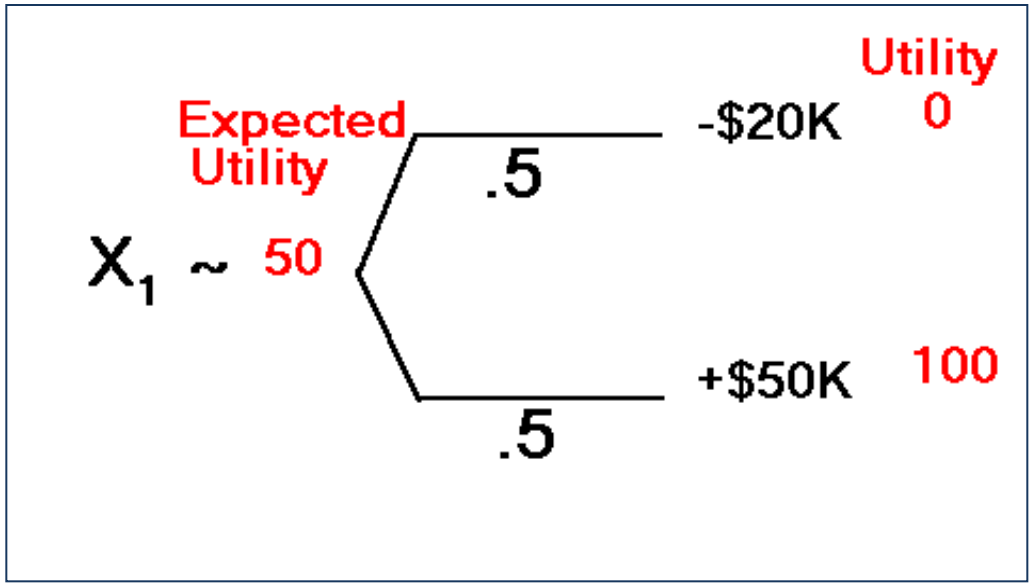
- How to: pick a range; say, -$20K to +$50K
- Arbitrarily choose utility values for these:
  - Say, U(-$20K) = 0, U(+$50K)=100 for convenience

- Now find X1 to satisfy the indifference:



Empirical Utility (continued…)

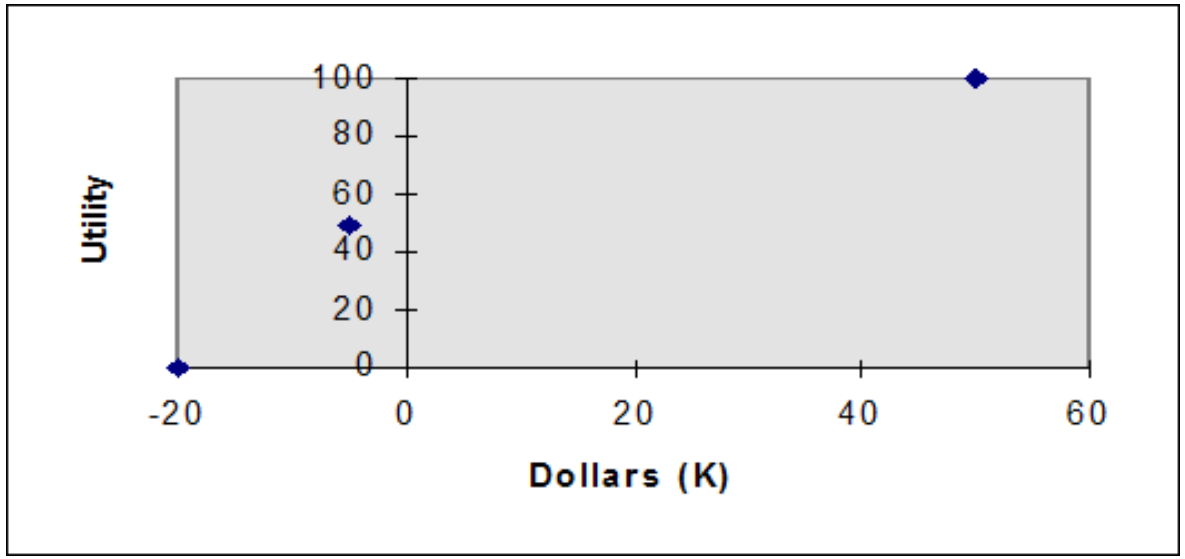
- Indifference implies that you place the same worth on each side… i.e. they have the same utility



- Suppose X1 = -$5K. Then U(-$5K) = 50

Empirical Utility (continued…)

We now have three points on the utility function:

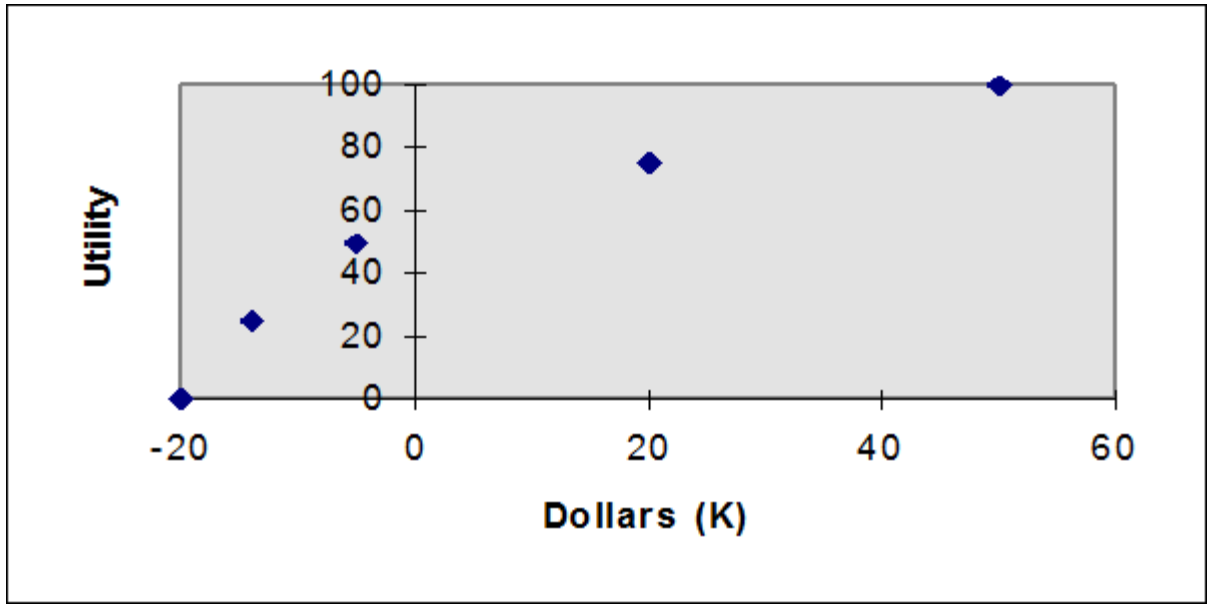


Now repeat with a 50/50 gamble {-$5K,+$50K}

Suppose X2 = +20K; by same logic, U($20K) = 75

Empirical Utility (continued…)

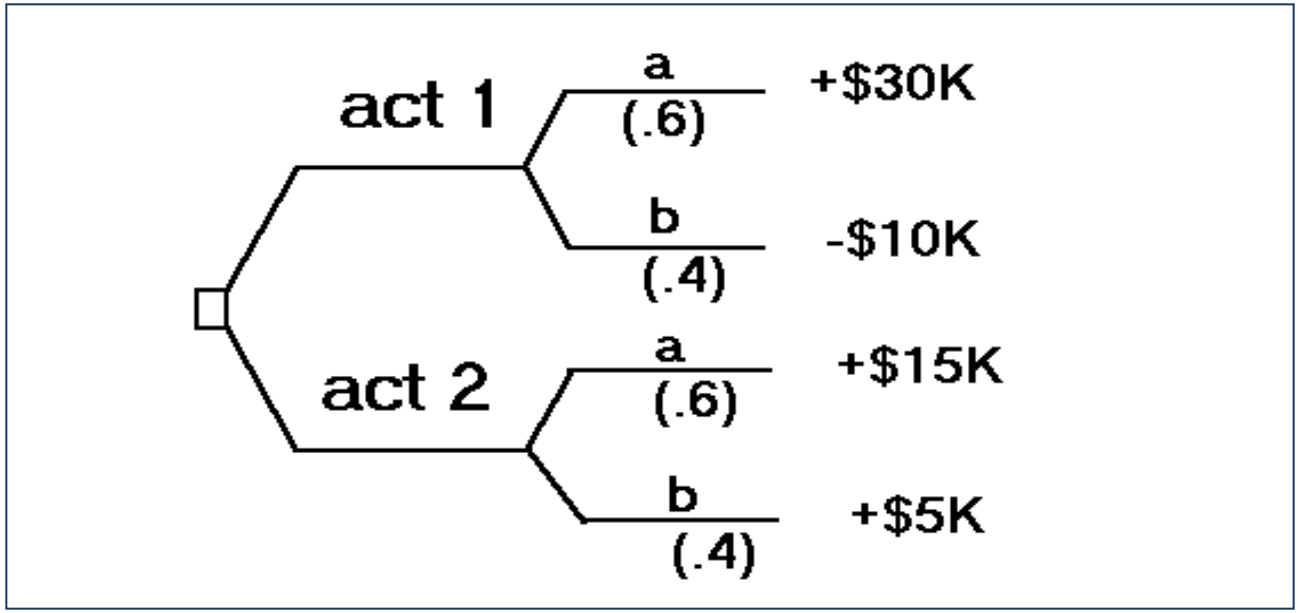
- Repeating with intermediate points, you will see a picture something like:



- Connect the dots, yielding a utility function

Empirical Utility (continued…)

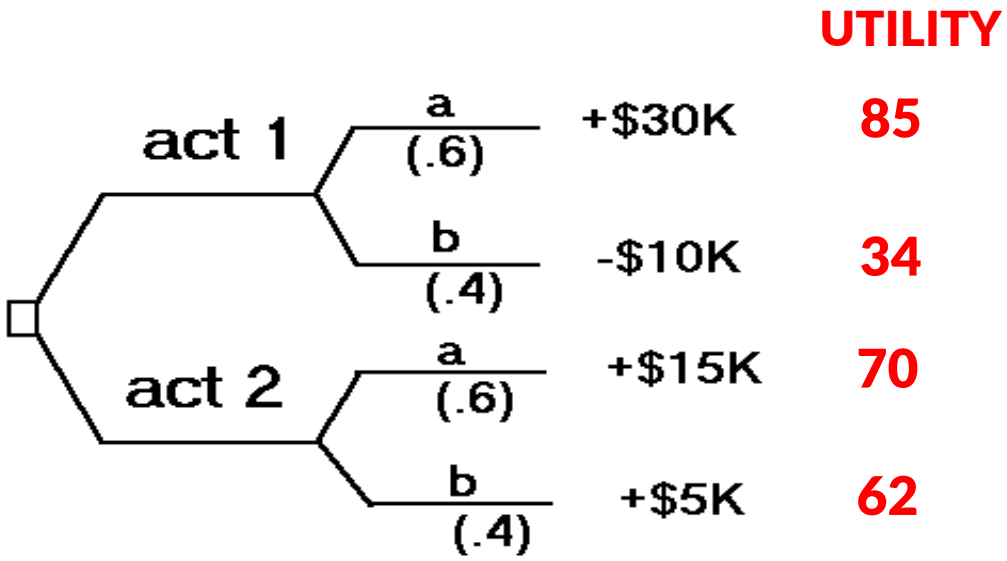
- Apply this U(.) to the following problem:



- Note that EV(act 1) = $14K, EV(act 2) = $11K

Empirical Utility (continued…)

- Reading the utility off the graph for the outcomes:



- So, expected utility for act 1 is .6×85+.4×34 = 64.6
- Expected utility for act 2 is .6×70+.4×62 = 66.8

Empirical Utility (continued…)

Thus although the expected value criterion indicates that act 1 is the best choice, since:

EV(act 1) = 14K > EV(act 2) = 11K

The expected utility criterion, by discounting for the higher risk involved with act 1, suggests that act 2 is the more preferred since:

EU(act 2) = 66.8 > EU(act 1) = 64.6.

But does such a utility function actually exist?

Exponential Utility

The exponential utility function finds the utility of any dollar amount x as:

$$U(x) = 1 - e^{-x/R}$$

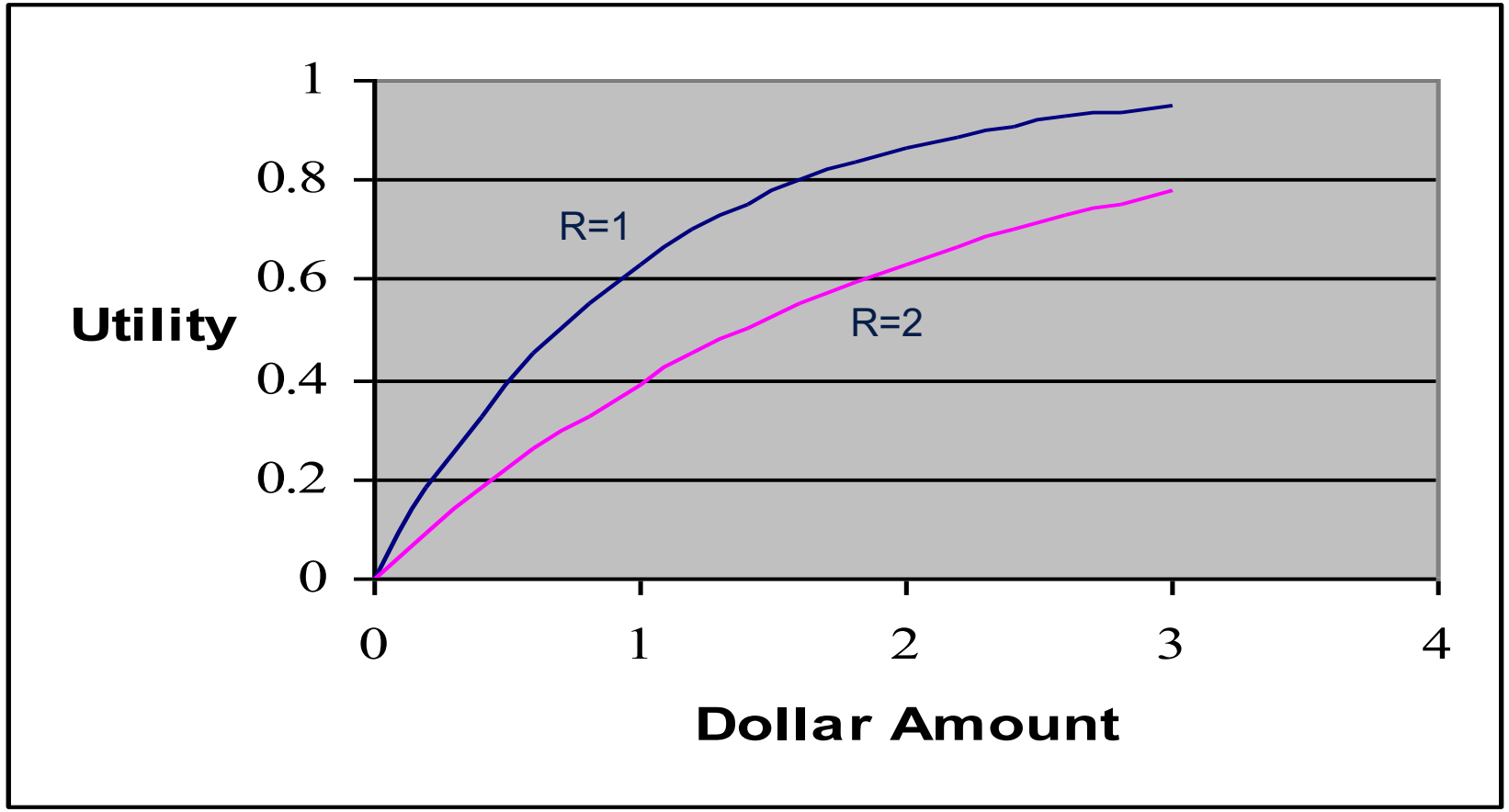
The parameter R is called the "risk tolerance" and measures the curvature of the function:

larger R => more tolerance => more linear U(x)

smaller R => more conservative => curved U(x)

This difference shows up graphically:

Typical Exponential Utility Curves



Describing Exponential Utility Functions

- Advantage: exponential utility captures risk aversion quite well, and is easy to assess
- To find R, here's a trick: what dollar amount would make you (the firm) indifferent between playing this game or not:

- The dollar amount Y for which this is true is actually the risk tolerance, R!

Describing Exponential Utility Functions (continued…)

- Why is this so? Suppose you have a risk tolerance of R. Then not playing the game gives you a utility of:

$$U(0) = 1 - e^{-0/R} = 0$$

- Playing the game will give an expected utility:

$$\begin{aligned} 0.5\,U(R) + 0.5\,U(-R/2) &= 0.5(1 - e^{-R/R}) + 0.5(1 - e^{R/2/R}) \\ &= 1 - 0.5e^{-1} - 0.5e^{0.5} = 0 \end{aligned}$$

- Since both options give the same utility, you are indifferent between them. This can only be true if you choose the $ value in the game as R.

Example

WILDCAT DYNAMICS CO.
(see handout)

Multiple Criteria Decision Analysis

Multi-Factor Decision Problems

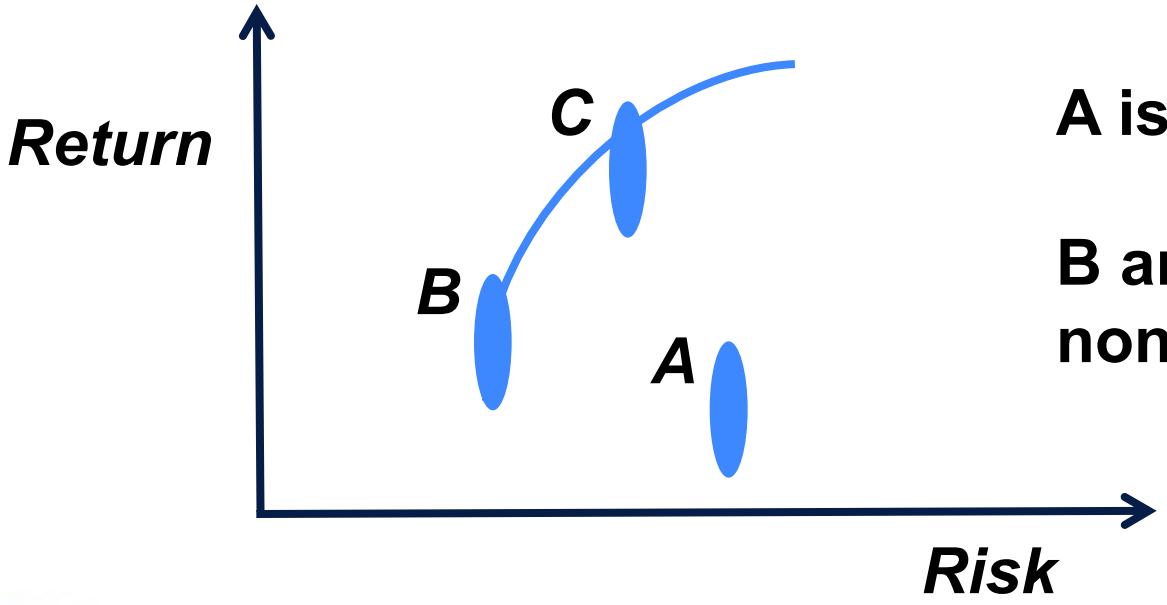
- We have seen some decisions that can be boiled down to a single criterion like cost or profit
  - Clear score for identifying 'better' solutions
- MANY decisions involve multiple competing factors
  - Health and safety
  - Environmental impact
  - Public perception
  - Cost or profit
  - etc.
- Particularly true of public policy decisions

Reasons for Analysis

- Make the 'best' decision
- Build a case for a decision that you already think is the correct one (due diligence)
- Build an argument against a decision that you think is the wrong one!
- Provide a framework for debate
- Expose assumptions and biases

Best Decisions

- Bad news: There is almost never a single best decision when multiple factors are involved
- Instead there are 'dominated' or 'non-dominated' decisions
- A classic example – investments risk/return:



A is dominated

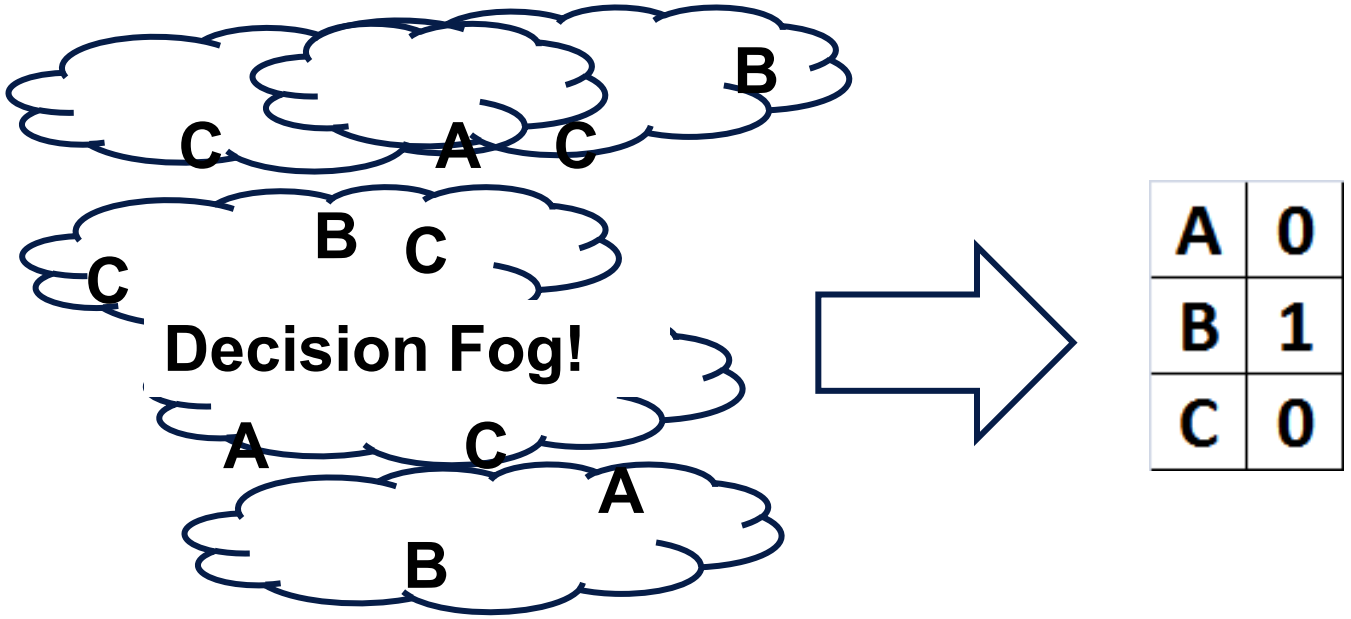
B and C are non-dominated

So, How to Choose?

- In the investment example, a choice can be made once the investors 'appetite for risk' is determined.
- This is much more difficult when there are more criteria and most of them are not quantifiable.
  - E.g. safety, architectural style, colour...
- Many decision-makers will argue that numerical scales are meaningless in these situations.
- Ironically, all decisions must eventually boil down to the simplest scale of all:
  0 (or No) for rejected choices
  1 (or Yes) for final decision

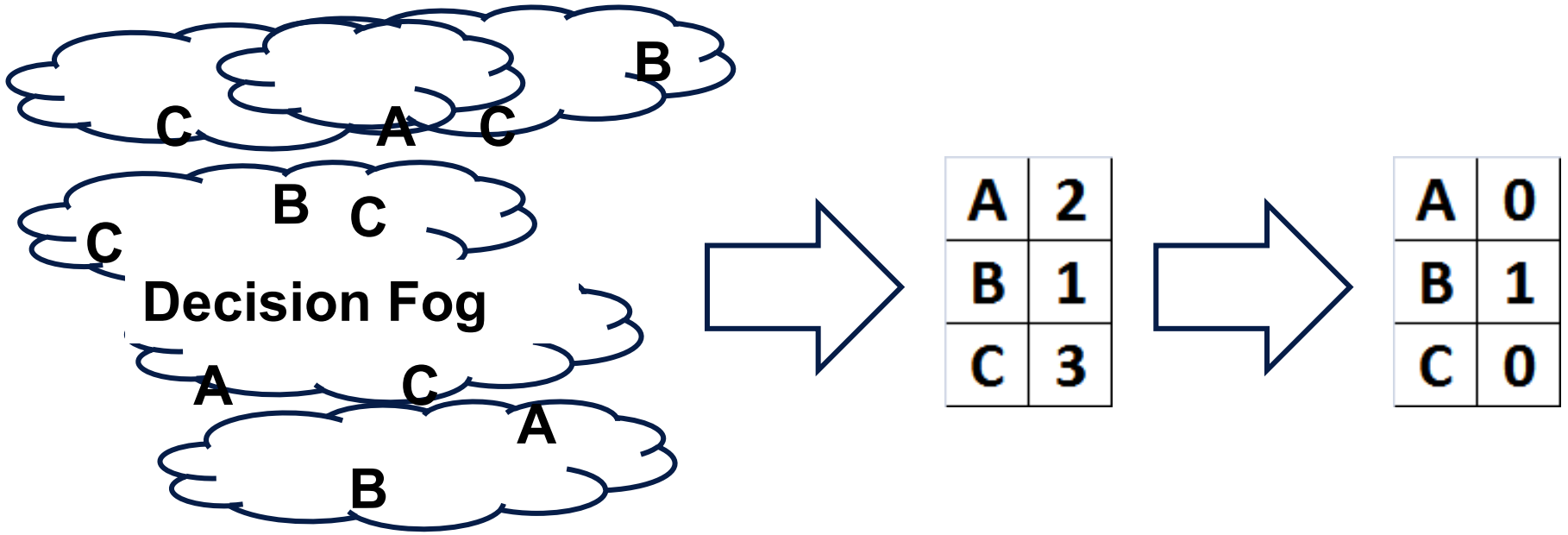
The Challenge

We need to get from complex 'fog' of competing opinions and indecision to a 0/1 scale

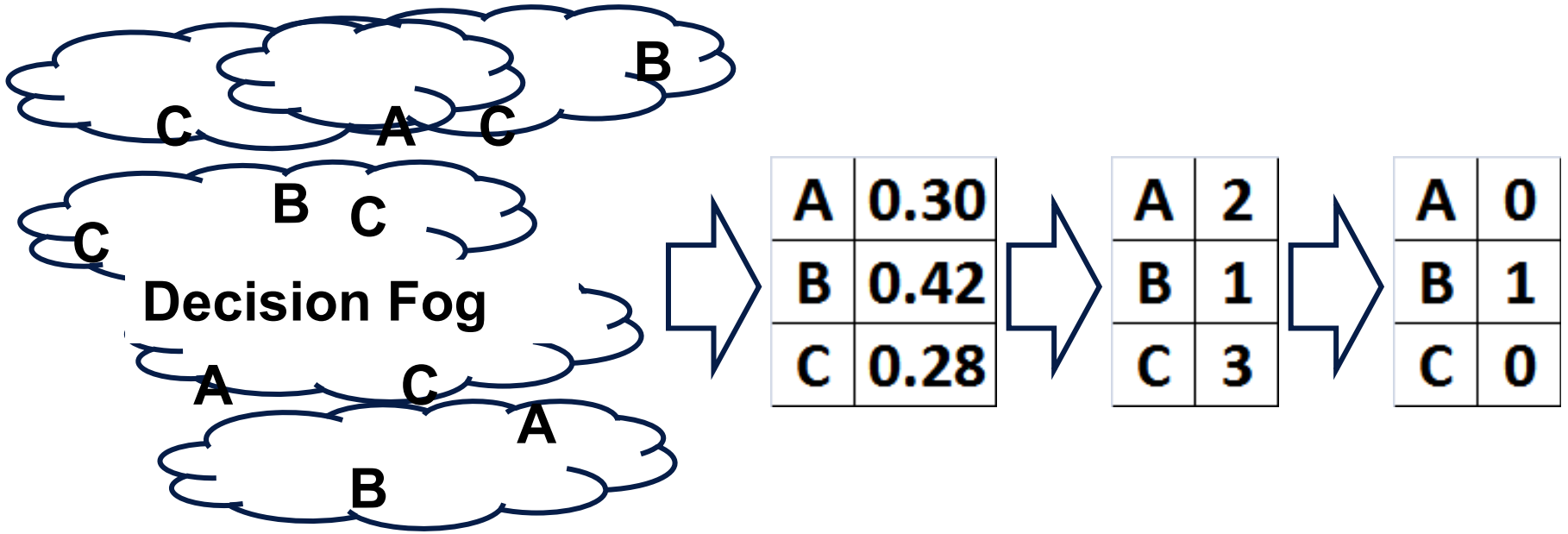


A	0
B	1
C	0

A Possible Intermediate Step: Rank-Ordering



Or… A Weighted Scale



- This is allowing more 'nuanced' comparisons
- More gradually bridging from intuition to final conclusion

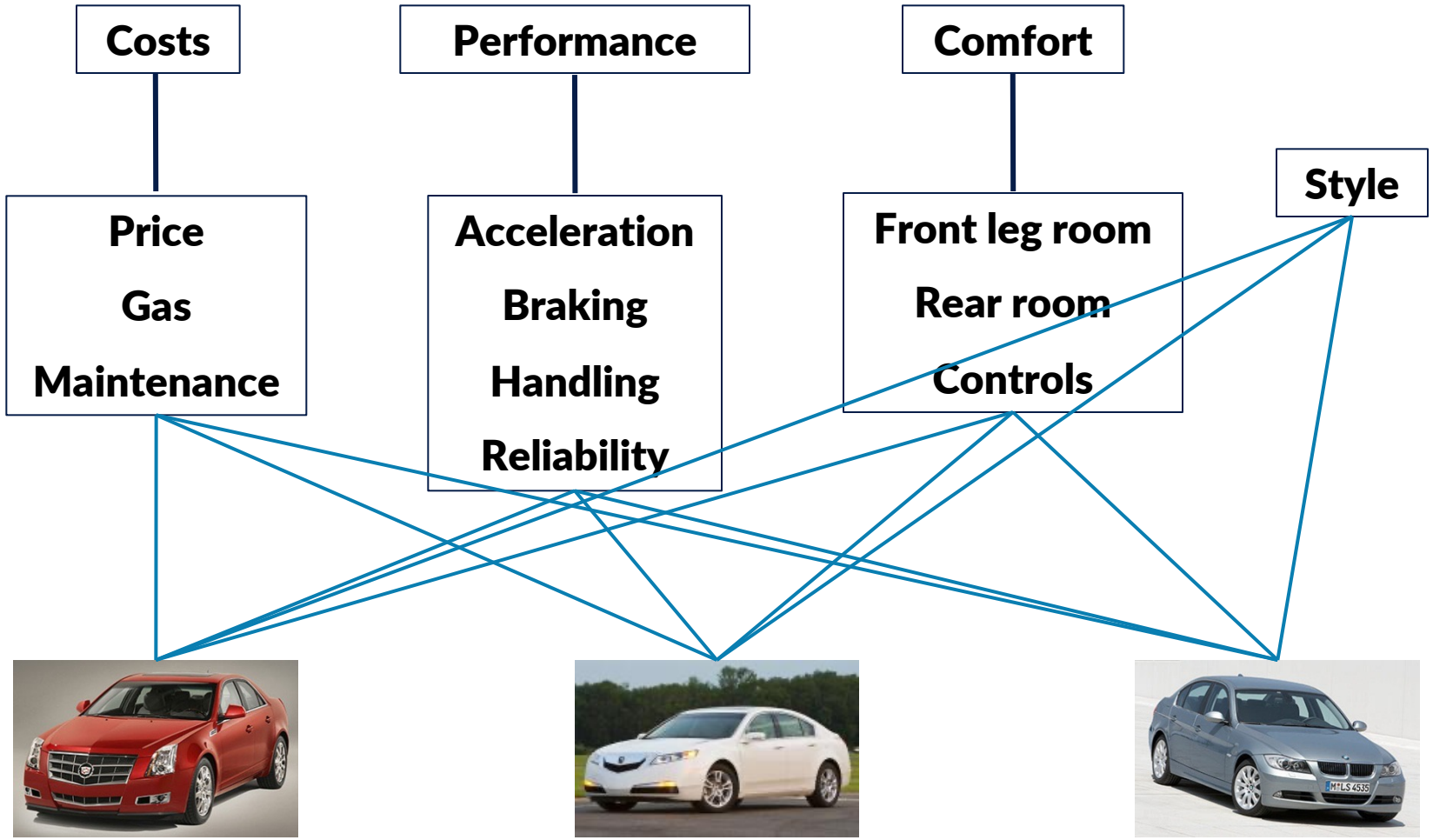
Some Choices

Cadillac CTS
$40K to $65K

Acura TL
$40K to $49K

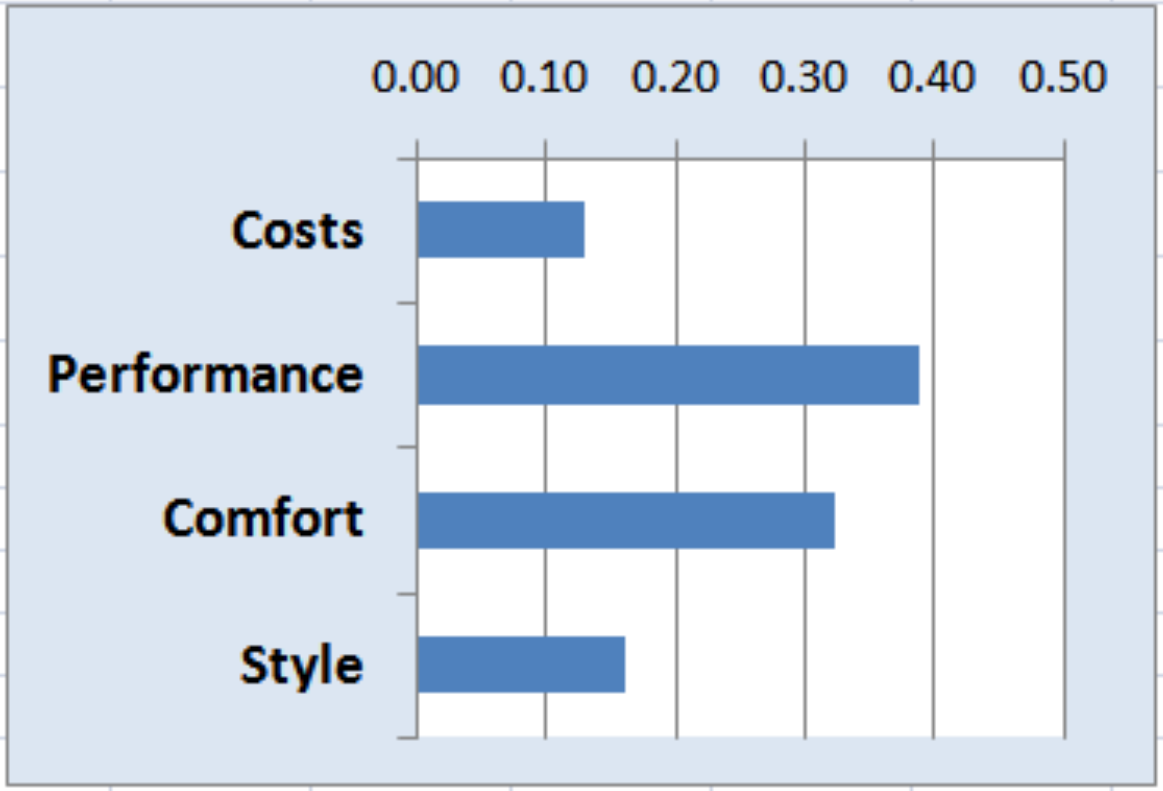
BMW 328i
$35K to $70K

A Hierarchy of Criteria



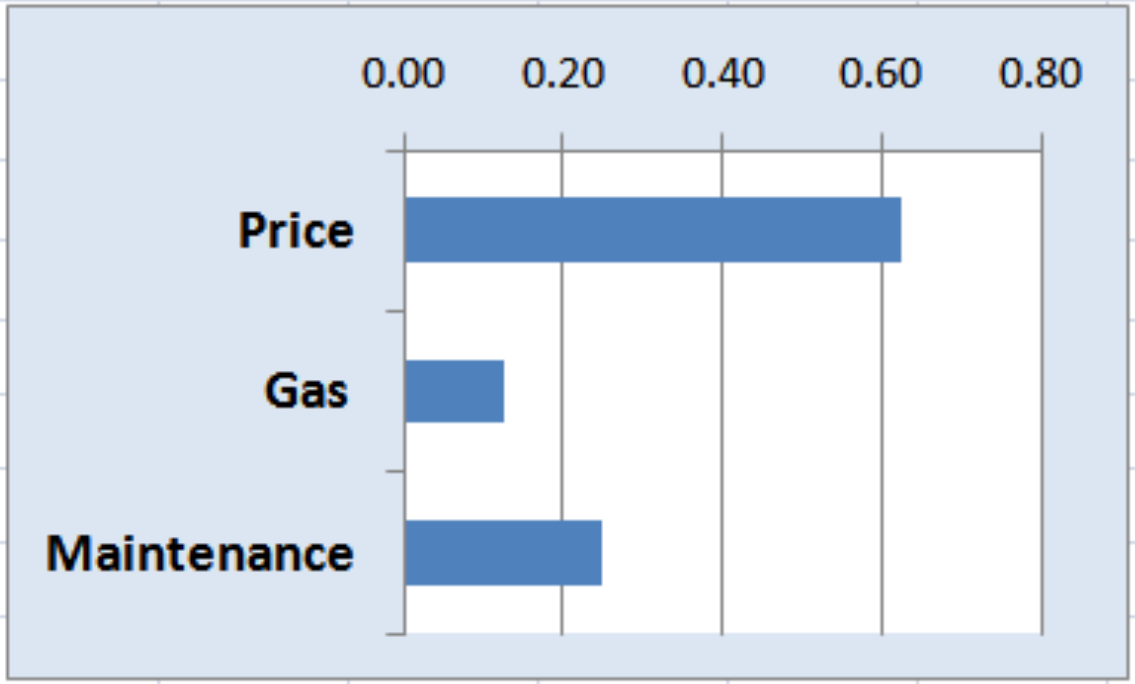
Direct Rating of the Level One Criteria

Costs	100	0.13
Performance	300	0.39
Comfort	250	0.32
Style	125	0.16
	775	



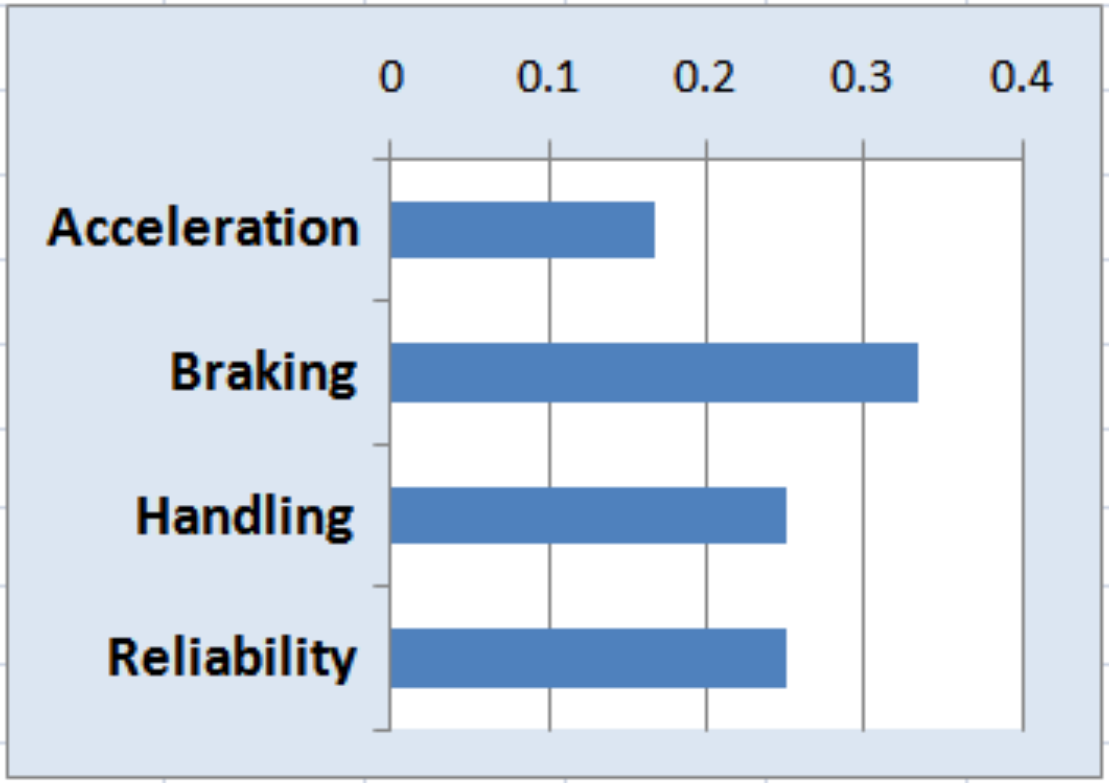
Rating Costs Criteria

Price	500	0.63
Gas	100	0.13
Maintenance	200	0.25
	800	



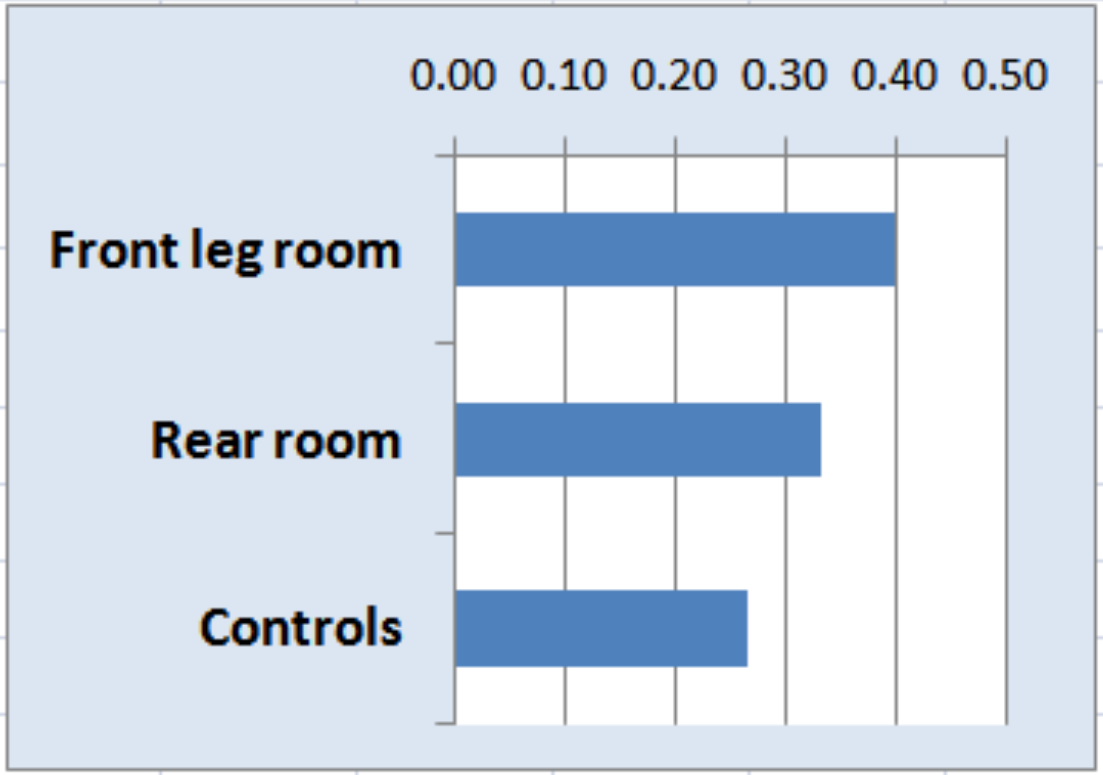
Rating Performance Criteria

Acceleration	100	0.17
Braking	200	0.33
Handling	150	0.25
Reliability	150	0.25
	600	



Rating Comfort Criteria

Front leg room	150	0.40
Rear room	125	0.33
Controls	100	0.27
	375	



Rating the Choices

		Choices			
		Cadillac CTS	Acura TL	BMW 3 Series	
Costs	Price	45	52	45	142
	Gas	19	23	23	65
	Maintenance	50	100	75	225
Performance	Acceleration	100	100	98	298
	Braking	128	131	130	389
	Handling	100	90	100	290
	Reliability	50	100	98	248
Comfort	Front leg room	44	42	40.5	126.5
	Rear room	28.5	28.5	27.5	84.5
	Controls	100	100	85	285
Style		90	80	100	270

The Final Rankings

					Choices		
					Cadillac CTS	Acura TL	BMW 3 Series
	Wt		Wt	Wt	**0.320**	**0.341**	**0.340**
Costs	0.129	Price	0.63	0.081	0.026	0.030	0.026
		Gas	0.13	0.016	0.005	0.006	0.006
		Maintenance	0.25	0.032	0.007	0.014	0.011
Performance	0.387	Acceleration	0.17	0.065	0.022	0.022	0.021
		Braking	0.33	0.129	0.042	0.043	0.043
		Handling	0.25	0.097	0.033	0.030	0.033
		Reliability	0.25	0.097	0.020	0.039	0.038
Comfort	0.323	Front leg room	0.40	0.129	0.045	0.043	0.041
		Rear room	0.33	0.108	0.036	0.036	0.035
		Controls	0.27	0.086	0.030	0.030	0.026
Style	0.161			0.161	0.054	0.048	0.060
	1.00			1.0000			

When Comparisons are Difficult – Full AHP

- Aside from the hierarchy structure, the previous method is really just a 'linear weighting scale' method
- Works fine if it is not hard to do simultaneous pair-wise comparisons
- When it is very difficult to do these, AHP uses comparisons one pair at a time…

A Scale for Comparisons

AHP provides a method to determine the scores of the choices on each criteria, and weighting factors among the criteria, using a scale like the following:

A and B are equally important (or preferred)	1	
A is somewhat more important than B	3	somewhat less (1/3)
A is more important than B	5	less (1/5)
A is much more important than B	7	much less (1/7)
A is very much more important than B	9	very much less (1/9)

Determining Relative Importance

- AHP calls for a series of these pair-wise comparisons to compare each criterion with each other in terms of relative importance
- Example: You might be asked to compare the relative importance of performance versus comfort
  - You might say, "Performance is moderately more important to me than Comfort".
- We could score performance to comfort as a 3 in this case
- This also means that comfort compared to performance would score 1/3

Example: Level One Comparisons

Pairwise Comparisons	Most Important	Assessment	Score
Costs - Performance	Performance	Cost is much less important than performance	1/7
Costs - Comfort	Comfort	Cost is less important than comfort	1/5
Costs - Style	Style	Cost is somewhat less important than style	1/3
Performance - Comfort	Performance	Performance is somewhat more important than comfort	3.2
Performance - Style	Performance	Performance is much more important than style	7
Comfort - Style	Comfort	Comfort is somewhat more important than style	3

fractional scores are allowed

Comparison Matrix

	Costs	Performance	Comfort	Style
Costs	1	1/7	1/5	1/3
Performance	7	1	3.2	7
Comfort	5	1/3.2	1	3
Style	3	1/7	1/3	1

Calculating Weightings

There are a few different ways to convert the comparison matrix to a single weighting scale; for example:

	Costs	Performance	Comfort	Style	Product	4th Root	Scores
Costs	1.000	0.143	0.200	0.333	0.010	0.312	0.05
Performance	7.000	1.000	3.200	7.000	156.8	3.539	0.60
Comfort	5.000	0.313	1.000	3.000	4.688	1.471	0.25
Style	3.000	0.143	0.333	1.000	0.143	0.615	0.10
					Total	5.937	

Weighting Scales from Two Different Methods

Scale from direct simultaneous comparison

Costs	100	0.13
Performance	300	0.39
Comfort	250	0.32
Style	125	0.16
	775	

Scale from pair-wise comparisons

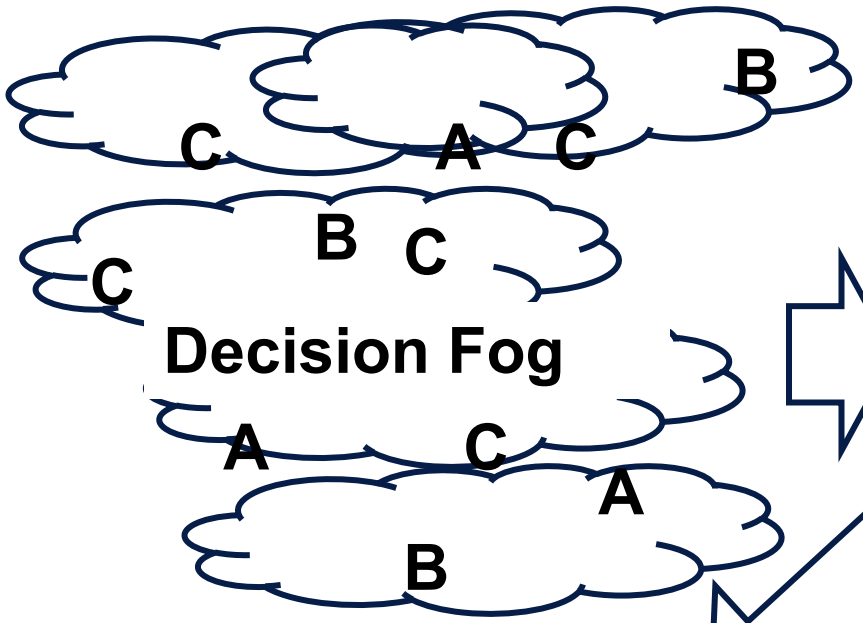
	Scores
Costs	0.05
Performance	0.60
Comfort	0.25
Style	0.10

Which one is better?
Depends on context

A Consistency Problem

- Pair-wise comparisons allow focusing on trade-offs one at a time but… inconsistencies can occur
  - Example:
    A to B has a rating of 3
    B to C has a rating of 2
    then A to C should be around 6 (i.e. 3×2)
  - If A to C were 3, for example, inconsistency would exist
- The AHP has a way to identify how substantial such inconsistencies are

AHP Provides Intermediate Steps to Final Decision



Pairwise Comparisons	Most Important	Assessment	Score
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Performance - Style	Performance	Performance is much more important than style	7
Comfort - Style	Comfort	Comfort is somewhat more important than style	3

	A	B	C			
A	1.00	0.50	1.60	0.8	0.93	0.30
B	2.00	1.00	1.05	2.1	1.28	0.42
C	0.63	0.95	1.00	0.6	0.84	0.28
					3.050	

A	0.30
B	0.42
C	0.28

A	2
B	1
C	3

A	0
B	1
C	0

Completion of the Full AHP

- To complete the AHP analysis requires:
  - Pair-wise comparison matrices for all level two criteria
  - Reduction to weighting scales
  - Checking and adjustment for consistency
  - Final calculation as before
- This can be very tedious for decision-makers
  - Some short cuts have been developed, but still requires a lot of pair-wise comparisons
- Direct comparison is preferable if possible!

Final Rankings (using original weights)

					Choices		
					Cadillac CTS	Acura TL	BMW 3 Series
	Wt		Wt	Wt	**0.320**	**0.341**	**0.340**
Costs	0.129	Price	0.63	0.081	0.026	0.030	0.026
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Car Choice Conclusion

- The process provides a basis for our purchase decision
- It shows the Acura is preferred, given our judgments about criteria, but…
  - Cadillac 3.20 Acura 3.41 BMW 3.40
  - Quite close rankings – may need to introduce additional criteria!
- The process can help decision makers gain insight into the trade-offs made in the decision process

Caveat One: Preference Reversal

- AHP uses a 'linear' weighting scale
- All such scaling methods are subject to 'preference reversal'
- Example:
  - Run AHP on choices A and B and find rank ordering B > A
  - Add in new choice C, re-run AHP, and wind up with A > B > C
  - A and B have reversed ranks!
- Can happen, for example, if new option C is close to B in ratings – similar to political vote-splitting

Fixes for Preference Reversal

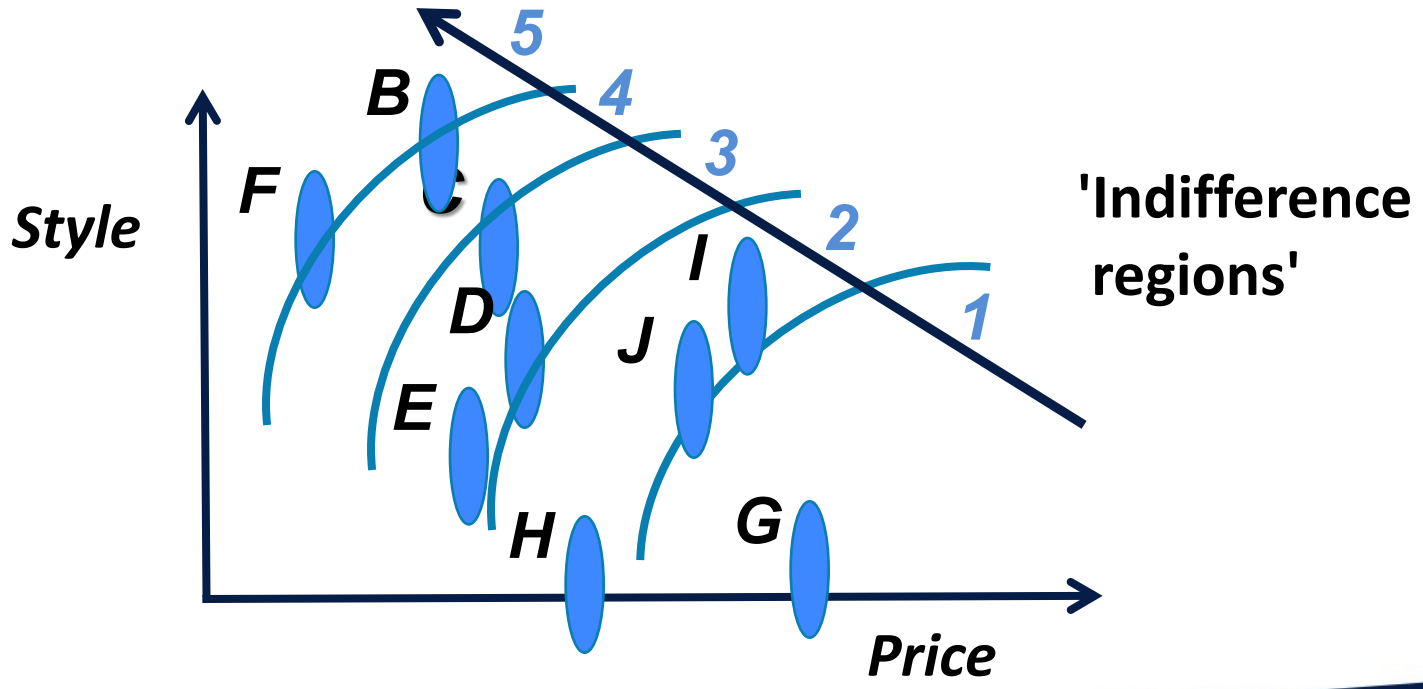
- Fortunately, there are remedial measures if preference reversal occurs, or is a possibility...
- If it is known that two choices are quite close:
  E.g. two different models of Acura
  - Include only one of the similar choices in an initial AHP
  - If that one is ranked first, then do a second AHP to choose between the two similar choices
- If the process is computerized, do all possible AHP's for two choices at a time!
  - Should produce a more genuine rank ordering

Caveat Two: Interactions Among Criteria

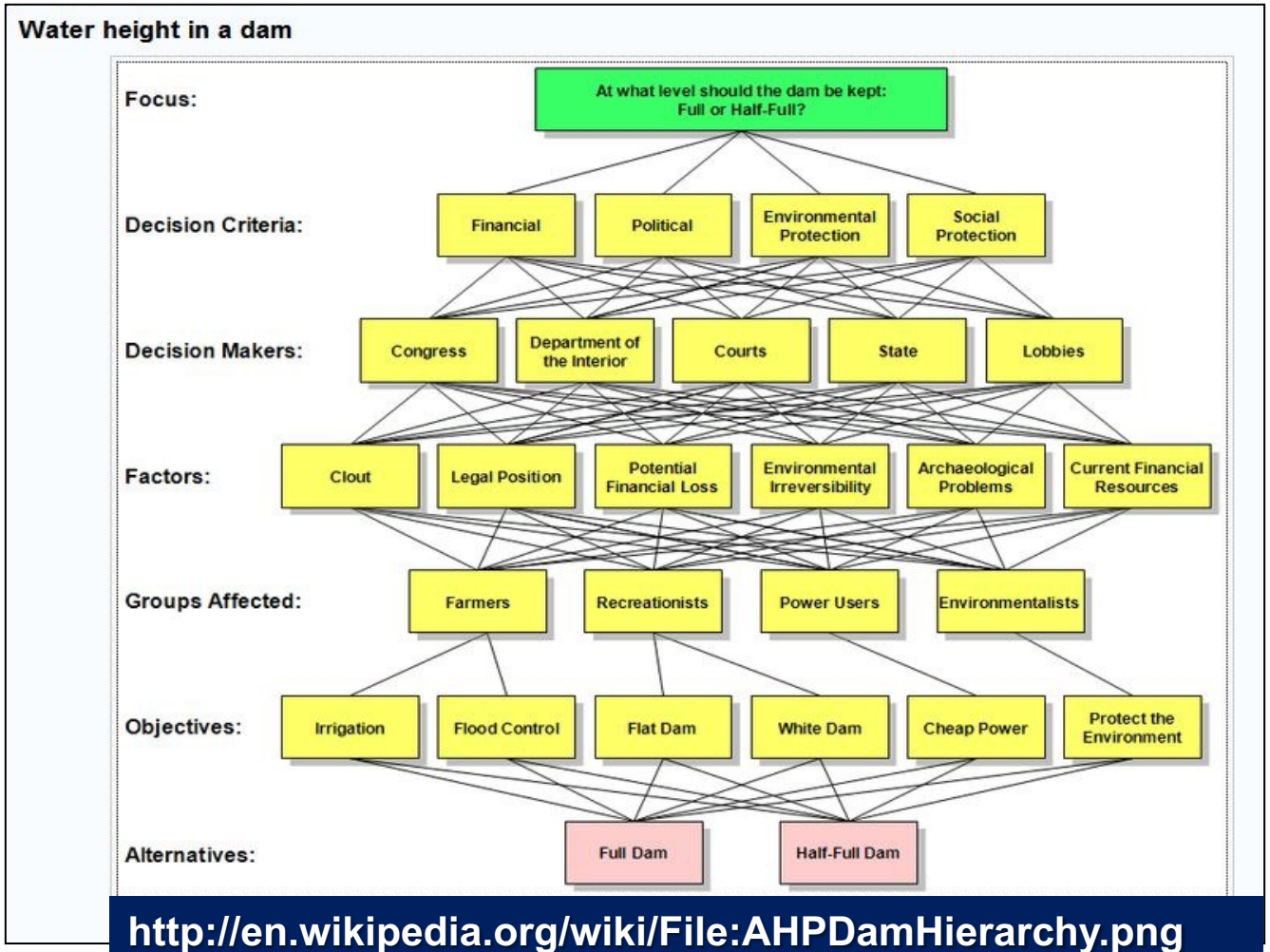
- Criteria do not always naturally fall into a neat hierarchy
- Example: Importance of style (one criterion) may depend on price (a different criterion)
  - Can ignore such fine points, but this weakens the credibility of the entire process
- One possible approach

Fixes for Interactions

- Replace price and style with single price-style criterion
- E.g. low price, good styling would land high on scale



A Real Hierarchy: Water Height in Dam



http://en.wikipedia.org/wiki/File:AHPDamHierarchy.png

AHP Applications

- Job selection (salary, advancement opportunities, learning opportunities, location, firm reputation)
- Employee hiring (education, experience, age, appearance, recommendations, interview performance, interpersonal skills, leadership skills, administrative skills)
- Site selection (labour market, tax environment, regulatory environment, construction costs, distance to market, corporate strategy issues)

AHP Applications (continued...)

- Railroad route selection (environmental issues, distance, topography, costs, nodal issues)
- Automated manufacturing technological choice (purchase cost, cycle times, throughput rates, scrap, product quality, manufacturing flexibility, technological risk)
- Military aircraft selection (cost, range, weapons load, mission profile, radar cross-section, maintenance cost)

Reverse AHP?

- What if your customer or client is the one making the choice?
- Run AHP backwards!
- With or without customers help, develop AHP for them – including your main competitors.
- Then determine what you need to do to place first!

Example

Job applicants (see handout)

Summary

- AHP and other analytical methods can help with organizing thinking about decisions
- Analysis allows people to focus attention on key values or assumptions rather than just 'debating' entire final decision
- There are no fool-proof methods!
- People make decisions, not computers!
  - But analysis can help if you let it