Assignment 2
Fuzzy Sets

All the questions must be implemented in Python/C/C++

1. Let A(x) and B(x) be two fuzzy sets of speed limit on a highway, defined by the following membership functions:
   a. Fuzzy set A: "Low speed limit"

$$\mu_{A(x)} = \begin{cases} 1, if\ x < 30\ (low\ speed) \\ \dfrac{x-30}{50-30}, if\ 30 \le x < 50\ (moderately\ low\ speed) \\ \dfrac{x-50}{70-50}, if\ 50 \le x < 70\ (increasing\ speed) \\ 0, if\ x \ge 70\ (not\ low\ speed) \end{cases}$$

   b. Fuzzy set B: "High speed limit"

$$\mu_{B(x)} = \begin{cases} 0, if\ x < 60\ (not\ high\ speed) \\ \dfrac{x-60}{80-60}, if\ 60 \le x < 80\ (moderately\ high\ speed) \\ \dfrac{x-80}{100-80}, if\ 80 \le x < 100\ (increasing\ speed) \\ 1, if\ x \ge 100\ (high\ speed) \end{cases}$$

Now perform the following operations to obtain the resultant set and plot the same:

i. Union: $\mu_{A\cup B}(x) = \max(\mu_{A(x)}, \mu_{B(x)})$
ii. Intersection: $\mu_{A\cap B}(x) = \min(\mu_{A(x)}, \mu_{B(x)})$
iii. Complement of both sets: $\mu_{A'(x)} = 1 - \mu_{A(x)}$, $\mu_{B'(x)} = 1 - \mu_{B(x)}$
iv. Difference of the two sets : $A(x) - B(x) = \{(x, \mu_{A-B}(x)), x \in X\}\ where\ \mu_{A-B}(x) = \mu_{A\cap B'}(x)$

2. Let C(x) and D(x) be two fuzzy sets of temperature (Celsius) in a city, defined by the following membership functions:
   a. Fuzzy set C: "Cold"

$$\mu_{C(x)} = \begin{cases} 0, if\ x < -10\ (extremely\ cold) \\ \dfrac{x-(-10)}{0-(-10)}, if\ -10 \le x < 0\ (very\ cold) \\ 1, if\ 0 \le x < 5\ (cold) \\ 0, if\ x \ge 5\ (not\ cold) \end{cases}$$

b. Fuzzy set D: “Warm”

$$\mu_{D(x)} = \begin{cases} 1, if\ x < 25\ (not\ warm) \\ \dfrac{x-25}{35-25}, if\ 25 \leq x < 35\ (moderately\ warm) \\ 1, if\ 35 \leq x < 40\ (warm) \\ 0, if\ x \geq 40\ (extremely\ warm) \end{cases}$$

Now, perform the following operations to obtain the result:

i. Max-Min Composition:

a. $C \circ D = \max_{x_i \in x} \left(\min \left(\mu_{C(x)}, \mu_R \left(\mu_{C(x)}, \mu_{D(x)} \right) \right) \right)\ where\ R\ is\ a\ fuzzy\ relation$

b. $C \circ D = \max_{x_i \in x} \left(\min \left(\mu_{D(x)}, \mu_R \left(\mu_{C(x)}, \mu_{D(x)} \right) \right) \right)\ where\ R\ is\ a\ fuzzy\ relation$

ii. Max-Product Composition:

a. $C \circ D = \max_{x_i \in x} \left(\mu_{C(x)} \times \mu_R \left(\mu_{C(x)}, \mu_{D(x)} \right) \right)\ where\ R\ is\ a\ fuzzy\ relation$

b. $C \circ D = \max_{x_i \in x} \left(\mu_{D(x)} \times \mu_R \left(\mu_{C(x)}, \mu_{D(x)} \right) \right)\ where\ R\ is\ a\ fuzzy\ relation$