assignment_09

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1 Team Members:

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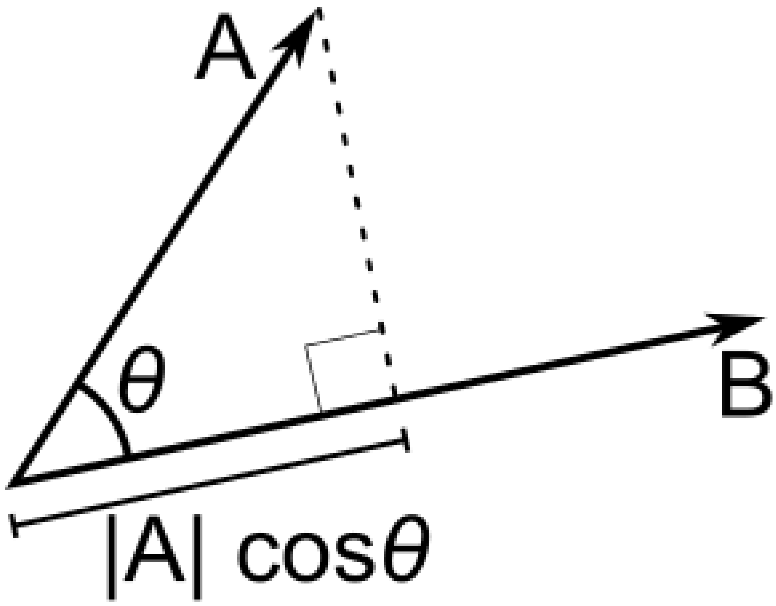
```
In [5]: import numpy as np
        import matplotlib.pyplot as plt
        import random
        import math
        from IPython.display import Image
        from IPython import display
        %matplotlib inline
        from tsp_solver.greedy import solve_tsp
```

2 Exercise 1

Show that in the SOM algorithm the winner neuron for an input x is that neuron k whose weight vector wk maximizes the inner product < wk; x > of x and wk, take x and wk as normalized.

```
In [6]: Image(filename ='../ravi/1.png')
```

```
Out[6]:
```



Then the inner product or dot product of these 2 vectors is equivalent to :

$$\mathbf{A}.\mathbf{B} = ||\mathbf{A}||\ ||\mathbf{B}||\ cos(\theta)$$

. Maximising this inner product implies $cos(\theta) = 1$ or $\theta = 0$. In other words, the euclidean distance between the 2 vectors is minimised. Since we need to minimize the distance between a winning neuron and the input vector, hence the inner product $< wk; x >$ needs to be maximised

3 Exercise 2

```
In [7]: class SOM:
            def __init__(self,input_,num_of_nodes,eta,initial_wts,threshold):
                self.input_ = input_
                self.num_of_nodes = num_of_nodes
                self.eta = eta
                self.current_wts = initial_wts
                self.t2 = 1000
                self.threshold = threshold

            def euclidean_distance_1d(self,x,y):
                return abs(x - y)

            def get_winning_neuron(self,x,W):
                winner = min([(self.euclidean_distance_1d(x,w), index) for index,w in enumerate(
                return winner

            def d_ij(self,winner):
```

```
        distance = [self.euclidean_distance_1d(self.current_wts[winner],\
                                               self.current_wts[i])\
                    for i in range(len(self.current_wts))]
        return distance

    def gaussian(self,sigma,distance):
        h = [np.exp(-(d**2)/(2*sigma**2)) for d in distance]
        return h

    def compute_width(self,initial_sigma,n,t1):
        return initial_sigma*np.exp(-n/t1)

    def weight_adaptation(self,current_wt,eta,h,x):
        new_wts = [(w + (eta*h*(x-w))) for w in current_wt]
        return new_wts

    def exponential_decay_update(self,initial_eta,n,t2):
        return initial_eta*np.exp(-n/t2)

    def compute_t1(self,t2):
        sigma=2
        return t2/np.log(sigma)

    def stopping_criteria(self,w_old,w_new):
        result = 0
        for i,w in enumerate(w_old):
            result += abs(w - w_new[i])

        if (result < self.threshold):
            print "Stopping criteria check:sum(w_old - w_new) : " \
            +str(result)+"< "+str(self.threshold)+". Iteration end for current input"
            return True
        else:
            print "Stopping criteria check :sum(w_old - w_new)  : " \
            +str(result)+"> "+str(self.threshold)
            return False

    def train(self):
        sigma = 2
        t1 = self.compute_t1(self.t2)
        n = 1
        for x in self.input_:
            while(True):
                win_idx = self.get_winning_neuron(x,self.current_wts)
                print "\nWinner neuron index : "+str(win_idx)+", weight :"+str(self.curr
                lateral_dist = self.d_ij(win_idx)
                h = self.gaussian(sigma,lateral_dist)
                updates_wts = self.weight_adaptation(self.current_wts,self.eta,h[win_idx
```

```
                        print "Updated weights :",updates_wts
                        if not self.stopping_criteria(self.current_wts,updates_wts):
                            self.current_wts =  np.array(updates_wts)
                            self.eta = self.exponential_decay_update(self.eta,n,self.t2)
                            n += 1
                        else:
                            break
                print "\nFinal adjusted weights :",self.current_wts
```

```
In [8]: initial_wts = np.array([[0.15,0.45],
                                [0.3,0.9]])
        inputs = [0.1,0.2,0.4,0.5]
```

```
In [9]: """
        Initial weights : [0.15 , 0.45]
        """
        som = SOM(inputs,2,0.1,initial_wts[0],0.01)
        som.train()
```

```
Winner neuron index : 0, weight :0.15
Updated weights : [0.14499999999999999, 0.41500000000000004]
Stopping criteria check :sum(w_old - w_new)  : 0.04> 0.01

Winner neuron index : 0, weight :0.145
Updated weights : [0.1433445425147285, 0.40341179760309959]
Stopping criteria check :sum(w_old - w_new)  : 0.0132436598822> 0.01

Winner neuron index : 0, weight :0.143344542515
Updated weights : [0.14275793792092928, 0.39930556544650508]
Stopping criteria check:sum(w_old - w_new) : 0.00469283675039< 0.01. Iteration end for current i

Winner neuron index : 0, weight :0.143344542515
Updated weights : [0.1441112907532954, 0.40065891827887118]
Stopping criteria check:sum(w_old - w_new) : 0.0035196275628< 0.01. Iteration end for current in

Winner neuron index : 1, weight :0.403411797603
Updated weights : [0.14681799641802767, 0.40336562394360342]
Stopping criteria check:sum(w_old - w_new) : 0.0035196275628< 0.01. Iteration end for current in

Winner neuron index : 1, weight :0.403411797603
Updated weights : [0.1481713492503938, 0.40471897677596957]
Stopping criteria check:sum(w_old - w_new) : 0.00613398590854< 0.01. Iteration end for current i

Final adjusted weights : [ 0.14334454  0.4034118 ]
```

```
In [10]: """
         Initial weights : [0.3,0.9]
```

```
    """
    som = SOM(inputs,2,0.1,initial_wts[1],0.01)
    som.train()
```

```
Winner neuron index : 0, weight :0.3
Updated weights : [0.27999999999999997, 0.82000000000000006]
Stopping criteria check :sum(w_old - w_new)  : 0.1> 0.01

Winner neuron index : 0, weight :0.28
Updated weights : [0.27337817005891402, 0.79351268023565624]
Stopping criteria check :sum(w_old - w_new)  : 0.0331091497054> 0.01

Winner neuron index : 0, weight :0.273378170059
Updated weights : [0.27103175168371713, 0.78412700673486879]
Stopping criteria check :sum(w_old - w_new)  : 0.011732091876> 0.01

Winner neuron index : 0, weight :0.271031751684
Updated weights : [0.27018023473230185, 0.78072093892920769]
Stopping criteria check:sum(w_old - w_new) : 0.00425758475708< 0.01. Iteration end for current i

Winner neuron index : 0, weight :0.271031751684
Updated weights : [0.27067810541598047, 0.78121880961288637]
Stopping criteria check:sum(w_old - w_new) : 0.00326184338972< 0.01. Iteration end for current i

Winner neuron index : 0, weight :0.271031751684
Updated weights : [0.27167384678333778, 0.78221455098024362]
Stopping criteria check:sum(w_old - w_new) : 0.00255455085425< 0.01. Iteration end for current i

Winner neuron index : 0, weight :0.271031751684
Updated weights : [0.27217171746701641, 0.7827124216639223]
Stopping criteria check:sum(w_old - w_new) : 0.00255455085425< 0.01. Iteration end for current i

Final adjusted weights : [ 0.27103175  0.78412701]
```

Results show that when starting from initial weights [0.15, 0.45], the network converges (i.e, the stopping criteria is satisfied) in a smaller number of iterations compared to when we start with the initial weights of [0.3, 0.9]. Furthermore, when using initial weights of [0.3, 0.9], the neuron with the initial weight 0.3 is selected as the winning neuron at every iteration.

4 Exercise 3

```
In [13]: class TravellingSalesMan:
             def __init__(self,_no_of_neurons,
                          _no_of_cities,
                          _lattice_radius,
                          _lattice_center,
```

```
                  _eta,
                  _no_epochs):

        self.no_of_neurons = _no_of_neurons
        self.no_of_cities = _no_of_cities
        self.lattice_radius = _lattice_radius
        self.lattice_center = _lattice_center
        self.eta = _eta
        self.no_of_epochs = _no_epochs

        self.current_weights = self.points_in_circle()
        print "Initial coordinates of neurons"
        print self.current_weights

        self.cities = self.get_cities()
        print "Initial coordinates of cities"
        print self.cities

        self.sigma = (self.lattice_radius * 2) + 5
        self.t_one = self.no_of_epochs/np.log(self.sigma)

        #city labels
        self.city_labels = ['wismar','schwerin','rostock',
                            'stralsund','greifswald',
                            'neubrandenberg']

    '''
    find euclidean distance between two coordinates
    '''
    def euclidean_distance_2d(self,x, y):
        return math.sqrt(pow(y[0]-x[0],2) + pow(y[1]-x[1],2))

    '''
    find euclidean distance between given input
    and all neurons weight, and return winning neuron
    with shortest distance
    '''
    def get_winner(self,x):
        return min([(self.euclidean_distance_2d(x,w),index)
                    for index,w in enumerate(self.current_weights)])[1]

    '''
    return city coordinates
    '''
    def get_cities(self):
        return np.array([[1.3,5.7],
                         [30.7,98.3],
                         [95.3,69.3],
```

```
                      [37.3,22.5],
                      [85.5,12.5],
                      [46.6,63.6]])

def compute_distance_for_TspSolve(self):

    a = self.get_cities()
    dist = np.zeros((0,a.shape[0]))
    for i in range(a.shape[0]):
        euclidean = []
        for j in a:
            euclidean.append(tsm.euclidean_distance_2d(a[i],j))
        dist = np.vstack((dist, np.hstack((euclidean))))
    return dist

'''
plot cities and neuron locations
'''
def plot_cities(self,epoch):

    #getting x,y coordinates of cities
    cities_x = self.cities[:,0:1]
    cities_y = self.cities[:,1:2]

    #configuration for plot
    fig, ax = plt.subplots()
    plt.xlim([1,120])
    plt.ylim([1,120])
    fig.set_figheight(9)
    fig.set_figwidth(15)
    plt.title("Sigma = " + str(self.sigma)+ \
              ", Learning rate = " + str(self.eta) + \
              ", Current epoch = " + str(epoch))
    plt.xlabel("x-coordinates of cities and neuron weights")
    plt.ylabel("y-coordinates of cities and neuron weights")


    #plot cities
    ax.plot(cities_x, cities_y,'ro')

    #plot neurons
    if epoch is 0:
        ax.add_patch(plt.Circle(self.lattice_center,
                                radius=self.lattice_radius,
                                color='g',
                                fill=False))

    #plot connections between neurons
```

```
        if epoch is not 0:
            ax.plot(self.current_weights[:,0:1],
                    self.current_weights[:,1:2],
                    marker='o', linestyle='--',
                    color='b')
        else:
            ax.plot(self.current_weights[:,0:1],
                    self.current_weights[:,1:2],'bo')

        #showing name of the cities in the plot
        for i, label in enumerate(self.city_labels):
            ax.annotate(label, (cities_x[i]+1,cities_y[i]+1))

        #showing name of the neurons in the plot
#          print current_weights
        for index, weight in enumerate(self.current_weights):
            ax.annotate(index, (weight[0]+0.5,weight[1]+1.5))


    '''
    generate points in circle lattice structure
    '''
    def points_in_circle(self):
        points = np.empty((0,2))
        circle_center = self.lattice_center
        radius = self.lattice_radius
        n = self.no_of_neurons
        for x in xrange(0,n):
            point = [circle_center[0]+np.cos(2*np.pi/n*x)*radius,
                     circle_center[1]+np.sin(2*np.pi/n*x)*radius]
            points = np.vstack([points,point])
        return points

    '''
    update weight function
    '''
    def weight_adaptation(self,neuron,winner_neuron,x):
        current_weight = self.current_weights[neuron]
        h_ij = self.calculate_H_i_j(neuron,winner_neuron)
        new_weight = (current_weight + (self.eta*(h_ij)*(x-current_weight)))
        self.current_weights[neuron] = new_weight
        return True

    '''
    update learning rate funtion
    '''
    def eta_update(self,n):
        self.eta = self.eta*np.exp(-n/1000000000.0)
```

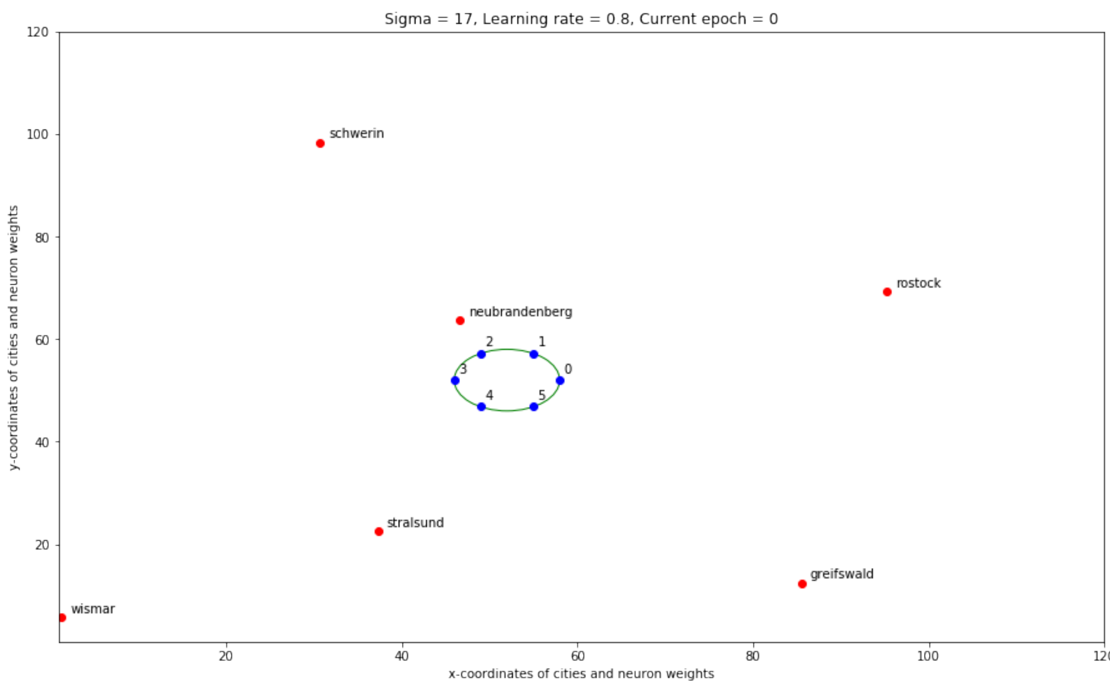
```python
'''
calcuate H_ij function
'''
def calculate_H_i_j(self,current_neuron,winner_neuron):
    distance = self.euclidean_distance_2d(self.current_weights[winner_neuron],
                                          self.current_weights[current_neuron])
    return np.exp(-(distance**2)/(2*(self.sigma**2)))

'''
sigma updation function
'''
def sigma_update(self,n):
    self.sigma = self.sigma * np.exp(-(n/1000000000.0))


'''
get neighbors of winning neuron
'''
def get_neighors(self,winner):
    neuron_positions = range(self.current_weights.shape[0])
    if winner is len(neuron_positions)-1:
        return [neuron_positions[winner],
                neuron_positions[winner-1],
                neuron_positions[0]]
    else:
        return [neuron_positions[winner],
                neuron_positions[winner-1],
                neuron_positions[winner+1]]


'''
sort the visit order
'''
def find_visit_order(self):
    order = []
    for city in self.cities:
        order.append(self.get_winner(city))

    self.sorted_order = []

    for i in range(len(order)):
        self.sorted_order.append(order.index(i))

    self.city_order = []
    for index in self.sorted_order:
        self.city_order.append(self.city_labels[index])
```

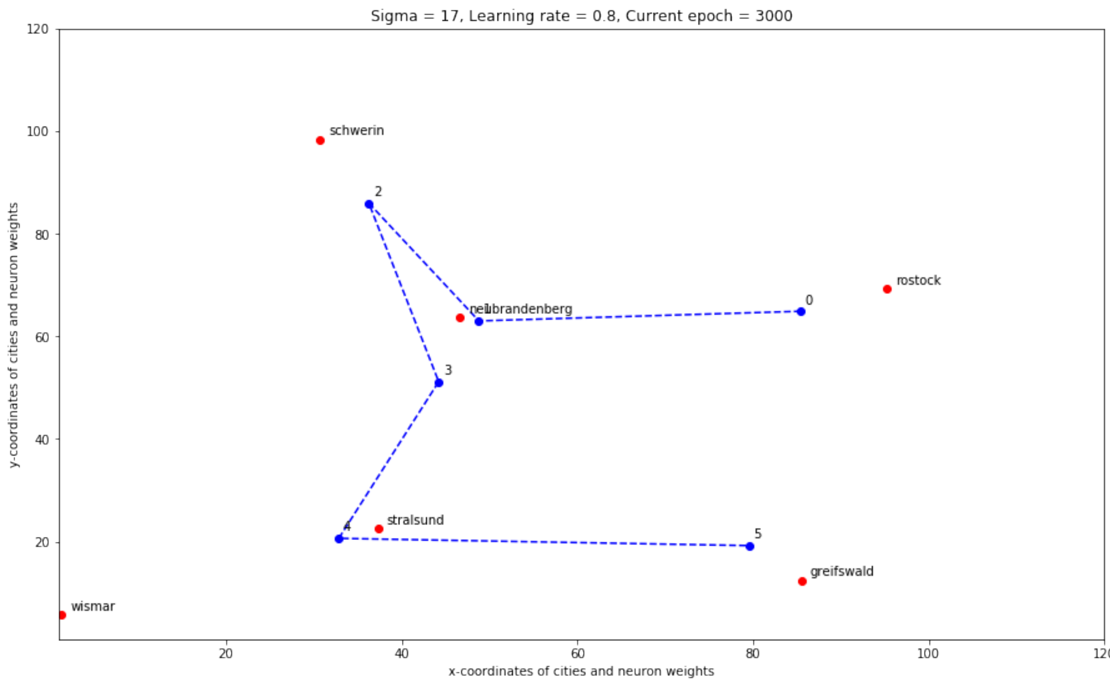
```
            return self.sorted_order

        '''
        calculate the distance to visit
        '''
        def calculate_total_path(self,order):
            distance = 0

            for index in range(len(order)-1):
                distance += self.euclidean_distance_2d(self.cities[order[index]],
                                                      self.cities[order[index+1]])
            distance += self.euclidean_distance_2d(self.cities[order[0]],
                                                   self.cities[order[-1]])
            self.total_distance = distance
            return self.total_distance

        '''
        training function
        '''
        def train(self):
            for epoch in range(self.no_of_epochs):
                for city in self.cities:
                    winner_neuron = self.get_winner(city)
                    winner_with_neighbors = self.get_neighors(winner_neuron)
                    for neuron in winner_with_neighbors:
                        self.weight_adaptation(neuron,winner_neuron,city)
                    self.eta_update(epoch)
                if epoch % 3000 is 0:
                    self.plot_cities(epoch+3000)
                self.sigma_update(epoch)

            print self.calculate_total_path(self.find_visit_order())
```

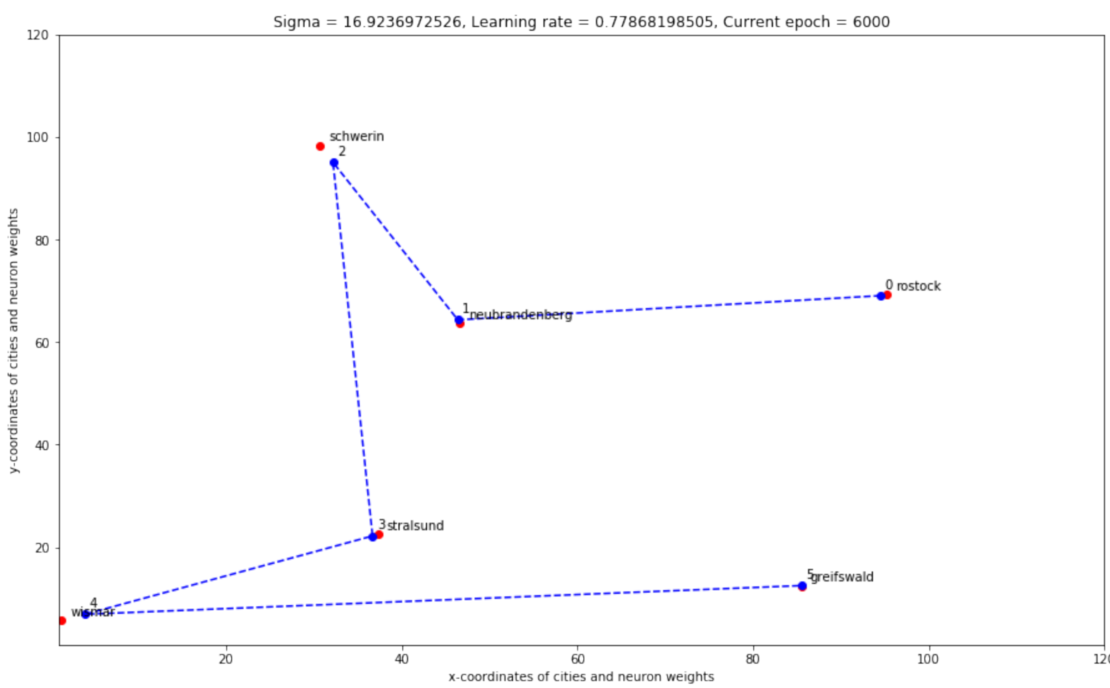
```
In [14]: '''
         Initialization
         '''
         no_of_neurons = 6
         no_of_cities = 6
         lattice_radius = 6
         lattice_center = (52, 52)
         eta = 0.8
         no_of_epochs = 30000

         tsm = TravellingSalesMan(no_of_neurons,
                                   no_of_cities,
                                  lattice_radius,
                                  lattice_center,eta,
                                  no_of_epochs)
```

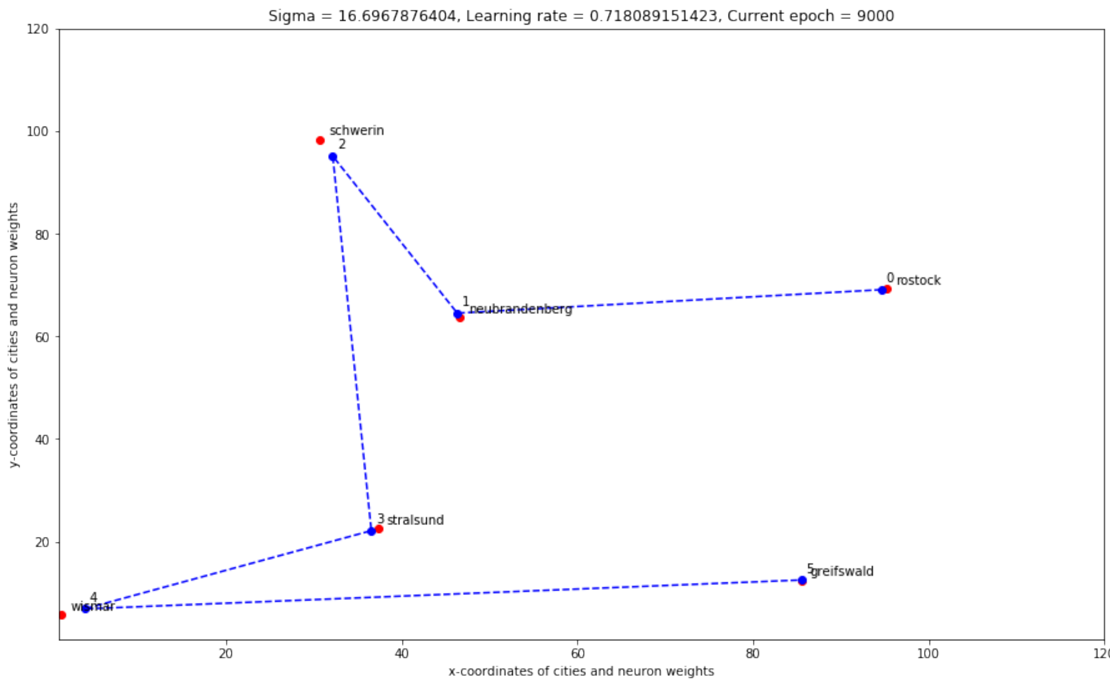
```
tsm.plot_cities(0)
tsm.train()
```

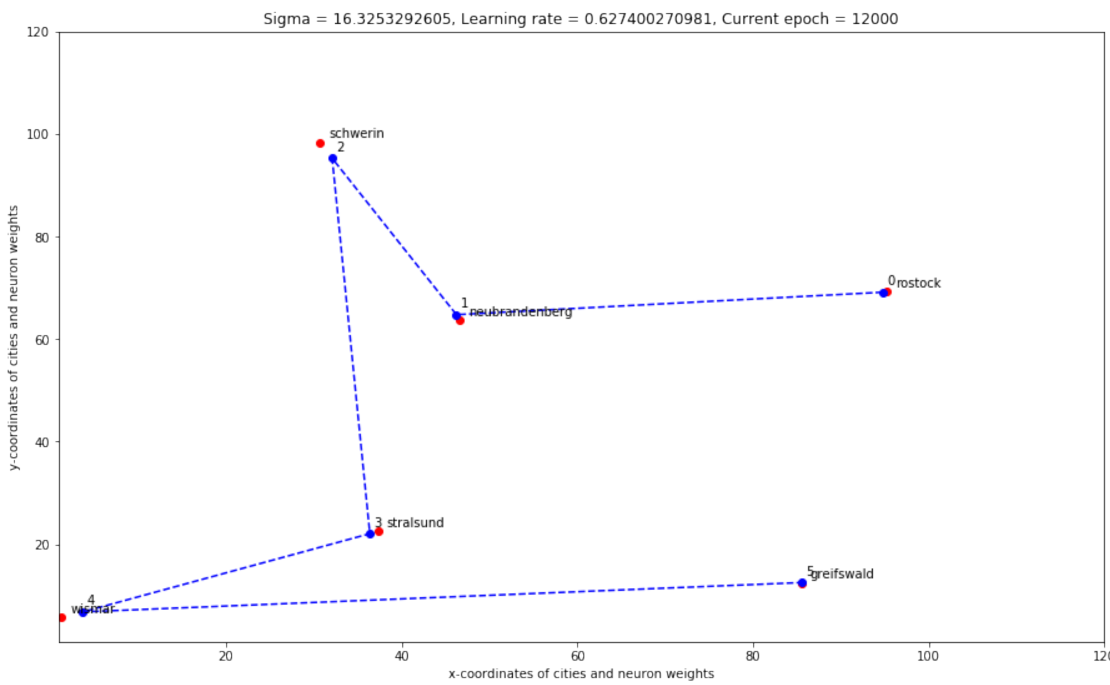
```
Initial coordinates of neurons
[[ 58.          52.        ]
 [ 55.          57.19615242]
 [ 49.          57.19615242]
 [ 46.          52.        ]
 [ 49.          46.80384758]
 [ 55.          46.80384758]]
Initial coordinates of cities
[[  1.3   5.7]
 [ 30.7  98.3]
 [ 95.3  69.3]
 [ 37.3  22.5]
 [ 85.5  12.5]
 [ 46.6  63.6]]
345.129019896
```

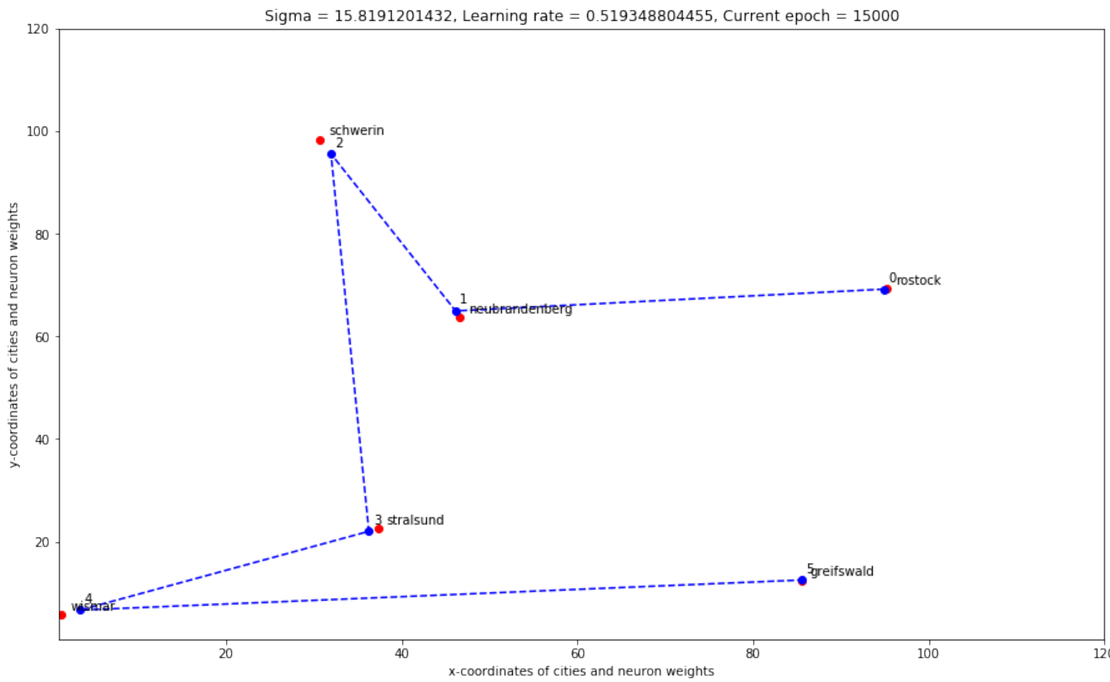


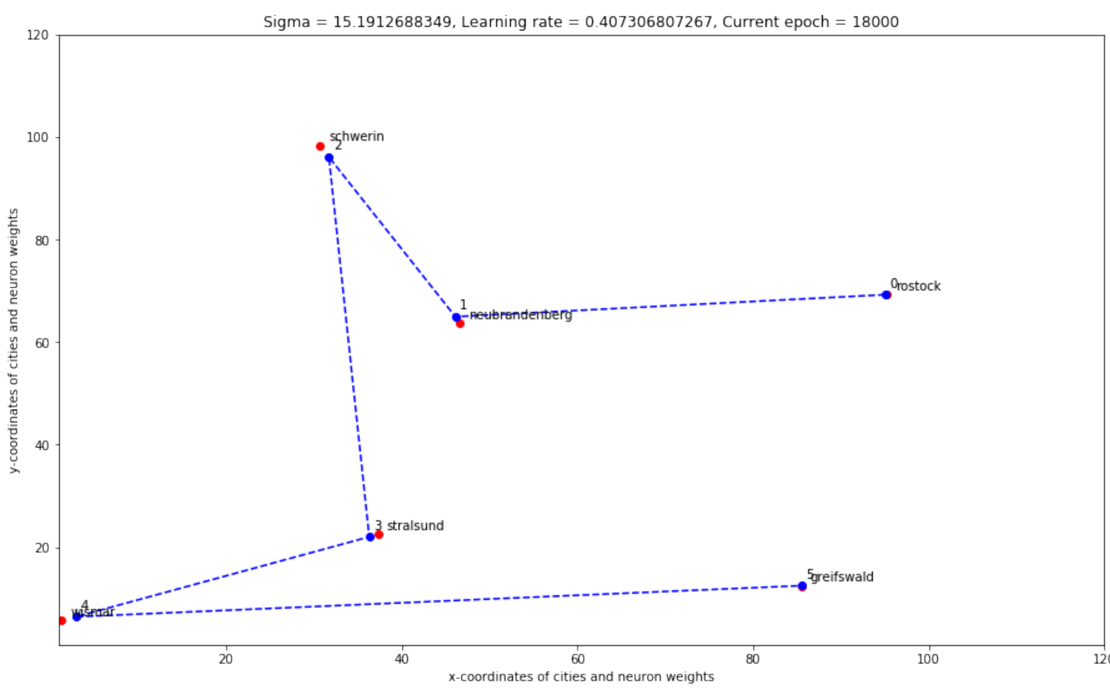

Sigma = 17, Learning rate = 0.8, Current epoch = 3000
schwerin
rostock
neubrandenberg
stralsund
greifswald
wismar
0
1
2
3
4
5
120
100
80
60
40
20
20
40
60
80
100
120
x-coordinates of cities and neuron weights
y-coordinates of cities and neuron weights

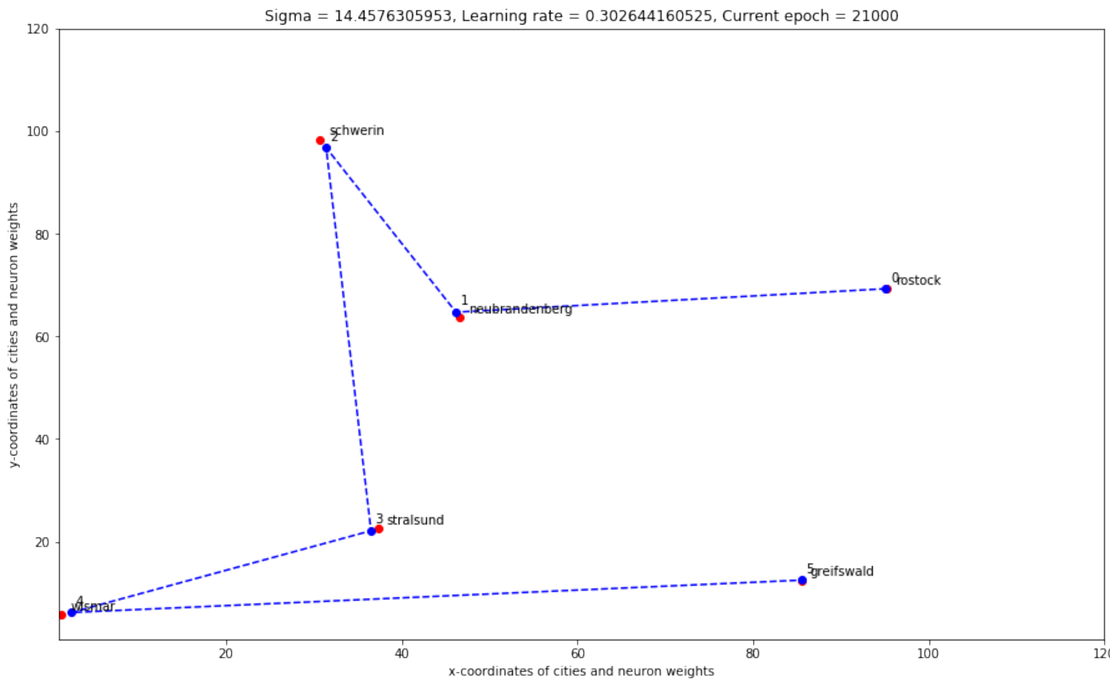

Sigma = 16.9236972526, Learning rate = 0.77868198505, Current epoch = 6000
schwerin
rostock
neubrandenberg
stralsund
greifswald
wismar
0
1
2
3
4
5
120
100
80
60
40
20
20
40
60
80
100
120
x-coordinates of cities and neuron weights
y-coordinates of cities and neuron weights

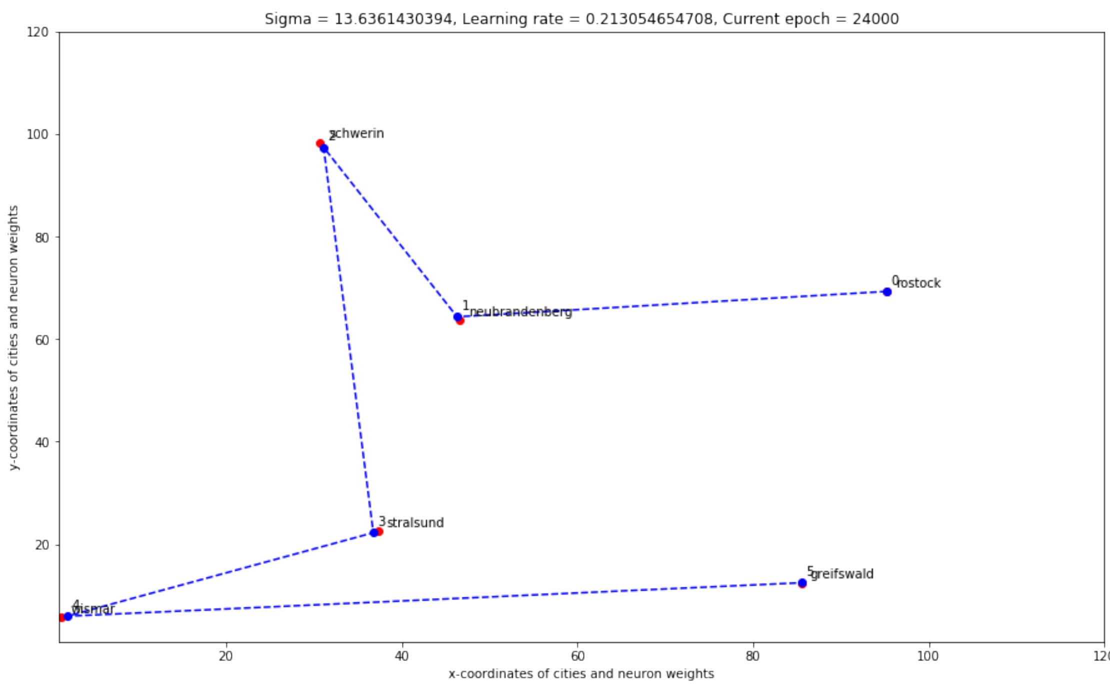

Sigma = 16.6967876404, Learning rate = 0.718089151423, Current epoch = 9000
schwerin
2
0
rostock
1
neubrandenberg
3
stralsund
5
greifswald
4
wismar
y-coordinates of cities and neuron weights
x-coordinates of cities and neuron weights
120
100
80
60
40
20

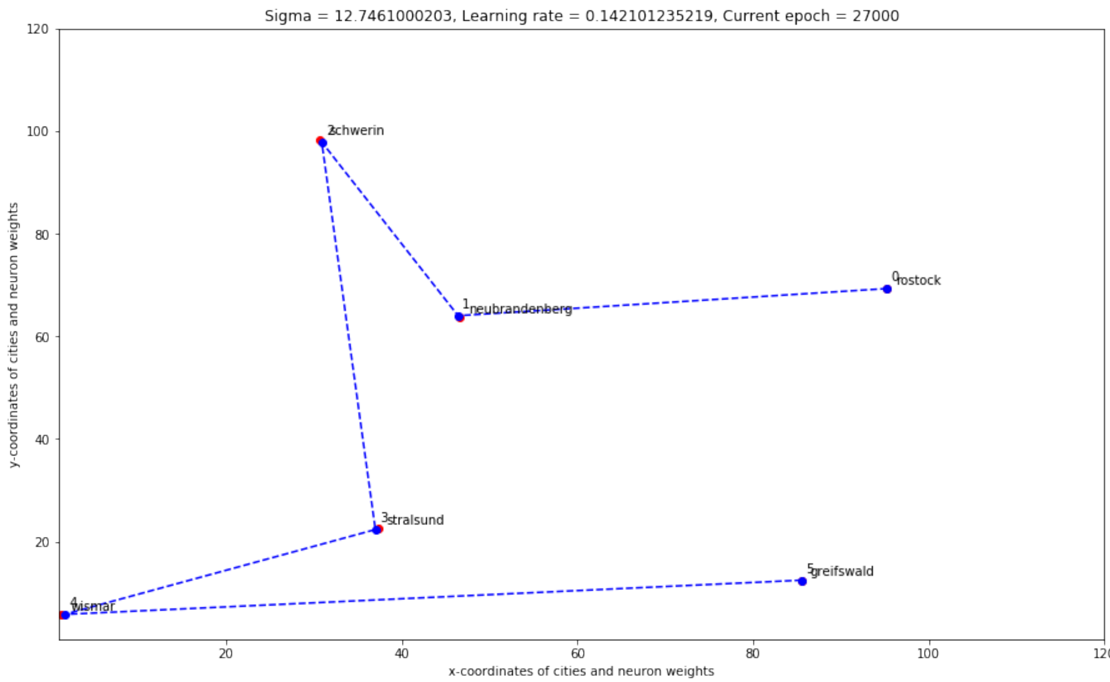

Sigma = 16.3253292605, Learning rate = 0.627400270981, Current epoch = 12000
schwerin
2
0
rostock
1
neubrandenberg
3
stralsund
5
greifswald
4
wismar
y-coordinates of cities and neuron weights
x-coordinates of cities and neuron weights
120
100
80
60
40
20

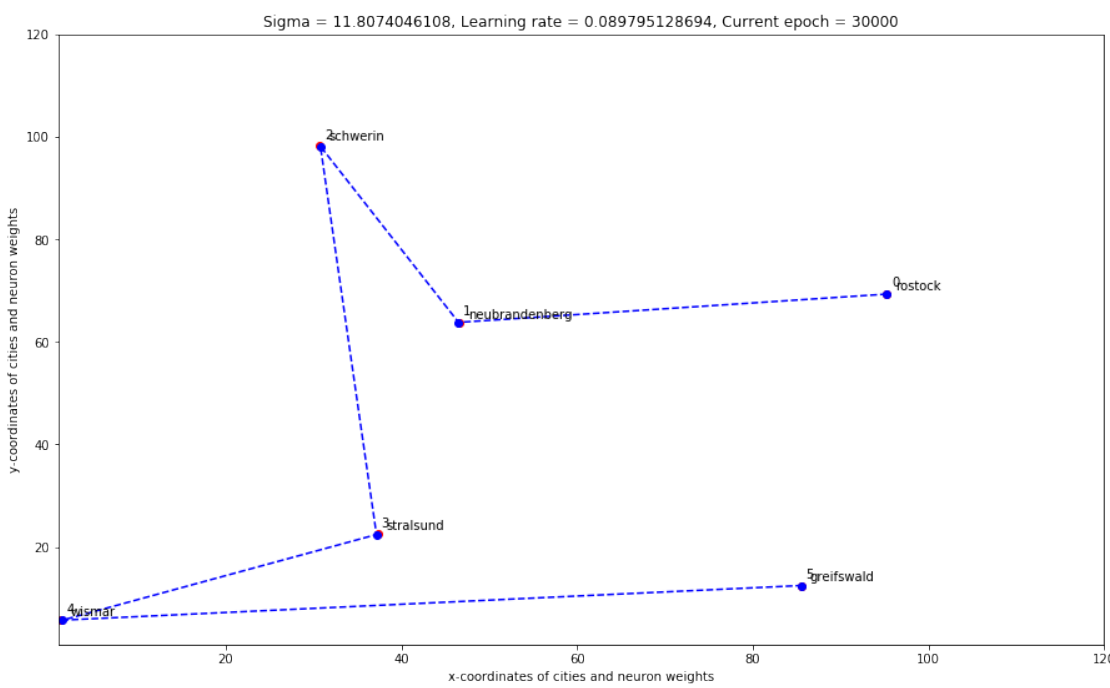

Sigma = 15.8191201432, Learning rate = 0.519348804455, Current epoch = 15000
y-coordinates of cities and neuron weights
x-coordinates of cities and neuron weights
schwerin
rostock
neubrandenberg
stralsund
greifswald
0
1
2
3
4
5


Sigma = 15.1912688349, Learning rate = 0.407306807267, Current epoch = 18000
y-coordinates of cities and neuron weights
x-coordinates of cities and neuron weights
schwerin
rostock
neubrandenberg
stralsund
greifswald
0
1
2
3
4
5


Sigma = 14.4576305953, Learning rate = 0.302644160525, Current epoch = 21000
y-coordinates of cities and neuron weights
x-coordinates of cities and neuron weights
schwerin
rostock
neubrandenberg
stralsund
greifswald
wismar


Sigma = 13.6361430394, Learning rate = 0.213054654708, Current epoch = 24000
y-coordinates of cities and neuron weights
x-coordinates of cities and neuron weights
schwerin
rostock
neubrandenberg
stralsund
greifswald
wismar


Sigma = 12.7461000203, Learning rate = 0.142101235219, Current epoch = 27000
y-coordinates of cities and neuron weights
x-coordinates of cities and neuron weights
0 rostock
1 neubrandenberg
2 schwerin
3 stralsund
4 wismar
5 greifswald


Sigma = 11.8074046108, Learning rate = 0.089795128694, Current epoch = 30000
y-coordinates of cities and neuron weights
x-coordinates of cities and neuron weights
0 rostock
1 neubrandenberg
2 schwerin
3 stralsund
4 wismar
5 greifswald

5 Initial order of cities before training

0.'wismar'
1.'schwerin'
2.'rostock'
3.'stralsund'
4.'greifswald'
5.'neubrandenberg'

6 After training

```
In [15]: print "Order of cities to visit: ", tsm.sorted_order
         print "Order of cities to visit: ", tsm.city_order
         print "Total travel distance : ", tsm.total_distance
```

```
Order of cities to visit:  [2, 5, 1, 3, 0, 4]
Order of cities to visit:  ['rostock', 'neubrandenberg', 'schwerin', 'stralsund', 'wismar', 'gre
Total travel distance :  345.129019896
```

7 Path using TSP library

```
In [16]: #compute path using TSP solve library
         distances = tsm.compute_distance_for_TspSolve()
         paths = solve_tsp(distances)

         travelled_distance =  tsm.calculate_total_path(paths)
         print "total travel distance ", travelled_distance
         # print "Cties are travelled in ", paths

         print "Cities are travelled in order "
         for path in paths:
             print " >> " + str(tsm.city_labels[path]),
```

```
total travel distance  332.959578276
Cities are travelled in order
 >> wismar  >> stralsund  >> neubrandenberg  >> schwerin  >> rostock  >> greifswald
```

8 Observations:

If two cities are in straight line and closer together, neurons stuck in the middle and cannot move towards the cities.

If we update only the winning neuron, sometimes observed one or two neurons stays in its initial position.

So we are updating the neighborhood neurons of winning neurons using H_ij function, then all neurons started to move towards cities.

Paths that we obtained using TSP library are different, as compare to algorithm that we implement. However, total distance is approximately closer.