PROBABILITY

N RAMESH

WHAT IS PROBABILITY?

- Probability is a value between 0 and 1 that a certain event will occur

EXAMPLE FOR PROBABLITY

- The probability that a fair coin will come up heads is 0.5
- Mathematically we write:

$$P(E_{heads}) = 0.5$$

WHAT ISPROBABILITY?

- In the above “heads” example, the act of flipping a coin is called a trial.
- Over very many trials, a fair coin should come up “heads” half of the time.

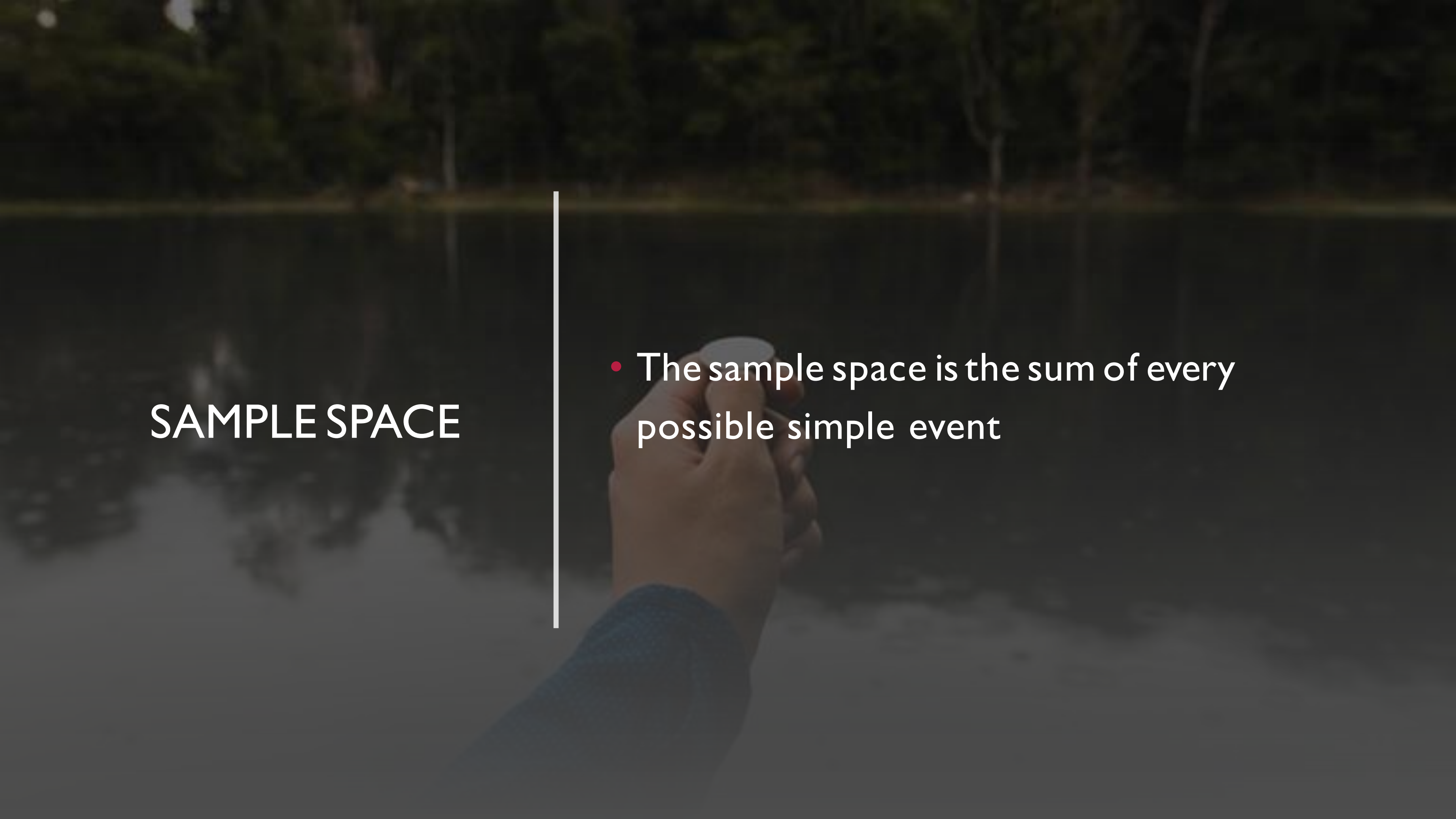
TRIALS HAVE NO MEMORY!

- If a fair coin comes up tails 5 times in a row, the chance it will come up heads is *still* 0.5

- Each trial is independent of all others

EXPERIMENTS

- Each trial of flipping a coin can be called an experiment
- Each mutually independent outcome is called a simple event



SAMPLE SPACE

- The sample space is the sum of every possible simple event

EXAMPLE FOR SAMPLE SPACE

- Consider rolling a six-sided die
- One roll is an experiment
- The simple events are:

  $E_1=1$ $E_2=2$ $E_3=3$

  $E_4=4$ $E_5=5$ $E_6=6$

- Therefore, the sample space is:
  - S = {E1,E2,E3,E4,E5,E6}

EXPERIMENTS

- The probability that a fair die will roll a six:

The simple event is:

$E_6 = 6$ (one event)

Total sample space:

$S = \{E_1, E_2, E_3, E_4, E_5, E_6\}$ *(six possible outcomes)*

The probability:

$\text{P(Roll Six)} = 1/6$

PROBABILITY EXERCISE

- A company made a total of 50 trumpet valves
- It is determined that one of the valves was defective
- If three valves go into one trumpet, what is the probability that a trumpet has a defective valve?

PROBABILITY EXERCISE

1. Calculate the probability of having a defective valve:

$$P(E_{defective\,valve}) = \frac{1}{50} = 0.02$$

PROBABILITY EXERCISE

2. Calculate the probability of having a defective trumpet:

$$P(E_{defective\ trumpet}) = 3 \times P(E_{defective\ valve})$$

$$= 3 \times 0.02 = \mathbf{0.06}$$

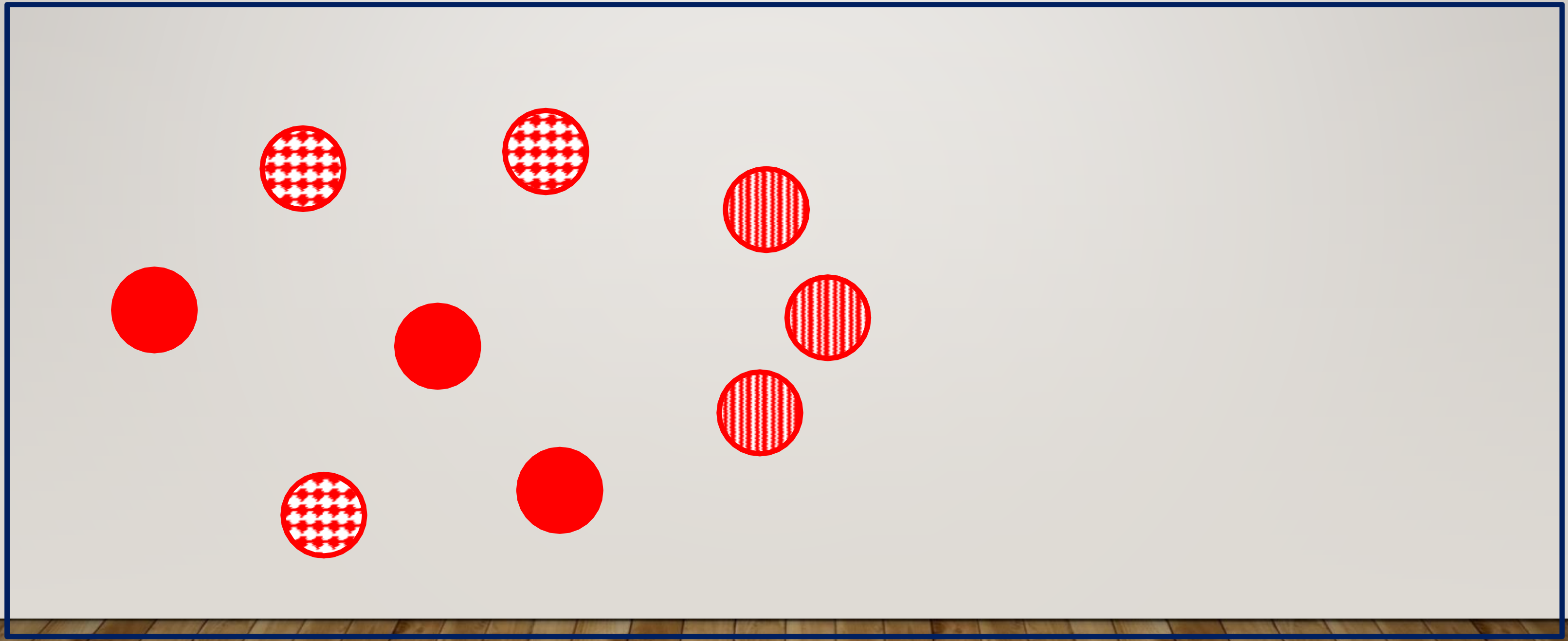
INTERSECTIONS, UNIONS & COMPLEMENTS

INTERSECTIONS

- In probability, an intersection describes the sample space where two events *both* occur.

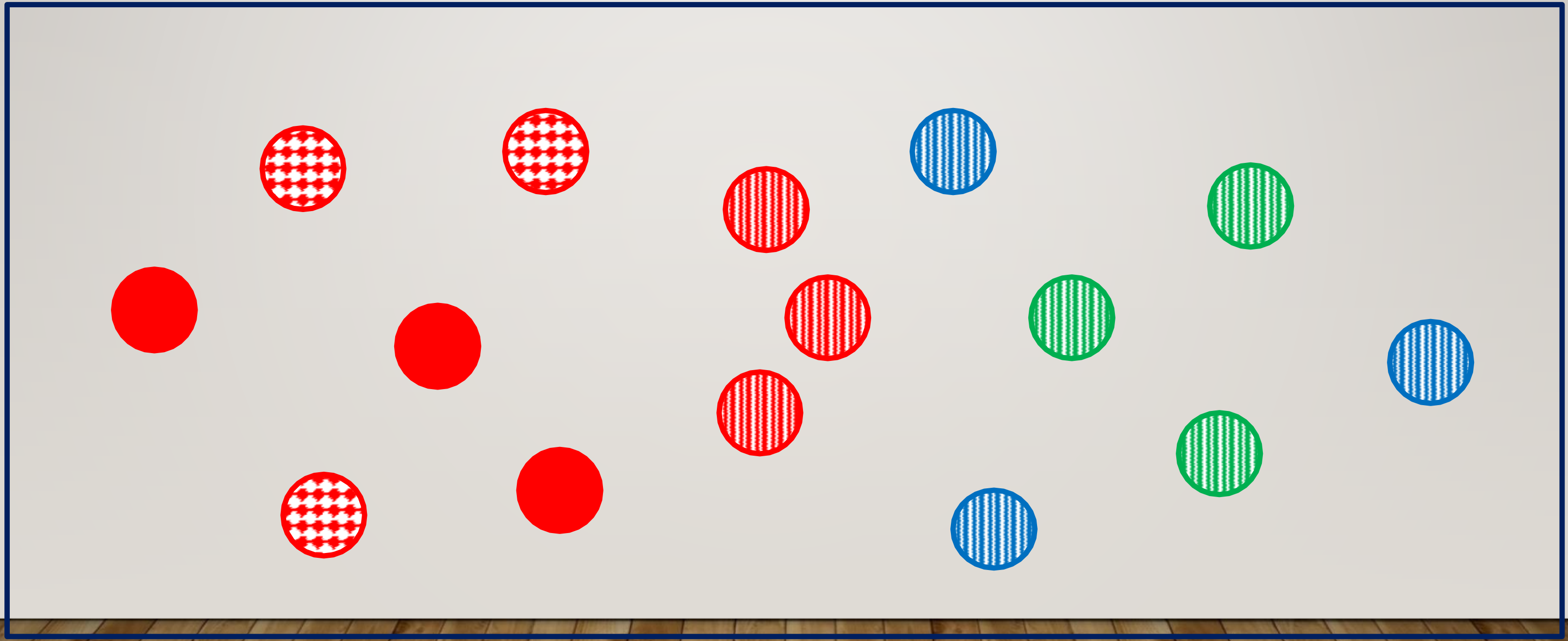
INTERSECTIONS

- 9 of the balls are red:



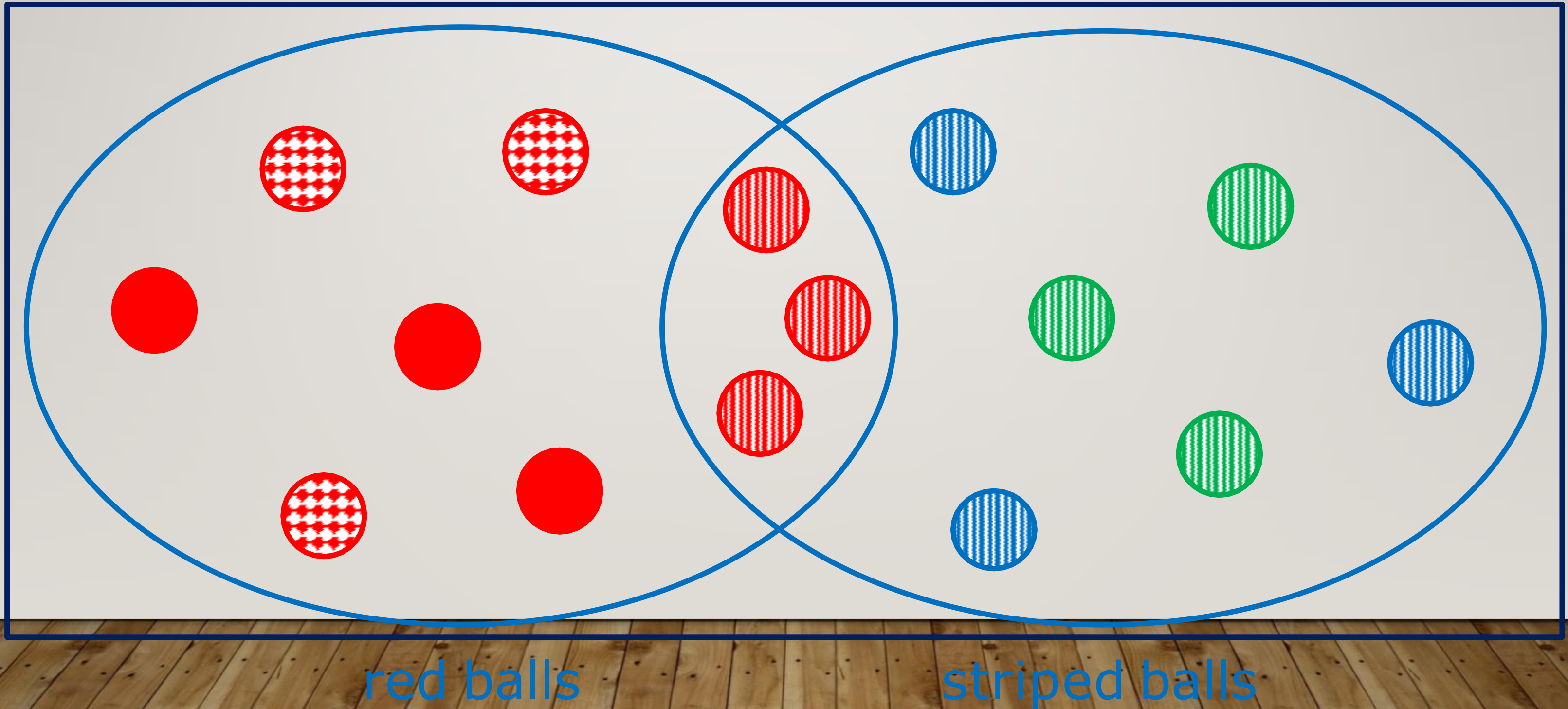
INTERSECTIONS

- 9 of the balls are striped:



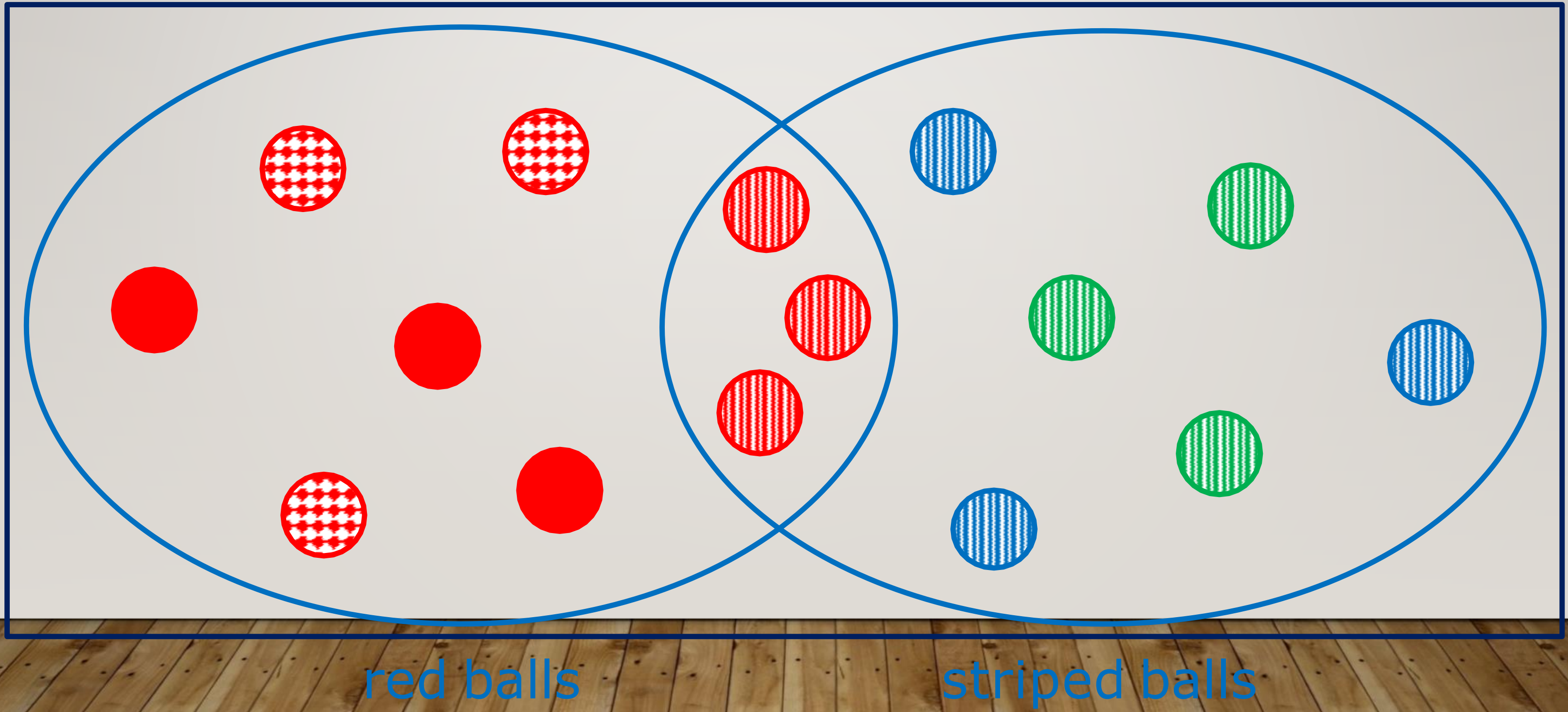
INTERSECTIONS

- 3 of the balls are both red and striped:

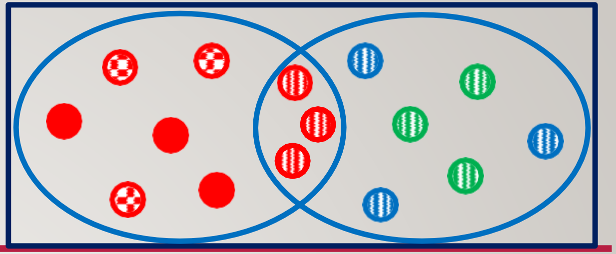


INTERSECTIONS

- What are the odds of a red, striped ball?



INTERSECTIONS



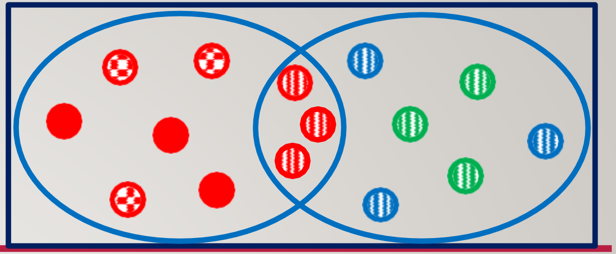
- If we assign A as the event of red balls, and B as the event of striped balls, the intersection of A *and* B is given as:

$$A \cap B$$

- Note that order doesn't matter:

$$A \cap B = B \cap A$$

INTERSECTIONS



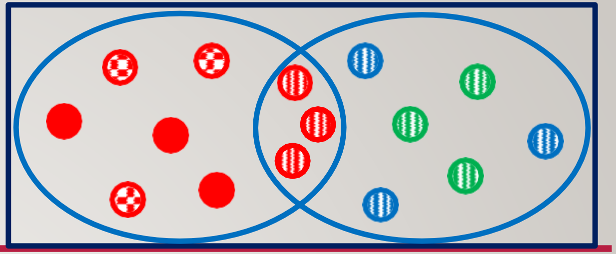
- The probability of A *and* B is given as

$$P(A \cap B)$$

- In this case:

$$P(A \cap B\ ) = \frac{3}{15} = \mathbf{0.2}$$

UNIONS



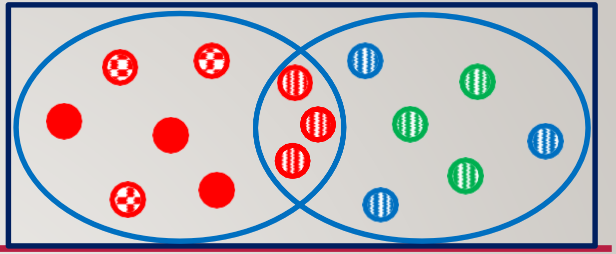
- The union of two events considers if A *or* B occurs, and is given as:

$$A \cup B$$

- Note again, order doesn't matter:

$$A \cup B = B \cup A$$

UNIONS



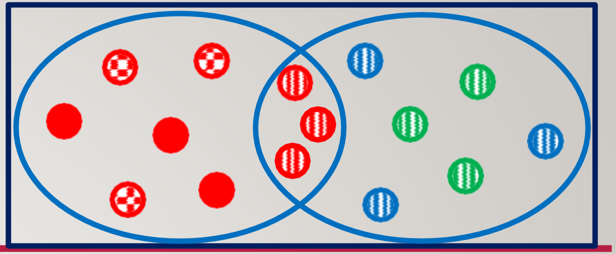
- The probability of A *or* B is given as:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- In this case:

$$P(A \cup B) = \frac{9}{15} + \frac{9}{15} - \frac{3}{15} = \frac{15}{15} = \mathbf{1.0}$$

COMPLEMENTS



- The complement of an event considers everything outside of the event, given by:

$$\bar{A}$$

- The probability of *not* A is:

$$P(\bar{A}) = 1 - P(A) = \frac{15}{15} - \frac{9}{15} = \frac{6}{15} = \mathbf{0.4}$$

INDEPENDENT

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DEPENDENT EVENTS

INDEPENDENT EVENTS

- An independent series of events occur when the outcome of one event has _no effect_ on the outcome of another.

EXAMPLE FOR INDEPENDENT EVENT

- An example is flipping a fair coin twice
- The chance of getting heads on the second toss is independent of the result of the first toss.

INDEPENDENT EVENTS

- The probability of seeing two heads with two flips of a fair coin is:

$$P(H_1 H_2) = P(H_1) \times P(H_2)$$

$$= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

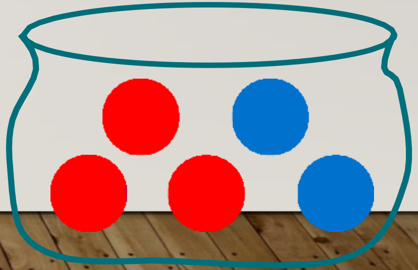
1st Toss	2nd Toss
H	H
H	T
T	H
T	T

DEPENDENT EVENTS

- A dependent event occurs when the outcome of a first event <u>does</u> affect the probability of a second event.
- A common example is to draw colored marbles from a bag *without replacement*.

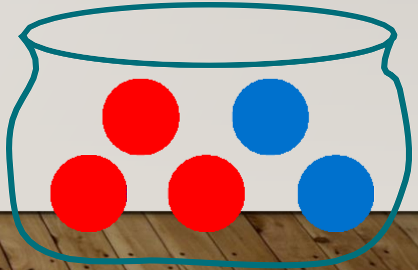
EXAMPLE FOR DEPENDENT EVENTS

- Imagine a bag contains 2 blue marbles and 3 red marbles.
- If you take two marbles out of the bag, what is the probability that they are both red?



EXAMPLE FOR DEPENDENT EVENTS

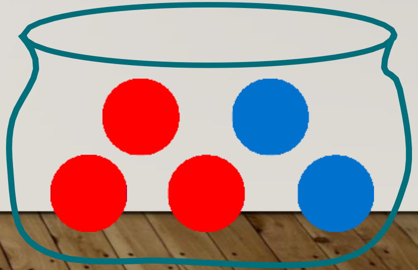
- Here the color of the first marble affects the the probability of drawing a 2nd red marble.



EXAMPLE FOR DEPENDENT EVENTS

- The probability of drawing a first red marble is easy:

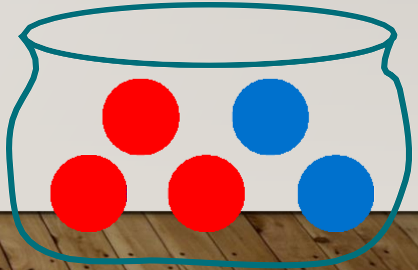
$$P(R_1) = \frac{3}{5}$$



EXAMPLE FOR DEPENDENT EVENTS

- The probability of drawing a second red marble *given that* the first marble was red is written as:

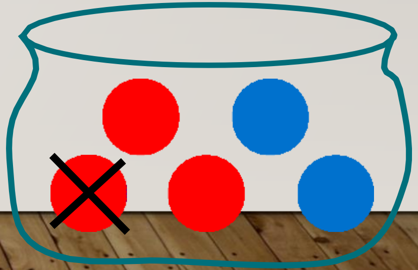
$$P(R_2|R_1)$$



EXAMPLE DEPENDENT EVENTS

- After removing a red marble from the sample set thisbecomes:

$$P(R_2|R_1) = \frac{2}{4}$$

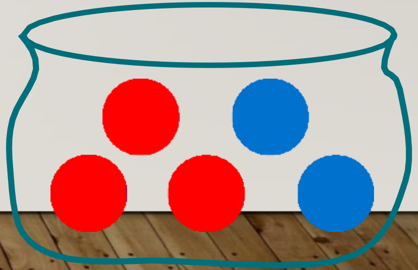


EXAMPLE FOR DEPENDENT EVENTS

- So the probability of two red marbles is:

$$P(R_1 \cap R_2) = P(R_1) \cdot P(R_2|R_1)$$

$$= \frac{3}{5} \times \frac{2}{4} = \frac{6}{20} = \mathbf{0.3}$$



CONDITIONAL PROBABILITY

CONDITIONAL PROBABILITY

- The idea that we want to know the probability of event A, *given* that event B has occurred, is conditional probability.
- This is written as $P(A|B)$

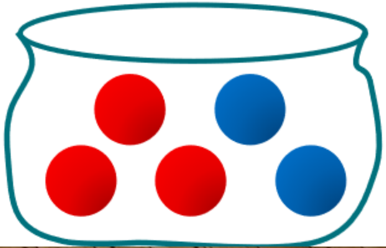
CONDITIONAL PROBABILITY

- Going back to dependent events, the probability of drawing two red marbles is:

$$P(R_1 \cap R_2) = P(R_1) \cdot P(R_2|R_1)$$

- The conditional in this equation is:

$$P(R_2|R_1)$$



CONDITIONAL PROBABILITY

- Rearranging the formula gives:

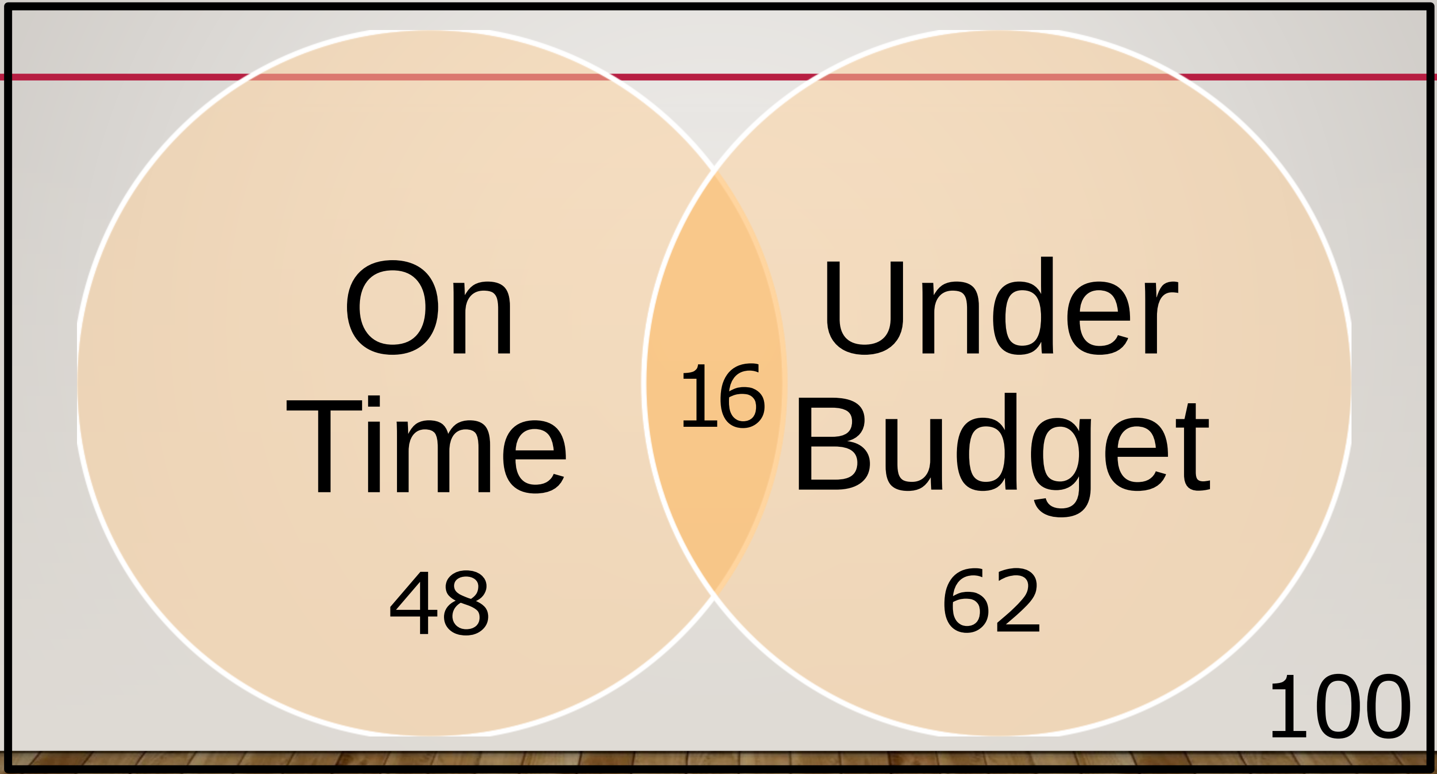
$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

- That is, the probability of A given B equals the probability of A and B divided by the probability of B

CONDITIONAL PROBABILITY EXERCISE

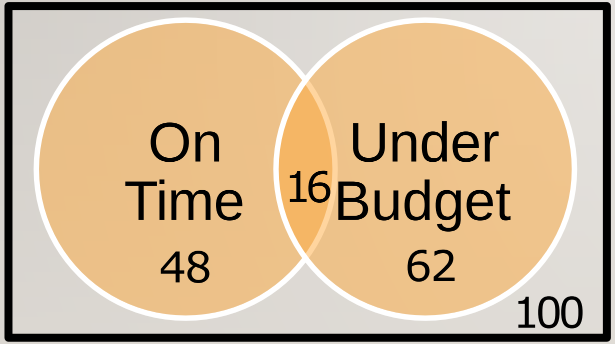
- A company finds that out of every 100 projects, 48 are completed on time, 62 are completed under budget, and 16are completed both on time and under budget.
- Given that a project is completed on time, what is the probability that it is under budget?

CONDITIONAL PROBABILITY EXERCISE



CONDITIONAL PROBABILITY EXERCISE

Given that a project is completed on time B, what is the probability that it is under budget A?



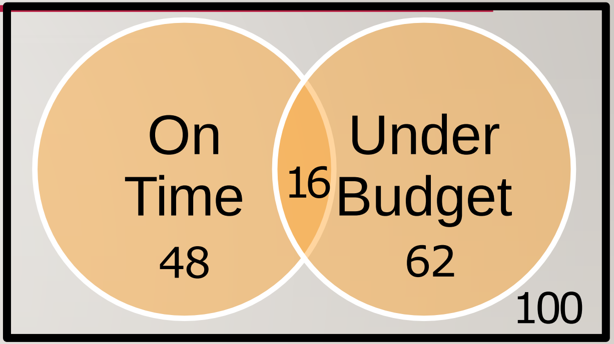
$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{16}{48} = \mathbf{0.33}$$

ADDITION & MULTIPLICATION RULES

ADDITION RULE

- From our project example, what is the probability of a project completing on time *or* under budget?



- Recall from the section on unions:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- This is the **addition rule**

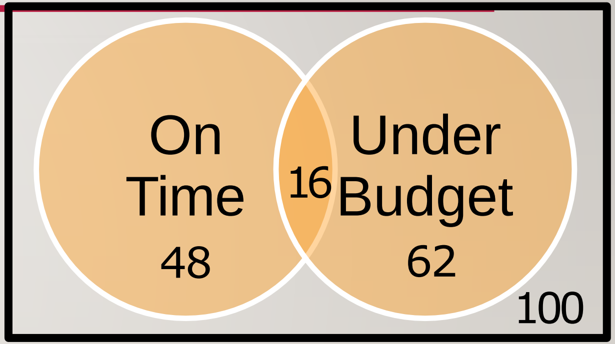
ADDITION RULE

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

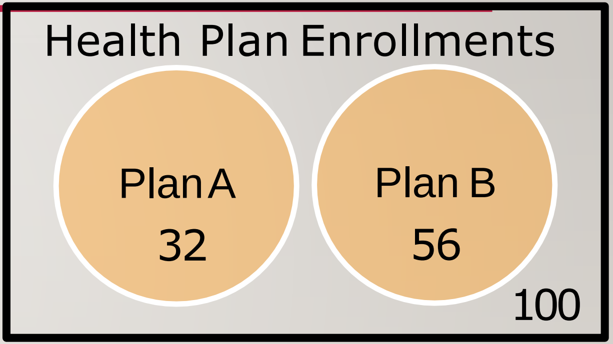
$$= \frac{48}{100} + \frac{62}{100} - \frac{16}{100}$$

$$= 0.48 + 0.62 - 0.16$$

$$= \mathbf{0.94}$$



ADDITION RULE FOR MUTUALLY EXCLUSIVE EVENTS



- When two events cannot both happen, they are said to be mutually exclusive.
- In this case, the addition rule becomes:

$$P(A \cup B) = P(A) + P(B) \cancel{-P(A \cap B)}$$

MULTIPLICATION RULE

- From the section on dependent events we saw that the probability of A and B is:

$$P(A \cap B) = P(A) \cdot P(B|A)$$

- This is the **multiplication rule**

BAYESTHEOREM

BAYESTHEOREM

- We've already seen conditional probability:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \; provided\ that\ P(B) > 0$$

$$P(A \cap B) = P(A) \cdot P(B|A) \; provided\ that\ P(A) > 0$$

BAYESTHEOREM

- We can then connect the two conditional probability formulas to get Bayes' Theorem:

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)} \ provided\ that\ P(A), P(B) > 0$$