Topic 5: The Capital Asset Pricing Model (CAPM)

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CAPM: Introduction

- Portfolio theory takes prices/expected returns as given
  - Given these expected returns, all investors should allocate wealth between risk-free asset and tangency portfolio

- Now: **What is the tangency portfolio? What determines asset prices/expected returns?**

- Answer: **CAPM**
  - predicts tangency portfolio
  - predicts relation between risk and expected return
  - underlies most of real-world financial decision making

The Market Portfolio

- p_i = price of one share of risky asset *i*
- n_i = number of shares outstanding for risky asset *i*
- Market Portfolio = portfolio in which each risky asset *i* has the following weight:

$$\omega_{i\mathrm{M}} = \frac{p_i \times n_i}{\sum_i p_i \times n_i} = \frac{\text{Market Capitalization of Security } i}{\text{Total Market Capitalization}}$$

CAPM Part I:
Tangency Portfolio = Market Portfolio

- If we add up all the investments of all investors, what do we get?
  – It must be the market portfolio
- But recall from portfolio theory:
  – All investors hold the tangency portfolio, which is the same for all investors
- **Therefore the tangency portfolio must be the market portfolio**

Tangency Portfolio = Market Portfolio

What would happen if tangency portfolio ≠ market portfolio?

Example

- Total wealth invested in risky assets is \$100B
- Apple has 4M shares outstanding at price P
- Tangency portfolio puts weight 2% on Apple

1. Total market cap in the economy should be total wealth = \$100B
   – Wealth has to go somewhere

2. If Apple stock price is P=\$250, market portfolio has 1% in Apple
   – This is less than investors want, they will buy more Apple

3. If Apple stock price is P=\$1000, market portfolio has 4% in Apple
   – This is more than investors want, they will sell Apple

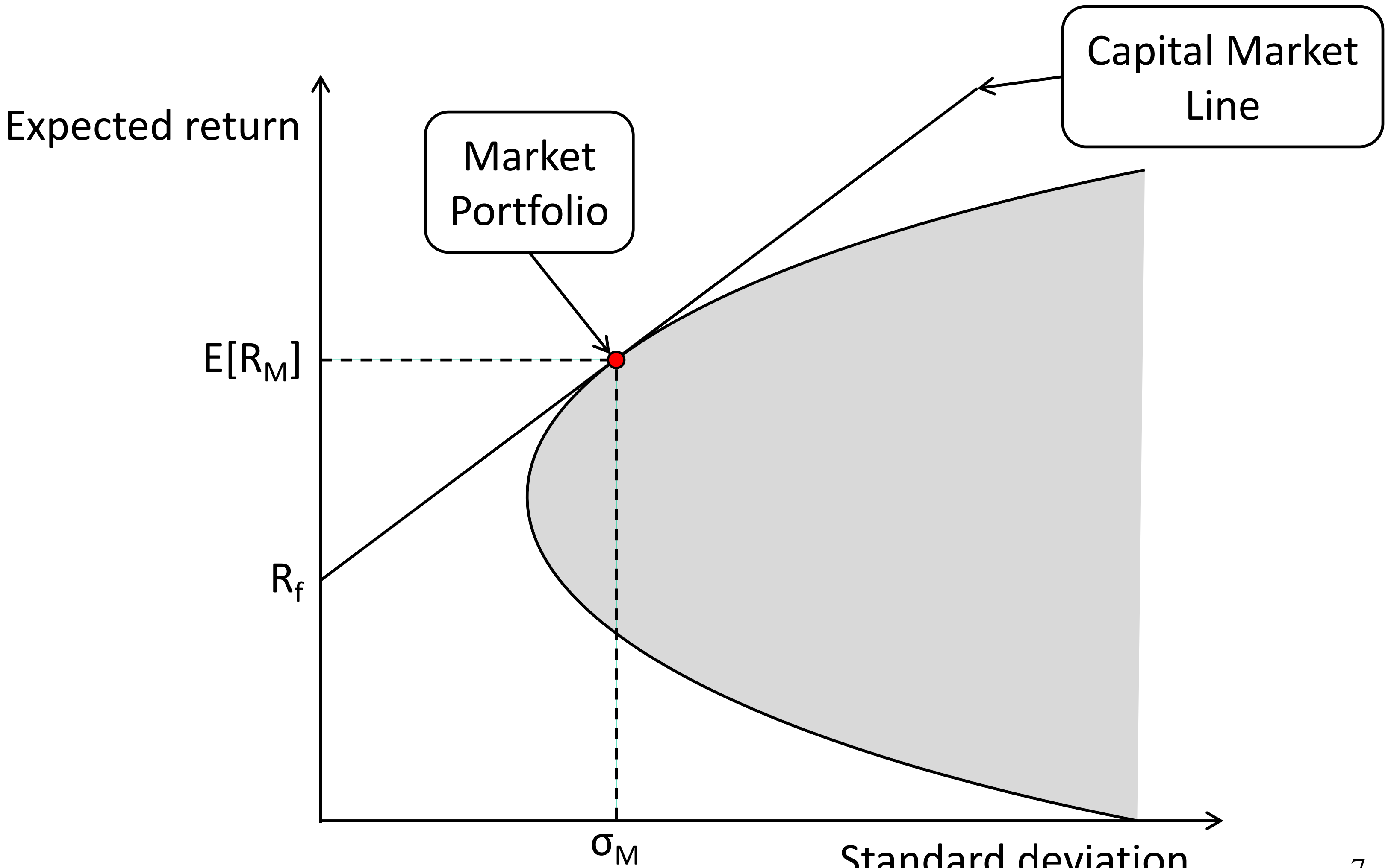
The Capital Market Line (CML)

- The market portfolio's capital allocation line is called the Capital Market Line (CML).

- The CML gives the risk-return combinations achieved by forming portfolios from the risk-free security and the market portfolio:

$$E(R_p) = R_f + \left(\frac{E(R_\mathrm{M}) - R_f}{\sigma_\mathrm{M}} \right) \sigma_p$$

The E(R)-S.D. Frontier and The Capital Market Line



CAPM Part II:
Expected Returns of Individual Securities

- We have taken risk and expected returns as given
  - but there should be a relation between risk and "equilibrium" expected return
  - What return do risk-averse investors "require" to absorb the supply of each security?
- The required return on any security is proportional to the risk contribution of that security to the overall portfolio
  - Very generic statement!
  - CAPM: how to measure "risk contribution" precisely

Security Market Line

- CAPM predicts that expected excess return of a security is linear in its **beta**:

$$E[R_i] = R_f + \beta_i E[R_M - R_f]$$

- This linear relation is the <u>Security Market Line</u> (SML).
- The beta measures the security's **systematic risk**, i.e., its sensitivity to market movements:

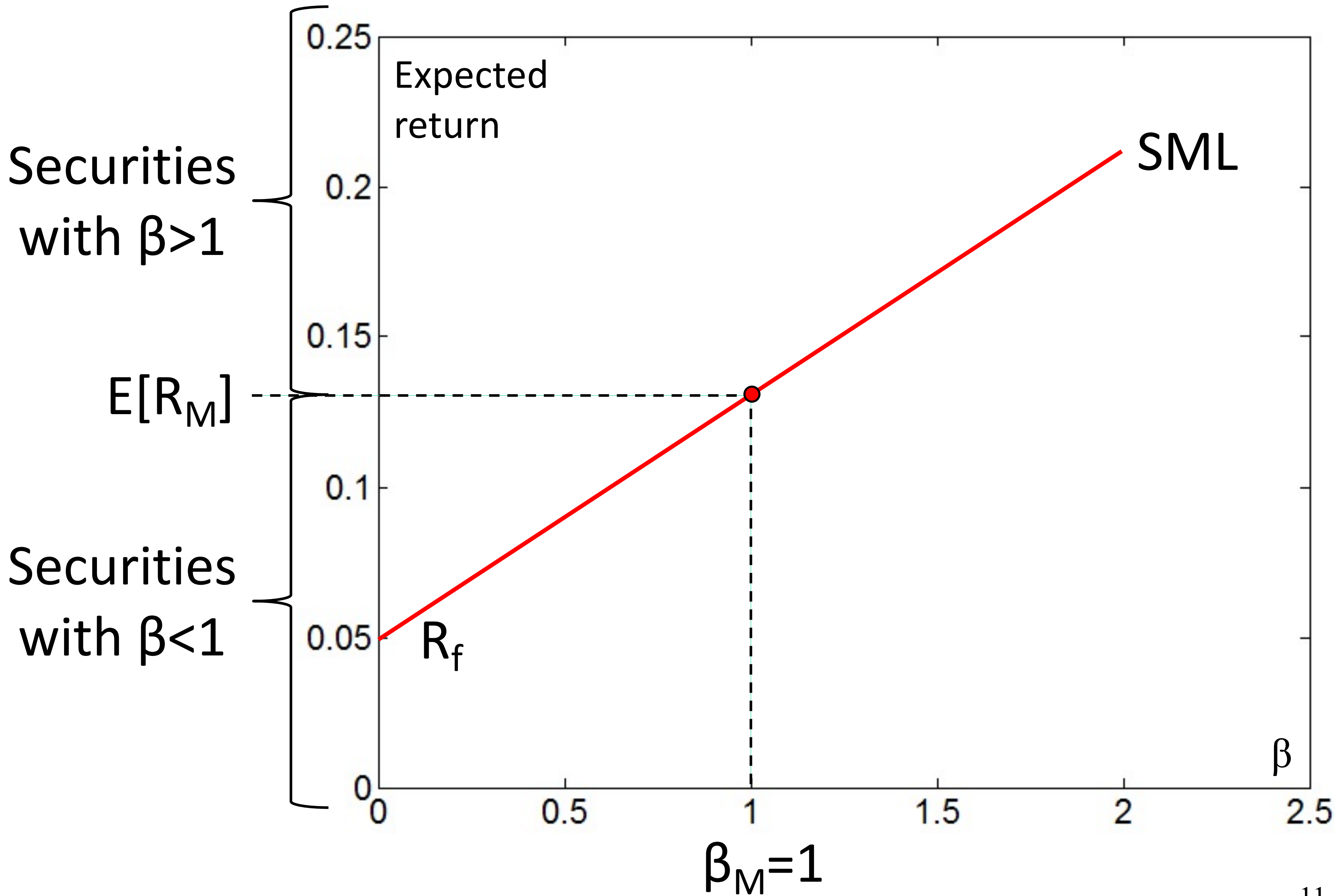
$$\beta_i = \frac{Cov(R_i, R_M)}{Var(R_M)}$$

Example

- Numerical example
  - Suppose that a stock has a beta of 1.5. The market portfolio has an expected return of 10% and the risk-free rate is 3%. What is the expected return on this stock?

$$E[R] = 3\% + 1.5 * (10\% - 3\%) = 3 + 1.5 * 7 = 13.5\%$$

Security Market Line (SML)



Sketch of proof of SML formula

$$E[R_i] = R_f + \beta_i E[R_M - R_f]$$

- Suppose you hold the market (=tangency) portfolio and add a <u>small</u> amount ∂w_i of security i while reducing the weight on the risk-free asset by ∂w_i.
  - New portfolio's Sharpe ratio

- In equilibrium, this must give you the same Sharpe ratio as the market portfolio itself. (Why?)

Sketch of proof of SML formula

$$E[R_i] = R_f + \beta_i E[R_M - R_f]$$

- Suppose you hold the market (=tangency) portfolio and add a small amount ∂w_i of security i while reducing the weight on the risk-free asset by ∂w_i.

- In equilibrium, this must give you the same Sharpe ratio as the market portfolio itself. Therefore

$$E[R_i] - R_f = \frac{1}{\sigma_M} \frac{\partial \sigma_M}{\partial w_i} (E[R_M] - R_f)$$

- Next: compute $\frac{1}{\sigma_M} \frac{\partial \sigma_M}{\partial w_i}$...

What is the risk contribution of security i?

- Recall the standard deviation of the market portfolio:

$$\sigma_M = \sqrt{\Sigma\Sigma w_i w_j Cov(R_i, R_j)}$$

- The risk contribution of security i depends on its <u>covariance with the market portfolio</u>:

$$\frac{\partial \sigma_M}{\partial w_i} = \frac{1}{\sigma_M} \sum w_j \, Cov(R_i, R_j) = \frac{1}{\sigma_M} Cov\left(R_i, \sum w_j \, R_j\right).$$

$$= \frac{1}{\sigma_M} Cov(R_i, R_M)$$

- Therefore $\boldsymbol{\frac{1}{\sigma_M} \frac{\partial \sigma_M}{\partial w_i} = \frac{Cov(R_i, R_M)}{{\sigma_M}^2} = \beta_i}$

Risk Premium

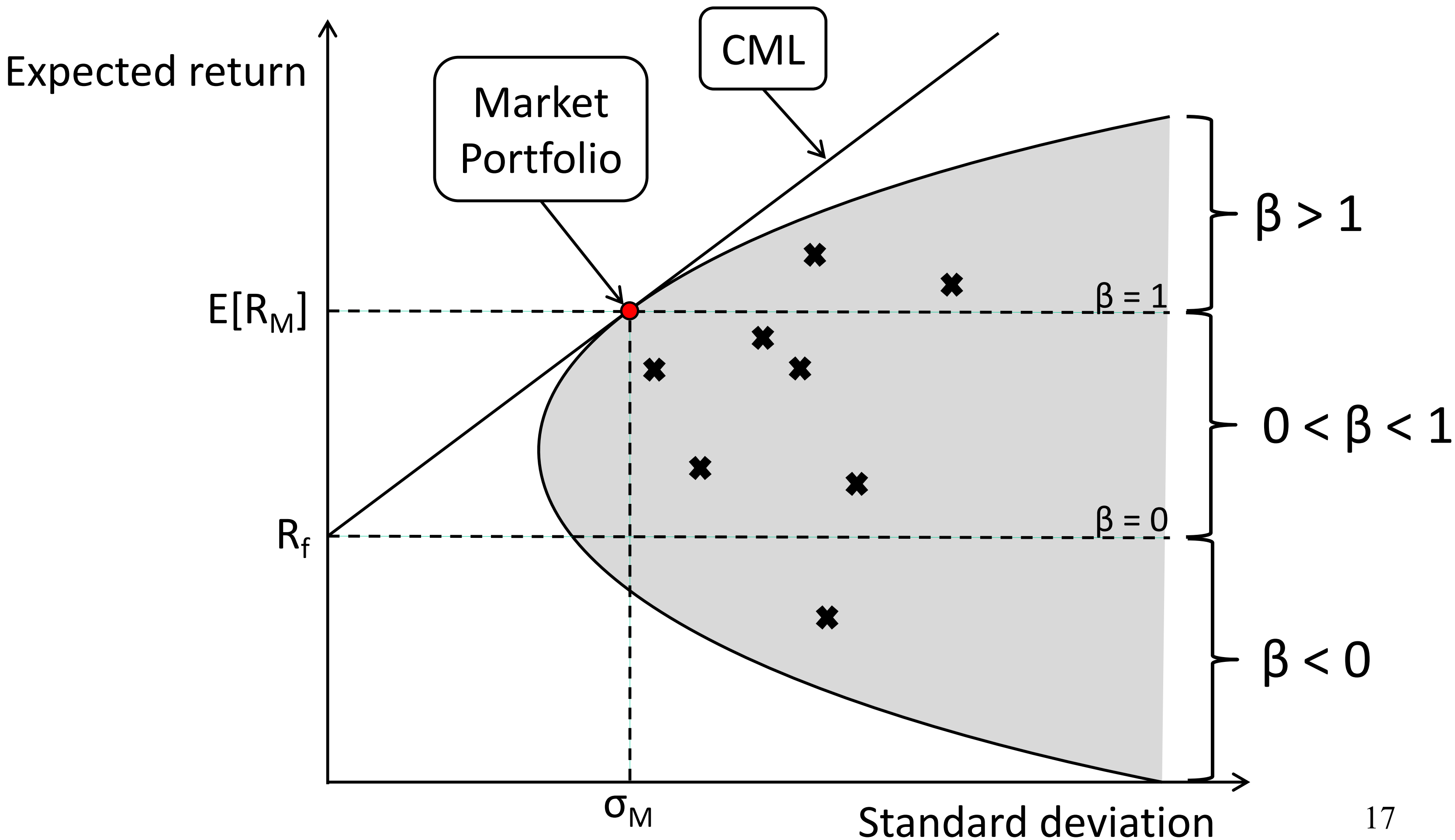
- SML: $E(R_i) = R_f + \beta_i [E(R_M) - R_f]$
- Stock *i*'s systematic or market risk is β_i
- Investors are compensated for holding systematic risk in the form of higher returns.
- The size of the compensation depends on the *Equilibrium Risk Premium*, $E(R_M) - R_f$.
- The equilibrium risk premium is **increasing** in:
  1. the variance of the market portfolio (volatile times)
  2. the degree of risk aversion of *average* investor

Betas for 49 industries, 1980-2016

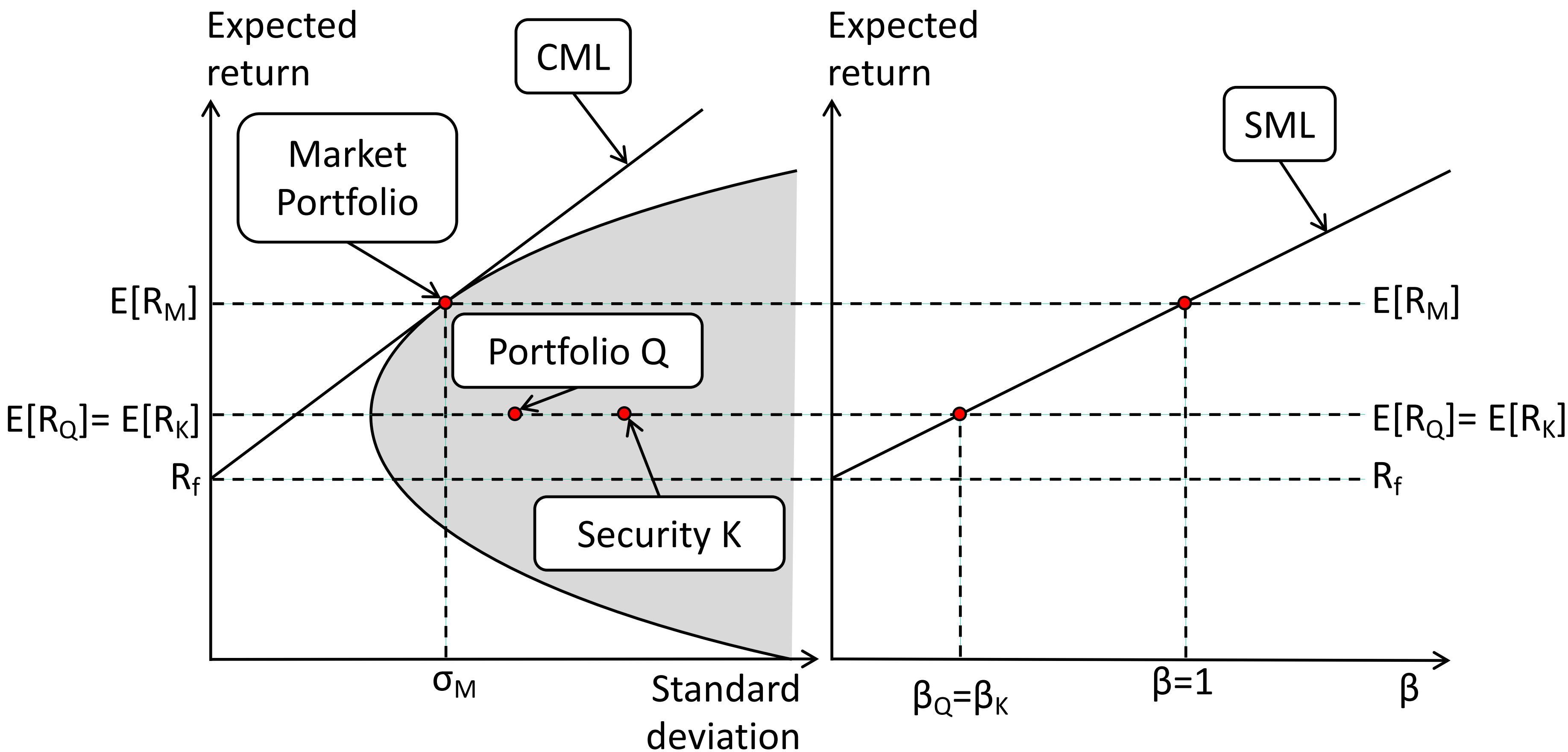
Low Beta				High Beta			
Industry	Beta	Industry	Beta	Industry	Beta	Industry	Beta
Util	0.44	Meals	0.85	Books	1.03	Coal	1.14
Gold	0.59	Telcm	0.85	Clths	1.03	BldMt	1.15
Food	0.60	MedEq	0.86	Rubbr	1.04	Autos	1.21
Smoke	0.63	Insur	0.90	Banks	1.04	ElcEq	1.23
Beer	0.64	Hlth	0.91	Aero	1.05	Mach	1.25
Guns	0.70	Paper	0.95	Chems	1.06	Cnstr	1.26
Hshld	0.72	Boxes	0.96	FabPr	1.07	Fin	1.26
Drugs	0.74	Rtail	0.96	Toys	1.07	Fun	1.28
Soda	0.74	Whlsl	0.97	Ships	1.09	LabEq	1.30
Oil	0.78	Trans	0.98	RlEst	1.09	Hardw	1.31
Agric	0.80	PerSv	0.99	BusSv	1.11	Steel	1.40
				Other	1.12	Chips	1.42
				Txtls	1.14	Softw	1.46
				Mines	1.14		

Why do stocks in different industries have high/low betas?

E(R)-S.D. Frontier and Betas



The Capital Market Line (CML) and the Security Market Line (SML)



Slope of the CML is

$$\frac{E[R_M] - R_f}{\sigma_M}$$

Slope of the SML is

$$E[R_M] - R_f$$

Systematic and Idiosyncratic Risk

- β_i measures security i's contribution to the total risk of a well-diversified portfolio, namely the market portfolio.

- Hence, β_i measures the systematic risk of the stock.

- Investors must be compensated for holding systematic risk. This explains the CAPM equation:

$$E(R_i) = R_f + \beta_i \left[E(R_M) - R_f \right], \; i = 1,\ldots,N.$$

Systematic and Idiosyncratic Risk

- The CAPM equation can be written as:

$$R_i = R_f + \beta_i \cdot (R_M - R_f) + error_i$$

$$\text{where } \beta_i = \frac{Cov\left(R_i, R_M\right)}{\sigma_M^2}$$

$$\mathrm{E}(error_i) = 0$$

$$Cov\left(error_i, R_M\right) = 0$$

- This implies the total risk of a security can be partitioned into two components:

$$\underbrace{\sigma_i{}^2}_{\substack{Var(R_i)\\ Total\ risk}} = \underbrace{\beta_i{}^2\ \sigma_M{}^2}_{\substack{Systematic\\ (market)\ risk}} + \underbrace{\bar{\sigma}_i{}^2}_{\substack{Var(error_i)\\ Idiosyncratic\ risk}}$$

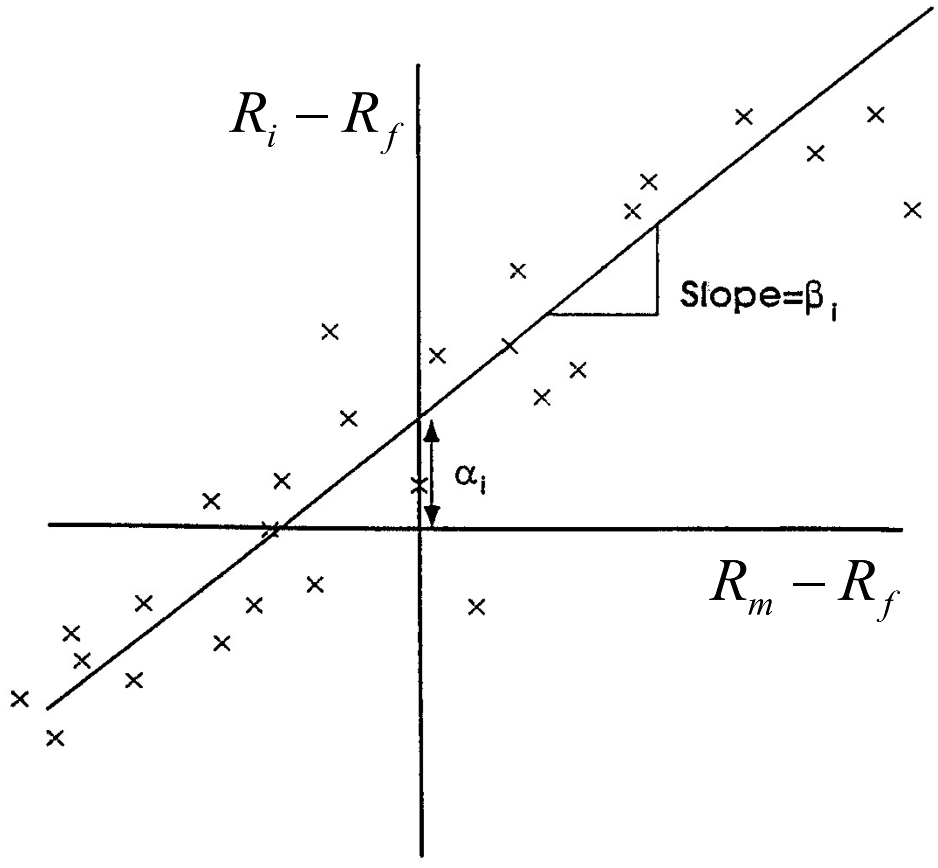
ABC stock has a volatility of 90% and a beta of 3. The market portfolio has an expected return of 14% and a volatility of 15%. The risk-free rate is 7%. What is the equilibrium expected return of ABC?

What is the proportion of ABC stock's variance which is diversified away in the market portfolio?

Security Characteristic Line (SCL)

The SCL is the "regression line":

$$R_i(t) - R_f = \alpha_i + \beta_i(R_M(t) - R_f) + error_i(t)$$



CAPM implies α_i=0 for all securities i!

Estimating Beta by Linear Regressions (OLS)

- Get data on *excess returns*:

$$R_i^e(t) = R_i(t) - R_f \qquad\qquad R_M^e(t) = R_M(t) - R_f$$

- Where R_f is the risk-free rate from time t-1 to time t.

- Estimate β_i by running the regression:

$$R_i^e(t) = \alpha_i + \beta_i R_m^e(t) + error_i(t)$$

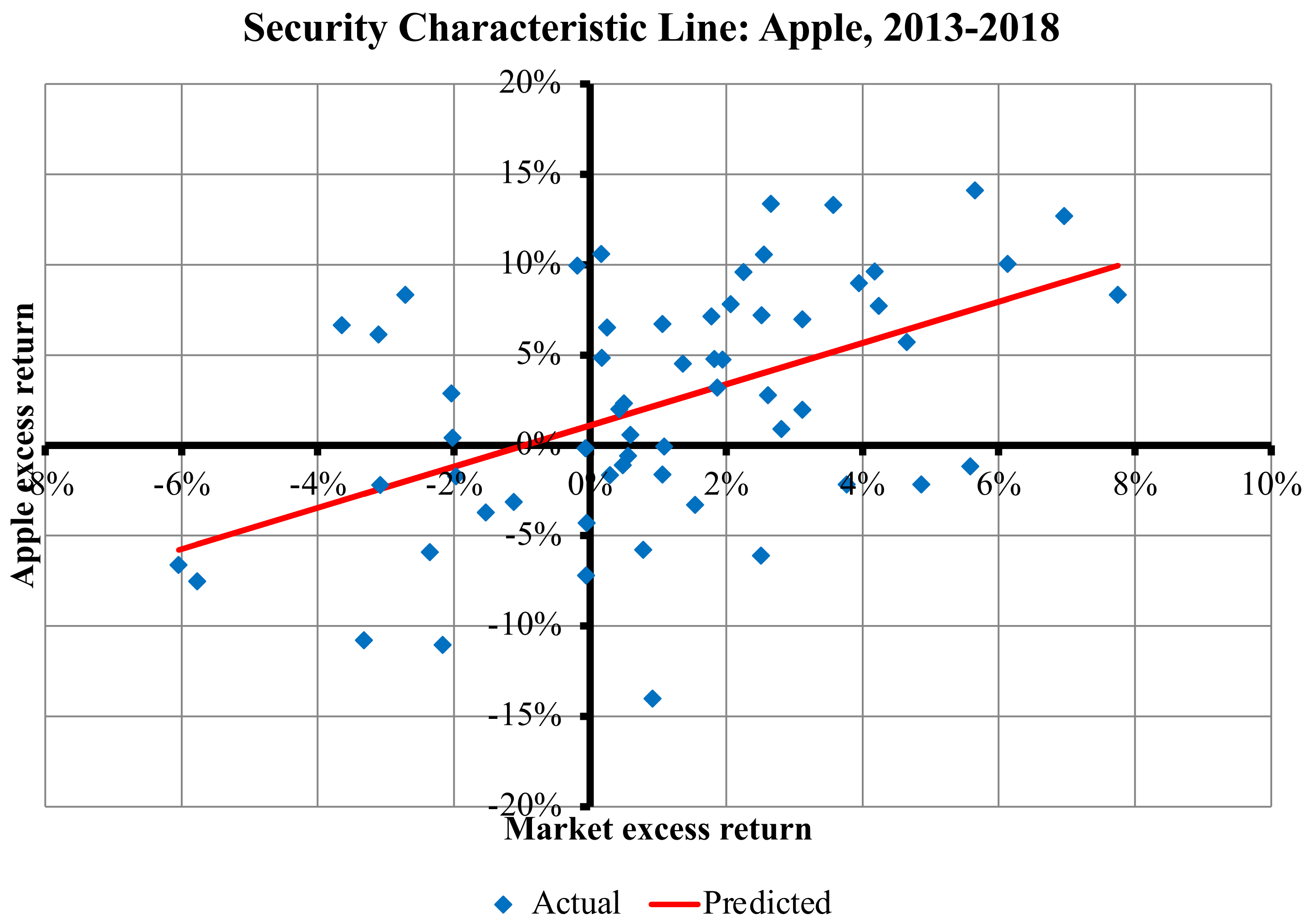
- Typically, 60 months of data are used.

Estimating Beta (Apple example)

- Take 5 years (2013-2018) of monthly data on Apple returns, Market returns and 1 month US T-bills.
- Construct excess returns
- Run the regression, for instance using Excel:
  - apply *Tools, Add-ins, Analysis ToolPak*
  - use *Tools, Data Analysis, Regression*
- The result is in the spreadsheet "betareg.xls" on the website
- Excel Regression output:

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0.011011	0.007964	1.382601	0.172088
X Variable 1	1.141364	0.257708	4.428895	4.25E-05

Estimating Beta: SCL for Apple



Applications of the CAPM

- Portfolio choice - done
- Shows what a “fair” security return is - done
- Gives benchmark for security analysis
- Required return used in capital budgeting

Application 1: Stock Selection

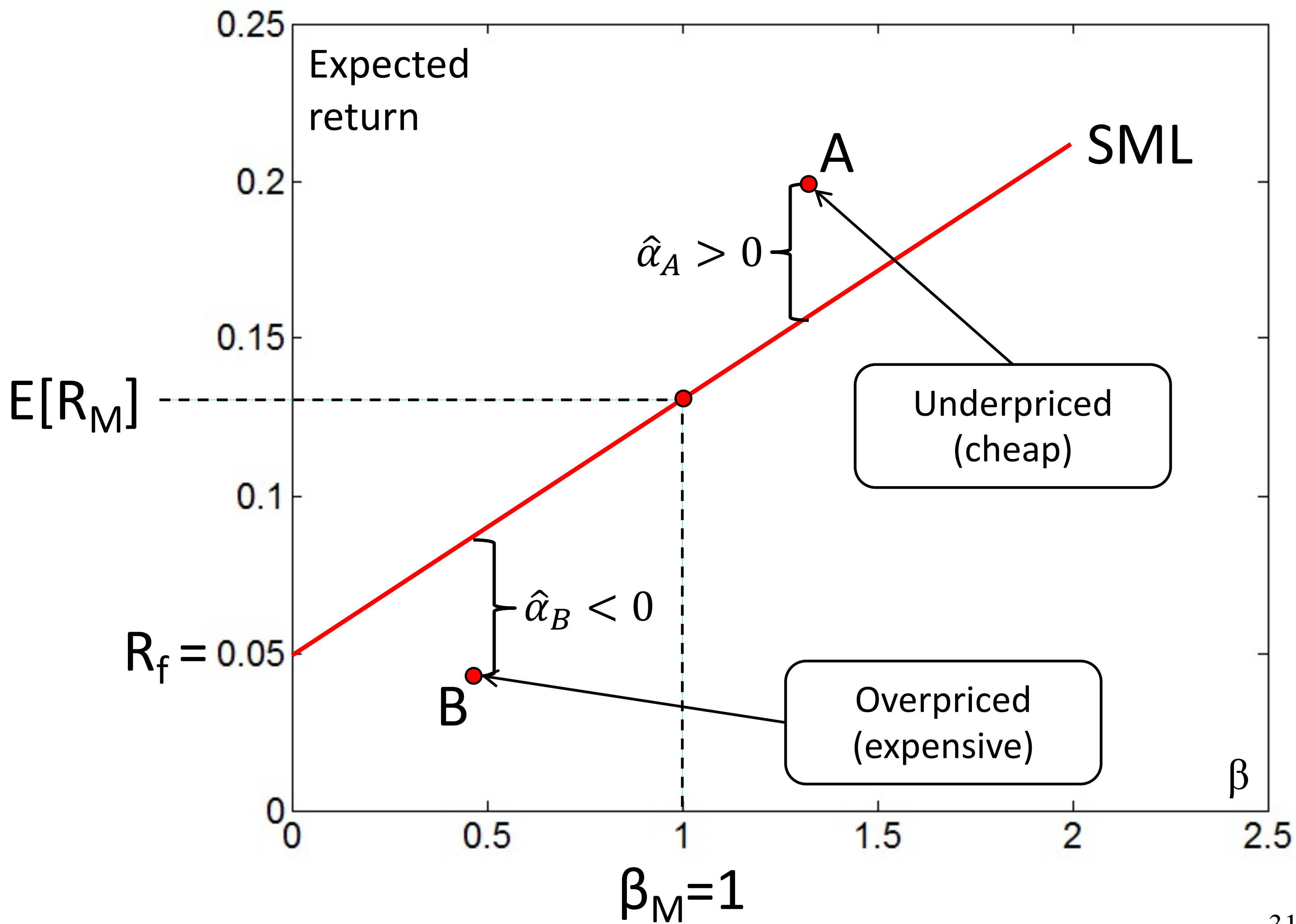
- One possible benchmark for *stock selection* is to find assets that are cheap relative to CAPM.
- A security i's *alpha* is defined as:

$$\alpha_i = E[R_i] - R_f - \beta_i \left[E[R_M] - R_f \right]$$

$$\text{where } \beta_i = \frac{\text{cov}[R_i, R_M]}{\sigma_M^2}$$

- Some fund managers try to buy positive-alpha stocks and (short) sell negative-alpha stocks.
- CAPM predicts that all alphas are **zero**.

Stock Selection



Active and Passive Strategies

- An **“active” strategy** tries to beat the market by stock picking, by timing, or by other methods.
- But the CAPM implies that
  - security analysis is not necessary
  - every investor should just buy a mix of the risk-free security and the market portfolio, a **“passive” strategy**
- Grossman-Stiglitz paradox: How can market be efficient if everyone uses a passive strategy?

Application 2: Capital Budgeting and NPV

- Should firm undertake long-term risky project?
- Calculate Net Present Value, using CAPM to calculate required rate of return.
- Manager's objective: increase value of firm. Only undertake projects with NPV>0.
- Compare with decision based on Internal Rate of Return on project.

Example NPV

- Project cash flows: \$1000 (0), \$300 (3), \$600 (4), \$900 (5)
- Discount rate? *Cost of capital* is given by CAPM:

$$E[R_i] = R_f + \beta_i E[R_M - R_f]$$

- If $\beta_i = 1.75$, $R_f = .04$, $E[R_M]=.12$, Then $E[Ri]=.18$.
- NPV = $-1000+300/(1.18)^3 + 600/1.18^4 + 900/(1.18)^5$
  $= -114.54 < 0$, so Don't undertake project!
- If $\beta_i = .50$, $E[R_i]=.08$ and NPV = $291.69 > 0$ and go.
- IRR is discount rate where NPV = 0. Here IRR = .147. Alternative rule: Undertake project when $E[R_i] < IRR$.
- Only appropriate for well diversified firm/project?

CAPM Summary

- Prediction 1: Everyone should hold a mix of the market portfolio and the risk-free asset. (That is, everyone should hold a **portfolio of stocks** on the CML.)
- Prediction 2: The expected return on an **individual stock** is a linear function of its beta. (That is, stocks should be on SML.)
- The beta is given by: $\beta_i = \dfrac{Cov(R_i, R_M)}{\sigma_M^2}$
- A stock's beta can be estimated using historical data by linear regression. (That is, by estimating the SCL.)