Portfolio Choice 3: General Case

Riskfree asset + multiple risky assets

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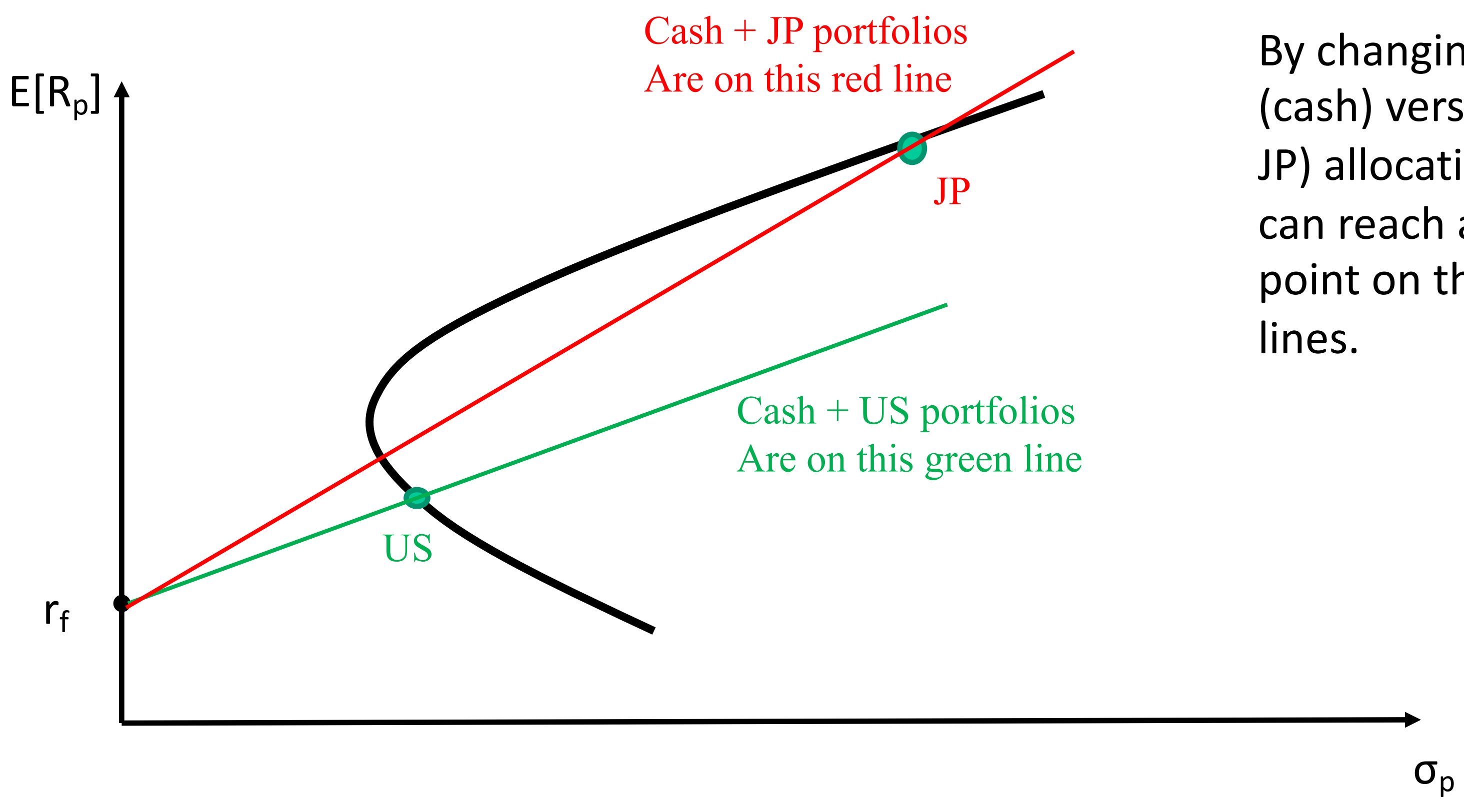
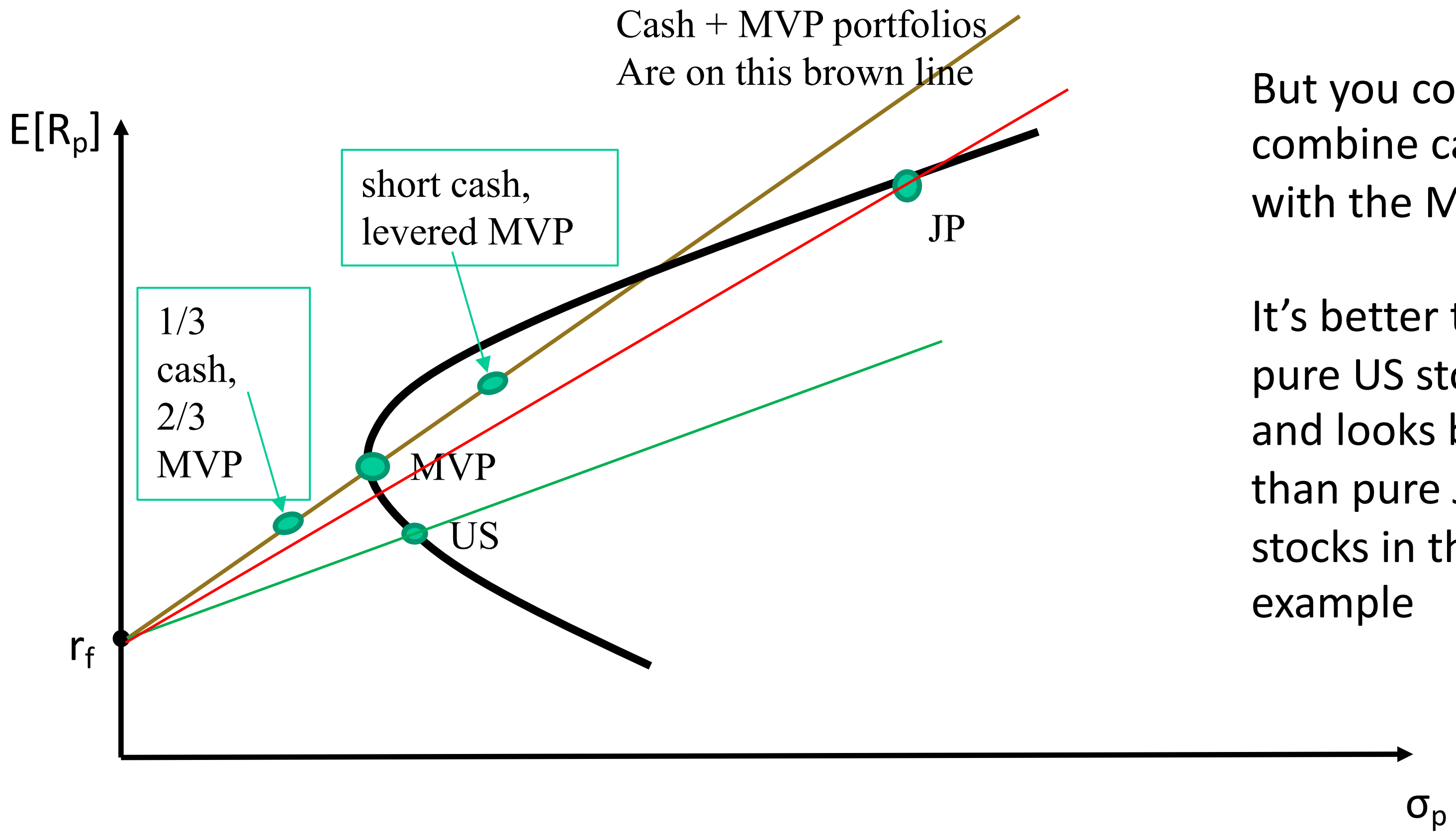
Riskless asset (Cash) + 2 Risky Assets



By changing the (cash) versus (US, JP) allocation you can reach any point on these lines.

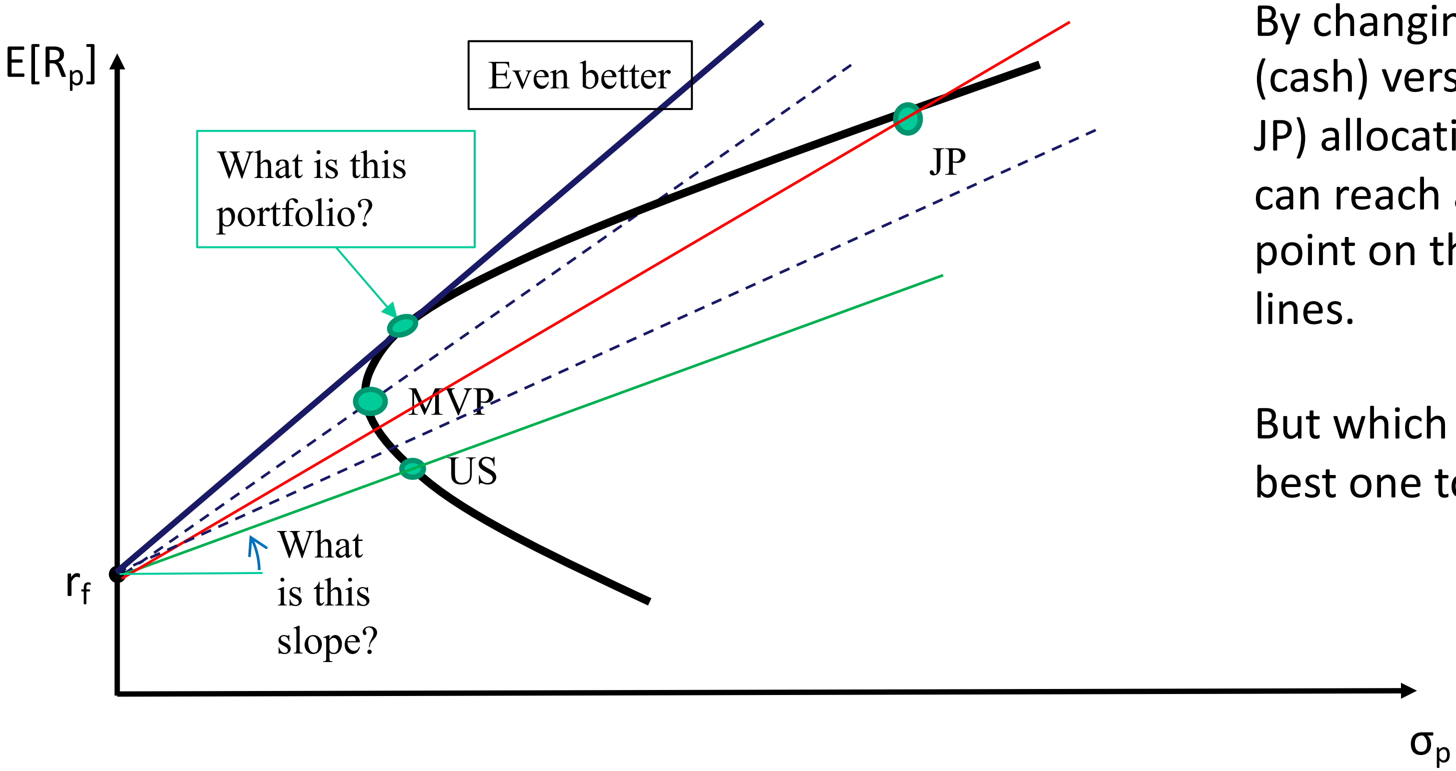
Cash + MVP



But you could also combine cash with the MVP

It's better than pure US stocks, and looks better than pure JP stocks in this example

CALs with Cash, US, and JP



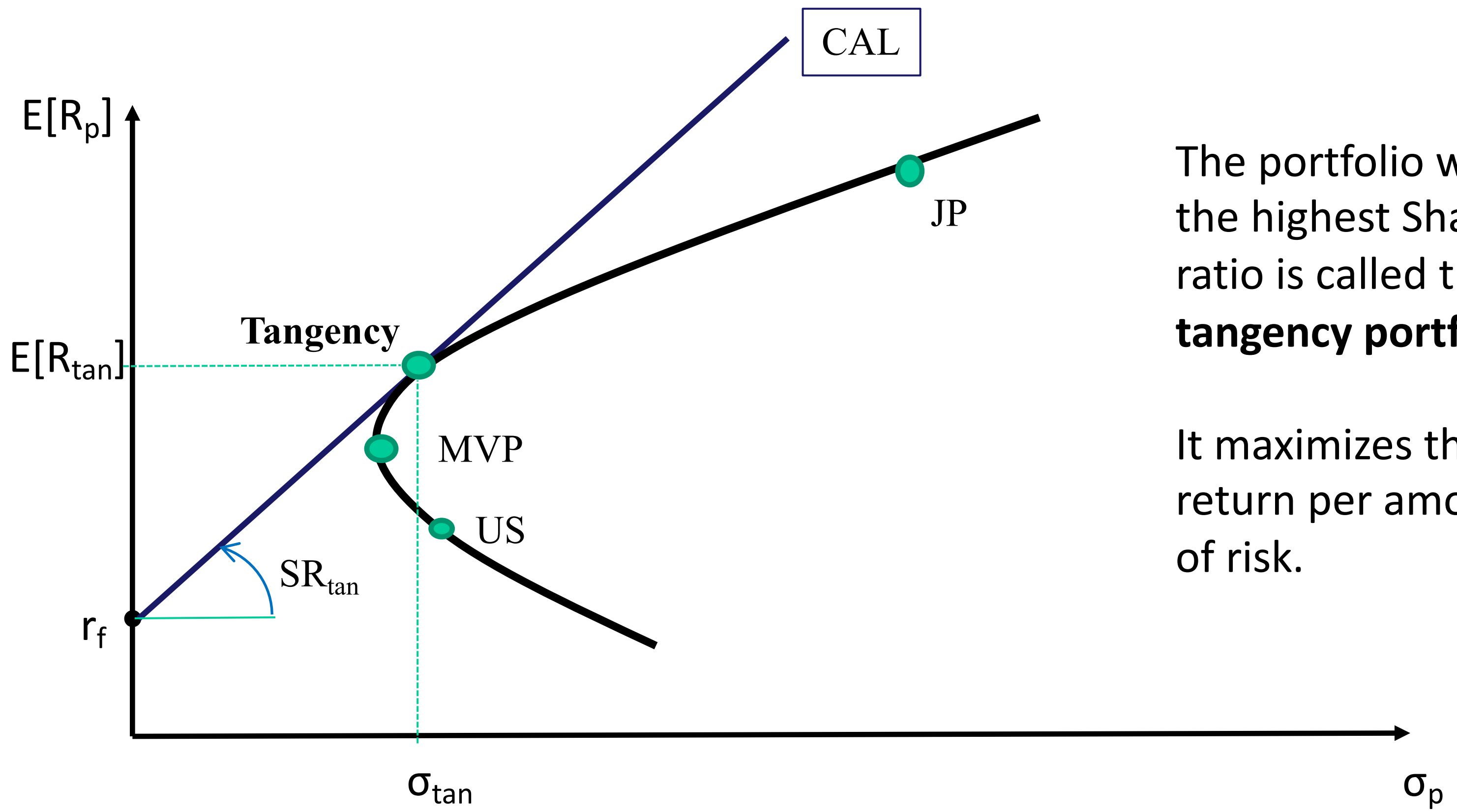
By changing the (cash) versus (US, JP) allocation you can reach any point on these lines.

But which is the best one to use?

Looking for Highest Sharpe

- We will want to combine cash with a portfolio of risky assets
- The question is : which one?
- From the previous figure it is apparent that the portfolio of risky assets with the highest Sharpe ratio will play a key role in our analysis.

Tangency Portfolio and CAL



The portfolio with the highest Sharpe ratio is called the **tangency portfolio.**

It maximizes the return per amount of risk.

Portfolio Selection with Two Risky and One Risk-Free Assets

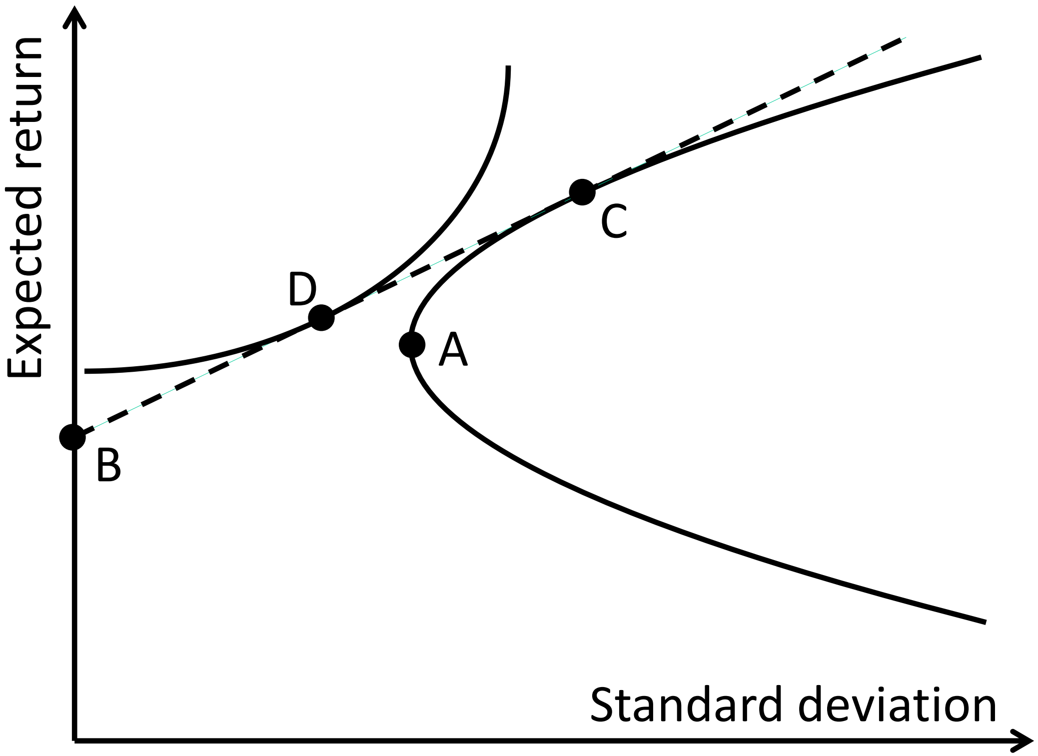
1. Create the set of possible mean-SD combinations from different portfolios of risky assets

2. Find the "**tangency portfolio**," aka **MVE** portfolio (mean-variance efficient) that is, the portfolio with the highest Sharpe ratio:

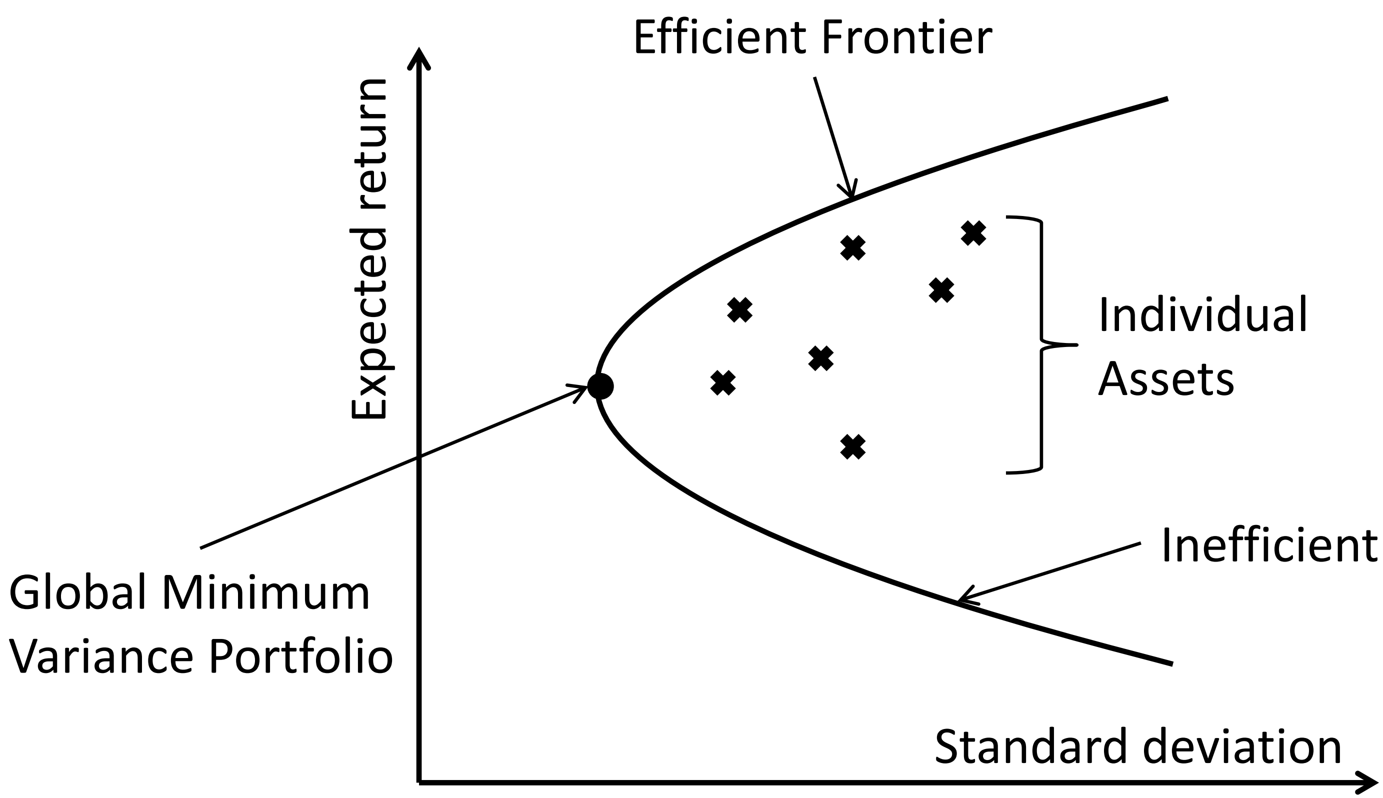
$$SR_i = \frac{E[R_i] - R_f}{\sigma_i}$$

3. Choose the combination of the tangency portfolio and the risk-free asset to suit your risk-return preferences.

Portfolio terminology



Investment Opportunity Set with Many Risky Assets



Portfolio Selection with Many Risky Assets and a Risk-Free Asset

1. Create the set of possible mean-s.d. combinations from different portfolios of risky assets

2. Find the "tangency portfolio," i.e. the portfolio with the highest Sharpe ratio:

$$SR_i = \frac{E[R_i] - R_f}{\sigma_i}$$

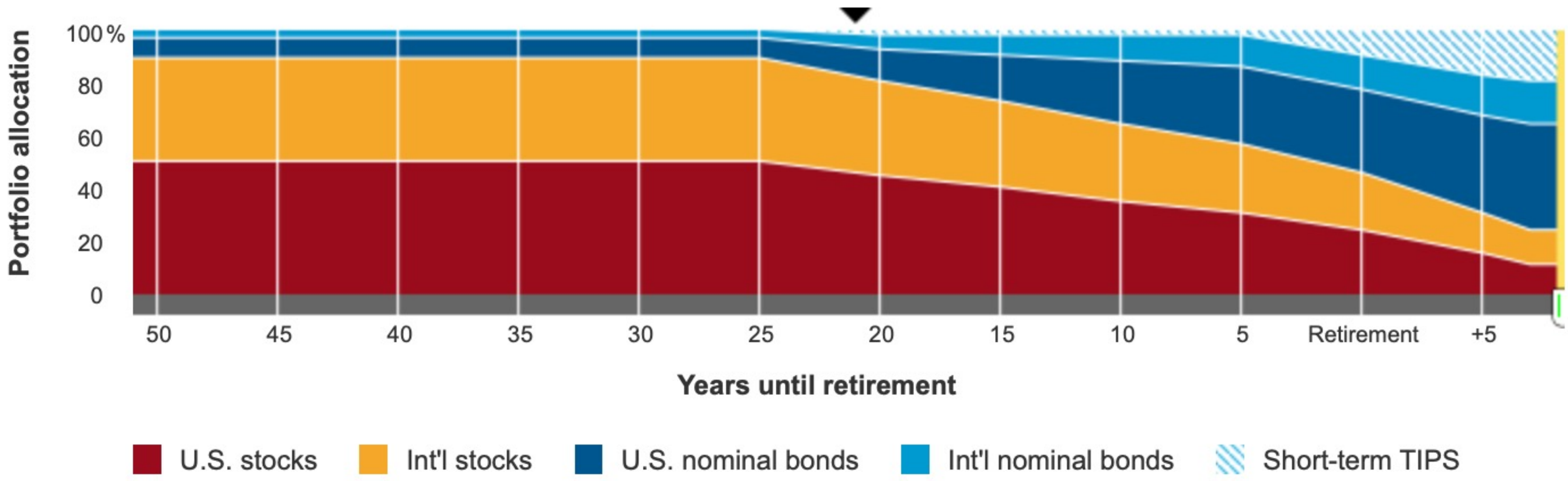
3. Choose the combination of the tangency portfolio and the risk-free asset to suit your risk-return preferences.

Two-Fund Separation

- All investors hold combinations of the same two "mutual funds":
  - The risk-free asset
  - The tangency portfolio
- An investor's risk aversion determines the fraction of wealth invested in the risk-free asset.
- But all investors should have the rest of their wealth invested in the same tangency portfolio!

Suppose the tangency portfolio has 60% US and 40% JP. The optimal portfolio of investor A has 60% risk-free asset. What is the composition of the rest of the optimal portfolio? Give w_{US}, w_{JP}.

Application: Target Retirement Funds (Vanguard)



- Consistent with the idea that you become more risk averse as you age. Ratio of US vs Int'l stock appears constant : consistent with fund separation theorem
- For bonds it's less clear. One reason is that risk free rate depends on the horizon: 1 month, 1 year, 5 years? This changes the meaning of what we call (riskless) "cash" and (risky) "bonds".

Strategic Allocation

John Boggle, founder of Vanguard Group

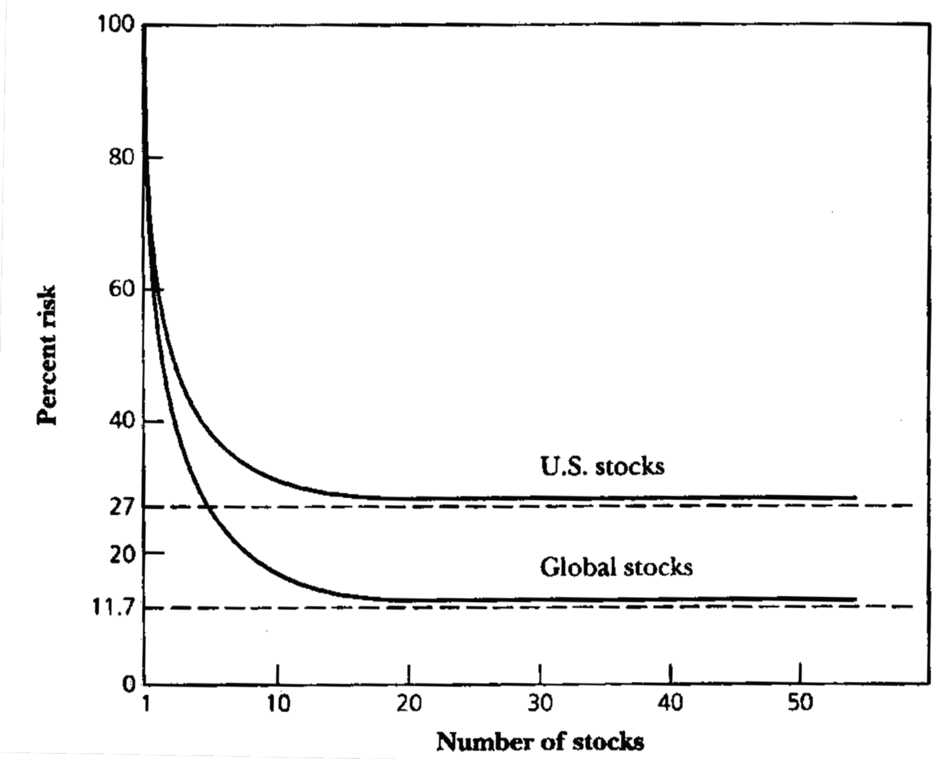
The fundamental decision of investing is the allocation of your assets: How much should you own in stock? How much should you own in bonds? How much should you own in cash reserves? That decision accounts for an astonishing 94% of the difference in total returns achieved by institutional investors. There is no reason to believe that the same relationship does not hold for individual investors.

Risk Reduction with Many Assets

Figure shows the percentage reduction in risk from adding stocks to our portfolio.

- 100 represents the typical vol of a US stock, around 44% annualized
- Volatility of holding 500 stocks, around 12%. Ratio is 12/44 = 0.27. **That is a 73% risk reduction!**
- N = 20 gets you most of the diversification

Diversifying globally reduces risk even more.



Diversification in Large Portfolios

- With 2 securities (N=2), the portfolio variance is:

$$\sigma_p^2 = \omega_1^2\sigma_1^2 + \omega_2^2\sigma_2^2 + 2\omega_1\omega_2\rho_{12}\sigma_1\sigma_2$$

- In general, the portfolio variance is:

$$\sigma_p^2 = \sum_{i=1}^{N}\omega_i^2\sigma_i^2 + 2\sum_{i=1}^{N}\sum_{j>i}^{N}\omega_i\omega_j\rho_{ij}\sigma_i\sigma_j$$

- The standard deviation of the portfolio is:

$$\sigma_p = \sqrt{\sigma_p^2}$$

Risk Reduction in Equally-Weighted Portfolios: Independent Returns

- Suppose we have an equally weighted portfolio (holding weights 1/N) of N <u>**independent**</u> stocks.
- The variance of the portfolio return is

$$\sigma_{\mathrm{p}}^{2} = \frac{1}{\mathrm{N}^{2}} \sum_{\mathrm{i}=1}^{\mathrm{N}} \sigma_{\mathrm{i}}^{2} = \frac{1}{\mathrm{N}} \left[\begin{array}{c} \text{Average} \\ \text{Variance} \end{array} \right]$$

- As the number of assets increase, the risk is completely **diversified away**
  - Next: what if stocks not independent?

Risk Reduction in Equally-Weighted Portfolios: Correlated returns

- Suppose we have an equally weighted portfolio (holding weights 1/N) of N stocks, **not independent.**

- The variance of the portfolio return is:

$$\sigma_p^2 = \frac{1}{N^2}\sum_{i=1}^{N}\sigma_i^2 + \frac{2}{N^2}\sum_{i=1}^{N}\sum_{j>i}^{N}\mathrm{cov}(R_i, R_j)$$

$$= \frac{1}{N}\left[\frac{1}{N}\sum_{i=1}^{N}\sigma_i^2\right] + \left(1 - \frac{1}{N}\right)\left[\frac{1}{N(N-1)/2}\sum_{i=1}^{N}\sum_{j>i}^{N}\mathrm{cov}(R_i, R_j)\right]$$

$$= \frac{1}{N}\begin{bmatrix}\text{Average}\\ \text{Variance}\end{bmatrix} + \left(1 - \frac{1}{N}\right)\begin{bmatrix}\text{Average}\\ \text{Covariance}\end{bmatrix}$$

Risk in Equally-Weighted Portfolios: The General Case

What happens when N goes to infinity?

- Variance of portfolio return

  → average covariance of returns

- Risk of portfolio → <u>non-diversifiable</u> risk

Idiosyncratic vs. Systematic Risks

- **Idiosyncratic risk** is diversified away when N becomes very large
  - e.g. individual company news.
- Risk of portfolio converges to <u>non-diversifiable</u> risk = **systematic risk**
  - e.g. market risk, macroeconomic risk, industry risk
- Key lessons:
  - The **volatility** of an individual asset is **not** a sufficient indicator of its riskiness
  - The riskiness of an asset (within a portfolio) depends on its **covariance** with the rest of the portfolio

Implementation issues with large N

- With N stocks, one needs:
  - N estimates of expected returns
  - N estimates of variances
  - N(N-1)/2 estimates of correlation between all pairs of returns
  - Not every set of estimates is internally consistent.
- For N = 500, this amounts to 125,750 parameters that need to be estimated.
- One solution is to use factor models: all systematic risk is captured by a few factors (monetary policy, growth, …)
  - Then only care about covariance of each stock with these factors

Summary & What's next?

- We understand the optimal allocation among a risk-free asset and any number of risky assets. We only need a few funds to satisfy **all investors**.
- Extremely complex problem boils down to a single decision: how much in risk-free asset and how much in tangency portfolio
- But there are many parameters to estimate. And we still do not know how assets are priced.
- <u>Next: Capital Asset Pricing Model (CAPM)</u>