# Assignment_09

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## 1 Team Members:

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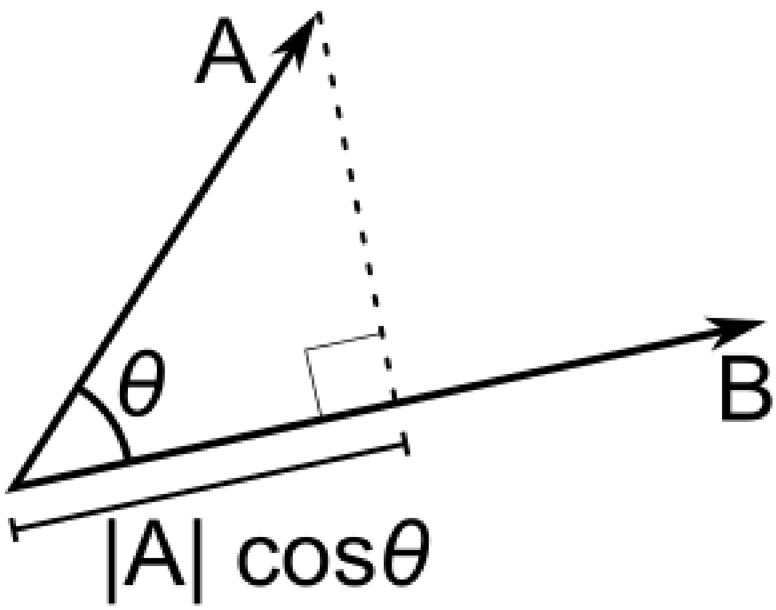
In [2]:
```
import numpy as np
import matplotlib.pyplot as plt
import random
import math
from IPython.display import Image
from IPython import display
%matplotlib inline
from tsp_solver.greedy import solve_tsp
```

## 2 Exercise 1

Show that in the SOM algorithm the winner neuron for an input x is that neuron k whose weight vector wk maximizes the inner product < wk; x > of x and wk, take x and wk as normalized.

In [32]:
```
Image(filename ='../ravi/1.png')
```

Out[32]:




## 3 Exercise 2

Then the inner product or dot product of these 2 vectors is equivalent to :

$$\mathbf{A}.\mathbf{B} = ||\mathbf{A}||\ ||\mathbf{B}||\ cos(\theta)$$

. Maximising this inner product implies $cos(\theta) = 1$ or $\theta = 0$. In other words, the euclidean distance between the 2 vectors is minimised. Since we need to minimize the distance between a winning neuron and the input vector, hence the inner product $< wk; x >$ needs to be maximised

```
In [33]: class SOM:
             def __init__(self,input_,num_of_nodes,eta,initial_wts,threshold):
                 self.input_ = input_
                 self.num_of_nodes = num_of_nodes
                 self.eta = eta
                 self.current_wts = initial_wts
                 self.t2 = 1000
                 self.threshold = threshold

             def euclidean_distance_1d(self,x,y):
                 return abs(x - y)

             def get_winning_neuron(self,x,W):
                 winner = min([(self.euclidean_distance_1d(x,w), index)
                               for index,w in enumerate(W)])[1]
                 return winner
```

```
def d_ij(self,winner):
    distance = [self.euclidean_distance_1d(self.current_wts[winner],\
                                           self.current_wts[i])\
                for i in range(len(self.current_wts))]
    return distance

def gaussian(self,sigma,distance):
    h = [np.exp(-(d**2)/(2*sigma**2)) for d in distance]
    return h

def compute_width(self,initial_sigma,n,t1):
    return initial_sigma*np.exp(-n/t1)

def weight_adaptation(self,current_wt,eta,h,x):
    new_wts = [(w + (eta*h*(x-w))) for w in current_wt]
    return new_wts

def exponential_decay_update(self,initial_eta,n,t2):
    return initial_eta*np.exp(-n/t2)

def compute_t1(self,t2):
    sigma=2
    return t2/np.log(sigma)

def stopping_criteria(self,w_old,w_new):
    result = 0
    for i,w in enumerate(w_old):
        result += abs(w - w_new[i])

    if (result < self.threshold):
        print "Stopping criteria check:sum(w_old - w_new) : " \
        +str(result)+"< "+str(self.threshold)+". Iteration end \
        for current input"
        return True
    else:
        print "Stopping criteria check :sum(w_old - w_new)  : " \
        +str(result)+"> "+str(self.threshold)
        return False

def train(self):
    sigma = 2
    t1 = self.compute_t1(self.t2)
    n = 1
    for x in self.input_:
        while(True):
            win_idx = self.get_winning_neuron(x,self.current_wts)
            print "\nWinner neuron index : "+str(win_idx)+", weight :'
```

```python
                              +str(self.current_wts[win_idx])
                        lateral_dist = self.d_ij(win_idx)
                        h = self.gaussian(sigma,lateral_dist)
                        updates_wts = self.weight_adaptation(self.current_wts,
                                                             self.eta,
                                                             h[win_idx],x)
                        print "Updated weights :",updates_wts
                        if not self.stopping_criteria(self.current_wts,
                                                      updates_wts):
                            self.current_wts =  np.array(updates_wts)
                            self.eta = self.exponential_decay_update(self.eta,
                                                                     n,self.t2)
                            n += 1
                        else:
                            break
                print "\nFinal adjusted weights :",self.current_wts
```

In [34]:
```python
initial_wts = np.array([[0.15,0.45],
                        [0.3,0.9]])
inputs = [0.1,0.2,0.4,0.5]
```

In [35]:
```python
"""
Initial weights : [0.15 , 0.45]
"""
som = SOM(inputs,2,0.1,initial_wts[0],0.01)
som.train()
```

```
Winner neuron index : 0, weight :0.15
Updated weights : [0.14499999999999999, 0.41500000000000004]
Stopping criteria check :sum(w_old - w_new)  : 0.04> 0.01

Winner neuron index : 0, weight :0.145
Updated weights : [0.1433445425147285, 0.40341179760309959]
Stopping criteria check :sum(w_old - w_new)  : 0.0132436598822> 0.01

Winner neuron index : 0, weight :0.143344542515
Updated weights : [0.14275793792092928, 0.39930556544650508]
Stopping criteria check:sum(w_old - w_new) : 0.00469283675039< 0.01. Iteration end

Winner neuron index : 0, weight :0.143344542515
Updated weights : [0.1441112907532954, 0.40065891827887118]
Stopping criteria check:sum(w_old - w_new) : 0.0035196275628< 0.01. Iteration end f

Winner neuron index : 1, weight :0.403411797603
Updated weights : [0.14681799641802767, 0.40336562394360342]
Stopping criteria check:sum(w_old - w_new) : 0.0035196275628< 0.01. Iteration end f
```

```
Winner neuron index : 1, weight :0.403411797603
Updated weights : [0.1481713492503938, 0.40471897677596957]
Stopping criteria check:sum(w_old - w_new) : 0.00613398590854< 0.01. Iteration end

Final adjusted weights : [ 0.14334454  0.4034118 ]
```

```
In [36]: """
         Initial weights : [0.3,0.9]
         """
         som = SOM(inputs,2,0.1,initial_wts[1],0.01)
         som.train()
```

```
Winner neuron index : 0, weight :0.3
Updated weights : [0.27999999999999997, 0.82000000000000006]
Stopping criteria check :sum(w_old - w_new)  : 0.1> 0.01

Winner neuron index : 0, weight :0.28
Updated weights : [0.27337817005891402, 0.79351268023565624]
Stopping criteria check :sum(w_old - w_new)  : 0.0331091497054> 0.01

Winner neuron index : 0, weight :0.273378170059
Updated weights : [0.27103175168371713, 0.78412700673486879]
Stopping criteria check :sum(w_old - w_new)  : 0.011732091876> 0.01

Winner neuron index : 0, weight :0.271031751684
Updated weights : [0.27018023473230185, 0.78072093892920769]
Stopping criteria check:sum(w_old - w_new) : 0.00425758475708< 0.01. Iteration end

Winner neuron index : 0, weight :0.271031751684
Updated weights : [0.27067810541598047, 0.78121880961288637]
Stopping criteria check:sum(w_old - w_new) : 0.00326184338972< 0.01. Iteration end

Winner neuron index : 0, weight :0.271031751684
Updated weights : [0.27167384678333778, 0.78221455098024362]
Stopping criteria check:sum(w_old - w_new) : 0.00255455085425< 0.01. Iteration end

Winner neuron index : 0, weight :0.271031751684
Updated weights : [0.27217171746701641, 0.7827124216639223]
Stopping criteria check:sum(w_old - w_new) : 0.00255455085425< 0.01. Iteration end

Final adjusted weights : [ 0.27103175  0.78412701]
```

Results show that when starting from initial weights [0.15, 0.45], the network converges (i.e, the stopping criteria is satisfied) in a smaller number of iterations compared to when we start with the initial weights of [0.3, 0.9]. Furthermore, when using initial weights of [0.3, 0.9], the neuron with the initial weight 0.3 is selected as the winning neuron at every iteration.

## 4 Exercise 3

In [37]:
```
class TravellingSalesMan:
    def __init__(self,_no_of_neurons,
                 _no_of_cities,
                 _lattice_radius,
                 _lattice_center,
                 _eta,
                 _no_epochs):

        self.no_of_neurons = _no_of_neurons
        self.no_of_cities = _no_of_cities
        self.lattice_radius = _lattice_radius
        self.lattice_center = _lattice_center
        self.eta = _eta
        self.no_of_epochs = _no_epochs

        self.current_weights = self.points_in_circle()
        print "Initial coordinates of neurons"
        print self.current_weights

        self.cities = self.get_cities()
        print "Initial coordinates of cities"
        print self.cities

        self.sigma = (self.lattice_radius * 2) + 5
        self.t_one = self.no_of_epochs/np.log(self.sigma)

        #city labels
        self.city_labels = ['wismar','schwerin','rostock',
                            'stralsund','greifswald',
                            'neubrandenberg']

    '''
    find euclidean distance between two coordinates
    '''
    def euclidean_distance_2d(self,x, y):
        return math.sqrt(pow(y[0]-x[0],2) + pow(y[1]-x[1],2))

    '''
    find euclidean distance between given input
    and all neurons weight, and return winning neuron
    with shortest distance
    '''
    def get_winner(self,x):
        return min([(self.euclidean_distance_2d(x,w),index)
                    for index,w in enumerate(self.current_weights)])[1]
```

```
'''
return city coordinates
'''
def get_cities(self):
    return np.array([[1.3,5.7],
                     [30.7,98.3],
                     [95.3,69.3],
                     [37.3,22.5],
                     [85.5,12.5],
                     [46.6,63.6]])

def compute_distance_for_TspSolve(self):

    a = self.get_cities()
    dist = np.zeros((0,a.shape[0]))
    for i in range(a.shape[0]):
        euclidean = []
        for j in a:
            euclidean.append(tsm.euclidean_distance_2d(a[i],j))
        dist = np.vstack((dist, np.hstack((euclidean))))
    return dist

'''
plot cities and neuron locations
'''
def plot_cities(self,epoch):

    #getting x,y coordinates of cities
    cities_x = self.cities[:,0:1]
    cities_y = self.cities[:,1:2]

    #configuration for plot
    fig, ax = plt.subplots()
    plt.xlim([1,120])
    plt.ylim([1,120])
    fig.set_figheight(9)
    fig.set_figwidth(15)
    plt.title("Sigma = " + str(self.sigma)+ \
              ", Learning rate = " + str(self.eta) + \
              ", Current epoch = " + str(epoch))
    plt.xlabel("x-coordinates of cities and neuron weights")
    plt.ylabel("y-coordinates of cities and neuron weights")


    #plot cities
    ax.plot(cities_x, cities_y,'ro')

    #plot neurons
```

```
        if epoch is 0:
            ax.add_patch(plt.Circle(self.lattice_center,
                                    radius=self.lattice_radius,
                                    color='g',
                                    fill=False))

        #plot connections between neurons
        if epoch is not 0:
            ax.plot(self.current_weights[:,0:1],
                    self.current_weights[:,1:2],
                    marker='o', linestyle='--',
                    color='b')
        else:
            ax.plot(self.current_weights[:,0:1],
                    self.current_weights[:,1:2],'bo')

        #showing name of the cities in the plot
        for i, label in enumerate(self.city_labels):
            ax.annotate(label, (cities_x[i]+1,cities_y[i]+1))

        #showing name of the neurons in the plot
#         print current_weights
        for index, weight in enumerate(self.current_weights):
            ax.annotate(index, (weight[0]+0.5,weight[1]+1.5))

        plt.savefig(str(epoch)+".png")


    '''
    generate points in circle lattice structure
    '''
    def points_in_circle(self):
        points = np.empty((0,2))
        circle_center = self.lattice_center
        radius = self.lattice_radius
        n = self.no_of_neurons
        for x in xrange(0,n):
            point = [circle_center[0]+np.cos(2*np.pi/n*x)*radius,
                      circle_center[1]+np.sin(2*np.pi/n*x)*radius]
            points = np.vstack([points,point])
        return points

    '''
    update weight function
    '''
    def weight_adaptation(self,neuron,winner_neuron,x):
        current_weight = self.current_weights[neuron]
        h_ij = self.calculate_H_i_j(neuron,winner_neuron)
```

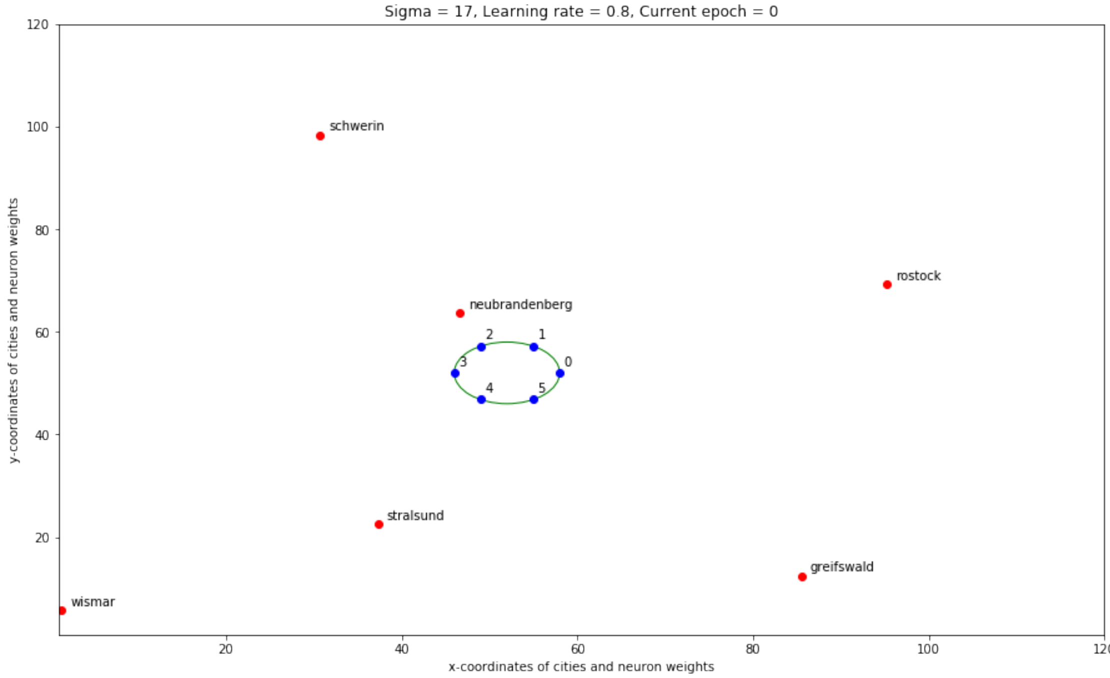
```
        new_weight = (current_weight + (self.eta*(h_ij)*
                                        (x-current_weight)))
        self.current_weights[neuron] = new_weight
        return True

    '''
    update learning rate funtion
    '''
    def eta_update(self,n):
        self.eta = self.eta*np.exp(-n/1000000000.0)

    '''
    calcuate H_ij function
    '''
    def calculate_H_i_j(self,current_neuron,winner_neuron):
        distance = self.euclidean_distance_2d(
            self.current_weights[winner_neuron],
            self.current_weights[current_neuron])
        return np.exp(-(distance**2)/(2*(self.sigma**2)))

    '''
    sigma updation function
    '''
    def sigma_update(self,n):
        self.sigma = self.sigma * np.exp(-(n/1000000000.0))


    '''
    get neighbors of winning neuron
    '''
    def get_neighors(self,winner):
        neuron_positions = range(self.current_weights.shape[0])
        if winner is len(neuron_positions)-1:
            return [neuron_positions[winner],
                    neuron_positions[winner-1],
                    neuron_positions[0]]
        else:
            return [neuron_positions[winner],
                    neuron_positions[winner-1],
                    neuron_positions[winner+1]]


    '''
    sort the visit order
    '''
    def find_visit_order(self):
        order = []
        for city in self.cities:
```

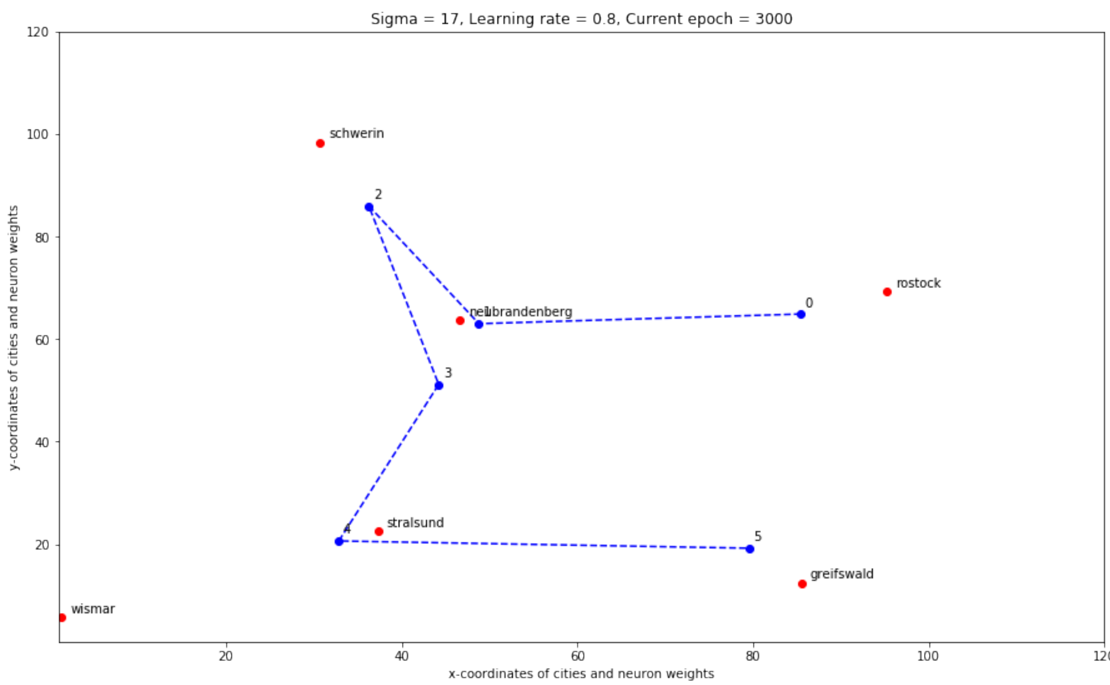
```
                order.append(self.get_winner(city))

            self.sorted_order = []

            for i in range(len(order)):
                self.sorted_order.append(order.index(i))

            self.city_order = []
            for index in self.sorted_order:
                self.city_order.append(self.city_labels[index])

            return self.sorted_order

        '''
        calculate the distance to visit
        '''
        def calculate_total_path(self,order):
            distance = 0

            for index in range(len(order)-1):
                distance += self.euclidean_distance_2d(self.cities[
                        order[index]],self.cities[order[index+1]])
            distance += self.euclidean_distance_2d(self.cities[order[0]],
                                                   self.cities[order[-1]])
            self.total_distance = distance
            return self.total_distance

        '''
        training function
        '''
        def train(self):
            for epoch in range(self.no_of_epochs):
                for city in self.cities:
                    winner_neuron = self.get_winner(city)
                    winner_with_neighbors = self.get_neighors(winner_neuron)
                    for neuron in winner_with_neighbors:
                        self.weight_adaptation(neuron,winner_neuron,city)
                    self.eta_update(epoch)
                if epoch % 3000 is 0:
                    self.plot_cities(epoch+3000)
                self.sigma_update(epoch)

            print self.calculate_total_path(self.find_visit_order())
```

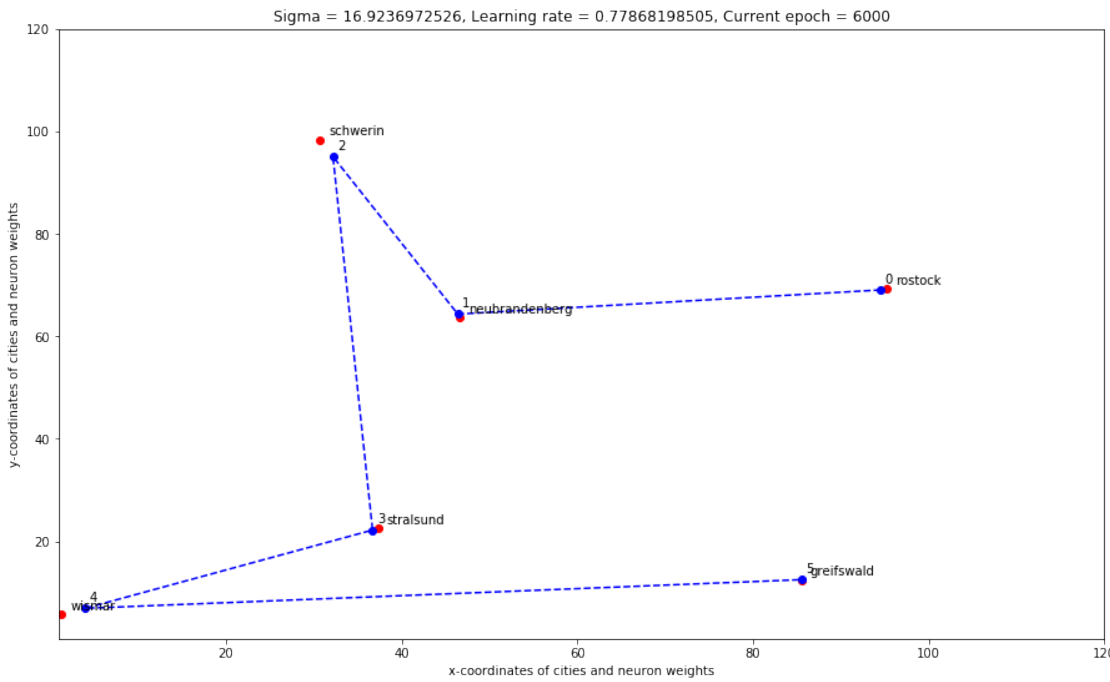
```
In [38]: '''
         Initialization
         '''
         no_of_neurons = 6
```

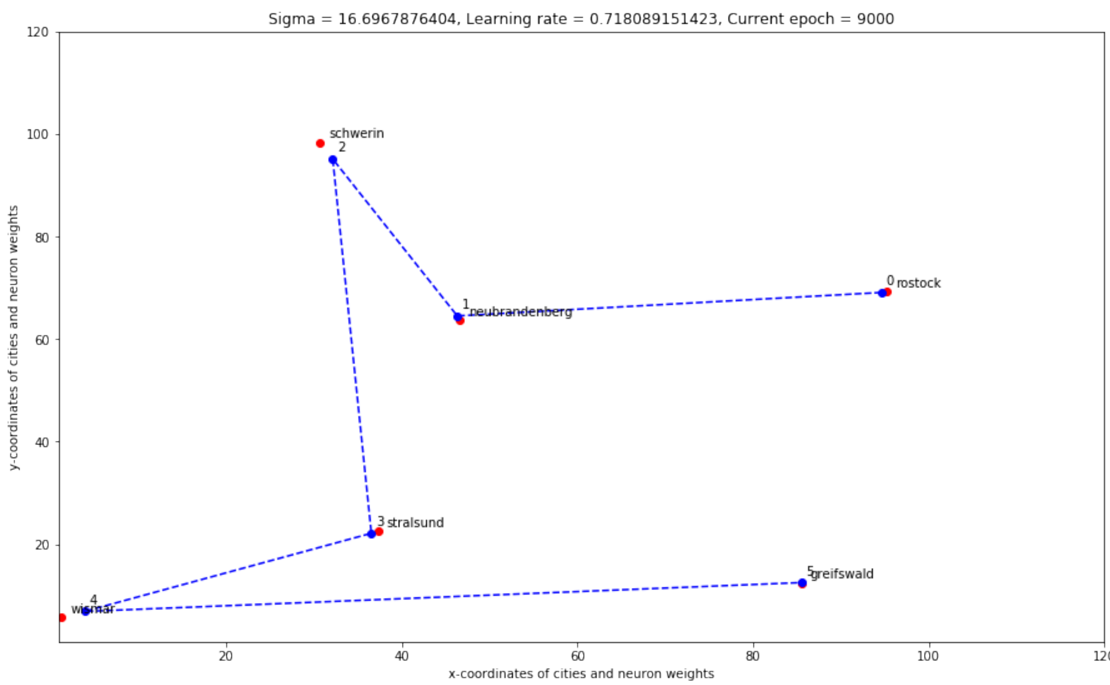
```
no_of_cities = 6
lattice_radius = 6
lattice_center = (52, 52)
eta = 0.8
no_of_epochs = 30000

tsm = TravellingSalesMan(no_of_neurons,
                         no_of_cities,
                        lattice_radius,
                        lattice_center,eta,
                        no_of_epochs)

tsm.plot_cities(0)
tsm.train()
```

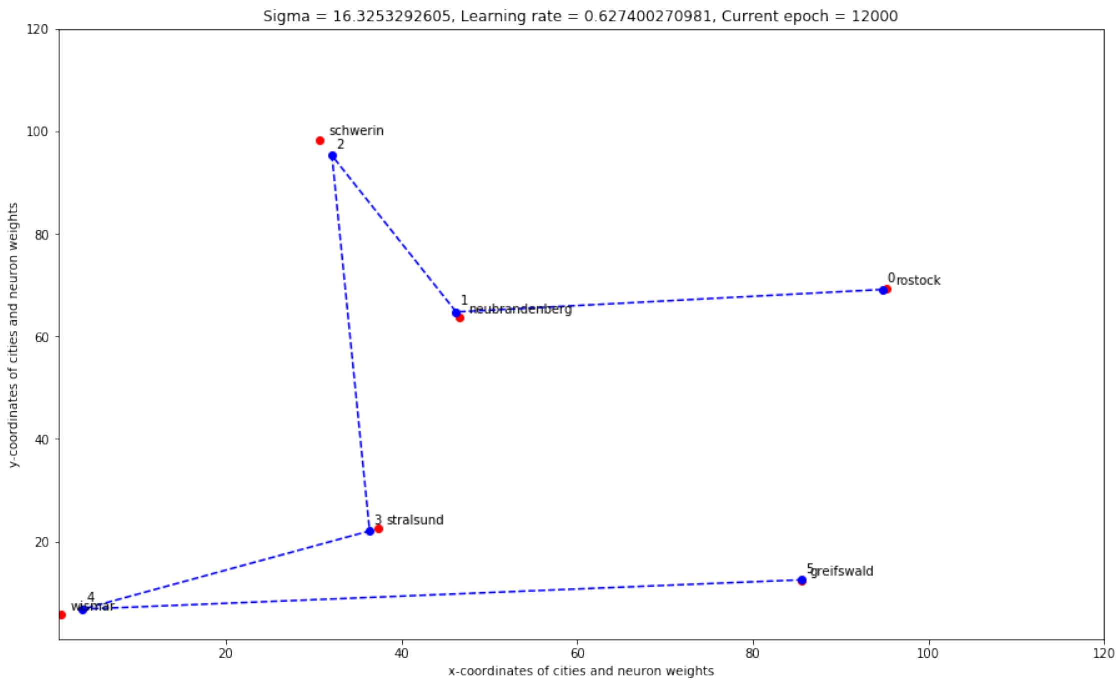
```
Initial coordinates of neurons
[[ 58.          52.        ]
 [ 55.          57.19615242]
 [ 49.          57.19615242]
 [ 46.          52.        ]
 [ 49.          46.80384758]
 [ 55.          46.80384758]]
Initial coordinates of cities
[[  1.3   5.7]
 [ 30.7  98.3]
 [ 95.3  69.3]
 [ 37.3  22.5]
 [ 85.5  12.5]
 [ 46.6  63.6]]
345.129019896
```

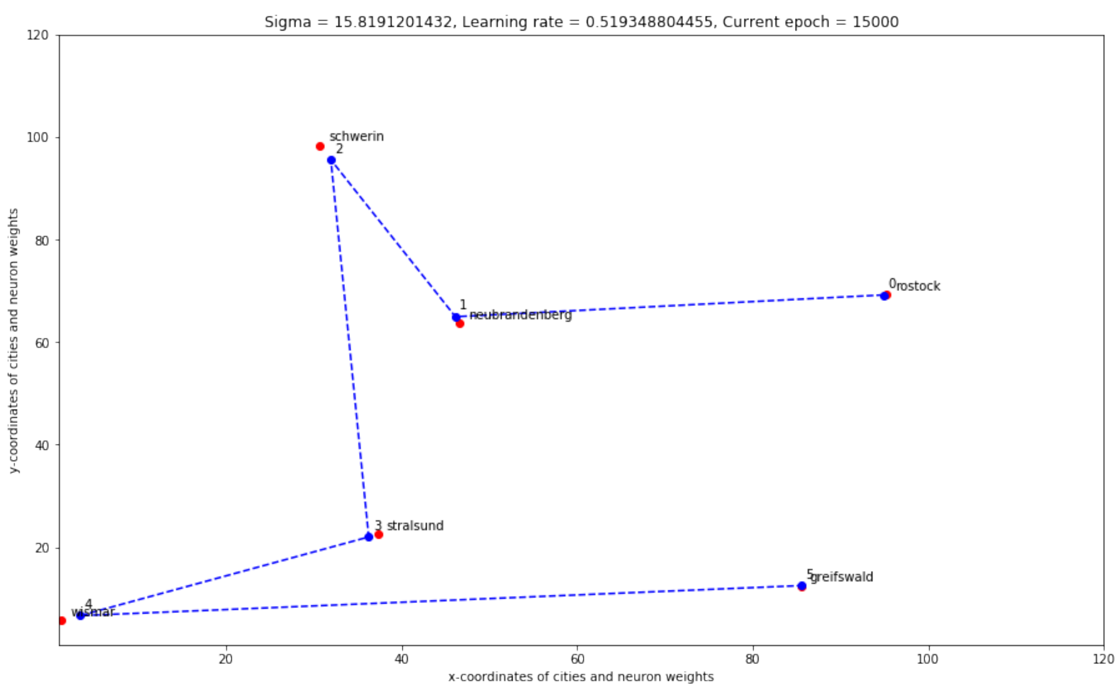

Sigma = 17, Learning rate = 0.8, Current epoch = 0
schwerin
rostock
neubrandenberg
stralsund
greifswald
wismar
0
1
2
3
4
5
y-coordinates of cities and neuron weights
x-coordinates of cities and neuron weights

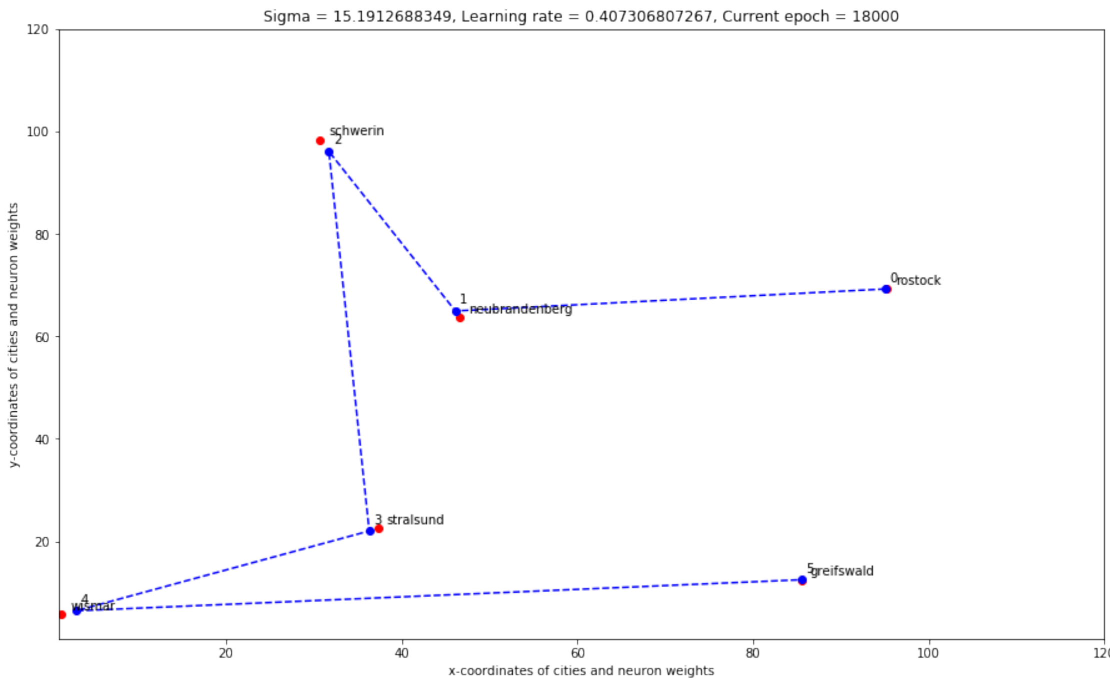

Sigma = 17, Learning rate = 0.8, Current epoch = 3000
schwerin
rostock
neubrandenberg
stralsund
greifswald
wismar
0
1
2
3
4
5
y-coordinates of cities and neuron weights
x-coordinates of cities and neuron weights

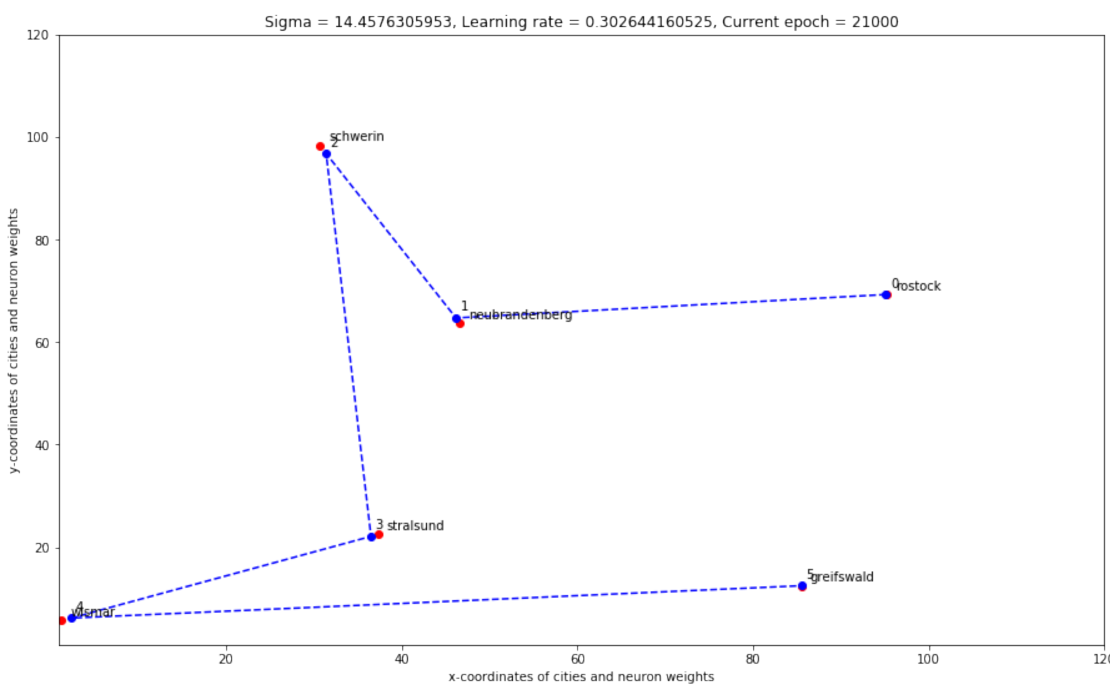

Sigma = 16.9236972526, Learning rate = 0.77868198505, Current epoch = 6000
y-coordinates of cities and neuron weights
x-coordinates of cities and neuron weights
schwerin
rostock
neubrandenberg
stralsund
greifswald
wismar

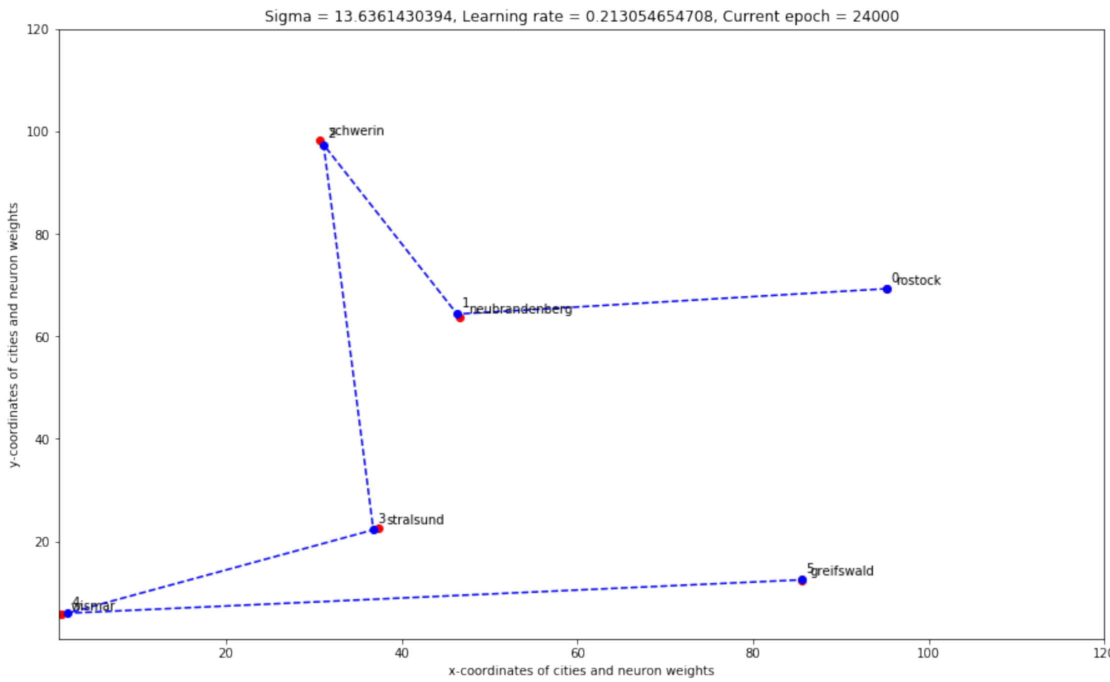

Sigma = 16.6967876404, Learning rate = 0.718089151423, Current epoch = 9000
y-coordinates of cities and neuron weights
x-coordinates of cities and neuron weights
schwerin
rostock
neubrandenberg
stralsund
greifswald
wismar

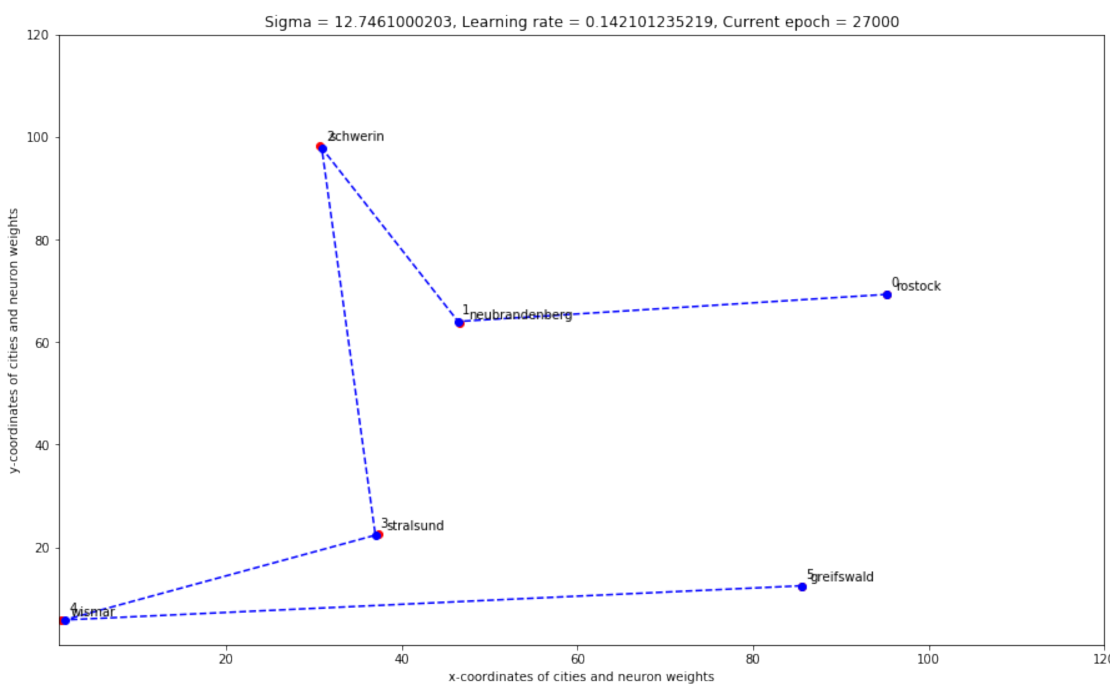

Sigma = 16.3253292605, Learning rate = 0.627400270981, Current epoch = 12000
schwerin
2
0
rostock
1
neubrandenberg
3
stralsund
5
greifswald
4
wismar
y-coordinates of cities and neuron weights
x-coordinates of cities and neuron weights
120
100
80
60
40
20
20
40
60
80
100
120

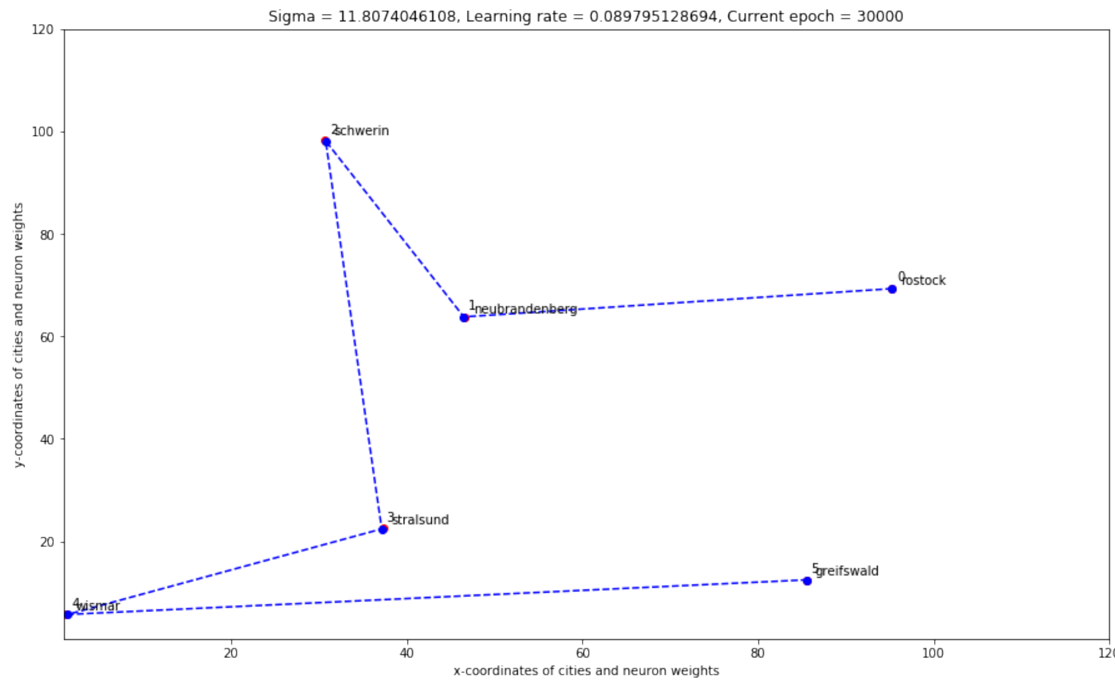

Sigma = 15.8191201432, Learning rate = 0.519348804455, Current epoch = 15000
schwerin
2
0
rostock
1
neubrandenberg
3
stralsund
5
greifswald
4
wismar
y-coordinates of cities and neuron weights
x-coordinates of cities and neuron weights
120
100
80
60
40
20
20
40
60
80
100
120


Sigma = 15.1912688349, Learning rate = 0.407306807267, Current epoch = 18000
y-coordinates of cities and neuron weights
x-coordinates of cities and neuron weights
schwerin
neubrandenberg
rostock
stralsund
greifswald
wismar
0
1
2
3
4
5
20
40
60
80
100
120


Sigma = 14.4576305953, Learning rate = 0.302644160525, Current epoch = 21000
y-coordinates of cities and neuron weights
x-coordinates of cities and neuron weights
schwerin
neubrandenberg
rostock
stralsund
greifswald
wismar
0
1
2
3
4
5
20
40
60
80
100
120


Sigma = 13.6361430394, Learning rate = 0.213054654708, Current epoch = 24000
x-coordinates of cities and neuron weights
y-coordinates of cities and neuron weights
schwerin
neubrandenberg
rostock
stralsund
greifswald
wismar

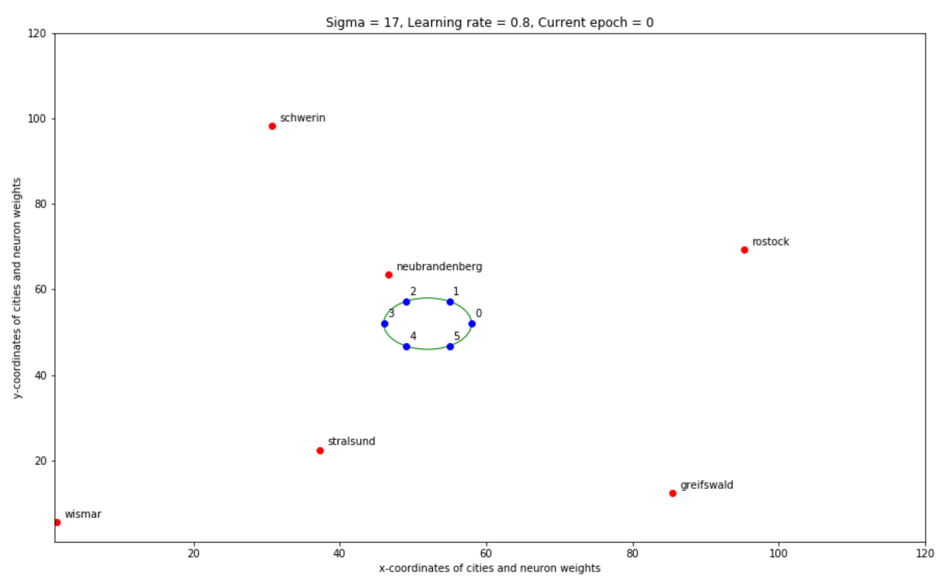

Sigma = 12.7461000203, Learning rate = 0.142101235219, Current epoch = 27000
x-coordinates of cities and neuron weights
y-coordinates of cities and neuron weights
schwerin
neubrandenberg
rostock
stralsund
greifswald
wismar




# 5 At epoch 0

```
In [39]: Image(filename ='0.png')
```

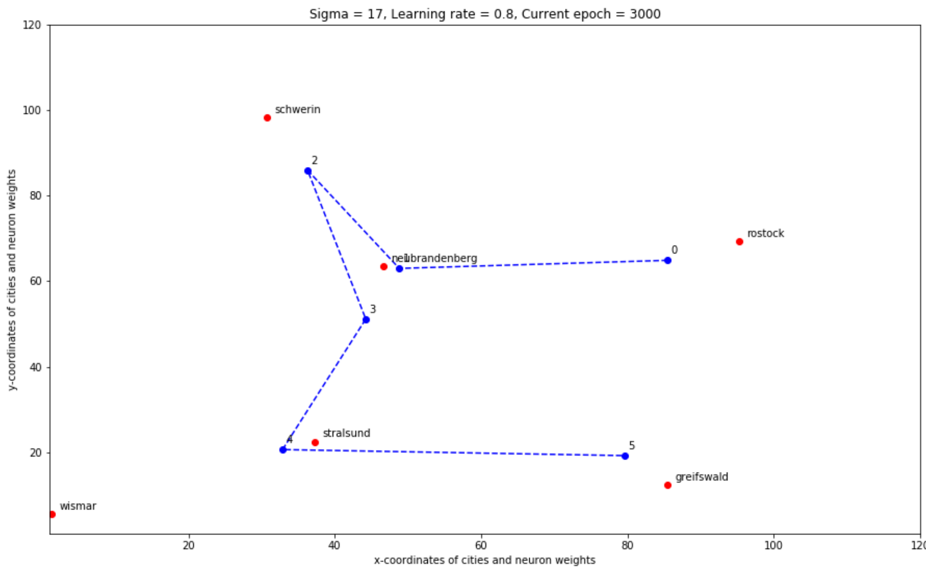
```
Out[39]:
```



# 6 At epoch 3000

```
In [40]: Image(filename ='3000.png')
```

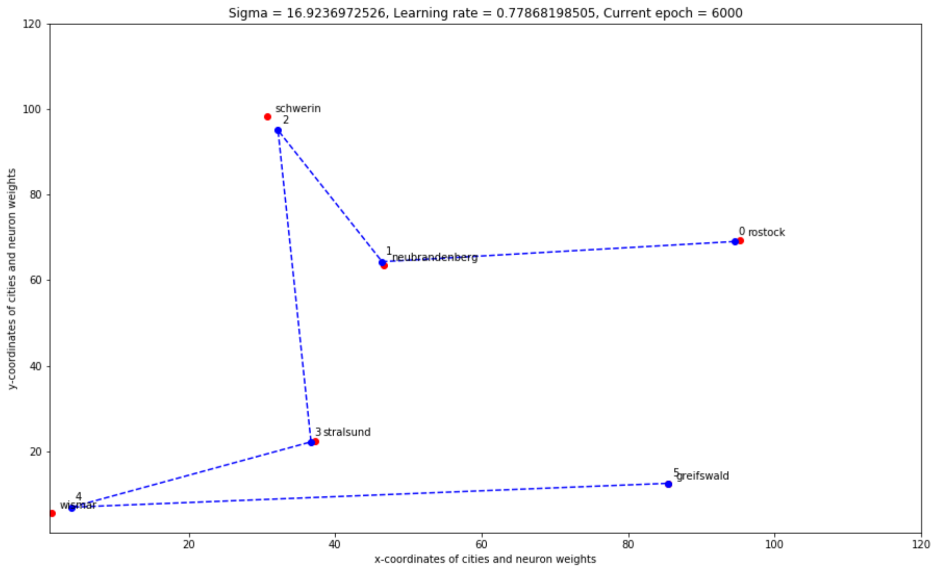
Out[40]:




# 7 At epoch 6000

```
In [41]: Image(filename ='6000.png')
```

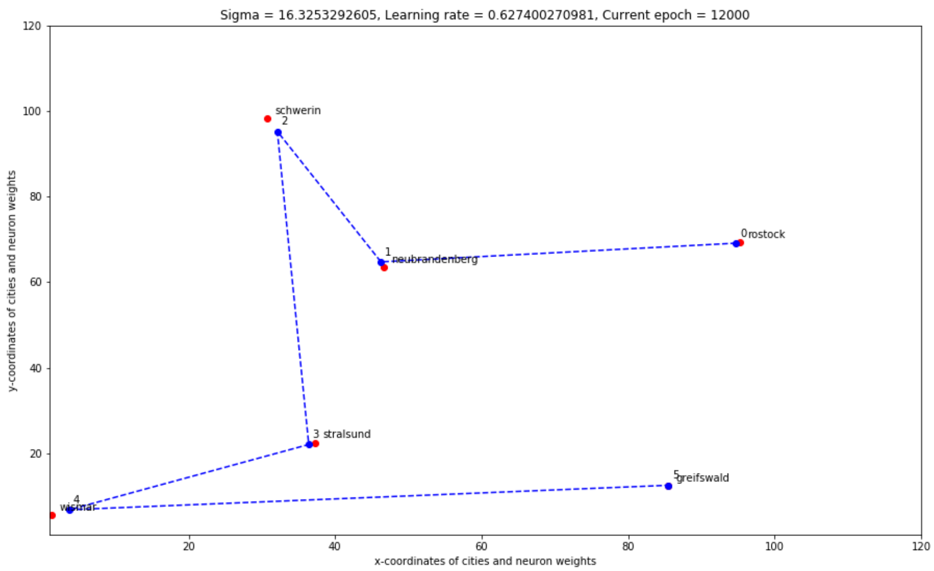
Out[41]:




# 8 At epoch 12000

```
In [42]: Image(filename ='12000.png')
```

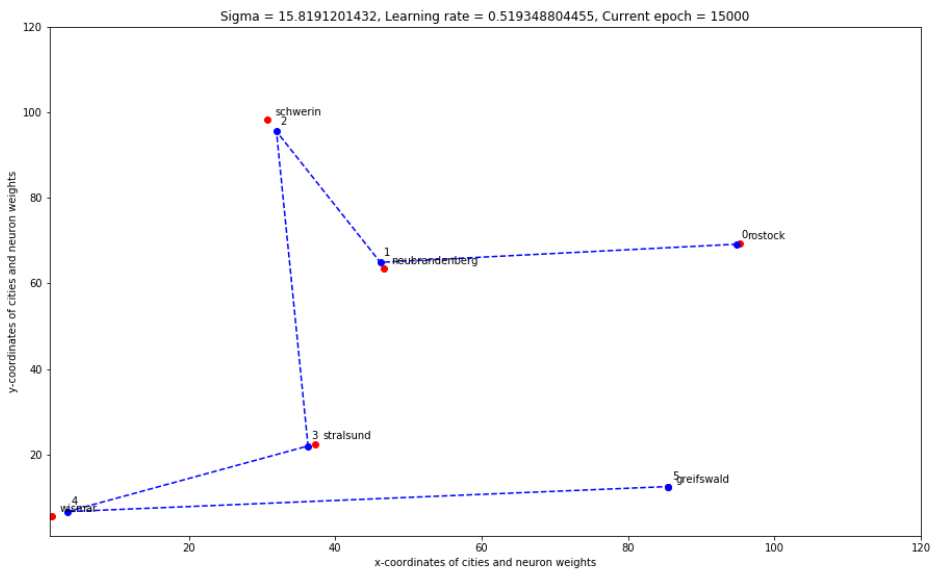
Out[42]:



# 9 At epoch 15000

```
In [43]: Image(filename ='15000.png')
```

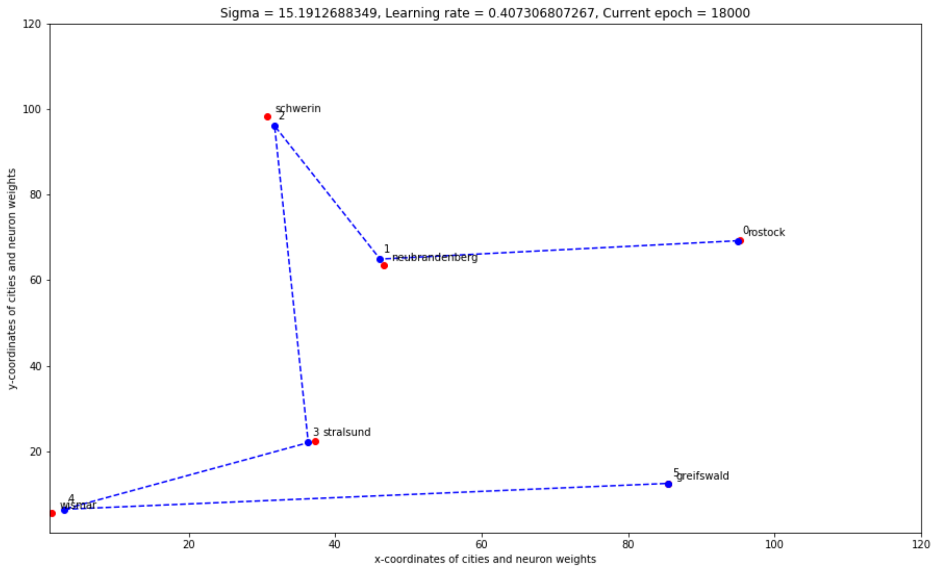
Out[43]:




# 10 At epoch 18000

```
In [44]: Image(filename ='18000.png')
```

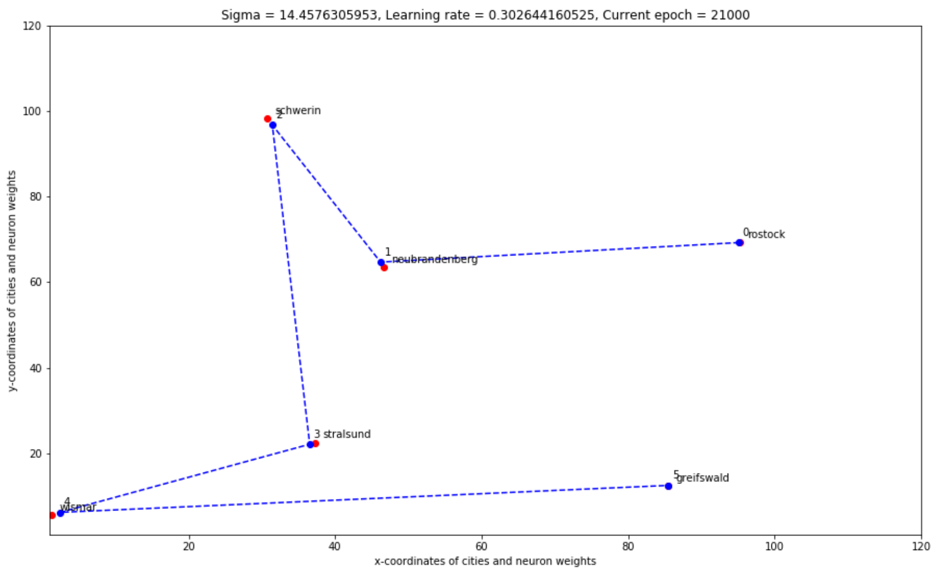
Out[44]:




# 11 At epoch 21000

```
In [45]: Image(filename ='21000.png')
```

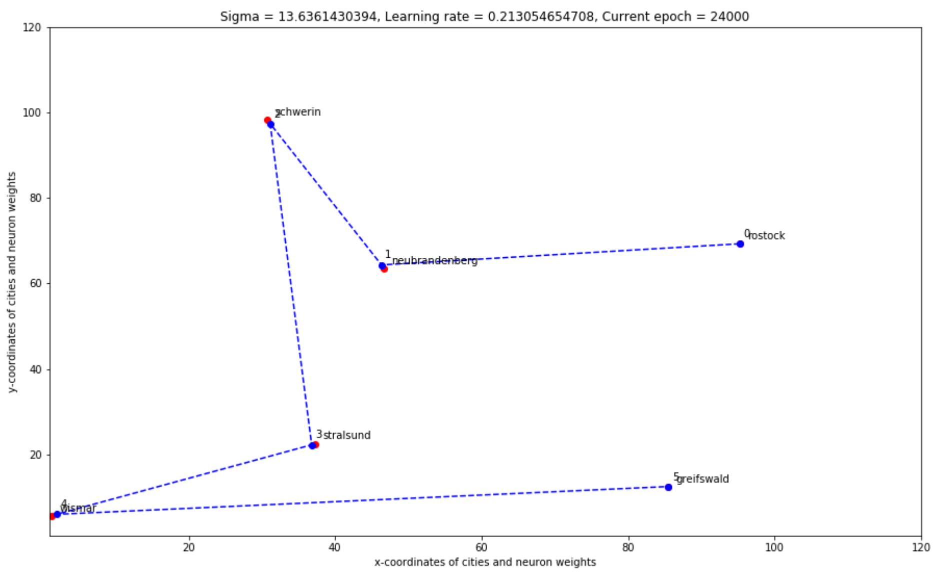
Out[45]:



## 12 At epoch 24000

```
In [46]: Image(filename ='24000.png')
```

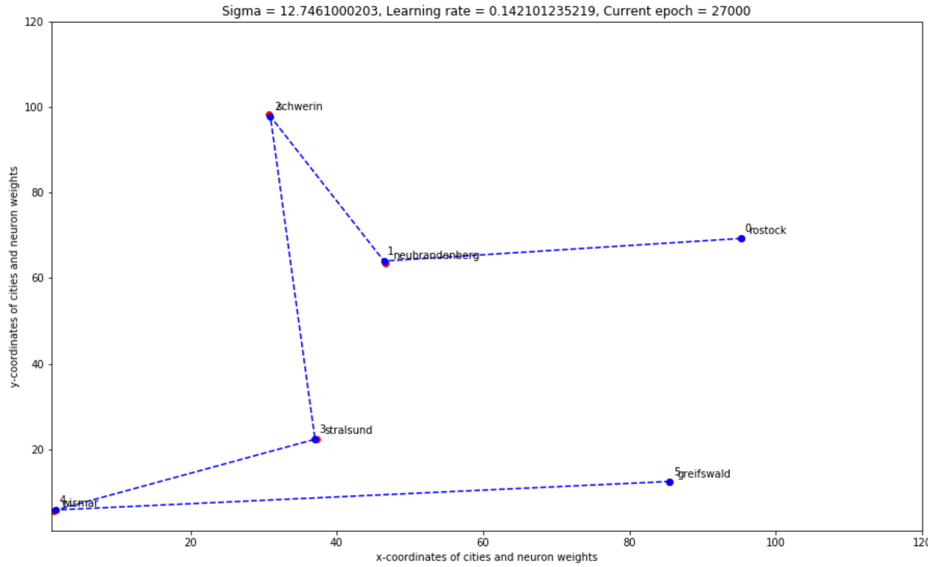
Out[46]:




## 13 At epoch 27000

```
In [47]: Image(filename ='27000.png')
```

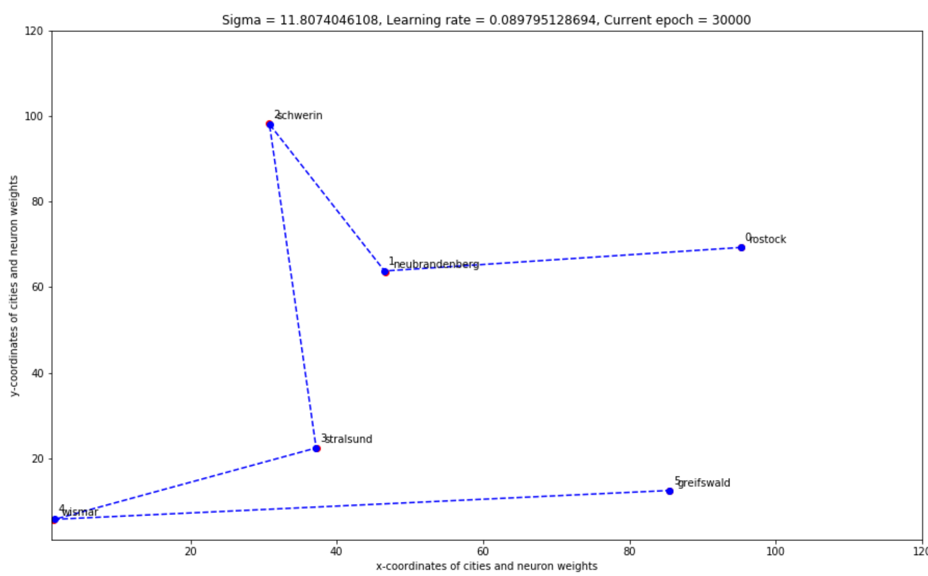
Out[47]:




# 14 At epoch 30000

```
In [48]: Image(filename ='30000.png')
```

Out[48]:



# 15 Initial order of cities before training

0.'wismar'
1.'schwerin'
2.'rostock'
3.'stralsund'
4.'greifswald'
5.'neubrandenberg'

# 16 After training

```
In [49]: print "Order of cities to visit: ", tsm.sorted_order
         print "Order of cities to visit: ", tsm.city_order
         print "Total travel distance : ", tsm.total_distance
```

```
Order of cities to visit:  [2, 5, 1, 3, 0, 4]
Order of cities to visit:  ['rostock', 'neubrandenberg', 'schwerin', 'stralsund', '
Total travel distance :  345.129019896
```

# 17 Path using TSP library

```
In [50]: #compute path using TSP solve library
         distances = tsm.compute_distance_for_TspSolve()
         paths = solve_tsp(distances)

         travelled_distance =  tsm.calculate_total_path(paths)
         print "total travel distance ", travelled_distance
         # print "Cties are travelled in ", paths

         print "Cities are travelled in order "
         for path in paths:
             print " >> " + str(tsm.city_labels[path]),
```

```
total travel distance  332.959578276
Cities are travelled in order
 >> wismar  >> stralsund  >> neubrandenberg  >> schwerin  >> rostock  >> greifswalc
```

# 18 Observations:

**If two cities are in straight line and closer together, neurons stuck in the middle and cannot move towards the cities.**

**If we update only the winning neuron, sometimes observed one or two neurons stays in its initial position.**

**So we are updating the neighborhood neurons of winning neurons using H_ij function, then all neurons started to move towards cities.**

**Paths that we obtained using TSP library are different, as compare to algorithm that we implement. However, total distance is approximately closer.**