# Quality Management Problems

**Problem 1**

An automatic filling machine is used to fill 2-litre bottles of cola. The machine's output is known to be approximately Normal with a mean of 2.0 litres and a standard deviation of 0.01 litre. Output is monitored using means of samples of five observations.

a) Determine the upper and lower control limits that will include roughly 95.5 percent of the sample means.
b) If the means for the six samples are 2.005, 2.001, 1.998, 2.002, 1.995, and 1.999, is the process in control?

*SOLUTION*

The alpha-risk ($\alpha$) is 0.955 = 0.045 (or 4.5%).

This is evenly divided between each of the distribution's two tails, or 2.25% on each tail. We are therefore seeking the $z$-value corresponding to either 2.25% on the lower tail, or 97.75% on the upper tail.

In Excel, "=norm.s.inv(0.9775)" yields approximately $z = 2$.

The control limits are:

$$UCL = \bar{\bar{x}} + z\sigma_{\bar{x}}$$

$$= \bar{\bar{x}} + z\frac{\sigma}{\sqrt{n}}$$

Recall also that we expect $\mu = \bar{\bar{x}}$, and therefore, the UCL is…

$$UCL = 2.0 + (2)\frac{0.01}{\sqrt{5}} \cong 2.009$$

$$LCL = 2.0 - (2)\frac{0.01}{\sqrt{5}} \cong 1.991$$

Since none of the six sample means are above the UCL, nor below the LCL, the process is "in-control".

**Problem 2**

Specification for the diameter of a metal shaft is much wider than the machine used to make the shafts is capable of. Consequently, the decision has been made to allow the cutting tool to wear a certain amount before replacement. The tool wears at the rate of 0.002 centimetre per metal shaft produced. The process has a natural variation, $\sigma$, of 0.01 centimetre and is Normally distributed. Specification for the diameter of the metal shafts is 15.0 to 15.2 centimetres. How many shafts can the process turn out before tool replacement becomes necessary (i.e., before the process makes an out-of-spec shaft)?

*SOLUTION*

Let:

$$LCL = LSL = 15.0 \text{ cm}$$

Furthermore, if we assume the LCL is 3 standard deviations below the mean and the UCL is 3 standard deviations above the mean, then the difference between LCL and UCL is 6 standard deviations:

$$\begin{aligned} UCL &= LCL + 6\sigma \\ &= 15.0 + 6(0.01) \\ &= 15.06 \end{aligned}$$

The USL is 15.2 cm.

The difference between the USL and UCL is 15.2 – 15.06 = 0.14 cm.

$$\frac{0.14 \text{ cm}}{0.004 \text{ cm/shaft}} = 35 \text{ shafts}$$

**Problem 3**

Six samples of five observations each have been taken of 80 kg concrete slabs produced by a machine, and the results are displayed below. Determine the UCL and LCL for sample mean and range and decide if the process is in control. Then suppose a new sample results in slab weights of 81.0, 81.0, 80.8, 80.6, and 80.5. Using the control limits identified earlier, does this new sample suggest the process is in control?

| Sample | | | | | |
|---|---|---|---|---|---|
| **1** | **2** | **3** | **4** | **5** | **6** |
| 79.2 | 80.5 | 79.6 | 78.9 | 80.5 | 79.7 |
| 78.8 | 78.7 | 79.4 | 79.4 | 79.6 | 80.6 |
| 80.0 | 81.0 | 80.4 | 79.7 | 80.4 | 80.5 |
| 78.4 | 80.4 | 80.3 | 79.4 | 80.8 | 80.0 |
| 81.0 | 80.1 | 80.8 | 80.6 | 78.8 | 81.1 |

*SOLUTION*

| | **Sample** | | | | | |
|---|---|---|---|---|---|---|
| | **1** | **2** | **3** | **4** | **5** | **6** |
| | 79.2 | 80.5 | 79.6 | 78.9 | 80.5 | 79.7 |
| | 78.8 | 78.7 | 79.4 | 79.4 | 79.6 | 80.6 |
| | 80 | 81 | 80.4 | 79.7 | 80.4 | 80.5 |
| | 78.4 | 80.4 | 80.3 | 79.4 | 80.8 | 80 |
| | 81 | 80.1 | 80.8 | 80.6 | 78.8 | 81.1 |
| $\bar{x}$ | 79.48 | 80.14 | 80.1 | 79.6 | 80.02 | 80.38 |
| $R$ | 2.6 | 2.3 | 1.4 | 1.7 | 2 | 1.4 |
| | | | | | | |
| $\bar{\bar{x}}$ | 79.95333 | | | | | |
| $\bar{R}$ | 1.9 | | | | | |

Given that $n = 5$, we have $d_2 = 2.326$; $D_3 = 0$; and $D_4 = 2.11$

For the $\bar{x}$-chart, we have:

$$\begin{aligned} UCL &= \bar{\bar{x}} + z\frac{\bar{R}}{d_2\sqrt{n}} \\ &= 79.95 + 3\left(\frac{1.9}{2.326\sqrt{5}}\right) \\ &= 81.046 \end{aligned}$$

and

$$\begin{aligned} LCL &= \bar{\bar{x}} - z\frac{\bar{R}}{d_2\sqrt{n}} \\ &= 79.95 - 3(s) \\ &= 78.85 \end{aligned}$$

For the $R$ chart, we have:

$$\begin{aligned} UCL &= D_4\bar{R} \\ &= 2.11(1.9) \\ &= 4.009 \end{aligned}$$

and

$$\begin{aligned} LCL &= D_3\bar{R} \\ &= 0(1.9) \\ &= 0 \end{aligned}$$

Among the six initial samples, the range never exceeds the UCL, nor does the sample mean exceed either the UCL or LCL and so the process is in control.

For the seventh sample, we have $\bar{x} = 80.78$ and $R = 0.5$. The process remains in control.

**Problem 4**

The design specifications for the concrete slab in the previous problem is between 78 and 81 kg. Is the specification being met? Is the process capable?

*SOLUTION*

There is an observed value of 81.1 so it would seem that the specifications are not *always* being met but this may be an outlier or it may be that the process isn't capable. One observation is not conclusive.

Recall that we've used an approximation for $\sigma_{\bar{x}}$ in the previous problem…

$$\frac{\bar{R}}{d_2\sqrt{n}} \cong \sigma_{\bar{x}}$$

or that,

$$\sigma_{\bar{x}} = \frac{1.9}{2.326\sqrt{5}}$$

$$= 0.365$$

We can then rearrange the following formula and solve for

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \;\rightarrow\; \sigma = \sigma_{\bar{x}}\sqrt{n} \;=\; (0.365)\left(\sqrt{5}\right) \;=\; 0.816$$

Recall that the specification for the weight of the slab is "80 kg" and must be between 78 and 81 kg. It does appear as though the process mean (as approximated by the mean of sample means) is close to 80 kg. Since the specification is asymmetric, we use $C_{pk}$

$$\frac{USL - \text{process mean}}{3\sigma} = \frac{81 - 80}{3(0.816)} = 0.41$$

$$\frac{\text{process mean} - LSL}{3\sigma} = \frac{80 - 78}{3(0.816)} = 0.817$$

$$\begin{aligned} C_{pk} &= \min\{0.41, 0.817\} \\ &= 0.41 \end{aligned}$$

Since this value is less than 1, the process is NOT capable.

In cases of symmetric process deviation (as in this case where the process deviation follows a normal distribution), and an asymmetric specification range, some organizations may choose to centre the process on the centre of the specification range rather than the nominal specification itself. In this case, we might centre the process on an expected value of 79.5 kg for the weight of the slab in order to squeeze more of our product inside the specification range. Although this is possible, the practice is often discouraged in industry (in this case we would be advertising the slab as 80 kg when we expect it to be 79.5 kg).

**Problem 4**

A company has just negotiated a contract to produce a part for another company. In the process of manufacturing the part, the inside diameter of successive parts becomes smaller and smaller as the cutting tool wears. However, the specification is so wide relative to machine capability that it is possible to set the diameter initially at a large value and let the process run for a while before replacing the cutting tool.

The inside diameter decreases at an average rate of 0.001 cm per part, and the process has a standard deviation of 0.01 cm and the variability is approximately Normally distributed. After how many parts must the tool be replaced if the design specification is between 3 cm and 3.5 cm, and the initial setting is three standard deviations below the upper design specification?

*SOLUTION*

$$LCL = LSL = 3.0 \text{ cm}$$

$$\sigma_{\bar{x}} = {}^{\sigma}/_{\sqrt{n}} = {}^{0.01}/_{\sqrt{1}} = 0.01 \text{ cm}$$

$$\begin{aligned} UCL &= LCL + 6\sigma_{\bar{x}} \\ &= 3 + 6(0.01) \\ &= 3.06 \text{ cm} \end{aligned}$$

$$USL - UCL = 3.5 - 3.06 = 0.44 \text{ cm}$$

$$\frac{0.44 \text{ cm}}{0.001 \text{ cm/part}} = 440 \text{ parts}$$

**Problem 5**

When Polaroid reduced the number of quality inspectors and increased operator responsibility for statistical process control in its R2 plant (which made instant film cartridges with 10 films in each) in Waltham, Massachusetts, in 1985, it encountered an unexpected result. Instead of quality improving, it actually got worse. To identify the problem, Bud Rolfs, quality control manager, asked an operator from each shift to sample six observations of the critical characteristics of the product and report these to him. One important characteristic of instant films is the pod weight. A pod is a small capsule at the end of each film that contains chemicals. When the film is pulled out, the pod bursts and released the chemicals that will develop the film. Too much chemical overdevelops the film and too little underdevelops it.

a) Using the following three samples of six observations each (in grams) from the first day of the data collection period to develop a sample mean and sample range control chart for the pod weight. Is the process in control?
b) The first sample from the second day is 2.841, 2.802, 2.802, 2.806, 2.807, and 2.807. Is the process still in control?

| Sample | Shift | 1 | 2 | 3 | 4 | 5 | 6 | Average |
|---|---|---|---|---|---|---|---|---|
| 1 | A | 2.800 | 2.799 | 2.760 | 2.802 | 2.805 | 2.803 | 2.795 |
| 2 | B | 2.750 | 2.820 | 2.850 | 2.740 | 2.850 | 2.790 | 2.800 |
| 3 | C | 2.768 | 2.807 | 2.807 | 2.804 | 2.804 | 2.803 | 2.799 |

*SOLUTION*

| Sample | Shift | 1 | 2 | 3 | 4 | 5 | 6 | Average | Range |
|---|---|---|---|---|---|---|---|---|---|
| 1 | A | 2.8 | 2.799 | 2.76 | 2.802 | 2.805 | 2.803 | 2.795 | 0.045 |
| 2 | B | 2.75 | 2.82 | 2.85 | 2.74 | 2.85 | 2.79 | 2.8 | 0.11 |
| 3 | C | 2.768 | 2.807 | 2.807 | 2.804 | 2.804 | 2.803 | 2.799 | 0.039 |
| | | | | | | | **Average:** | 2.798 | 0.064667 |

$n = 6$
$d_2 = 2.534$
$D_3 = 0$
$D_4 = 2.000$

<u>Sample Mean Control Limits:</u>

$$\text{UCL} = \bar{\bar{x}} + z\frac{\bar{R}}{d_2\sqrt{n}}$$

$$= 2.798 + 3\left(\frac{0.064667}{2.534\sqrt{6}}\right)$$
$$= 2.829$$

$$\text{LCL} = \bar{\bar{x}} - z\frac{\bar{R}}{d_2\sqrt{n}}$$

$$= 2.798 - 3\left(\frac{0.064667}{2.534\sqrt{6}}\right)$$
$$= 2.7667$$

Range Control Limits:

$$\begin{aligned} \text{LCL} &= D_3\bar{R} \\ &= (0)(0.064667) \\ &= 0 \end{aligned}$$

$$\begin{aligned} \text{UCL} &= D_4\bar{R} \\ &= (2.000)(0.064667) \\ &= 0.129334 \end{aligned}$$

For the new sample, average is 2.8 (in-control) and range is 0.039 (also still in-control).

**Problem 6**

An appliance manufacturer wants to contract with a repair shop to handle authorized repairs. The company has set an acceptable range of repair time of 50 minutes to 90 minutes. Two firms have submitted bids for the work. In test trials, one firm had a mean repair time of 74 minutes with a standard deviation of 4.0 minutes, and the other firm had a mean repair time of 72 minutes with a standard deviation of 5.1 minutes. Which firm would you choose and why?

*SOLUTION*

We can answer this by expecting a $6\sigma$ process and a sample size of 1 then establish the UCL and LCL to determine if the control limit falls outside the specification range.

Firm 1:

$$\begin{aligned} UCL &= \bar{\bar{x}} + 3\sigma_{\bar{x}} \\ &= 74 + 3(4) \\ &= 86 \text{ minutes} \end{aligned}$$

$$\begin{aligned} LCL &= \bar{\bar{x}} - 3\sigma_{\bar{x}} \\ &= 74 - 3(4) \\ &= 62 \text{ minutes} \end{aligned}$$

The first firm can be expected to complete the process within specifications more than 99% of the time.

Firm 2:

$$\begin{aligned} UCL &= \bar{\bar{x}} + 3\sigma_{\bar{x}} \\ &= 72 + 3(5.1) \\ &= 87.3 \text{ minutes} \end{aligned}$$

$$\begin{aligned} LCL &= \bar{\bar{x}} - 3\sigma_{\bar{x}} \\ &= 72 - 3(5.1) \\ &= 56.7 \text{ minutes} \end{aligned}$$

Although both firms have control limits within the specification range, I would be inclined to hire the first firm (assuming only a single firm is required). Recall that the USL is 90 minutes. Of the two firms, the second encroaches closer on this limit. The LSL is 50 minutes. Again, of the two firms, the second encroaches closest. Therefore, more of the process variability from the first firm will fall within specifications. This question could also be answered using $C_{pk}$ to identify the most "capable" process.

**Problem 7**

As part of an insurance company's training program, participants learn how to conduct a fast but effective analysis of clients' insurability. The goal is to have participants achieve a time less than 45 minutes. There is no minimum time but the quality of assessment should be acceptable. Test results for three participants were Armand, a mean of 38 minutes and a standard deviation of 3.0 minutes; Jerry, a mean of 37 minutes and a standard deviation of 2.5 minutes; and Melissa, a mean of 37.5 minutes and a standard deviation of 2.5 minutes. Which of the participants would you hire?

*SOLUTION*

Armand:

$$\begin{aligned} UCL &= \bar{\bar{x}} + 3\sigma_{\bar{x}} \\ &= 38 + 3(3) \\ &= 47 \text{ minutes} \end{aligned}$$

Armand's UCL exceeds the USL and he is therefore not a suitable hire.

Jerry:

$$\begin{aligned} UCL &= \bar{\bar{x}} + 3\sigma_{\bar{x}} \\ &= 37 + 3(2.5) \\ &= 44.5 \text{ minutes} \end{aligned}$$

Jerry's UCL is below the USL and he is therefore "capable"

Melissa:

$$\begin{aligned} UCL &= \bar{\bar{x}} + 3\sigma_{\bar{x}} \\ &= 37.5 + 3(2.5) \\ &= 45 \text{ minutes} \end{aligned}$$

Although Melissa's UCL is equal to the USL, we should be very careful on the wording of the problem, the goal is to achieve a time of "less than" 45 minutes.

**Problem 8**

Output from a process contains 2% defective units. Defective units that go undetected into final assemblies cost $25 to replace. An inspection process, which would detect and remove all defectives,

can be established to test these units. However, the inspector, who can test 20 units per hours, is paid \$8 per hour, including benefits. Should an inspection station be established to test all units?
a) What is the cost to inspect each unit?
b) What is the benefit (or loss) from the inspection process?

*SOLUTION*

Inspection Costs: \$8/hr

Failure Costs: (.02)(20 units/hr)(\$25/defect) = .4 units/hr × \$25/unit = \$10/hr

Since the inspection costs are cheaper than the failure costs, the inspector should be hired.

**Problem 9**

There is a 3 percent error rate at a specific point in a production process. If an inspector is placed at this point, all the errors can be detected and eliminated. However, the inspector is paid \$8 per hour and can inspect units in the process at the rate of 30 per hour. If no inspector is used and defects are allowed to pass this point, there is a cost of \$10 per unit to correct the defect later on. Should an inspector be hired?

*SOLUTION*

Inspection Costs: \$8/hr

Failure Costs: (0.03)(30/hr)(\$10)= \$9/hr

Inspection Costs are cheaper than Failure Costs. Therefore, hire the inspector.