1 Financial Markets

The starting bid price $ = 102.25 with ask of $102.50 and zero inventory

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1: Day One

On day 1, there we recieve orders of buy - 10,000 shares sell - 4,000 shares

On the 4000 shares bought and sold,

$$4000 * 102.25 = 409000 \text{ and } 4000 * 102.50 = 410000$$

Thus, $\$410000 - \$409000 = \$1000$ with the remaining being -6000 shares since we have more buy orders.
We can use the value in between ask/bid prices, being 102.375. Thus, inventory at day-end is

$$I = -6000 * \frac{102.5 - 102.25}{2} = -6000 * 102.375 = -\$614250$$

You can also use other ways to value inventory such as only using either the bid or ask price. With the bid price, inventory is $-6000 * 102.25 = -\$613500$ and in contrast inventory with the ask price $-6000 * 102.5 = -\$615000$.

1: Day Two

Prices change jump to 110.25 =bid price and 110.5 =ask price. And we have -6000 shares of inventory. On the 2nd day we recieve, market buy order of 2,000 shares such that $2000 * 110.50 = \$221000$ and market sell order of 8,000 shares such that $8000 * 110.25 = \$ - 882000$
Using the ask price (since they were sold from us) and shares from day one, $-6000 * 102.5 = -\$615000$.

$$\text{total-profit} = \text{day2}(221000 - 882000) + \text{day1}(615000) = -46000$$

Since, the inventory of day one is -6000 and day two is $8000 - 2000 = 6000$, thus invenotry is 0.

1: Market Makers/Dealers

The price of the market maker's (dealers) services is the bid-ask spread. Thus, they take risk in inventory instead of predicting prices/speculating. Yes we could have stopped taking (or not filled) buy orders from day one, such that we are not short -6000 in inventory. This action would leave us with $1000 profit for the day and improve overall performance for day one and two. This action is consistent with market makers.

2 Return Measures

$return/day = 1\% = r(t)$ with 250 trading days per year

2: Geometric Average

EAR (effective annual rate) - percentage increase in funds invested over T horizon (1 year - 250 days)

$$(1 + r(t))^t = [1 + EAR] \rightarrow (1 + .01)^{250} - 1 = 11.0321557 = EAR$$

$$(1 + r(t))^t = [1 + EAR] \rightarrow (1 + .01)^{250} - 1 = 11.0321557 = EAR = 1103.216\%$$

And $(EAR * 100) + 100$ is the final return + initial investment after a year.
The EAR is the geometric average return $1.01^{250} - 1 = 11.032$

2: Simple/Arithmetic Average

With a 1% return rate and 0% interest rate from the checking account (= no reinvesting).
$R_n = R_1 + R_2 + R_3 + \cdots + R_T$ with R_i (the daily return) being \$1 and $1 * 250$ earning after the end of a year with 250 trading days.

2: Arithmetic vs. Geometric

arithmetic - It is proper to use simple when there is no-reinvesting or interest is not applied, such as 2B

geometric - It is proper to use geometric average when we can rinvest the earnings at a periodic rate (e.g. 1 percents per trading day) since geometric allows for compounding

3: Alternatives

i. 8% compounded annually

$$((1 + .08))^1 - 1) = EAR = .08$$

ii. 8% compounded daily

$$((1 + .08))^{250} - 1) = EAR_{daily} = 226954537.4\%$$

4: *Excel Question*

For my data, the start date is 1/1/1988 with price \$120.43 and the final date is 02/18/22 with $P_1 = 2208.78$

$$HPR = \frac{P_1 + C}{P_0} - 1 = \frac{2208.78 + 0}{120.43} = 18.34077$$

Thus the we can compute the annualized HPR as

$$HPR_{ann} = (1 + HPR)^{1/T} - 1 = 19.34077^{1/34} - 1 = 1.091 = 0.1091\%$$

3 Time Value of Money

5: Present-Value of Bonds

Determing the present-value price of a 5yr-bond with \$1000 face value and \$5% yield

$$present - value = \frac{future - value}{(1 + yield)^T} = \frac{1000}{(1 + 0.05)^5} = \frac{1000}{1.27628} = 783.5262$$

5: Maturity

Now, price = \$325.57 thus the maturity of the bond is

$$future - value = price(1 + yield)^T \rightarrow 1000 = 325.57(1.05)^T \rightarrow 3.071536 = 1.05^T$$

Solving for T

$$Log(3.07154) = Log(1.05) * T \rightarrow 0.487355 = 0.0211892991 * T$$

Thus,

$$T = \frac{0.487355}{0.0211892991} = 23.00008217$$

Maturity for the 5-year bond at price \$325.57 is 23 years.

6: Investment Choice

If you were not to reinvesting your annual returns (or removing money from your account), option A is better. Otherwise the we can use HPR

$$(1 + r(t))^t = [1 + EAR] \rightarrow (1.55)^{10} - 1 = EAR = 79.0418249$$

Then,

$$79.0418249 * \$550 = 43473.0037$$

Since, the returns from the investment into Chase is greater than that of a 10-yr bond and the interest is guaranteed, I would prefer option A.

7: Annuities and Perpetuities

For the present-value of annuity, with periodic-payment = $10,000$,rate = 0.05, and period = 6

$$P * (\frac{1 - \frac{1}{(1+r)^n}}{r}) = present - value$$

$$10000 * (\frac{1 - \frac{1}{(1.05)^6}}{.05}) = 50,756.92068$$

Next the present-value of the 6-year annuity at rate=0.10.

$$10000 * (\frac{1 - \frac{1}{(1.1)^6}}{.1}) = 43,552.61$$

For the present-value of perpetuity, with periodic-payment $= 10,000$ and rate $= 0.05$,

$$PV = C/R = 10000/0.05 = 200000$$

Since this is the value of the perpetuity after 10 years, we work backwards, to find today's present value,

$$FV/(1+rate)^{periods} = present - value = 200,000/(1.05)^10 = 200,000/1.62889 = 122782.6507$$

Finally, the present-value of the perpetuity at rate $= .1$.

$$PV = C/R = 10,000/0.1 = 100,000$$

and the present-value of the perpetuity at rate $= .1$ today, (instead of 10 years from now) is

$$PV = \frac{FV}{(1+r)^n} = 100,000/1.1^{10} = 100,000/2.593742 = 38554.33$$

Thus, after all the computation, it seems that the perpetuity of 5% should be chosen as it has the higher present-value. However, the annuity should be chosen at the rate, 10% for the same reason.