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```
In [1]: import numpy as np
        import sympy as sp
```

1 Exercise 3.2 part (a)

```
In [24]: wo,w1 = sp.symbols("w0,w1")

         w = sp.Matrix([wo,w1])
         r = sp.Matrix([0.8182,0.354])
         R = sp.Matrix([[1,0.8182],
                        [0.8182,1]])
         print "The weight vector w ="
         sp.pprint(w)
         print "\n r_xd = "
         sp.pprint(r)
         print "\nAnd R_x = "
         sp.pprint(R)
```

```
The weight vector w =
w

w

 r_xd =
0.8182

0.354

And R_x =
  1     0.8182

0.8182    1
```

$\xi(\omega) = \frac{1}{2}\sigma^2 - r_{xd}^T\omega + \frac{1}{2}\omega^T R_x \omega$. Substituting the values of r_{xd} and R_x, the values of $r_{xd}^T\omega$ and $\frac{1}{2}\omega^T R_x \omega$ is :

```
In [13]: rw = r.T * w
         sp.pprint(rw)

[0.8182w + 0.354w]
```

```
In [14]: w_tRw = 0.5*w.T*R*w
         sp.pprint(w_tRw)

[w(0.5w + 0.4091w) + w(0.4091w + 0.5w)]
```

Thus $\xi(\omega) = \frac{1}{2}\sigma^2 - [0.8182\omega_0 + 0.354\omega_1] + [\omega_0(0.5\omega_0 + 0.4091\omega_1) + \omega_1(0.4091\omega_0 + 0.5\omega_1)]$
$\xi(\omega) = \frac{1}{2}\sigma^2 + 0.5.\omega_0^2 + 0.5.\omega_1^2 + 0.8182.\omega_0.\omega_1 - 0.8182.\omega_0 - 0.354.\omega_1$

```
In [15]: sigma = sp.Symbol('sigma')
         xi = 0.5*sigma**2 + (0.5*wo**2) + (0.5*w1**2) + (0.8182*wo*w1) - (0.8182*wo) - (0.354*w
         eqn1 = sp.diff(xi,wo)
         eqn2 = sp.diff(xi,w1)
         gradient = sp.Matrix([eqn1,eqn2])
         print "Gradient :"
         sp.pprint(gradient)

Gradient :
1.0w + 0.8182w - 0.8182

0.8182w + 1.0w - 0.354
```

```
In [16]: w_optimal = sp.solve([eqn1,eqn2],[wo,w1])
         print "The optimal weight vector = ",w_optimal

The optimal weight vector =  {w0: 1.59902944424901, w1: -0.954325891284541}
```

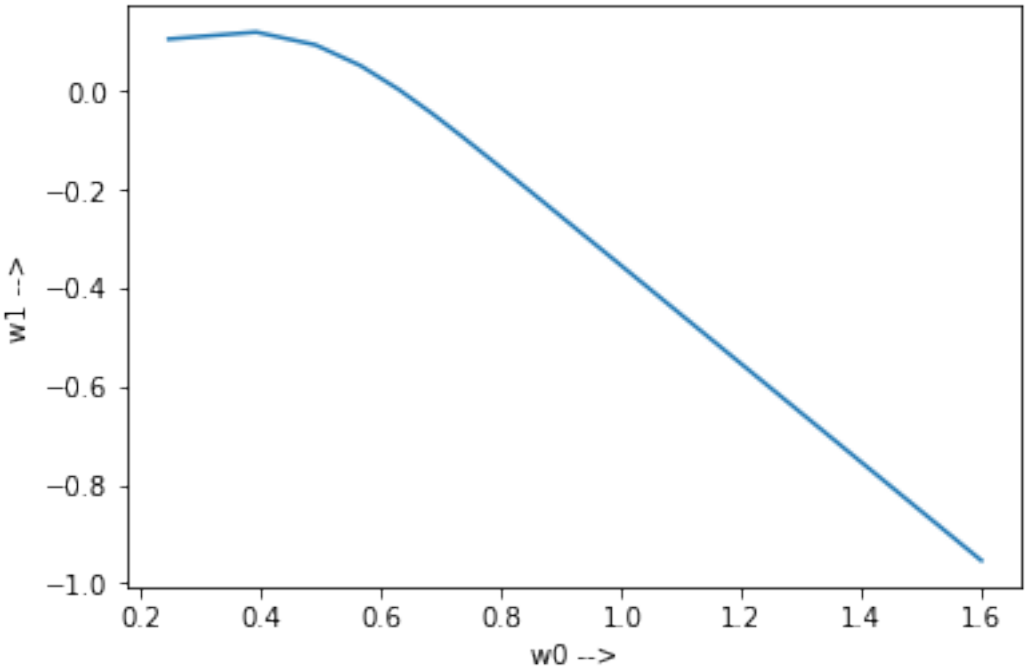
2 Exercise 3.2 part (b)

```
In [17]: import matplotlib.pyplot as plt
         %matplotlib inline


         def steepest_descent(eta, xi , gradient):
             w_init = np.array([0,0])

             w_n = w_init.T
             xi_n = xi.subs({wo:w_n[0],w1:w_n[1]})
             gradient_n = gradient.subs({wo:w_n[0],w1:w_n[1]})
```

```
    w_0 = []
    w_1 = []

    for i in range(200):
        eta_gn  =  np.asarray(eta*gradient_n)
        w_new = w_n - np.array([eta_gn[0,0],eta_gn[1,0]])
        #Update the weight vector, the gradient and corresponding value of xi

        gradient_n = gradient.subs({wo:w_new[0],w1:w_new[1]})
        xi_new = xi.subs({wo:w_new[0],w1:w_new[1]})

        xi_n = xi_new
        w_n = w_new

        w_0.append(w_n[0])
        w_1.append(w_n[1])

    w_0 = np.asarray(w_0)
    w_1 = np.asarray(w_1)

    plt.plot(w_0,w_1)
    plt.xlabel('w0 -->')
    plt.ylabel('w1 -->')
    return w_n
```

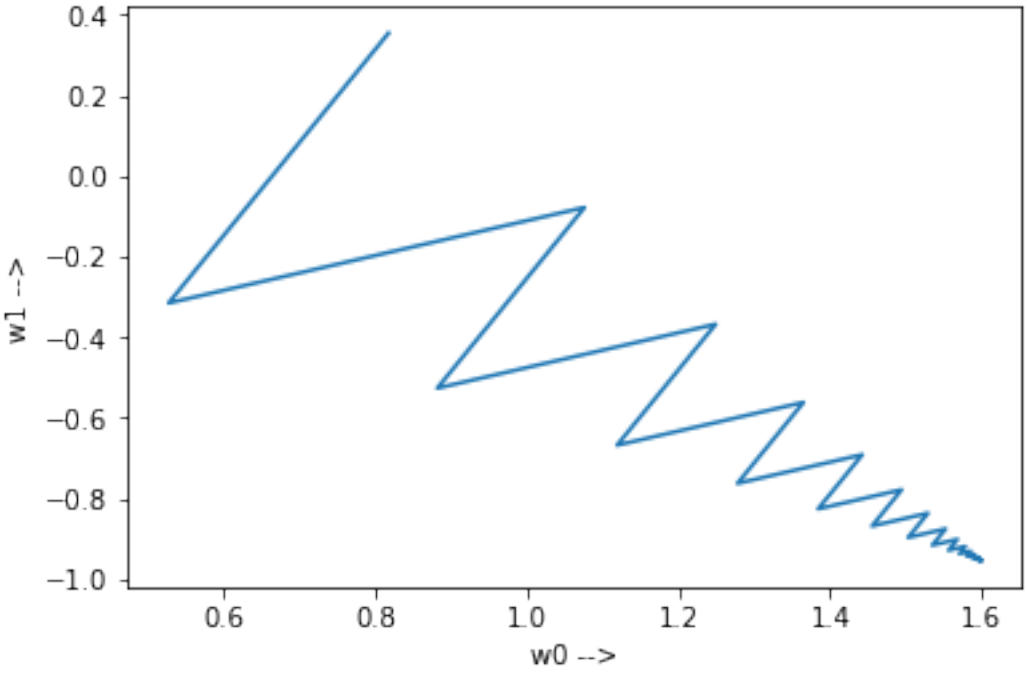
In [18]:
```
#Learning rate = 0.3
sp.pprint(steepest_descent(0.3,xi,gradient))
```

```
[1.59901227650625 -0.954308723541781]
```



In [19]:
```
#Learning rate = 1.0
sp.pprint(steepest_descent(1.0,xi,gradient))
```

```
[1.59902944424901 -0.954325891284542]
```



3 Exercise 3.4

Given correlation matrix R_x of the input vector $x(n)$ in the LMS algorithm

$$\mathbf{R_x} = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$$

Define the range of values for the learning-rate parameter η of the LMS algorithim for it to be convergent in the mean square.

4 Solution

The learning rate η and the input vector x(n) that determine the trasmittance of the feedback loop. So, the convergence behaviour of the LMS algorithm is influenced by the statistical characteristics behaviour of the input vector and the value of learning parameter η.

As shown in Haykin (2006), that the LMS is convergant in the mean square provided that η satisfies the following condition, $0 < \eta < \frac{2}{\lambda_{max}}$, where λ_{max} is the largest eigen value of the correlation matrix R_x.

But in some typical applications of the LMS algorithm, the knowledge of λ_{max} is not available, so to avoid this problem we may take trace of R_X which is a conservative estimate of λ_{max}. So to be convergent in the mean square, the condition should be $0 < \eta < \frac{2}{tr[R_x]}$

```
In [21]: #using trace of correlation method
         correlation_matrix = np.array([[1,0.5],[0.5,1]])
         #trace calculation
         correlation_matrix_trace = np.trace(correlation_matrix)
         upper_range = 2/correlation_matrix_trace
         print "Upper Range: " + str(upper_range)
```

```
Upper Range: 1.0
```

For the given correlation matrix, the learning rate parameter range from $0 < \eta < 1$