Topic 2: Return measures

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Holding Period Return

- The holding period return (HPR) is the most basic measure of performance
  - The “period” can be as short as a few minutes or as long as 10 years.
- At time 0
  - Pay $\$P_0$ to buy the asset at time 0. Assume zero bid/ask spread for simplicity.
- At time 1
  - Receive a cash payment (coupon or dividend) $\$C_1$
  - Resell the asset for $\$P_1$

Holding Period Return

- Your holding period return (return for short) is

$$R_1 = \frac{C_1 + P_1}{P_0} - 1$$

- Compute the HPR
  - You buy a stock at $35 and you resell it a week later for $36
  - You buy a bond with a $5 semi-annual coupon for $101. You sell it 6 months later for $99 after receiving one coupon.

Examples

- Stock over one week
  - P_0= $ 35, P_1= $ 36, C_1=0
  - HPR = 36/35 – 1 = 0.0286 = 2.86%
- Bond over 6 months
  - P_0= $ 101, P_1= $ 99, C_1= $5
  - HPR = (5+99)/101 – 1 = 0.0297 = 2.97 %

Capital Gains and Dividend Yields

- HPR is made of two components

$$HPR = Capital\ Gain + Dividend\ Yield$$

- $Capital\ Gain = \frac{P_1}{P_0} - 1$
  - Positive if the price goes up; negative if the price goes down
- $Dividend\ Yield = \frac{C_1}{P_0}$
  - Usually positive, unless you are paying to keep your position open

Comparing Returns over Different Periods

- Stock 2.857% in a week. Bond 2.970% in 6 months. How can you compare them?
- Suppose that
  - at time 0, you buy an investment for V*(0)*
  - you re-invest all intermediate cash-flows until date T
  - at time T, you sell the investment ***and the proceeds from the re-invested cash-flows*** for a total price of *V(T)*
- The **annualized/annual holding period return** (ann. *HPR*) is

$$ann.HPR = \left(\frac{V(T)}{V(0)} \right)^{1/T} - 1$$

HPR: Stock example

- Suppose dividend paid at the end. Holding period return:

$$HPR = \frac{\text{ending price} + \text{cash dividend}}{\text{beginning price}} - 1$$

- Annualized holding period return for a holding period of *T* years:

$$\text{ann. } HPR = (1 + HPR)^{1/T} - 1$$

$$= \left(\frac{\text{ending price} + \text{cash dividend}}{\text{beginning price}} \right)^{1/T} - 1$$

HPR and Annualized HPR

You bought shares for $42.39 **six months ago** and sell them now for $44.30. Suppose there was no dividend payment in these six months.

What is the HPR?
What is the ann. HPR?

HPR and Annualized HPR

You bought shares for $39.63 **two years ago** and sell them now for $42.37. Assume that the only dividend of $1.12 was paid at the end of **year 2.**

What is the HPR?
What is the ann. HPR?

HPR and Annualized HPR

You bought shares for $39.63 **two years ago** and sell them now for $42.37. Assume that the only dividend of $1.12 was paid at the end of **year 1** and reinvested at R=10%.

What is the HPR?
What is the ann. HPR?

Multi-Period Returns

- Suppose you invest $100 in an emerging markets fund. Your return is +100% in the first year and -50% in the second year.
- What is your annual HPR if you withdraw your first year gains (and invest them at zero interest rate)?
- What is your annual HPR if you reinvest your first year gains in the fund?

Multi-period returns: Simple Arithmetic Average

If you make a sequence of returns and **you do not reinvest your returns**, then your total return is approx. the simple (= arithmetic) average return:

- Start with $1 invested over several periods *t=1..T*. You make R_1 in the first period and save it in your zero-interest checking account.
- At time t, you make R_t and save it in your checking account. And so on. At the end you have $1 + R_1 + R_2 .. + R_T$.
- Your annual HPR is approximately the arithmetic average $\frac{R_1 + R_2 .. + R_T}{T}$

Arithmetic vs Geometric Average

- But what if **<u>you reinvest the proceeds</u>** at the same rate as that of your initial investment?
- At the end you have $(1+R_1)(1+R_2)..(1+R_T)$ and your ann. HPR is the geometric average

$$\left((1+R_1)(1+R_2)..(1+R_T)\right)^{\frac{1}{T}}-1$$

- Geometric average allows you to see how much your wealth grows, **including re-investments**.

APR vs EAR

- Borrowing rates are often quoted as **Annual Percentage Rate (APR)**
  - If you pay N times per year, your per period rate is APR/N.
  - Ex: 12% compounded semi-annually = 6% every 6 months, 12% compounded monthly = 1% every month
- APR may not represent the true cost of borrowing. The **Effective Annual Rate (EAR)** is

$$1 + EAR = \left(1 + \frac{APR}{N}\right)^N$$

- Which loan is cheapest?
  1. 10%, compounded semi-annually
  2. 10%, compounded quarterly
  3. 10%, compounded daily

Real Returns & Inflation

- Time t=0
  - An apple costs $1. You invest in a financial asset
- Time t=1
  - Your financial return is 10%. The price of an apple is $1.05
  - What is your return measured in apples?
  - Suppose you have $1. At time 0 you can buy 1 apple.
  - If you save it, it grows to $1.1. Then you can buy 1.1/1.05 = 1.0476 Apples
  - You real – apple based – return is 4.76%.

Real Returns & Inflation

- Inflation
  - Instead of considering a particular good (apples) we consider a broad basket of goods, including durable goods, energy prices, and so on.
  - The inflation rate π is the percentage change in the price index
- The real return is

$$1 + r^{real} = \frac{1 + r^{\$}}{1 + \pi}$$

- Treasury issues bonds indexed to inflation, guarantee real return…
  - Can use to infer market participants' inflation expectations!

Treasury Inflation Protected Securities (TIPS)