# H4
# Annual Return on a Zero Sold Before Maturity

Foundations of Finance

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1. The formulas for Yield to Maturity and Annual Return on a zero are as follows:

| | | |
|---|---|---|
| $F$ | = | Face Amount |
| $P$ | = | Current Price |
| $t$ | = | Years the bond is held |
| $V'$ | = | Value at the End |
| $V$ | = | Value at the Beginning |

Table 1: Notation

Yield to Maturity (annual compounding) is defined (with $t$ = years to maturity) as:

$$YTM = \left(\frac{F}{P}\right)^{1/t} - 1.$$

The Annual Return is defined (with $t$ equal to any number) as:

$$Ann.Ret. = \left(\frac{V'}{V}\right)^{1/t} - 1.$$

For a zero coupon bond sold after $t'$ years (with $t'$ less than maturity) at a price of $P'$ the annual return becomes (by substitution):

$$Ann.Ret. = \left(\frac{P'}{P}\right)^{1/t'} - 1.$$

*Based on the notes of Profs. Alexi Savov and William Silber.

When the bond is sold prior to maturity the annual return is sometimes called the holding period yield (HPY). The HPY is not necessarily equal to the YTM. This is best illustrated by a numerical example.

2. Suppose you buy a 10 year maturity zero at a price of \$435 per \$1000 face value. The yield to maturity (annual compounding) is:

$$YTM = \left(\frac{1000}{435}\right)^{1/10} - 1 = 0.0868.$$

The YTM is known with certainty because the value at the end, \$1000, is known with certainty (since this is a liability of the US Treasury).

3. If you sell this bond after 1 year at a price $P'$, its Annual Return during that one-year holding period, also known as its one year Holding Period Yield (HPY), is:

$$HPY = \frac{P'}{435} - 1.$$

Thus the HPY on this bond is uncertain. It depends on what $P'$ turns out to be, i.e. on the price you realize when selling the bond in one year. Thus, even though the YTM on a zero, which is the liability of the US government, is known with certainty, the HPY when selling before maturity is uncertain. (Note also that your one-year HPY would also be uncertain even if you do not actually sell the bond but are simply calculating what you earned that year based on the market price of the bond at the end of the year).

4. In one year this bond will have 9 years to maturity. Thus the HPY that you earn over this one year period depends on the price you can sell this 9 year bond for in one year. Three possibilities illustrate the main relationships.

   A. Suppose you could sell this bond in one year at a price of \$472.758. This price is, in fact, the only one that will make your one year holding period yield equal to 0.0868. It is derived by solving for the price, $x$ in the formula below, that makes the one-year HPY equal to 0.0868:

$$\begin{aligned} \frac{x}{435} - 1 &= 0.0868 \\ x &= 472.758. \end{aligned}$$

   Note that the YTM on the 9-year bond to those who buy it from you at a price of

472.758 and hold to maturity is:

$$YTM = \left(\frac{1000}{472.758}\right)^{1/9} - 1 = .0868.$$

This means that if yields to maturity remain the same (at 8.68%) from year 10 to year 9 your one-year HPY will equal the YTM you would have earned had you held the bond to maturity.

B. Suppose the price of the bond in one year is \$480 per \$1000. With $P = 435$ and $P' = 480$, we know the following:

$$HPY = \frac{480}{435} - 1 = 0.1034.$$

Thus, your one-year HPY (10.34%) is greater than the YTM you would have earned had you held the bond to maturity (8.68%). We also know the YTM on this 9-year bond (to the people who bought the bond from you at the end of one year at the price of \$480):

$$YTM = \left(\frac{1000}{480}\right)^{1/9} - 1 = 0.0850.$$

Thus if you sell your bond after its yield to maturity has fallen (in this case from 8.68% to 8.50%) your HPY is greater than the YTM calculated as of the day you bought the bond. The reason: The fall in the YTM from 8.68% to 8.5% between year 10 and 9 means that "anxious investors" have pushed up the price of the bond faster than 8.68% during the year.

C. Suppose the price of the bond in one year is only \$460 per \$1000. If you do the same calculations you find that your HPY is now 0.057 and the YTM of the people who bought the bond from you would be 0.0901. Thus if you sell your bond after its yield to maturity has risen (in this case from 0.0868 to 0.0901) your HPY is lower than the YTM calculated as of the day you bought the bond. The reason: The increase in YTM from 8.68% to 9.01% means that the price of the bond rose slower than 8.68% during the year you held the bond.

5. Now you can answer the following question: Which bond has a one-year holding period return that is known with certainty?