3.24pt

Design and Analysis of Algorithms CSE2012

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Algorithm Examples

- Algorithm for summation of N numbers

Algorithm Sum(a,n)

```
1: {
2: sum ← 0;
3: for i ← 1 to n do
4:     sum ← sum + a[i];
5: end for
6: }
```

Algorithm Examples

- Algorithm for Finding Largest Number in a given set

Algorithm Max(a,n)

```
1: {
2: larger ← a[0];
3: for i ← 1 to n do
4:     if (a[i] > larger) then
5:         larger ← a[i];
6:     end if
7: end for
8: return larger
9: }
```

Algorithm Examples

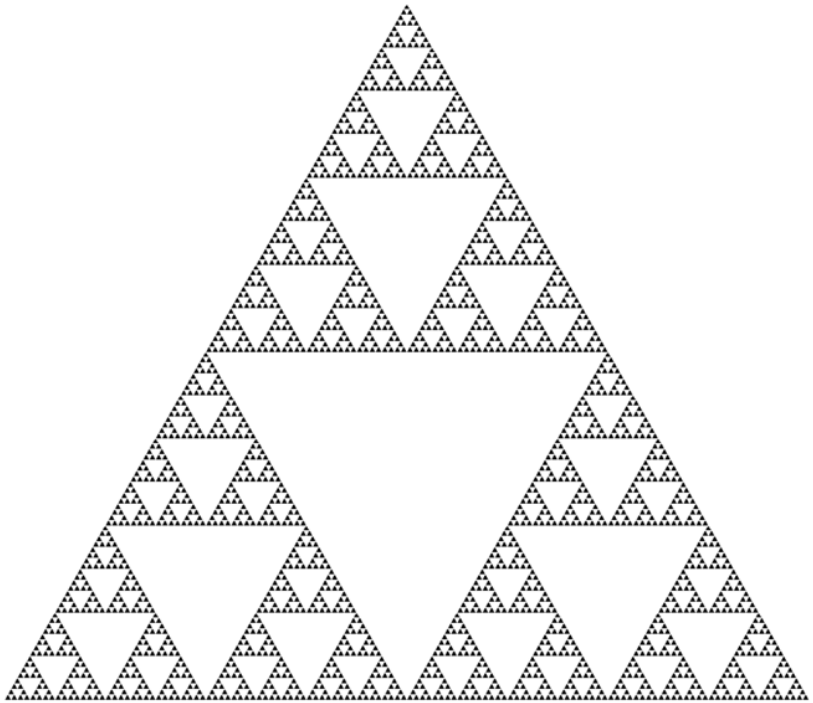
- Algorithm for Matrix Addition

Algorithm MatrixAdd(a,b,n,m)

```
1: {
2: for i ← 1 to n do
3:     for j ← 1 to m do
4:         c[i][j] ← a[i][j] + b[i][j];
5:     end for
6: end for
7: }
```

Recursive Algorithm Example

- Analyse the process of drawing of a following picture



Recursive Algorithm

- An algorithm is said to be recursive if the same algorithm is invoked in the body of the algorithm
- It solves the problem by possibly using the result of applying itself to a simpler problem
- Properties of Recursive Calls
  - It solves the large problem by using its solution to a simpler sub-problem.
  - Example: Divide and Conquer Approach
  - Eventually the sub-problem is simple enough that it can be solved without applying the algorithm to it recursively. This is called Base Case

Recursive Algorithm

- **Base Case**
  - There should be at least one base case
  - it is used to avoid the infinite looping
  - Every possible chain of recursive calls must eventually reach a base case.
- **Recursive Call**
  - Calls to the current method
  - Each recursive call should be defined so that it makes progress towards a base case.
- **Two types of Recursive Algorithm**
  - Direct Recursive Algorithm:
    - An algorithm that, directly calls it self is called direct recursive
  - Indirect Recursive Algorithm:
    - An algorithm - A said to be indirect recursive, if it calls another algorithm which in turn calls Algorithm - A

Recursive Algorithm Examples

- Recursive Algorithm for Finding Largest Number in a given set

Algorithm RecurMax(a,n)

```
1: {
2: if (n = 1) then
3:     return a[0];
4: else
5:     max{ RecurMax(a,n-1),a(n-1) };
6: end if
7: }
```

Towers of Hanoi

- Tower of Hanoi is a mathematical puzzle invented by a French Mathematician Edouard Lucas in 1883.
- The game starts by having few discs stacked in increasing order of size. The number of discs can vary, but there are only three Towers.
- The goal of Towers of Hanoi is to move all the disks from the leftmost Tower to the rightmost Tower, adhering to the following rules:
  1. Move only one disk at a time.

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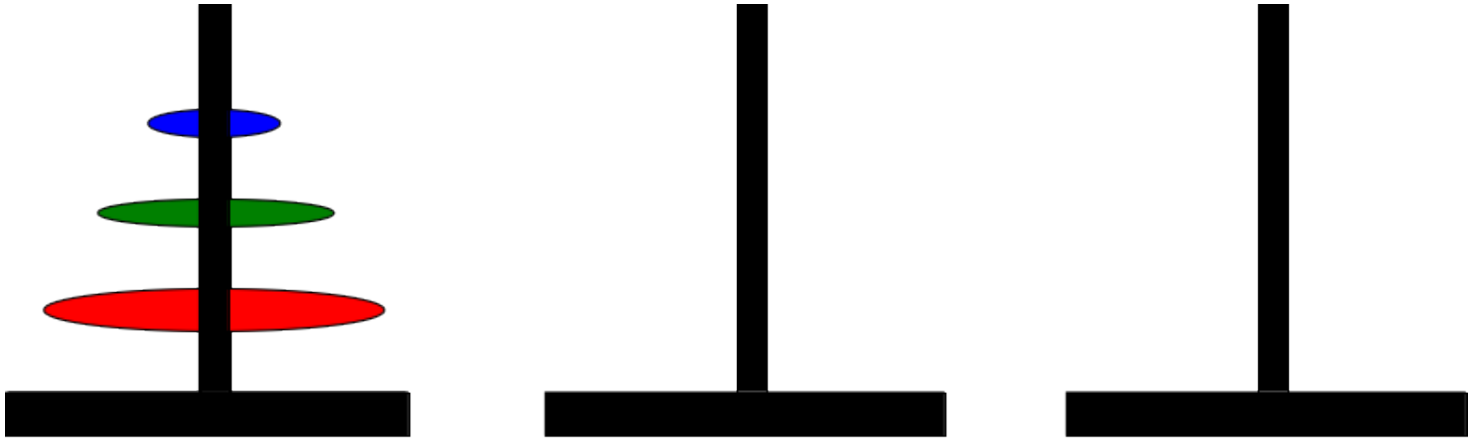
  1. Move only one disk at a time.
  2. A larger disk may not be placed on top of a smaller disk.
  3. All disks, except the one being moved, must be on a Tower.

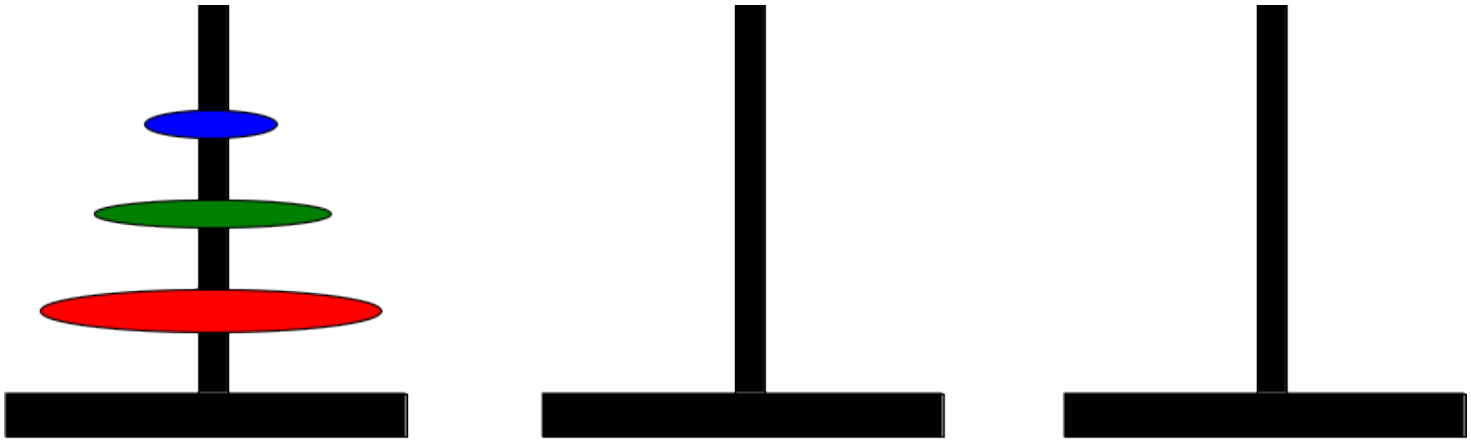
Recursive Algorithm Example

- Towers of Hanoi Problem : Initial Position



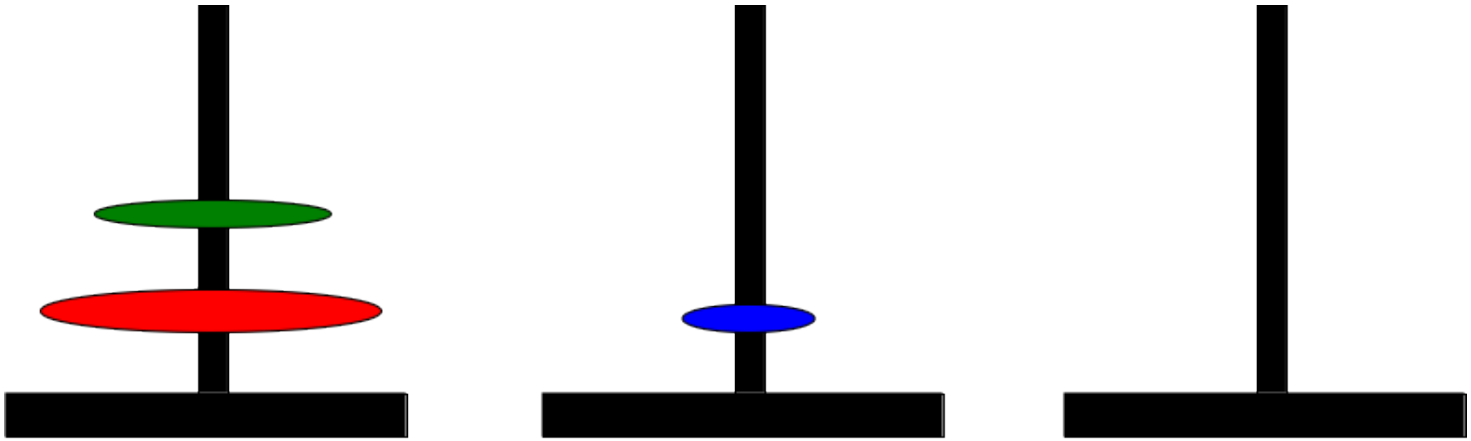
Recursive Algorithm Example

- Towers of Hanoi Problem : Step - 1



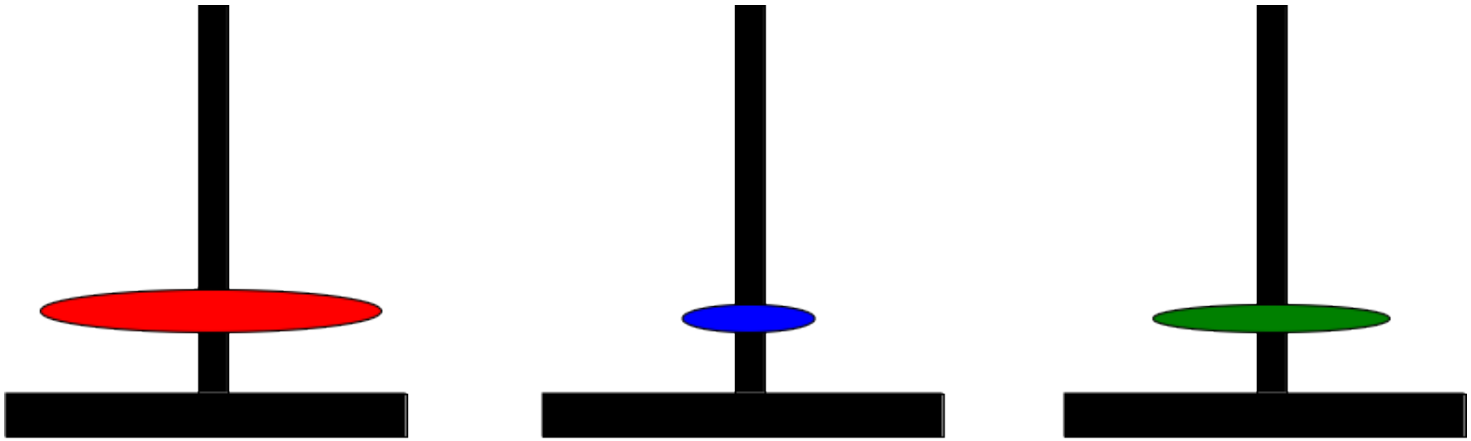
Recursive Algorithm Example

- Towers of Hanoi Problem : Step - 2



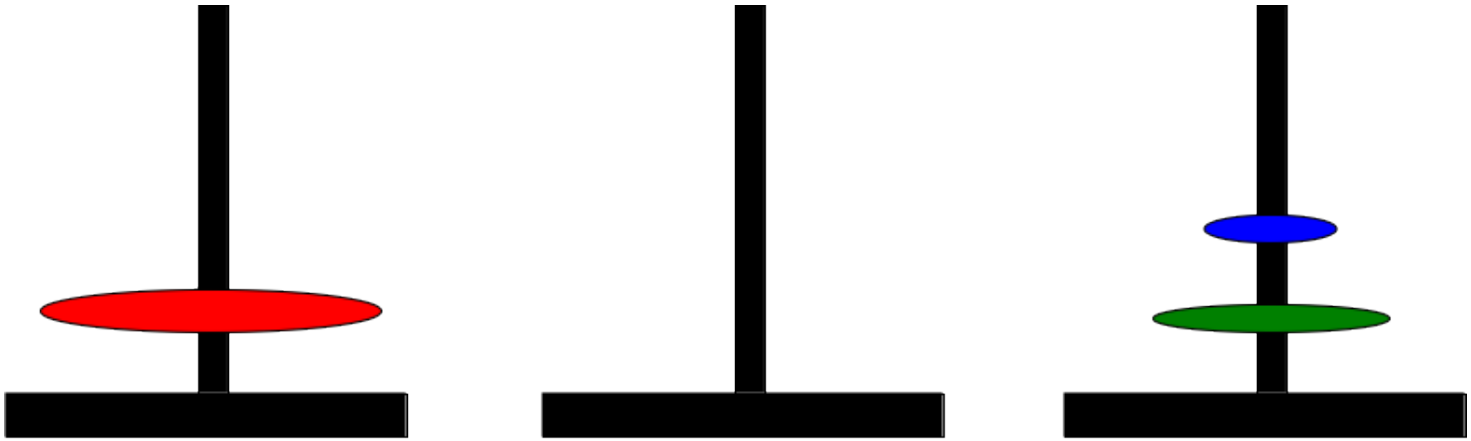
Recursive Algorithm Example

- Towers of Hanoi Problem : Step - 3



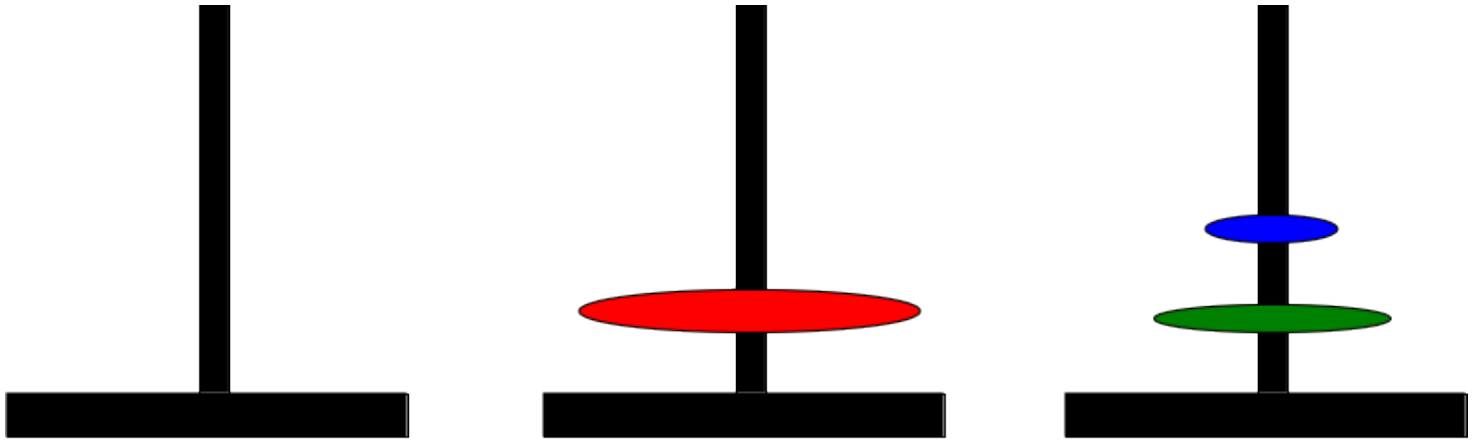
Recursive Algorithm Example

- Towers of Hanoi Problem : Step - 4



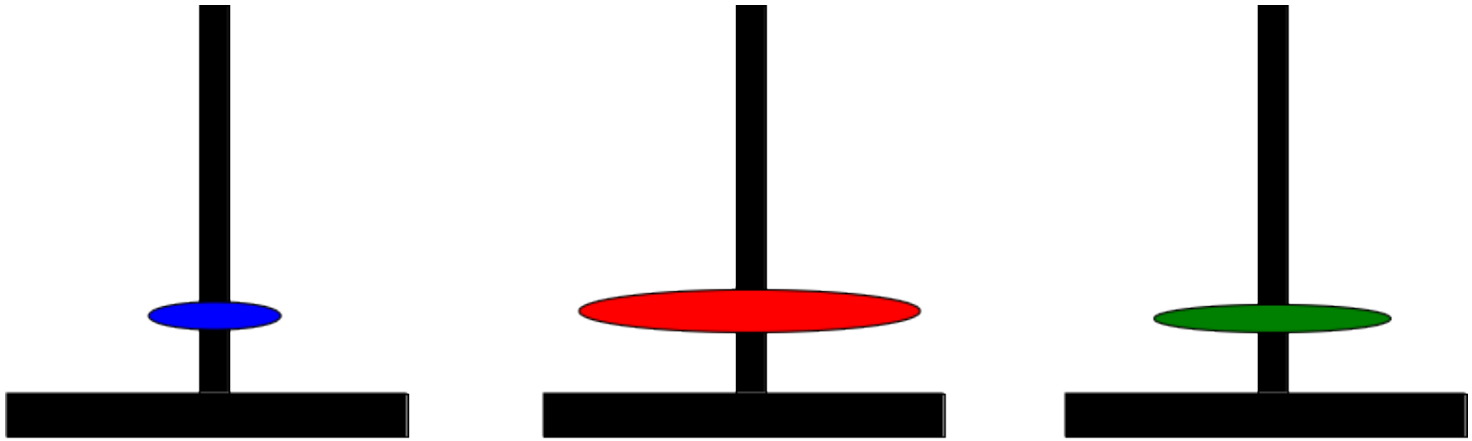
Recursive Algorithm Example

- Towers of Hanoi Problem : Step - 5



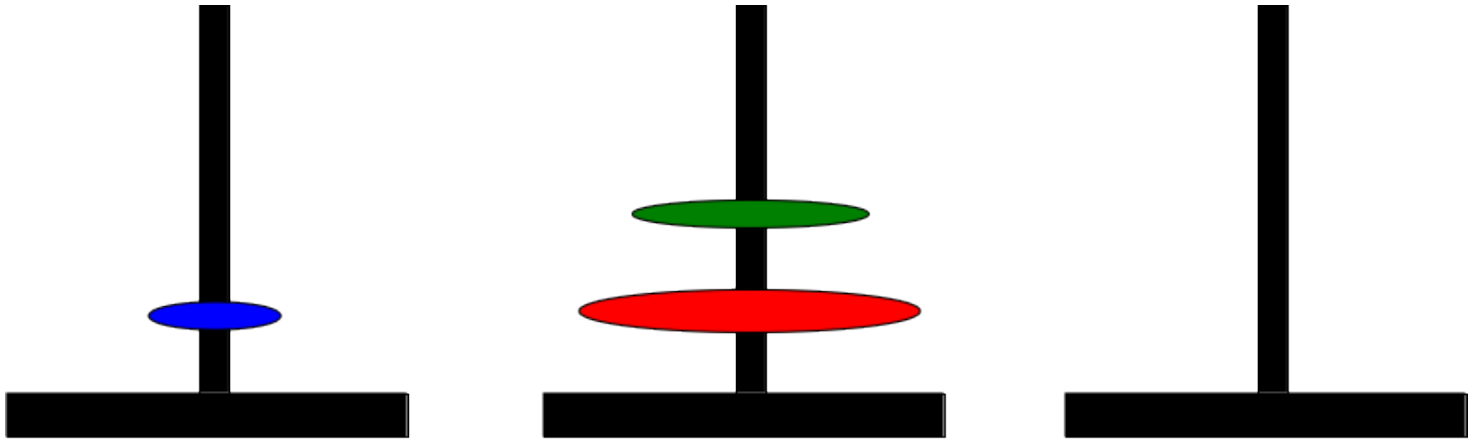
Recursive Algorithm Example

- Towers of Hanoi Problem : Step - 6



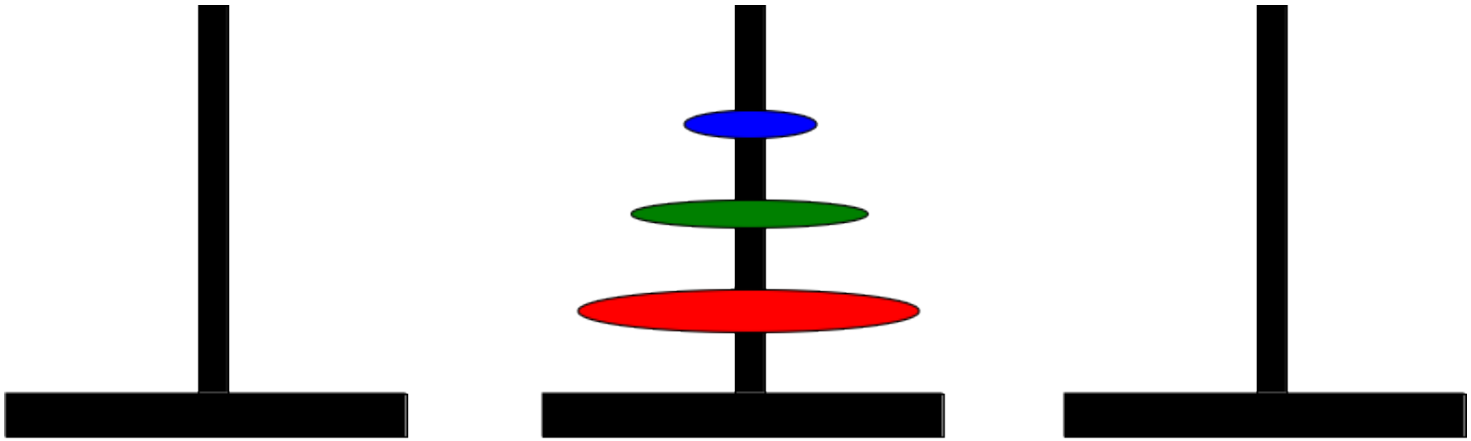
Recursive Algorithm Example

- Towers of Hanoi Problem : Step - 7



Recursive Algorithm Example

- Towers of Hanoi Problem : Step - 8



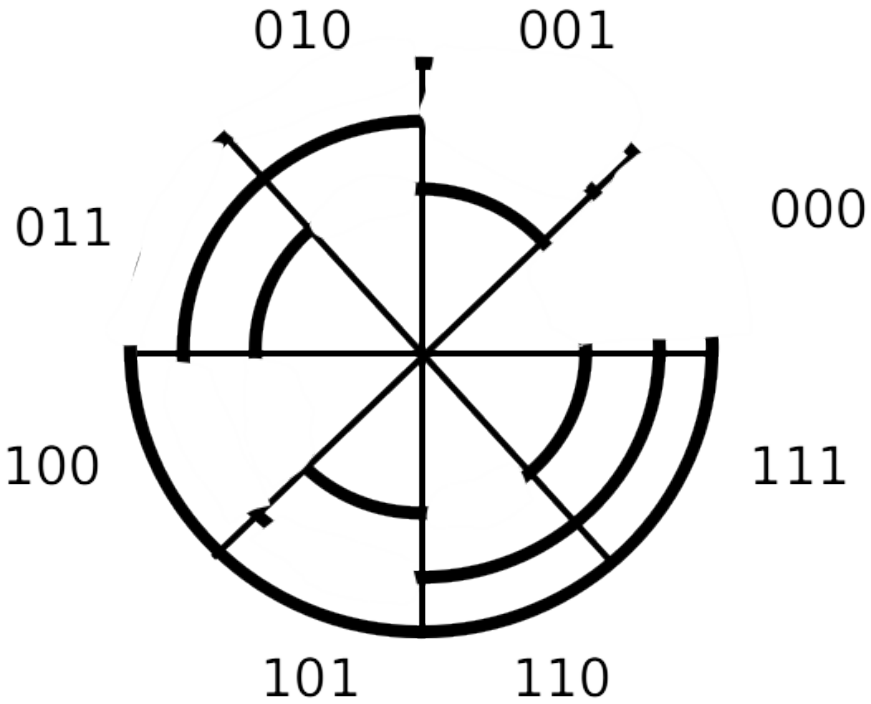
- Recursive Algorithm for Towers of Hanoi

Algorithm ToH(n,x,y,z)

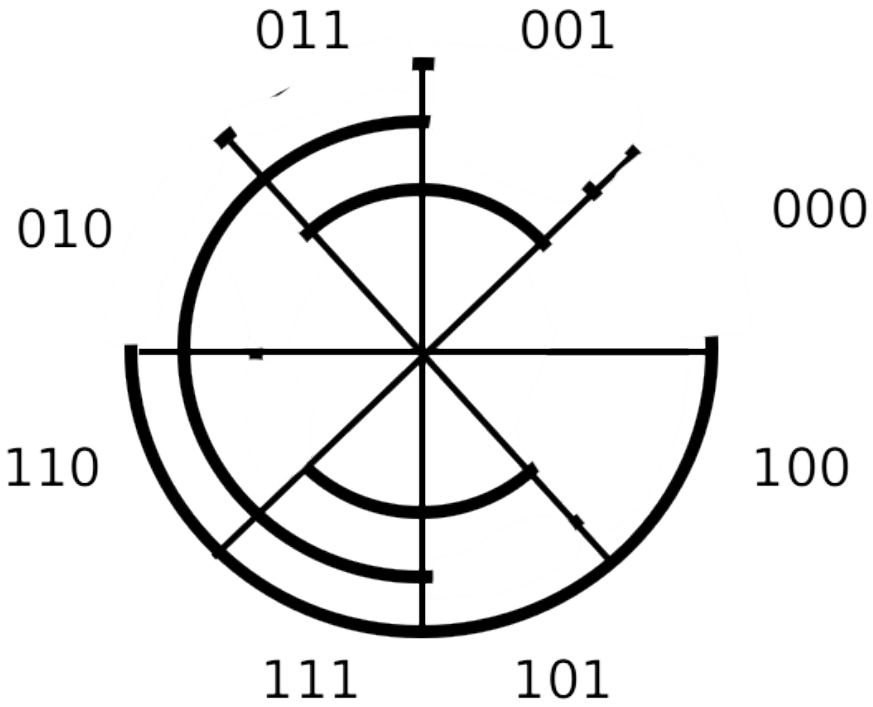
```
1: {
2: if (n ≥ 1) then
3:     ToH(n-1,x,z,y);
4:     Write("Move to disk from tower" ,x, "to top of tower", y);
5:     ToH(n-1,z,y,x);
6: end if
7: }
```

- **Gray Code**
  - An n-bit Gray code is a 1-1 onto mapping from [0..2^n-1] such that the binary representation of consecutive numbers differ by exactly one bit.
  - Invented by Frank Gray for a shaft encoder $\rightarrow$ it is a wheel with concentric strips and a conducting brush which can read the number of strips at a given angle.
  - The idea is to encode 2^n different angles, each with a different number of strips, corresponding to the n-bit binary numbers.

- **Shaft Encoder (Counting Order)**
  - Consecutive angles can have an abrupt change in the number of strips (bits) leading to potential detection errors.



- **Shaft Encoder (Gray Code)**
  - Since a Gray code is used, consecutive angles have only one change in the number of strips (bits).

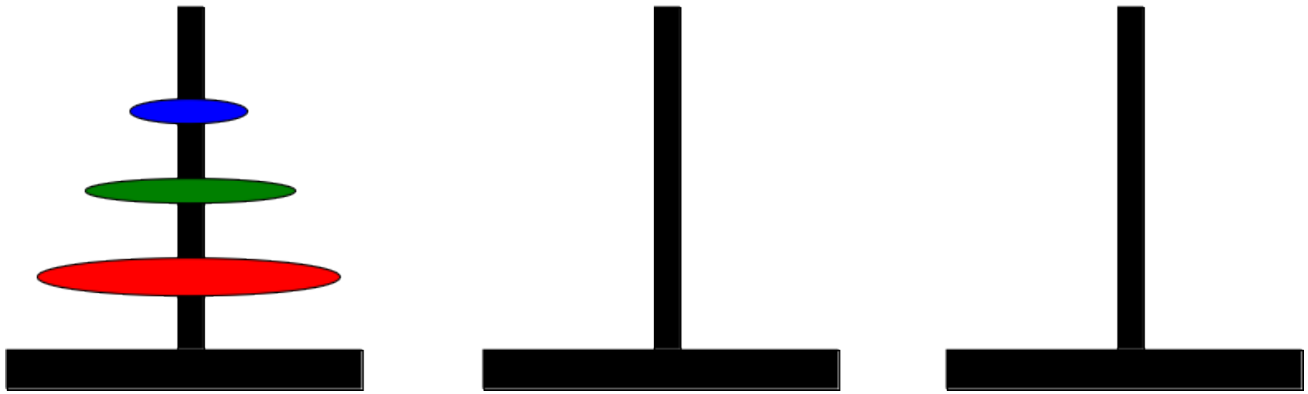


- **Gray Code and Towers of Hanoi**
  - Assume the coordinates $(d_0, \ldots, d_{n-1})$, where $d_i \in \{0, 1\}$
  - Associate d_i with the i^{th} disk
  - Initialize to (0,..,0) and flip the i^{th} coordinate when the i^{th} disk is moved
  - The sequence of coordinate vectors obtained from the Tower of Hanoi solution is a Gray code. **why??**

Recursive Algorithm Example

- Gray Code and Towers of Hanoi Problem

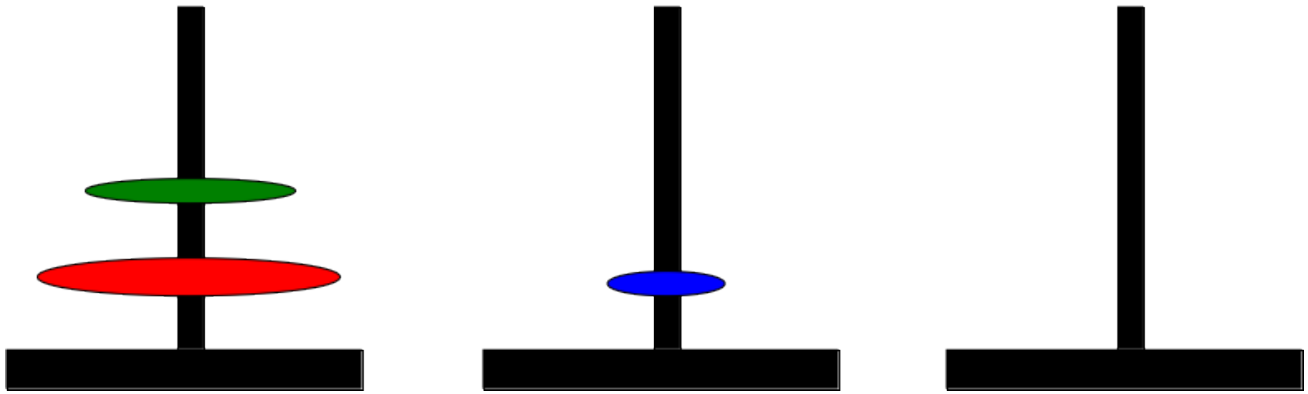
(0,0,0)



Recursive Algorithm Example

- Gray Code and Towers of Hanoi Problem

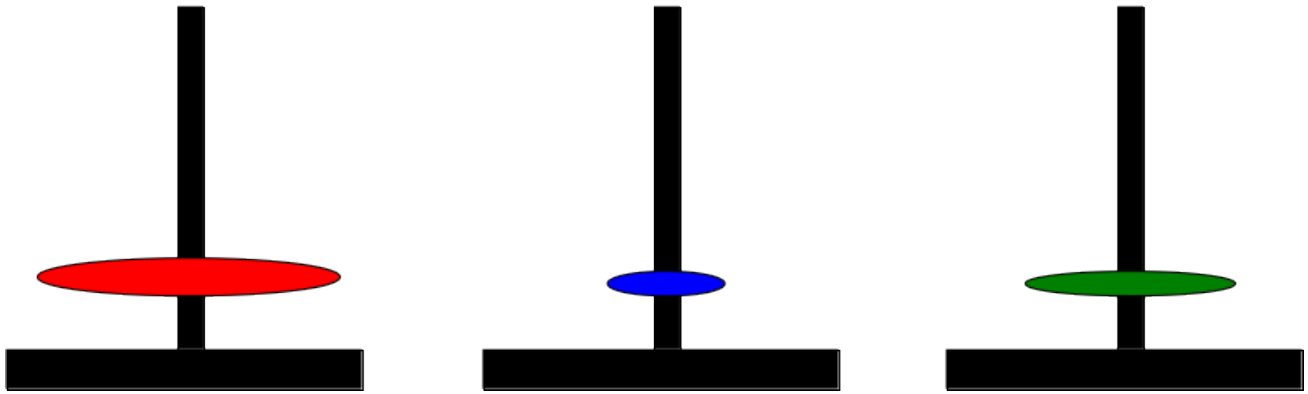
(0,0,1)



Recursive Algorithm Example

- Gray Code and Towers of Hanoi Problem

(0,1,1)



Recursive Algorithm Example

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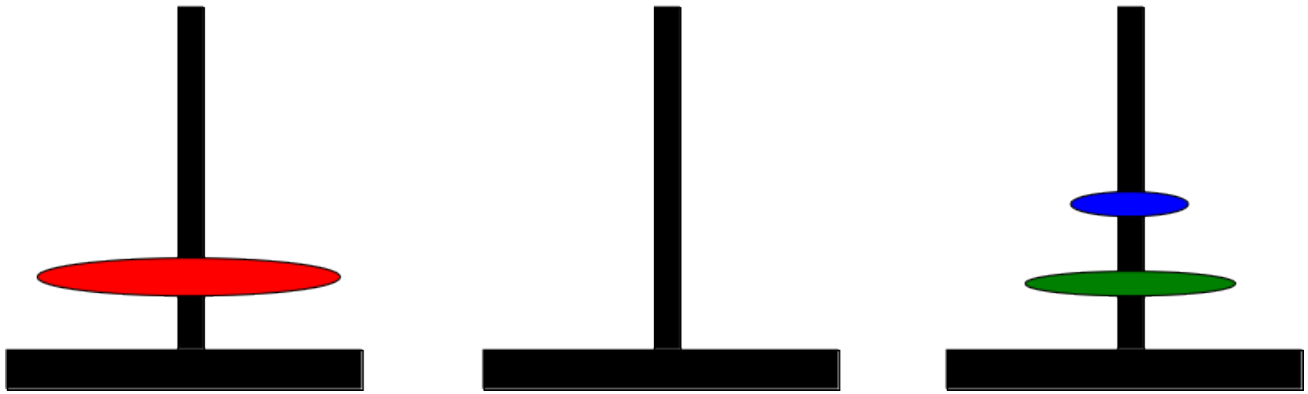
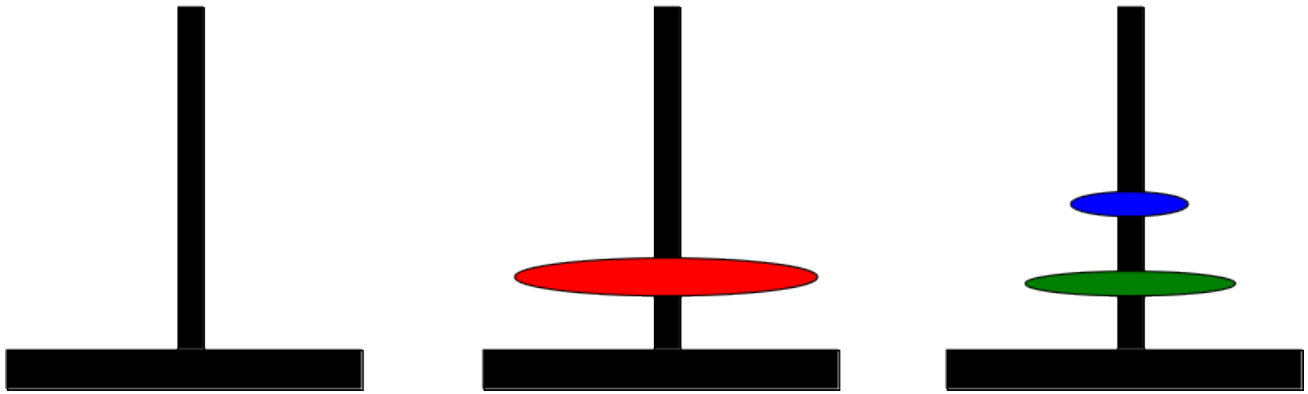


Recursive Algorithm Example

- Gray Code and Towers of Hanoi Problem

(1,1,0)



Recursive Algorithm Example

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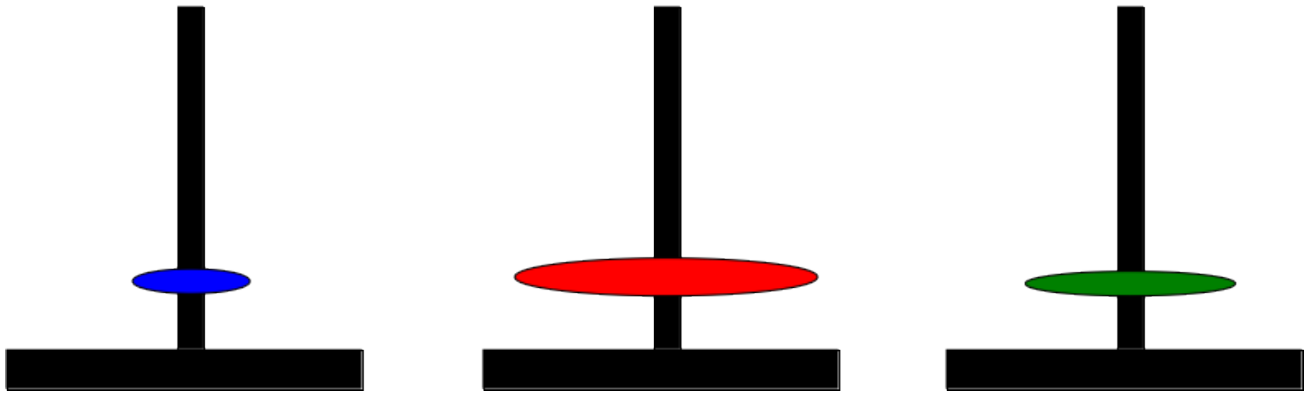

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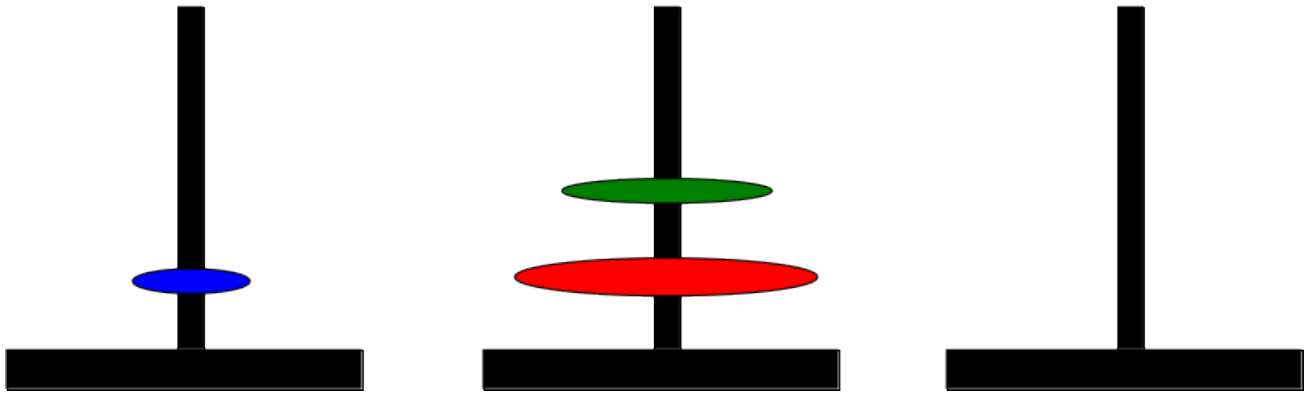

Recursive Algorithm Example

- Gray Code and Towers of Hanoi Problem

(1,0,0)