

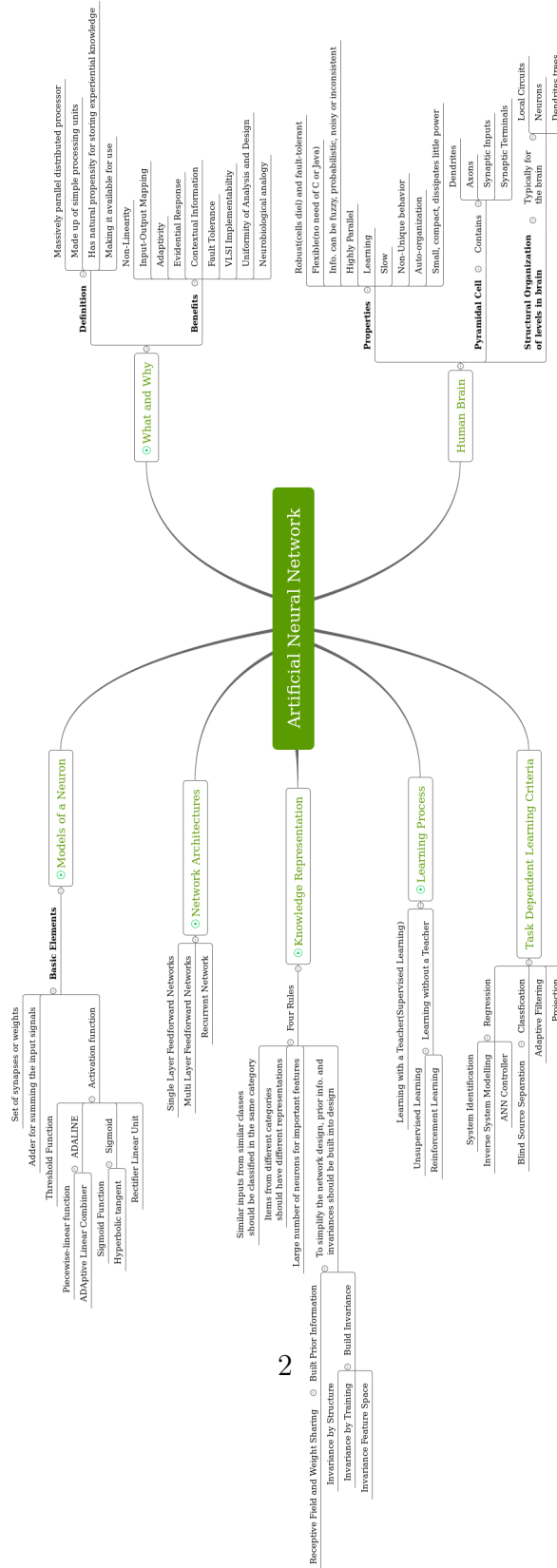
Neural Networks

Assignment 1

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1 Mindmap



2 Models of a neuron

2.1 Exercise 1.1

```
In [1]: from sympy import *
import sympy as sp
sp.init_printing(use_latex=True)

a, v = sp.symbols('a, v')
sigmoid = 1/(1+sp.exp((-a)*(v)))
sigmoid_derivative = sigmoid.diff(v)
sigmoid_derivative
```

Out[1]:

$$\frac{ae^{-av}}{(1 + e^{-av})^2}$$

At the origin, $v = 0$

```
In [16]: sigmoid_derivative.subs({v:0}).evalf()
```

Out[16]:

$$0.25a$$

2.2 Exercise 1.6

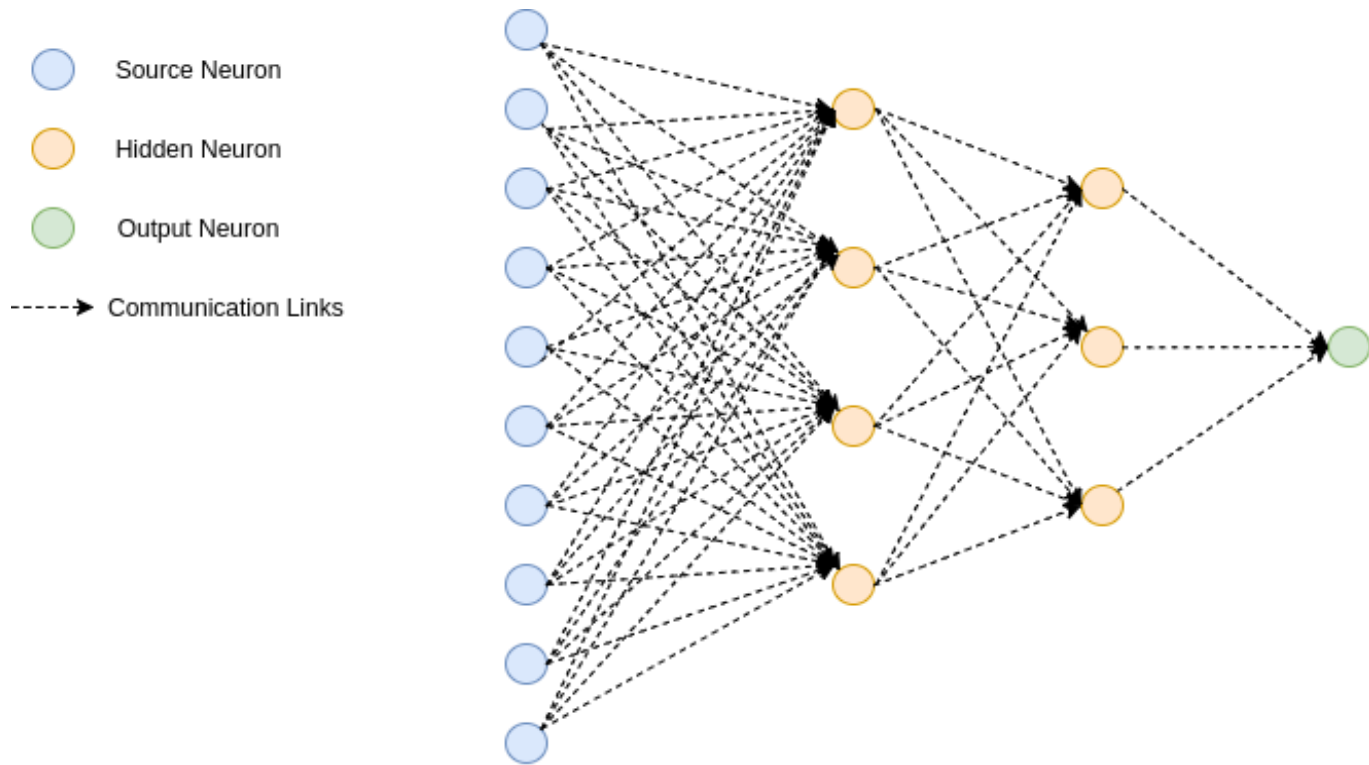
(a.) The given graph is similar to the representation of piecewise linear function. Hence we can formulate $\varphi(v)$ as:

$$\varphi(v) = \begin{cases} b & \text{if } v \geq 0.5a \\ v + 0.5b & \text{if } -0.5a < v < 0.5a \\ 0 & \text{if } v \leq -0.5a \end{cases} \quad (1)$$

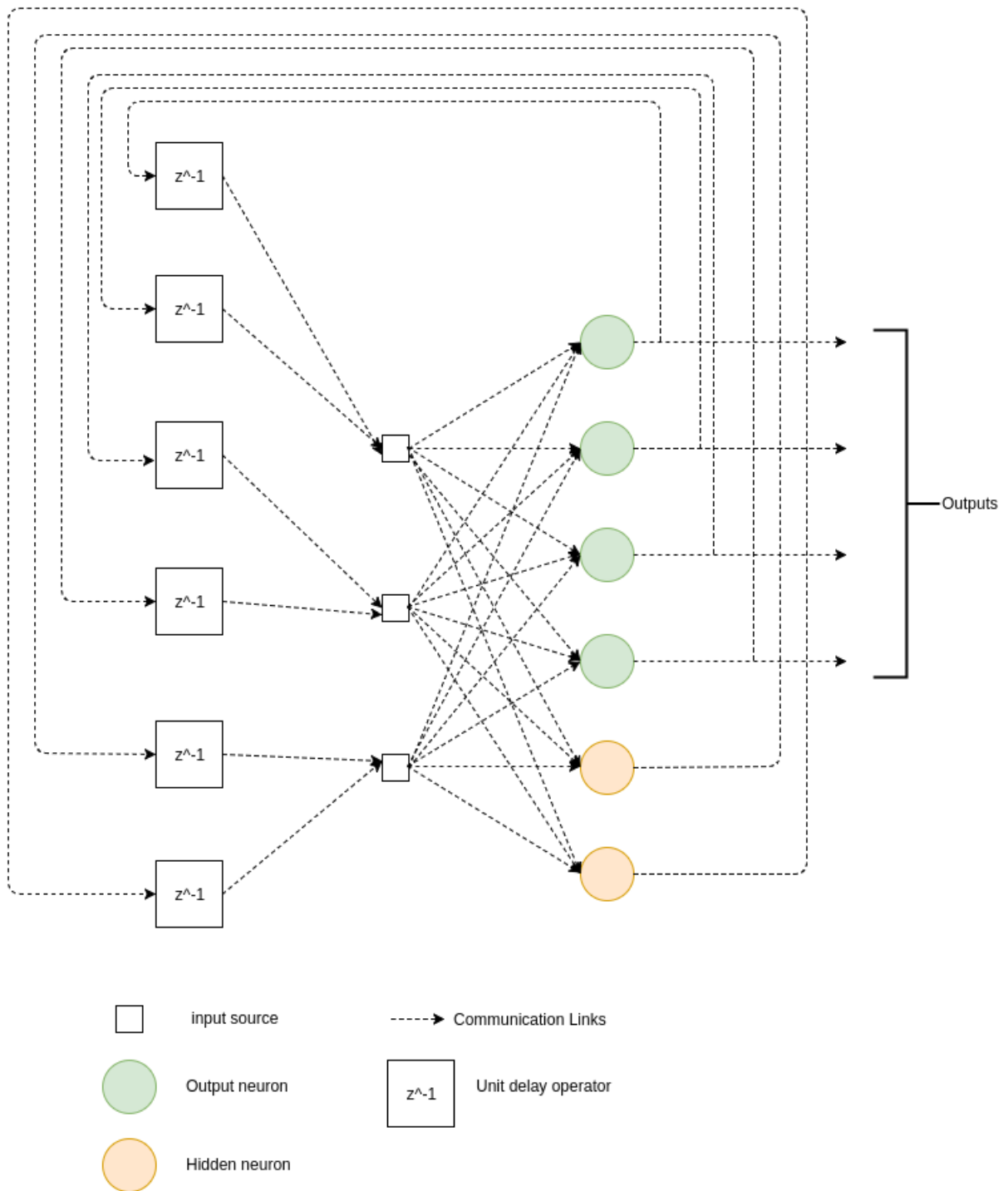
(b.) As a approaches zero, v approaches zero and the value of $\varphi(v)$ tends to $0.5b$

3 Network architectures

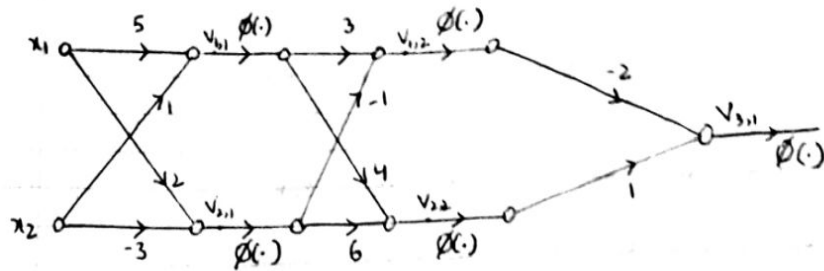
3.1 Exercise 1.12



3.2 Exercise 1.19



3.3 Exercise 1.13 (a)



$V_{1,1}$ and $V_{2,1}$ can be computed as:

$$V_{1,1} = \phi(5x_1 + x_2)$$

$$V_{2,1} = \phi(2x_1 - 3x_2)$$

We consider x_1 and x_2 as inputs

Similarly, $V_{1,2}$ and $V_{2,2}$ can be computed as

$$V_{1,2} = \phi(3V_{1,1} - V_{2,1}) = \phi(3(\phi(5x_1 + x_2)) - \phi(2x_1 - 3x_2))$$

$$V_{2,2} = \phi(6V_{2,1} + 4V_{1,1}) = \phi(6(\phi(2x_1 - 3x_2)) + 4(\phi(5x_1 + x_2)))$$

and $V_{3,1}$ can be computed as

$$V_{3,1} = \phi(-2V_{1,2} + V_{2,2})$$

$$V_{3,1} = \phi(-2(\phi(3(\phi(5x_1 + x_2)) - \phi(2x_1 - 3x_2))) + \phi(6(\phi(2x_1 - 3x_2)) + 4(\phi(5x_1 + x_2))))$$

$$V_{3,1} = \phi(-2(\phi(3(\phi(5x_1 + x_2)) - \phi(2x_1 - 3x_2))) + \phi(6(\phi(2x_1 - 3x_2)) + 4(\phi(5x_1 + x_2))))$$

3.4 Exercise 1.13 (b)

We consider x_1 and x_2 as inputs, $v_{1,1}$ and $v_{2,1}$ are local fields of first layer, $v_{1,2}$ and $v_{2,2}$ are local fields of second layer and $v_{3,1}$ is output. If the output neuron operates in its linear region, then logistic activation function is replaced by linear activation function, and input-output mapping can be written as:

$$\begin{aligned}v_{1,1} &= \varphi(5x_1 + x_2) = 5x_1 + x_2 \\v_{2,1} &= \varphi(2x_1 - 3x_2) = 2x_1 - 3x_2 \\v_{1,2} &= \varphi(3v_{1,1} - v_{2,1}) = 13x_1 + 6x_2 \\v_{2,2} &= \varphi(6v_{2,1} - 4v_{1,1}) = 32x_1 - 14x_2 \\v_{3,1} &= \varphi(-2v_{1,2} + v_{2,2}) = 6x_1 - 26x_2\end{aligned}$$

4 Knowledge Representation:

4.1 Exercise 1.21

We can apply Taylor series for very small values of α to get $S(\alpha, x)$,

$$S(\alpha, x) = S(0, x) + \alpha \frac{\partial S(\alpha, x)}{\partial \alpha}$$

Since tangent vector V is defined by partial derivative $\frac{\partial S(\alpha, x)}{\partial \alpha}$, we can substitute this in the above equation,

$$\begin{aligned}S(\alpha, x) &= S(0, x) + \alpha V \\V &= \frac{S(\alpha, x) - S(0, x)}{\alpha}\end{aligned}$$

The tangent vector will be locally invariant for very small α because the tangent distance becomes independent of α .