Data Visualisation
CSE2002

Scalar Visualization Technique

- **Scalar Function**
    - $f: R \to R$
        - 1-D, Histograms
    - $f: R^2 \to R$
        - 2-D, color mapping, contouring, height plot
    - $f: R^3 \to R$
        - 3-D, isosurface, slicing, volume visualization

Scalar Visualization Technique

- Visualizing scalar data is frequently encountered in science, engineering, and medicine.
- Mathematical Representation of Scalar Datasets or scalar fields
  - f:D$\rightarrow$ R, where D is a subset of R^2 or R^3.
- There exist many scalar visualization techniques, both for 2D and 3D datasets.
- Few Scalar Visualization Techniques:
  - color mapping
  - contouring (Isoline and Isosurfaces)
  - height plots
  - Volume Rendering (3D)
- scalar fields represent a quantity associated with a single (scalar) number, such as voltage, temperature, the magnitude of velocity, etc.

Scalar Visualization Technique

- Color mapping is probably the most widespread visualization method for scalar data.
- color mapping associates a color with every scalar value.
- Color Mapping is in mapping function phase in Visualization pipeline. i.e m:D $\rightarrow$ D_V. i.e with a color that depends on the scalar data defined on D
- For every point of the domain of interest D, color mapping applies a function c : R $\rightarrow$ **Colors** that assigns to that point a color c(s) $\in$ **Colors** which depends on the scalar value -s at that point.

Scalar Visualization Technique

- several ways to define such a scalar-to-color function c.
    - Color look-up tables
    - Transfer functions

Scalar Visualization Technique

- **Color Look-Up Tables:**
  - This is technique is simple to implement color mapping.
  - It uses Color look-up table or colormap
  - Colormap is a uniform sampling of the color-mapping function c:

$$C = \{c_i\}_{i=1..N}, \qquad \text{where} \quad c_i = c\left(\frac{(N-i)f_{\min} + i f_{\max}}{N}\right).$$

  - The above equation implemented as a table of N-colors C_1, C_2, C_3, .. C_N, which are associated with the scalar dataset values f, assumed to be in the [f_{min} and f_{max}].
  - Knowing the scalar range is important
  - Rule - 1: the colors c_i with low indices-i in the colormap represent low scalar values close to f_{min}.
  - Rule - 2: colors with indices close to N in the colormap represent high scalar values close to f_{max}

Scalar Visualization Technique

- **Color Look-Up Tables:**
  - f $_{min}$ and f $_{max}$ can be determined automatically by examining the sampled dataset values or be prescribed by the user.
  - Dataset values are outside the range?? $\rightarrow$ clamped to the prescribed range in the given colormap. Sometime scaling the dataset values to the prescribed range.
  - The scalar mapping is implemented by indexing into a color lookup table.
  - Scalar values then serve as indices into this lookup table.
  - Simply, the lookup table holds an array of colors.
  - Scalar values greater than the maximum are clamped to the maximum color
  - scalar values less than the minimum are clamped to the minimum value

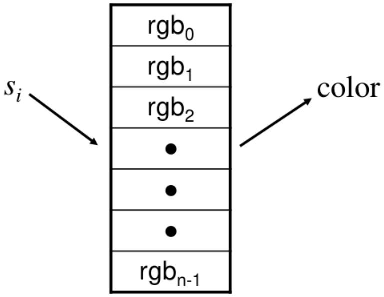
Scalar Visualization Technique

- **Color Mapping:**
  Then, for each scalar value s_i, the index i into the color able with n entries is given as:

$$s_i < \min \quad : \quad i = 0$$

$$s_i > \max \quad : \quad i = n - 1$$

$$\text{otherwise:} \quad i = n \cdot \left(\frac{s_i - \min}{\max - \min} \right)$$



Scalar Visualization Technique

- **Color Mapping: Case Study**
  - An application where we want to visualize a time-dependent scalar field f(t) with t $\in$ [t_{min} and t_{max}].
  - Visualize the color-mapped values of the scalar field f(t) for consecutive values of t in [t_{min} , t_{max}]
  - However, if we do not know f(t) for all values-t before we start the visualization, we cannot compute the absolute scalar range [f_{min}, f_{max}] to apply in look-up table equation.
  - If the range [$f_{min}(t_i)$, $f_{max}(t_i)$] of a time step $t_i \in$ [t_{min}, t_{max}] is much smaller than the absolute range [f_{min},f_{max}] $\rightarrow$ normalize f to absolute range.
  - Better solution might be to normalize the scalar range separately for every time frame f(t) $\rightarrow$ different color legends for every time frame

Scalar Visualization Technique

- **Transfer Function:**
  - A more general form of the lookup table is called transfer function.
  - In this function c is defined analytically.
  - we have RGB or HSV(Hue-Saturation-Value) color systems. c $= (c_R, c_G, c_B)$ where $c_R : R \to R$, $c_G : R \to R$ and $c_B : R \to R$
  - c_R, c_G and c_B are also called transfer functions
  - A transfer function is any expression that maps scalar values into a **color specification**.
  - Example: a function can be used to map scalar values into separate intensity values for the red, green, and blue components.

Scalar Visualization Technique

- **Design Effective Colormaps:**
  - The main challenge for visualizations using color mapping is to design an effective colormap **C**
  - What meant by effective colormap? $\rightarrow$ easily and accurately make statements about the original scalar dataset that was color mapped, by looking at the generated colors
    - **Absolute values** Tell the absolute data values at all points in the displayed dataset.
    - **Value ordering** Given two points in the displayed dataset, tell which of the corresponding two data values is greater. $\rightarrow$ how much that is not possible
    - **Value difference** Given two points in the displayed dataset, tell what is the difference of data values at these points. $\rightarrow$ how far apart each other, but not which one is small and large

Scalar Visualization Technique

- **Design Effective Colormaps:**
  - **Selected values** particular data value, tell which points in the displayed data take the respective value. $\rightarrow$ not discussing the values different than interested values
  - **Value change** Tell the speed of change, or first derivative, of the data values at given points in the displayed dataset $\rightarrow$ not able to tell absolute value
- Designing a colormap is a challenging task by considering all these goals in equal proposition.
- Focus on optimizing a subset of the above goals at the possible expense of the others

Scalar Visualization Technique

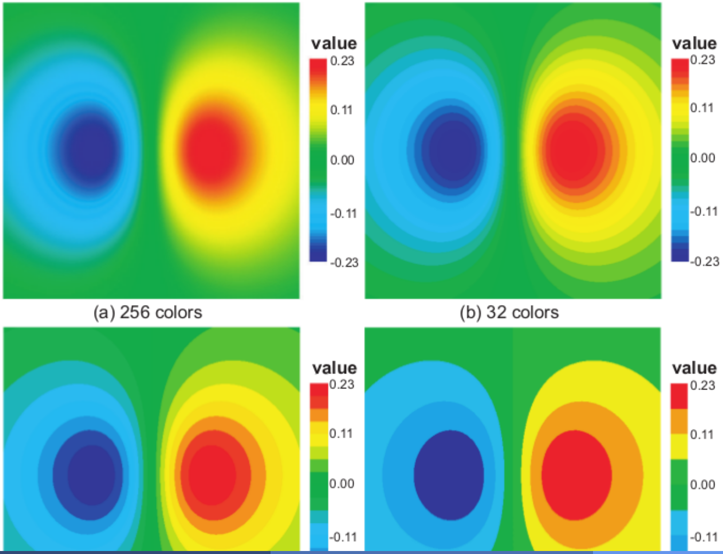
- **Construction of Colormaps**
  - **Color Legends:**
    - **Absolute values** goal can be achieved using color legends
    - a color strip containing all the colors c_i in colormap, annotated with labels that indicate the scalar values-f for all or a number of the depicted colors.
    - we are able to infer the scalar values of the depicted dataset at desired points in the drawn image.
    - color-mapping function in color legend must be injective $\rightarrow$ every scalar value in the range is associated with unique color.
    - we are able to visually distinguish separate regions having different colors with the variation of scalar data.
    - color legend supports Absolute values, Value ordering (how colors are ordered with respect to the ordering of the data values), value difference (to show which differences are small and which are large), selected values (which is the color showing the value of interest) and value change (color legend for magnitude of the gradient of scalar signal)

Scalar Visualization Technique

- **color banding**
  - Another important aspect in colormap design is the choice of the number of colors N
  - Choosing a small N would inevitably lead to the color banding effect.
  - color banding can be avoided by increasing the number of colors in the colormap and ensuring there are no sharp perceptual transitions in the colormap.
  - data varies quickly in spatial domain $\rightarrow$ avoid colorbanding
  - Typical scalar visualization applications $\rightarrow$ use $\rightarrow$ between 64 to 256 different colors in colormap
  - colorbanding usage in visualization of categorical data
    - First, we want to show the identity of each data value, i.e., the category i implied by the data value.
    - Second, if these values imply distinct categories, we cannot interpolate between the values.

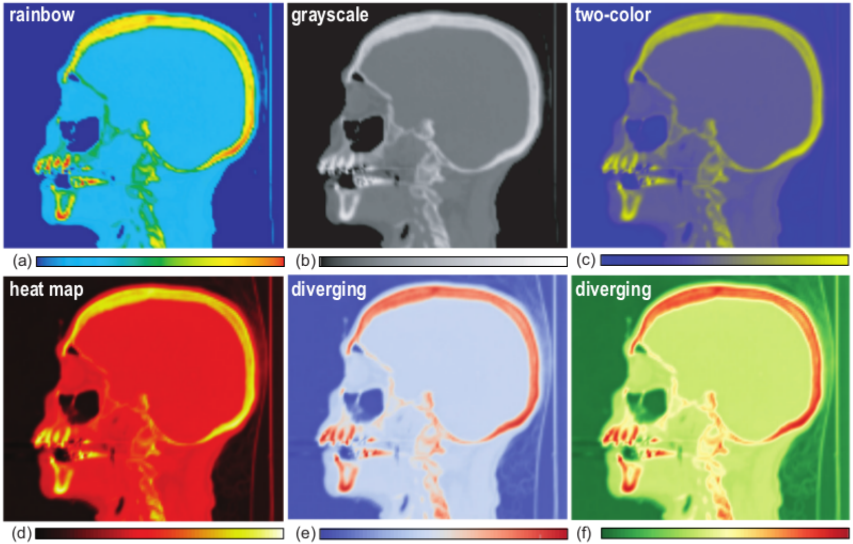
Scalar Visualization Technique

- Color banding on a dataset which ranges from -0.23 to 0.23



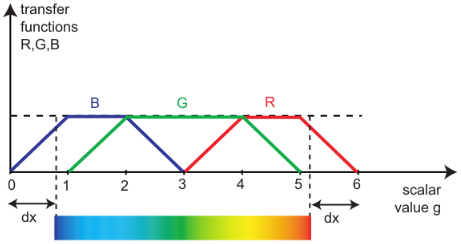
Scalar Visualization Technique

- Scalar visualization with color legends using various colormaps



Scalar Visualization Technique

- **Different types of Colormaps**
  - **Rainbow colormap or blue-to-red colormap**
    - red → hot color and high value
    - blue → cold color and low value
    - mostly in weather forecast applications
    - R,G,B can be used to construct rainbow colormap



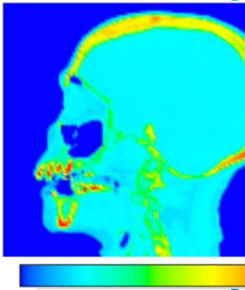
Scalar Visualization Technique

- **Rainbow colormap or blue-to-red colormap**
  - three trapezium-shaped transfer functions R, G, and B, ranging from zero to one.
  - The functions are centered at different locations on the scalar value axis, $\rightarrow$ which determines the blue-to-red colormap structure.
  - The functions overlap almost everywhere, which makes the hues in the colormap vary smoothly.
  - The parameter dx $\in$[0, 1] controls the amount of pure blue and red used at the beginning and the end of the colormap, respectively,
  - The transfer functions are linear basis functions to interpolate between their corresponding primary colors

Scalar Visualization Technique

- **Rainbow colormap Example**

- rainbow colormap applied to a 2D slice from a computer tomography (CT) volumetric dataset
- Data values indicate tissue density
- high data values corresponding to hard structures like bones
- low data values corresponding to soft tissues like brain, skin, fat, and muscles
- The lowest data values corresponds to the air
- harder tissues → color mapped to yellow, orange, and red
- the air → color mapped to dark blue



Scalar Visualization Technique

- **Rainbow colormap or blue-to-red colormap**
    - **Limitations:**
        - Focus
        - Luminance
        - Context
        - Ordering
        - Linearity

Scalar Visualization Technique

- Other colormap designs:
  - Grayscale
  - Two-hue
  - Heat-map
  - Diverging
  - Zebra colormap
  -

Scalar Visualization Technique

- **Contouring**
  - Using of few colors in a colormap leads to undesired color-banding effects
  - color banding is related to fundamental and widely used visualization technique called contouring.
  - A natural extension to color mapping is contouring
  - Think of the meaning of the sharp color transitions that separates the color bands. $\rightarrow$ gives the understanding of contour exactly.
  - the transition between the yellow and orange bands in next figure
  - All points in the figure that are on the border separating these two colors.
  - The dataset used for this image does not exhibit a sudden "jump" localized exactly on the extent of the border itself.

Scalar Visualization Technique

- **Contouring**

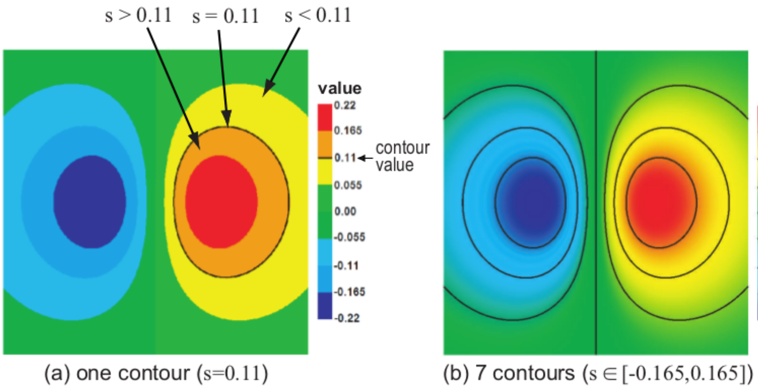


Figure 5.7. Relationship between color banding and contouring.

Scalar Visualization Technique

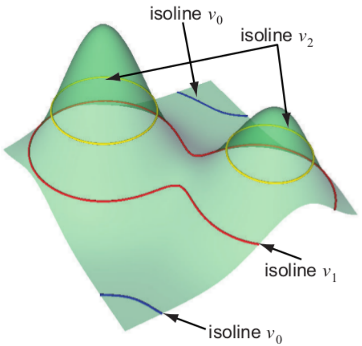
- **Contouring**
  - Points located on such a color border are called a contour line, or isoline.
  - Formally, a contour line C is defined as all points p in a dataset D that have the same scalar value, or isovalue s(p) = x
  - Mathematical Representation is: C(x) = $\{p \in D | s(p) = x\}$
  - Contour line stem came from cartography
  - Contour lines are drawn on land maps to explicitly indicate all points that have the same altitude.
  - The above equation can be used in higher dimensions as well
  - Example: weather maps annotated with lines of constant temp
  - For 3D datasets, contours are 2D surfaces called **isosurfaces** $\rightarrow$ can be approximated by many polygonal primitives.
  - Example: Medical Image Intensity $\rightarrow$ body tissues such as skin, bone, etc
  - combine the advantages of contours and color mapping $\rightarrow$ rich colormap

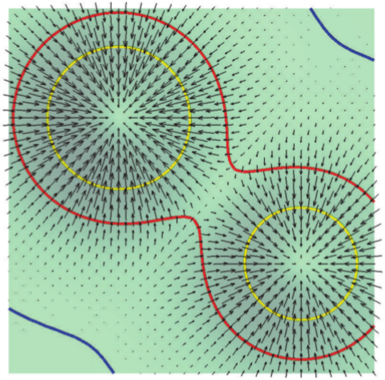
Scalar Visualization Technique

- **Contouring properties**
  - isolines can be either closed curves. $\rightarrow$ Isolines never stop inside the dataset itselfthey either close upon themselves, or stop when reaching the dataset border
  - an isoline never intersects (crosses) itself, nor does it intersect an isoline for another scalar value $\rightarrow$ isolines for different values are "nested" inside each other
  - The gradient of a function is the direction of the function's maximal variation, whereas contours are points of equal function value, so the tangent to a contour is the direction of the function's minimal (zero) variation.

Scalar Visualization Technique

- **Contouring Properties**





Scalar Visualization Technique

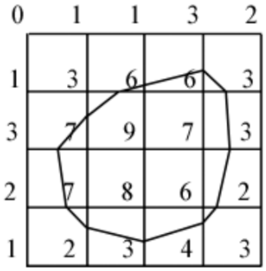
- **Contouring Computation**
  - How can we compute contours, given a discrete, sampled dataset D?
  - Since this dataset is defined as a set of cells carrying node or cell scalar data and additional basis functions $\rightarrow$ it is natural to try to construct contours in the same discrete cell space.
  - Reconstruction of a sampled dataset with piece-wise linear basis functions is a piecewise linear function itself.
  - The graph of this function is piecewise linear, too
  - Since an isoline has topological dimension 1, this is a polyline.

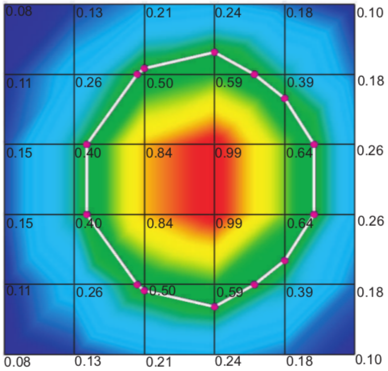
Scalar Visualization

- **Contouring Computation**
  - Basic algorithm for constructing an isoline
  - Consider a regular grid with scalar values assigned to the grid nodes.
  - Contouring always begins by selecting a scalar value that corresponds to the contour lines or surface generated.
  - Assumption: linear interpolation on the regular grid
  - we can identify those locations on the edges of the regular grid where the data assumes the isovalue.
  - Once the points on all edges are generated, we can connect these points into contours using a few different approaches.
  - One approach detects an edge intersection, i.e. the contour passes through an edge, and then tracks this contour as it moves across cell boundaries

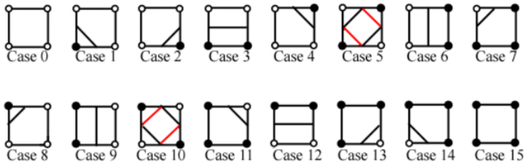
Scalar Visualization Technique

- **Contouring Computation**



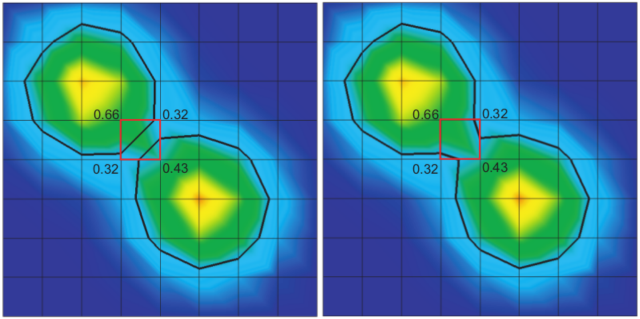


- **Contouring Computation**
  - We know that if a contour edge enters a cell, it must exit a cell as well.
  - The contour is tracked until it closes back on itself, or exits a data set boundary.
  - **Marching Squares**
    - This uses a divide and conquer technique, treating cells independently.
    - This marching squares algorithm assumes that a contour can only pass through a cell in a finite number of ways due to the linear interpolation used.



Scalar Visualization Technique

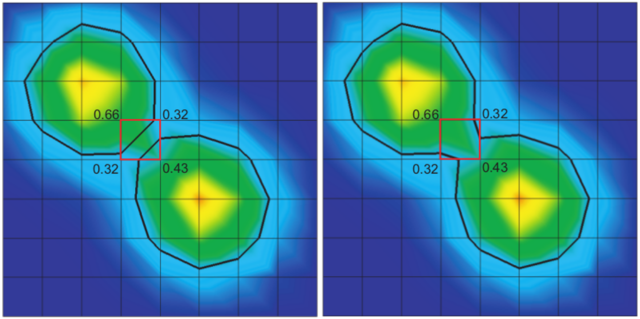
- **Contouring Computation**
  - Ambiguities in marching squares



(a) (b)

Scalar Visualization Technique

- **Contouring Computation**



(a) (b)

Scalar Visualization Technique

- **Contouring Computation**
  - **Marching Cubes**
    - Marching Cubes (MC) is an efficient method for extracting isosurfaces from scalar data set defined on a regular grid.
    - Similar to marching squares, surface segment is computed for each cell of the grid that approximates the isosurface.

Scalar Visualization Technique

- **Height Plots**
  - Also called elevation or carpet plots
  - Given 2D surface $D_s \in D$, part of a scalar dataset D, height plot can be described by the mapping operation $m: D_s \rightarrow D$, $m(x) \rightarrow x + s(x)n(x)$, $\forall\, x \in D_s$.
  - where s(x) is the scalar value of D at the point x and n(x) is the normal to the surface D_s at x.
  - Simply, the height-plot mapping operation "warps" a given surface D_s included in the dataset along the surface normal, with a factor proportional to the scalar values.
  - Height plots are a particular case of displacement, or warped, plots

Scalar Visualization Technique

- **Height Plots:**

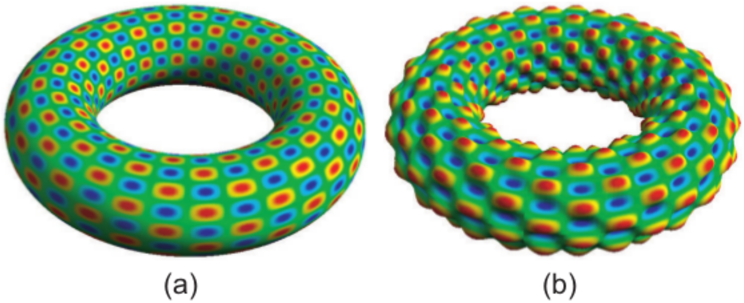


Figure 5.18. (a) Non-planar surface. (b) Height plot over this surface.

Scalar Visualization Technique

- Both height and color encode the scalar value. $\rightarrow$ bumps are red and valleys are blue

Scalar Visualization Technique

- **Height Plots:**

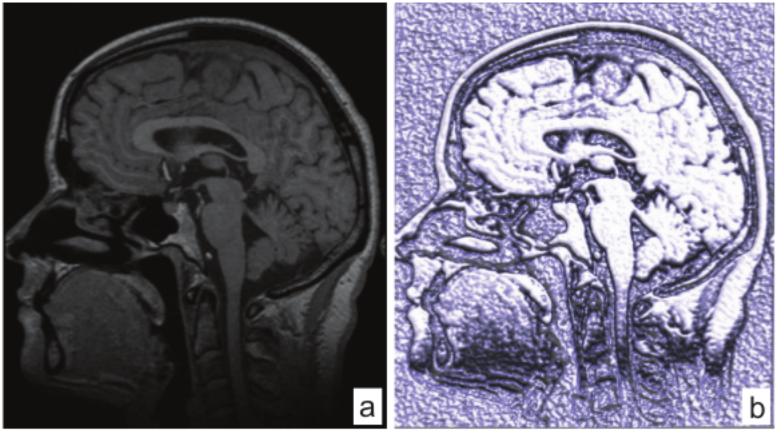


Figure 5.19. (a) Grayscale color mapping of scalar dataset. (b) Height plot
dataset, emphasizing fine-grained data variations.

Scalar Visualization Technique

- **Height Plots**
  - The left image shows a 2D scalar plot of a brain CT slice, displayed with a grayscale colormap.
  - Here, scalar values encode tissue density, with white corresponding to the hardest structures (bone).
  - The black color corresponds the air in the image
  - High-contrast borders between regions of different densities are seen easily, due the usage of linear scalar-to-grayscale colormap
  - Large differences of absolute data values are easily seen $\rightarrow$ the hardest and softest tissues within the given scan by looking for the brightest.
  - However, small-scale data variations are not easily visible. $\rightarrow$ 256 different colors

Scalar Visualization Technique

- **Height Plots**
  - By tuning the material specular and diffuse properties of the rendered 3D plot surface, we can now emphasize small-scale data variations much better.
  - However, we now can much better discern local, relatively small-scale, data variations, which map to luminance variations in the plot in height plots
  - absolute data values are now not visible.
  - shading differences do not depend on absolute, but relative data variations.