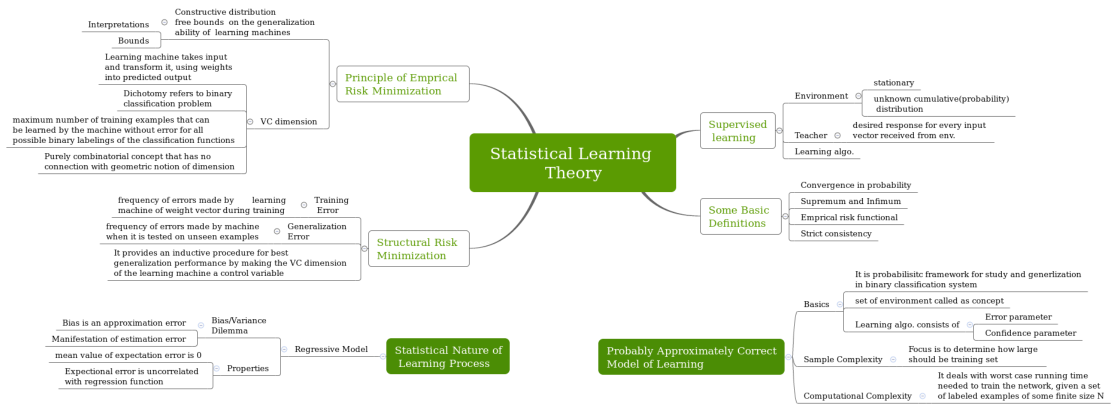
assignment4

November 4, 2017

1 Exercise 1

```
In [80]: from IPython.display import Image
         Image(filename='map.png')
```

```
Out[80]:
```



Statistical Learning Theory
Principle of Emprical Risk Minimization
Interpretations
Constructive distribution free bounds on the generalization ability of learning machines
Bounds
Learning machine takes input and transform it, using weights into predicted output
Dichotomy refers to binary classification problem
VC dimension
maximum number of training examples that can be learned by the machine without error for all possible binary labelings of the classification functions
Purely combinatorial concept that has no connection with geometric notion of dimension
Structural Risk Minimization
frequency of errors made by learning machine of weight vector during training
Training Error
frequency of errors made by machine when it is tested on unseen examples
Generalization Error
It provides an inductive procedure for best generalization performance by making the VC dimension of the learning machine a control variable
Supervised learning
Environment
stationary
unknown cumulative(probability) distribution
Teacher
desired response for every input vector received from env.
Learning algo.
Some Basic Definitions
Convergence in probability
Supremum and Infimum
Emprical risk functional
Strict consistency
Statistical Nature of Learning Process
Regressive Model
Bias/Variance Dilemma
Bias is an approximation error
Manifestation of estimation error
Properties
mean value of expectation error is 0
Expectional error is uncorrelated with regression function
Probably Approximately Correct Model of Learning
Basics
It is probabilisitc framework for study and generlization in binary classification system
set of environment called as concept
Learning algo. consists of
Error parameter
Confidence parameter
Sample Complexity
Focus is to determine how large should be training set
Computational Complexity
It deals with worst case running time needed to train the network, given a set of labeled examples of some finite size N

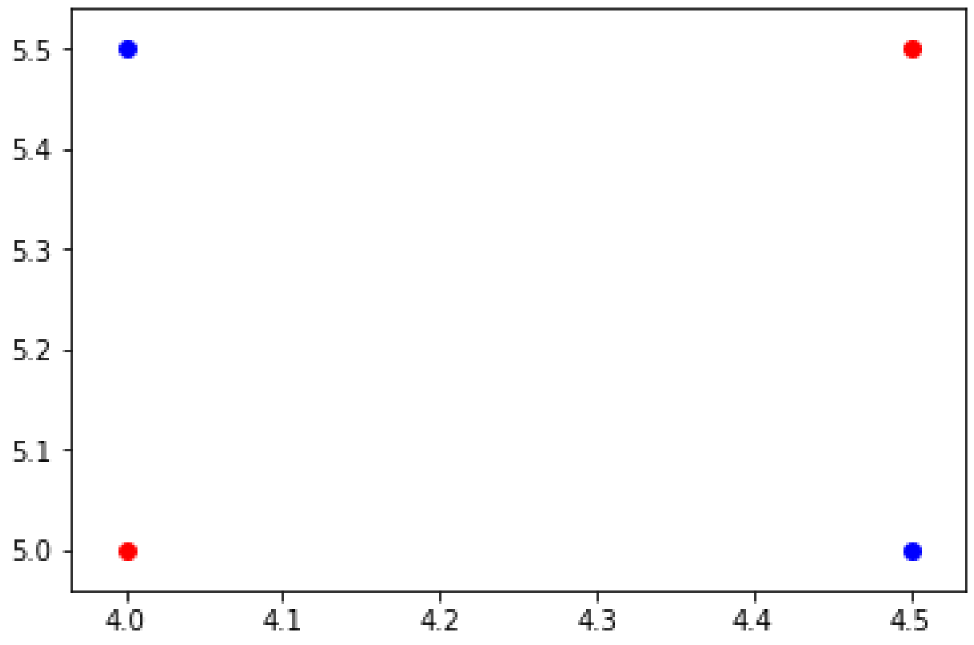
2 Exercise 2

1. If the given set of rectangles Hr is axis aligned, then the VC(Hr) = 4. This is because, there exists at least one configuration of points (such as $(1,0),(0,),(1,0),(0,1)$) that can be shattered. But a configuration of 5 points cannot be shattered by an axis alligned rectangle.

2. For the set of all circles Hc in the x,y plane, VC(Hc)=3 . If you see the below figure with 4 points, red belongs to class 1 and blue samples belongs to class 2. In this case, it is not possible to shatter them with single circle. So we can conclude that the VC dimention for circle is 3.

```
In [81]: plt.scatter([4,4.5],[5,5.5],color='r')
         plt.scatter([4,4.5],[5.5,5.0],color='b')
```

```
Out[81]: <matplotlib.collections.PathCollection at 0x7f0ab2762290>
```



3.For the set Ht of all triangles in the x,y plan, the VC(Ht) = 7. Consider 7 points in a circle lables in any order, a triangle can be used to shatter them but if there are 8 points in a circle, a triangle cannot shatter them.

3 Exercise 3 - Consistent learner

```
In [46]: import numpy as np
         import random
         import matplotlib.pyplot as plt
         %matplotlib inline


         def contained_in_rectangle(rect_bounds, point):
             a = rect_bounds[0]
             b = rect_bounds[1]
             c = rect_bounds[2]
             d = rect_bounds[3]

             if point[0] > a and point[0] < b and point[1] > c and point[1] < d:
                 return True
             else:
                 return False

         def generate_sample_point(rect_bounds):

             x = random.uniform(0, 100)
```

```
            y = random.uniform(0, 100)

            if contained_in_rectangle(rect_bounds, [x,y]):
                sample = {'input': np.array([[1.0,x,y]]),'label':True}
            else:
                sample = {'input': np.array([[1.0,x,y]]),'label':False}
                a
            return sample

        def generate_sample_set(rect_bounds, num_of_samples):
            sample_set = []
            for i in range(num_of_samples):
                sample_point = generate_sample_point(rect_bounds)
                sample_set.append(sample_point)
            return sample_set

        def logistic(v):
            return 1.0/(1 + np.exp(-v))
```

```
In [50]: #Training network with the generated sample set

        samples = generate_sample_set([10,50,10,50],100)

        #intial configurations
        bias = 0.5
        learning_rate = 0.1
        weights = np.array([[bias,0.0,0.0]])

        for i in range(10000):
            miss_classified_count = 0
            for sample in samples:
                v = (np.dot(sample['input'],weights.T))
                sigmoid_result = logistic(v)
                desired_result = sample['label']
                current_result = sigmoid_result >= 0.5
                if current_result != desired_result:
                    error = desired_result - sigmoid_result
                    weights = weights + (learning_rate*error*sample['input'])
                    miss_classified_count += 1

        #updated bias,weights and number of times wrongly classified
        print weights,miss_classified_count
```

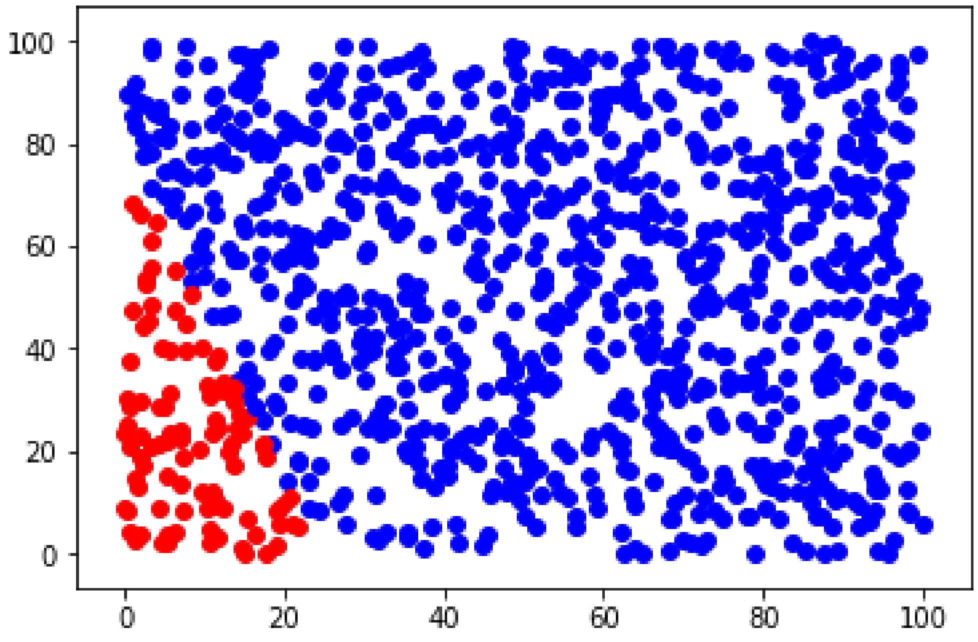
```
/home/ruby/anaconda2/lib/python2.7/site-packages/ipykernel_launcher.py:38: RuntimeWarning: overf


[[ 276.13976073  -11.29991441   -3.53380694]] 20
```

```
In [51]: #testing phase
         test_samples = generate_sample_set([10,50,10,50],1000)
         print len(test_samples)
         for sample in test_samples:
             v = (np.dot(sample['input'],weights.T))
             sigmoid_result = logistic(v)
             if sigmoid_result >= 0.5:
                 plt.scatter(sample['input'][0][1],sample['input'][0][2],c='r')
             else:
                 plt.scatter(sample['input'][0][1],sample['input'][0][2],c='b')
```

```
1000
```

```
/home/ruby/anaconda2/lib/python2.7/site-packages/ipykernel_launcher.py:38: RuntimeWarning: overf
```



4 Exercise 4

Consider training a two-input perceptron. Give an upper bound on the number of training examples sufficient to assure with 90% confidence that the learned perceptron will have true error of at most 5%. Does this bound seem realistic?

5 Solution

For a single preceptron with two-inputs, we can compute number of training examples as:

$$m \geq \frac{1}{\epsilon}\left(4\log_2\left(\frac{2}{\delta}\right) + 8\,(r+1)\log_2\left(\frac{13}{\epsilon}\right)\right)$$

Where: $\epsilon = 5\% = 0.05$
$\delta = 10\% = 0.10$
number of inputs = r = 2
Substituting in above equation:

```
In [53]: import numpy as np
         epsilon = 0.05
         delta = 0.10
         r = 2
         m = (4 * np.log2(2/delta) + 8*(r + 1) * np.log2(13/epsilon))/epsilon
         print "Number of training examples will be equal or greater than ", round(m)
```

```
Number of training examples will be equal or greater than  4196.0
```

6 Exercise 5

Consider the class C of concepts of the form (a <= x <= b) AND (c<=y <= d), where a ,b , c , and d are integers in the interval (0,99). Note each concept in this class correspondsto a rectangle with integer-valued boundaries on a portion of the x, y plane. Hint: Given a region in the plane bounded by the points (0,0) and (n -1 , n - I), the number of distinct rectangles with integer-valued boundaries within this region is 1/4 * n(n+1)ˆ2

a. Give an upper bound on the number of randomly drawn training examples sufficient to assure that for any target concept c in C, any consistent learner using H = C will, with probability 95%, output a hypothesis with error at most 0.15 !

b. Now suppose the rectangle boundaries a, b, c, and d take on real values instead of integer values. Update your answer to the first part of this question.

7 Solution

For $a \leq x \leq b$, $c \leq x \leq d$ and $x \in \{0, 1, ..., 99\}$
Number of possible combinations can be computed as:

$$\frac{n(n+1)}{2} = \frac{99 \cdot 100}{2} = 4950$$

Similarly, $c \leq y \leq d$ also has 4950 possible combinations
Therefore, number of distict rectangles can be computed as:

$$\frac{n(n+1)^2}{4} = \frac{99 \cdot 100^2}{4} = 247500$$

a. We compute hypothesis size and apply upper bound as:

$$N \leq \frac{1}{\epsilon}[\ln |H| + \ln \frac{1}{\delta}]$$

Since $\epsilon = 0.15$ and $\delta = 0.05$ is given.
Therefore, substituting in above equation

```
In [55]: n = 99
         epsilon = 0.15
         delta = 0.05
         rectangles = (n*((n+1)**2))/4
         # subsitute values in above formula, hypothesis size can be computed as
         N = round((1/epsilon)*(np.log(rectangles) + np.log(1/delta)))
         print "We need atleast " + str(N) + " examples to compute hypothesis size"
```

```
We need atleast 103.0 examples to compute hypothesis size
```

b. If we take rectangle boundaries such a,b,c,d as real values instead of integer values, then number of examples can be computed as:

$$N \leq \frac{1}{\epsilon}[8VC \log_2 \frac{13}{\epsilon} + 4 \log_2 \frac{2}{\delta}]$$

Since VC dimension of previous problem(a) is 4 as it is rectangle. Therefore, above equation can be further solved as:

```
In [56]: VC = 4
         N = round((4 * np.log2(2/delta) + 8*VC * np.log2(13/epsilon))/epsilon)
         print "we need atleast ", N, "examples"
```

```
we need atleast  1515.0 examples
```