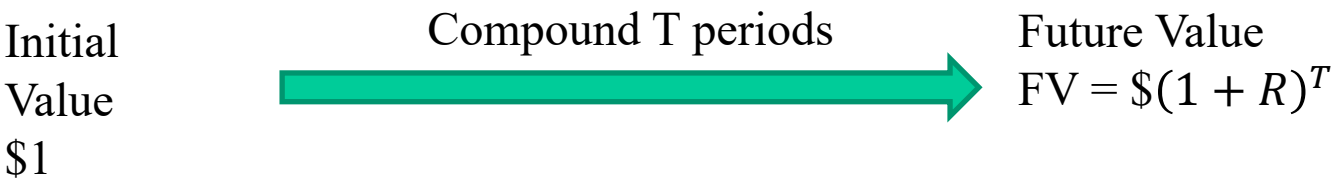
Topic 3: Time Value of Money

Prof. Olivier Wang

Compounding and Future Value

- You invest one dollar at rate ***R***
  - $1 is your initial investment, or initial value
  - You keep reinvesting principal and interest for T periods
- After T periods, you have the **Future Value :**

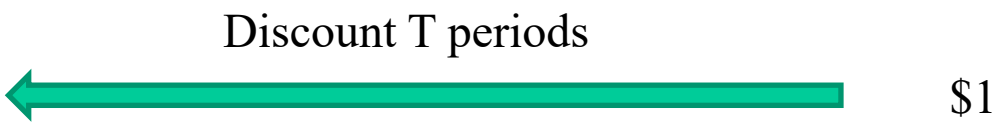


Discounting and Present Value

- $1 today gives you a future value of $(1+R)^T$ in T periods
- Then how much is $1 in T periods worth today?
- The **Present Value** is:

Present Value

$$PV = \frac{\$1}{(1+R)^T}$$



Present Value examples

- To receive $1000 in one year, how much should I invest today at a rate of 5%?
  - $PV = \frac{1000}{(1+.05)} = \$952.38$
- To receive $1000 in two years, how much should I invest today at a rate of 5%?
  - $PV = \frac{1000}{(1+.05)^2} = \$907.03$

Going back and forth

$$FV = PV \times (1 + R)^T$$

- If you invested $1,000 and received $1,109 after 3 years, what was the interest rate?

- If you invested $1,000 at 6% and received $2,133, for how long did you invest?

PV, FV, R, T tied together

- $FV = PV \times (1 + R)^T$
- $PV = \frac{FV}{(1+R)^T}$
- $R = \left(\frac{FV}{PV}\right)^{1/T} - 1$
- $T = \frac{\log\left(\frac{FV}{PV}\right)}{\log(1+R)}$

No arbitrage pricing principle

Price = PV

- Consider a bond paying \$1000 in a year. The interest rate is 5% so $PV = \frac{1000}{(1+.05)} = \$952.38$
- What do you do if the bond is priced at \$950?
- What do you do if the bond is priced at \$960?

Pricing securities: Zero Coupon Bonds

Simplest security: single payment *F* (face value) at maturity *T*

- Pricing: $\text{Price} = PV = F/(1+R)^T$
  - *R* is the **yield (aka yield to maturity/YTM)**
  - Ex: 2-year ZCB with F=$1000 and yield 4% has price P=1000/1.04^2 = $924.6

- Lower *R* <-> Higher price
- Higher *T* → Higher price (*pull to par*)
  - Handout H1

Pricing securities with multiple payments

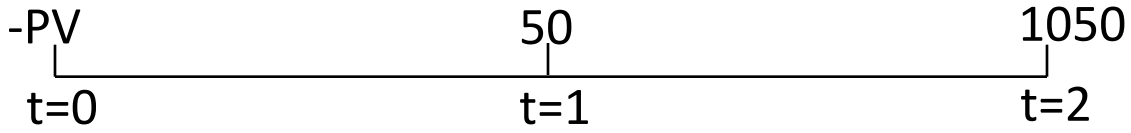
- Timeline
- Present value of cash flows C*(0), C(1), …, C(T)*

$$PV = \mathrm{C}(0) + \mathrm{C}(1)\frac{1}{(1+R)} + \ldots + C(T)\frac{1}{(1+R)^T}$$

- Superposition of zero coupon bonds!

Example: Pricing a coupon bond

- What is the PV of the following 5% coupon-bearing bond at 6% interest rate?



- What is the present value at 5%?
  - Could you have guessed this outcome?

Example: PV of New Machine

- A machine produces goods worth $10 in one year, $8 in two years, $6 in three years and then it breaks down. The rate is 5%.
- Should you pay $21 for it?

Net Present Value (NPV)

- The net present value of a project is the present value of cash flows minus the present value of costs, including the initial (setup) cost

$$NPV = PV(revenues) - PV(costs)$$

- It is efficient to invest in a project when the NPV is positive.
  - Consider the earlier machine example. The PV of future profits is $21.96. If the initial cost if $21, the NPV is $0.96.

Annuities (Handout H2)

- Definition: Pays a **fixed cash flow *C*** for *T* periods

$$PV = \frac{C}{1+R} + \frac{C}{(1+R)^2} + \cdots + \frac{C}{(1+R)^T} = C \underbrace{\left[\frac{1 - \frac{1}{(1+R)^T}}{R} \right]}_{PV\ Factor}$$

- Example: Which car can you afford?
  - You have no cash on hand.
  - You can afford $800 per month.
  - You can borrow at 2% per month.
  - You want to have paid the loan in full in 5 years.

Perpetuities

- Perpetuities
  - Definition: Annuity with infinite *T*

    Pays a fixed cash flow, *C*, every period forever
  - Example: consol bond in UK, or many 100-year bonds "almost perpetuities"
  - Pricing: $PV = C / R$
- For annuities/perpetuities: Careful about the timing!
  - First cash flow paid one period from now

U.K. to Repay First World War Bonds

Consol Bonds Worth Nearly $350 Million to be Repaid in February

Crowds in Trafalgar Square, London, in December 1917 during a rally to promote war bonds.
GETTY IMAGES

By Tommy Stubbington and Ben Edwards
Updated Oct. 31, 2014 10:45 am ET

SAVE PRINT TEXT

LONDON—Britain will repay a chunk of old debt. Really old debt, some of which was used to finance World War I.

That bit, along with other scraps incurred as far back as the Crimean War and the collapse of the South Sea Company were bundled together in 1927 into a 4% callable, perpetual note. For nearly a century, it was a good deal for the government. But after rates tumbled this year, that is no longer the case.

The U.K. Treasury said Friday it plans to repay the £218 million note ($348.8 million) which was issued by the then Chancellor of the Exchequer Winston Churchill in 1927. But it has also revived talk that the government could repay a further £2 billion of outstanding war debt, mostly in the form of the £1.9 billion "War Loan" dating from 1932.

Perpetuity example: Suppose that you are a donor and want to endow a student fellowship that pays $10,000 every year, forever. The interest rate is 5% per year. How much money should you give the university?

Types of bonds (cash flow and price)

Name of Bond	1	.. t ..	T	…	Price at time 0
Discount, Zero-Coupon	0	0	F		$P = \frac{F}{(1+r)^T}$
Perpetuity/consol	C	C	C	C..	$P = \frac{C}{r}$
Annuity, mortgage	C	C	C		$P = \frac{C}{r}\left(1 - \frac{1}{(1+r)^T}\right)$
Regular Coupon-Paying	C	C	C+F		$P = \frac{C}{r}\left(1 - \frac{1}{(1+r)^T}\right) + \frac{F}{(1+r)^T}$

Relationship between interest rates and asset prices

- For perpetuities?
- For other bonds?
  - Later in class: duration.
- For stocks?

Markets: The FED

- Jerome “Jay” Powell
  - Chair of the Board of Governors of the United States Federal Reserve

- Markets are constantly trying to forecast future Fed decisions

  https://www.cmegroup.com/trading/interest-rates/countdown-to-fomc.html