Mining Social Network Graphs

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Social Network

No introduction required

Really?

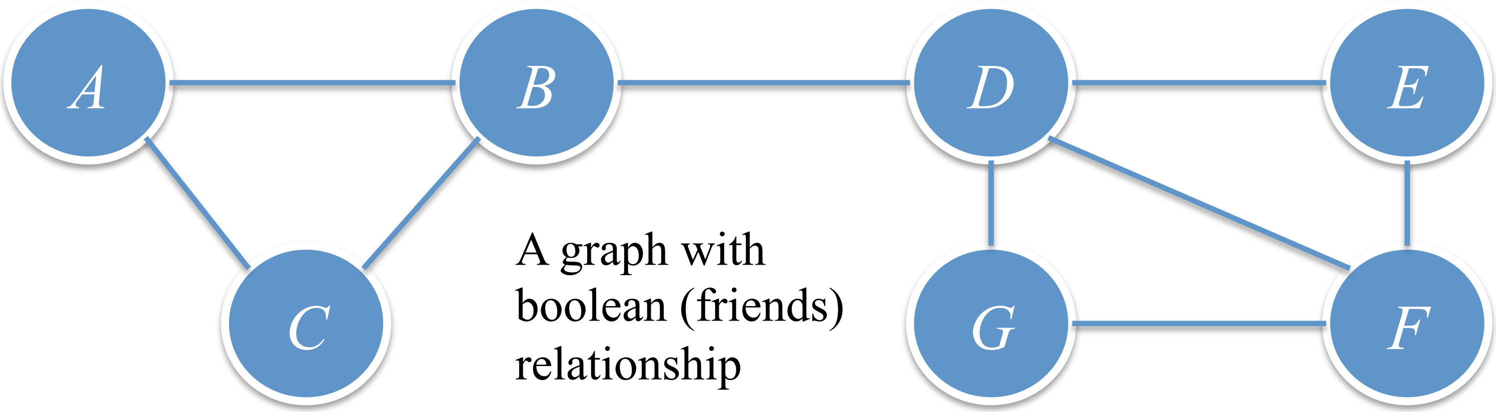
We still need to understand a few properties

disclaimer: the brand logos are used here entirely for educational purpose

Social Network

- A collection of **entities**
  - Typically people, but could be something else too
- At least one **relationship** between entities of the network
  - For example: friends
  - Sometimes **boolean**: two people are either friends or they are not
  - May have a **degree**
  - **Discrete** degree: friends, family, acquaintances, or none
  - Degree – **real number**: the fraction of the average day that two people spend talking to each other
- An assumption of **nonrandomness** or **locality**
  - Hard to formalize
  - Intuition: that relationships tend to cluster
  - If entity A is related to both B and C, then the probability that B and C are related is higher than average (random)

Social Network as a Graph



- Check for the non-randomness criterion
- In a random graph (V,E) of 7 nodes and 9 edges, if XY is an edge, YZ is an edge, what is the probability that XZ is an edge?
  - For a large random graph, it would be close to $|E|/({}^{|V|}C_2) = 9/21 \sim 0.43$
  - Small graph: XY and YZ are already edges, so compute within the rest
  - So the probability is $(|E|-2)/({}^{|V|}C_2-2) = 7/19 = 0.37$
- Now let's compute what is the probability for this graph in particular

Example courtesy: Leskovec, Rajaraman and Ullman

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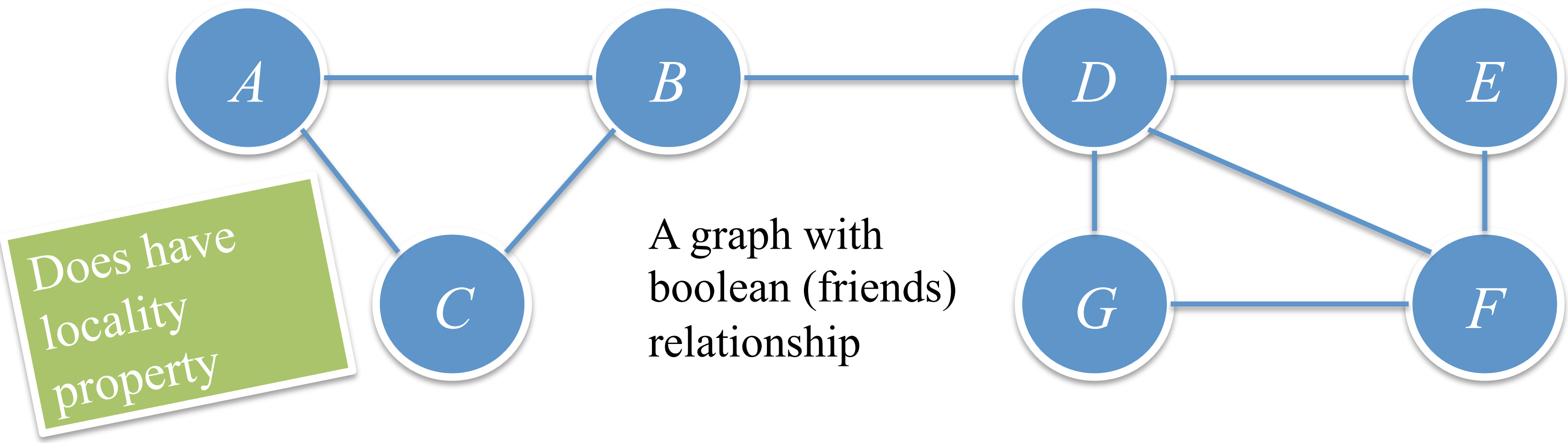


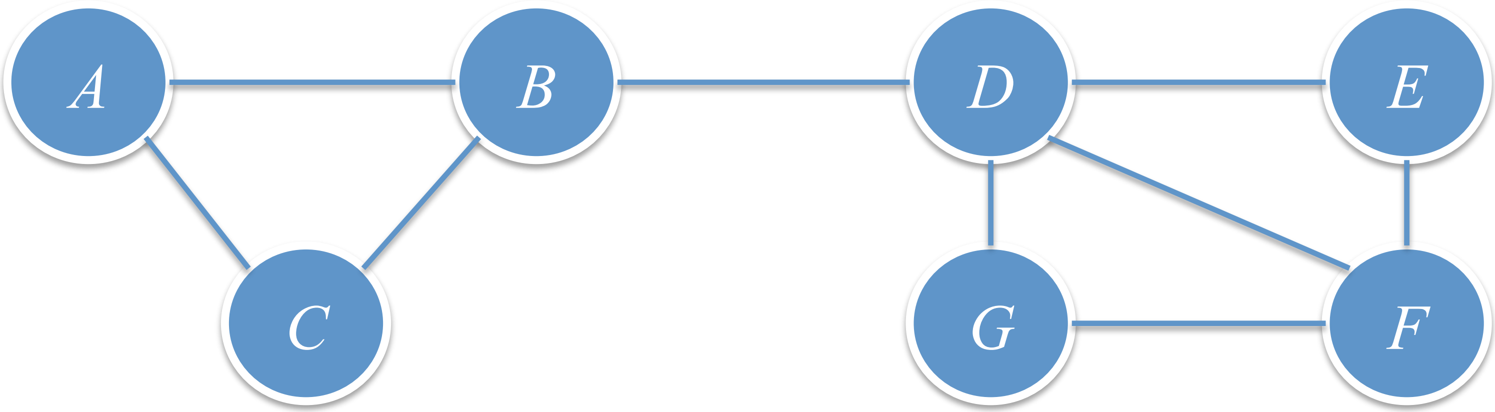
A graph with boolean (friends) relationship

- For each *X*, check possible *YZ* and check if *YZ* is an edge or not
- Example: if *X* = A, YZ = {BC}, it is an edge

X=	***YZ=***	***Yes/Total***
A	*BC*	1/1
B	*AC, AD, CD*	1/3
C	*AB*	1/1
D	*BE,BG,BF,EF, EG,FG*	2/6

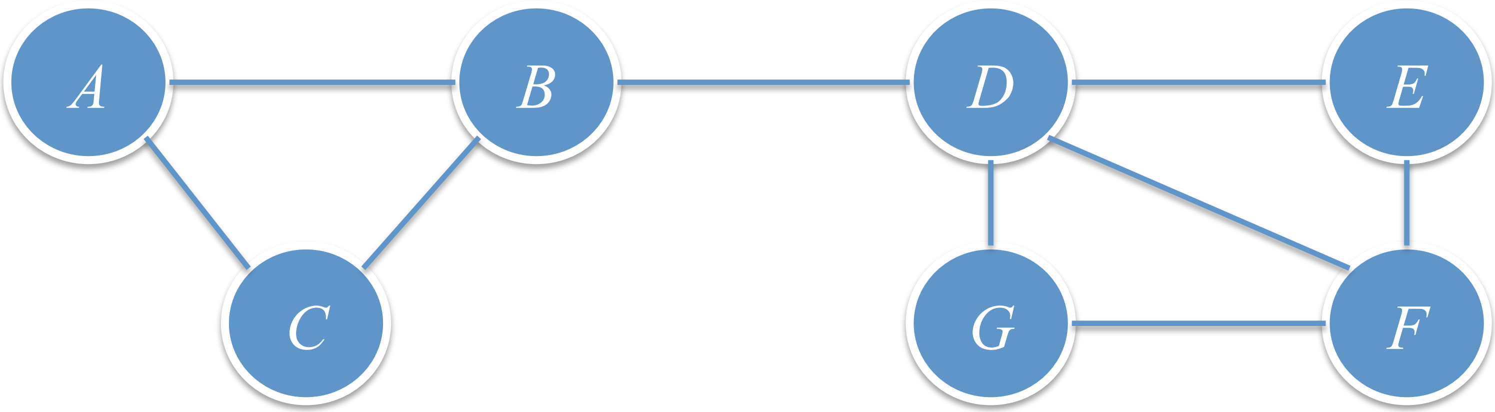
X=	***YZ=***	***Yes/Total***
E	*DF*	1/1
F	*DE,DG,EG*	2/3
G	*DF*	1/1
Total		9/16 ~ **0.56**

Types of Social (or Professional) Networks



- Of course, the "social network". But also several other types
- Telephone network
- Nodes are phone numbers
- *AB* is an edge if *A* and *B* talked over phone within the last one week, or month, or ever
- Edges could be weighted by the number of times phone calls were made, or total time of conversation

Types of Social (or Professional) Networks

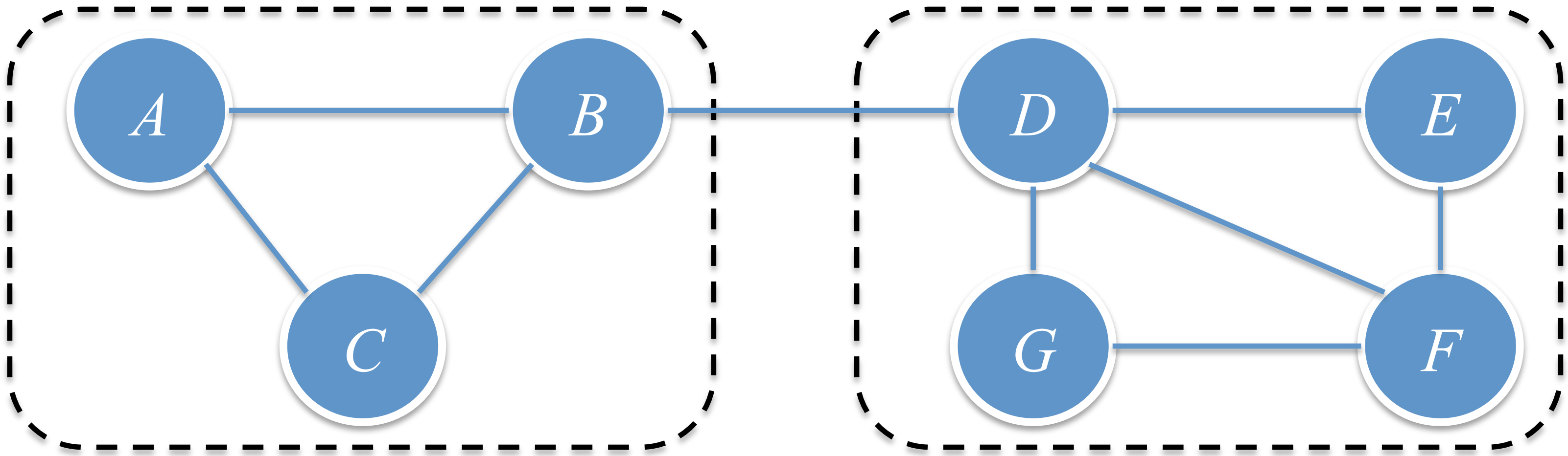


- Email network: nodes are email addresses
- AB is an edge if A and B sent mails to each other within the last one week, or month, or ever
  - One directional edges would allow spammers to have edges
- Edges could be weighted
- Other networks: collaboration network – authors of papers, jointly written papers or not
- Also networks exhibiting locality property

Clustering of Social Network Graphs

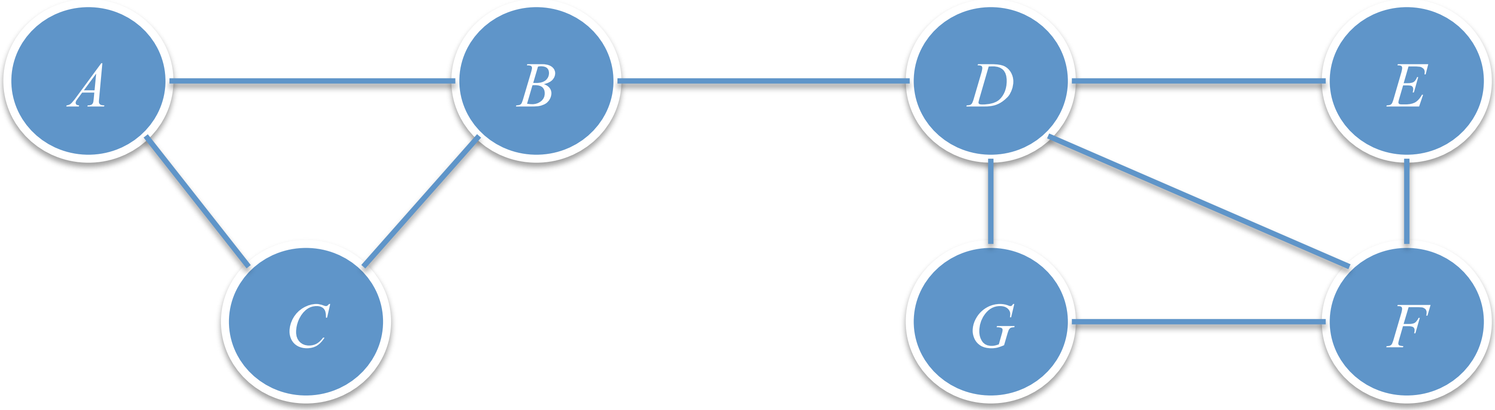
- Locality property → there are clusters
- Clusters are communities
  - People of the same institute, or company
  - People in a photography club
  - Set of people with "Something in common" between them
- Need to define a distance between points (nodes)
- In graphs with weighted edges, different distances exist
- For graphs with "friends" or "not friends" relationship
  - Distance is 0 (friends) or 1 (not friends)
  - Or 1 (friends) and infinity (not friends)
  - Both of these violate the triangle inequality
  - Fix triangle inequality: distance = 1 (friends) and 1.5 or 2 (not friends) or length of shortest path

Traditional Clustering



- Intuitively, two communities
- Traditional clustering depends on the distance
  - Likely to put two nodes with small distance in the same cluster
  - Social network graphs would have cross-community edges
  - Severe merging of communities likely
- May join *B* and *D* (and hence the two communities) with not so low probability

Betweenness of an Edge



- Betweenness of an edge AB: #of pairs of nodes (X,Y) such that AB lies on the shortest path between X and Y
  - There can be more than one shortest paths between X and Y
  - Credit AB the fraction of those paths which include the edge AB
- High score of betweenness means?
  - The edge runs "between" two communities
- Betweenness gives a better measure
  - Edges such as *BD* get a higher score than edges such as *AB*
- Not a distance measure, may not satisfy triangle inequality. Doesn't matter!

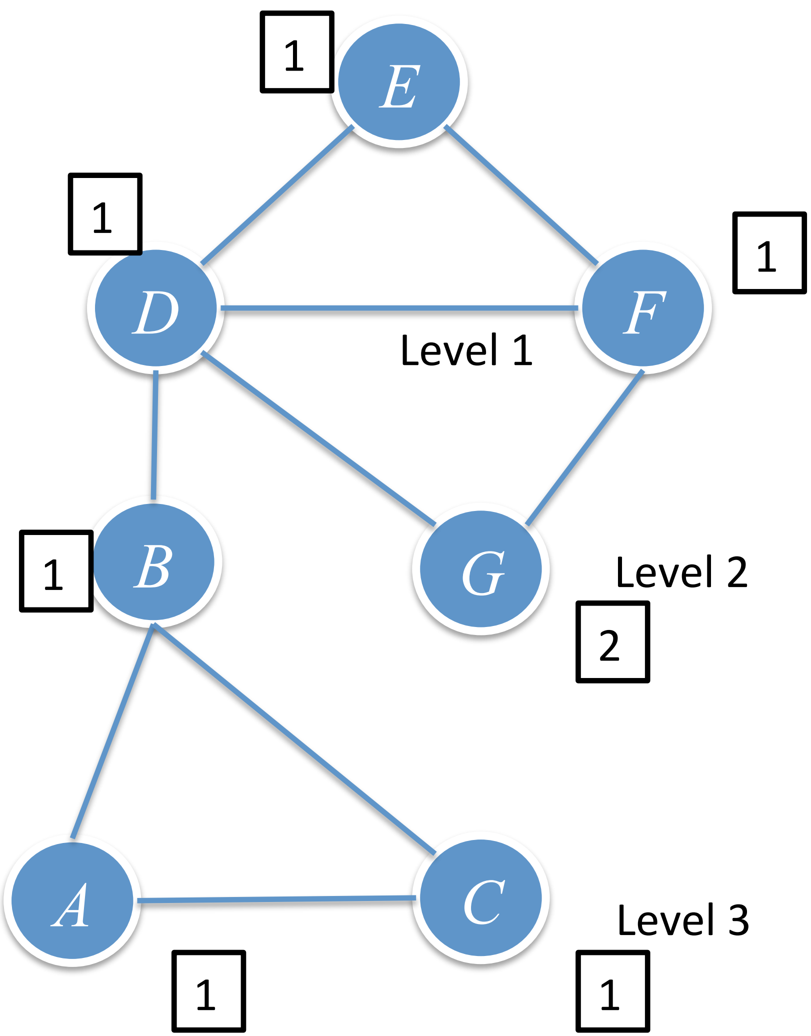
The Girvan – Newman Algorithm

- Step 1 – BFS: Start at a node X, perform a BFS with X as root

- Observe: level of node Y = length of shortest path from X to Y
- Edges between level are called “DAG” edges
  - Each DAG edge is part of at least one shortest path from X

- Step 2 – Labeling: $\boxed{\text{Label}}$ each node Y by the number of shortest paths from X to Y

Calculate *betweenness* of edges



The Girvan – Newman Algorithm

Step 3 – credit sharing:

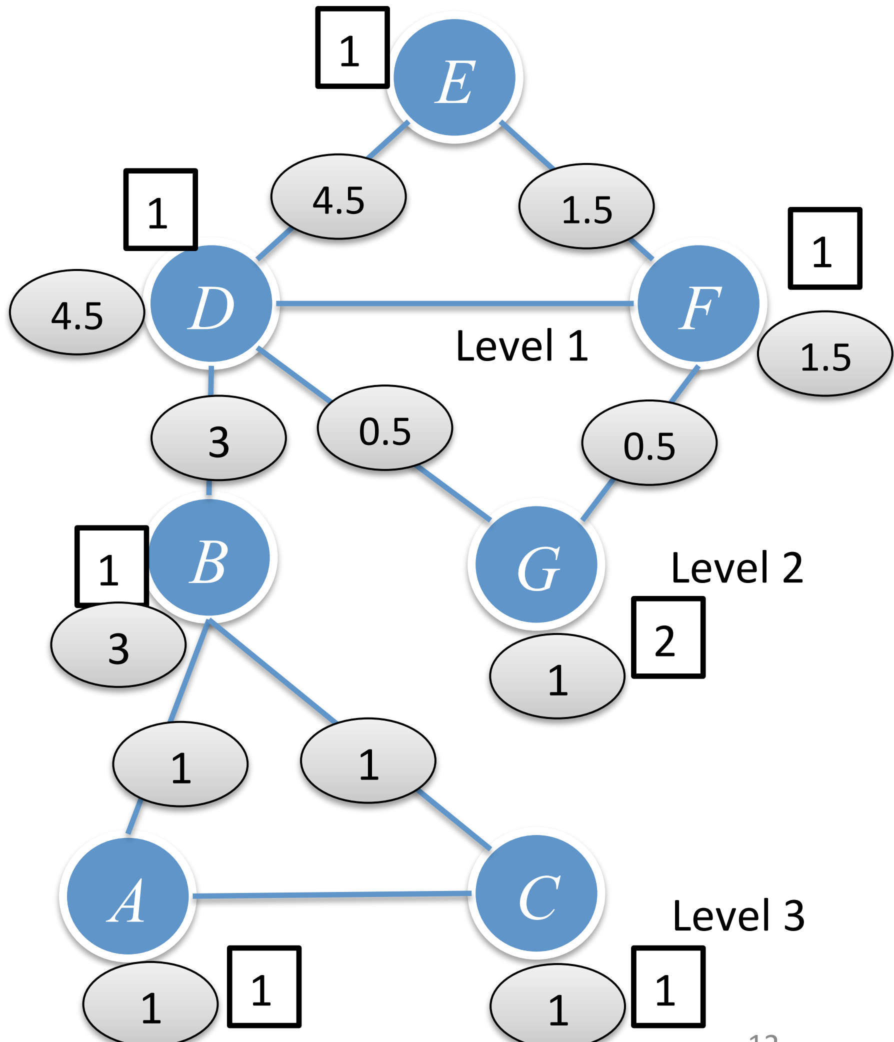
- Each leaf node gets credit 1
- Each non-leaf node gets 1 + sum(credits of the DAG edges to the level below)
- Credit of DAG edges: Let Y_i (i=1, … , k) be parents of Z, p_i = label(Y_i)

$$credit(Y_i, Z) = \frac{credit(Z) \times p_i}{(p_1 + \cdots p_k)}$$

- Intuition: a DAG edge Y_iZ gets the share of credit of Z proportional to the #of shortest paths from X to Z going through Y_iZ

Finally: Repeat Steps 1, 2 and 3 with each node as root. For each edge, betweenness = sum credits obtained in all iterations / 2

Calculate betweenness of edges



Computation in practice

- Complexity: n nodes, e edges
  - BFS starting at each node: $O(e)$
  - Do it for n nodes
  - Total: $O(ne)$ time
  - Very expensive
- Method in practice
  - Choose a random subset W of the nodes
  - Compute credit of each edge starting at each node in W
  - Sum and compute betweenness
  - A reasonable approximation

Finding Communities using Betweenness

Method 1:

- Keep adding edges (among existing ones) starting from lowest betweenness
- Gradually join small components to build large connected components

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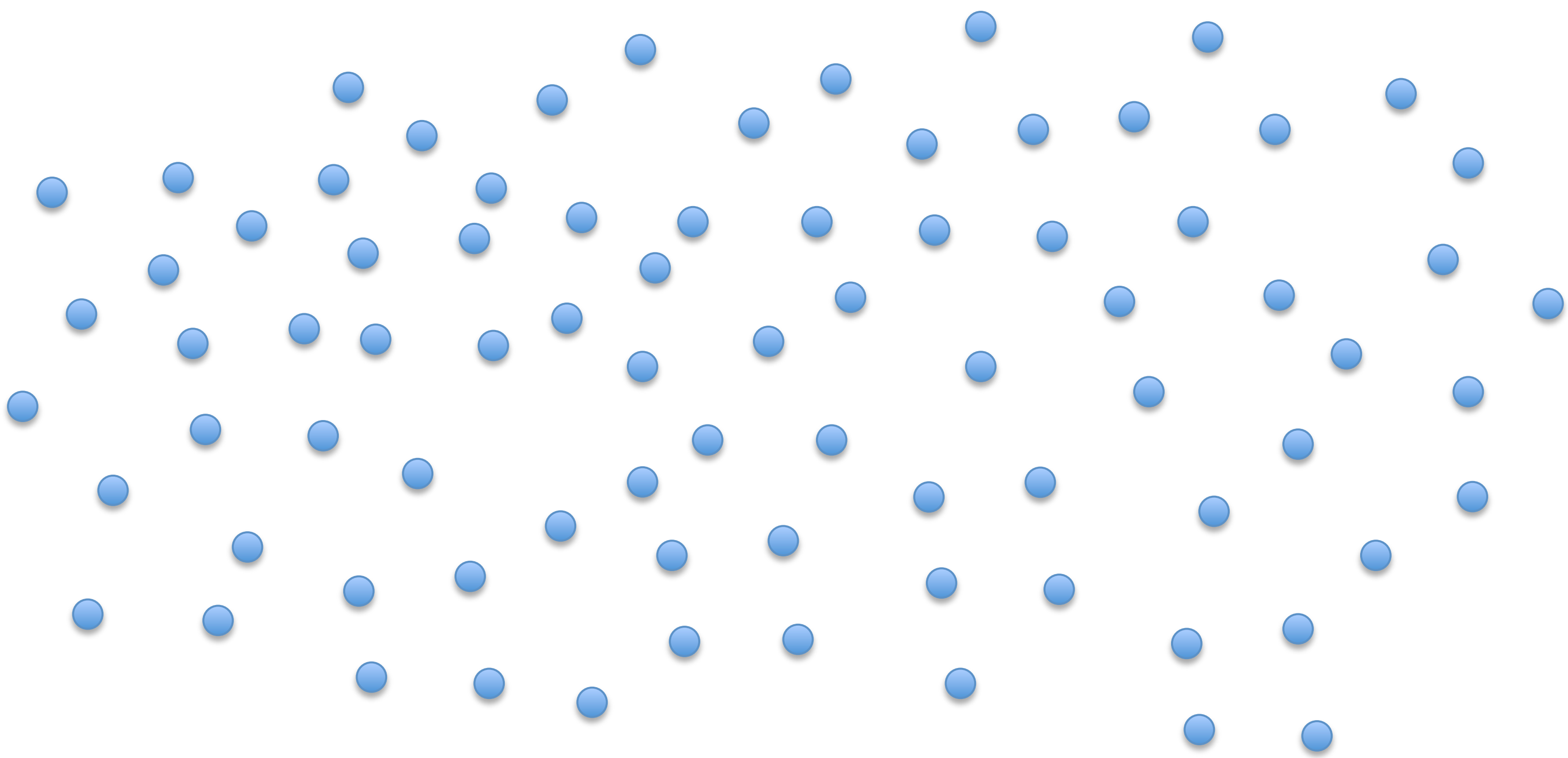

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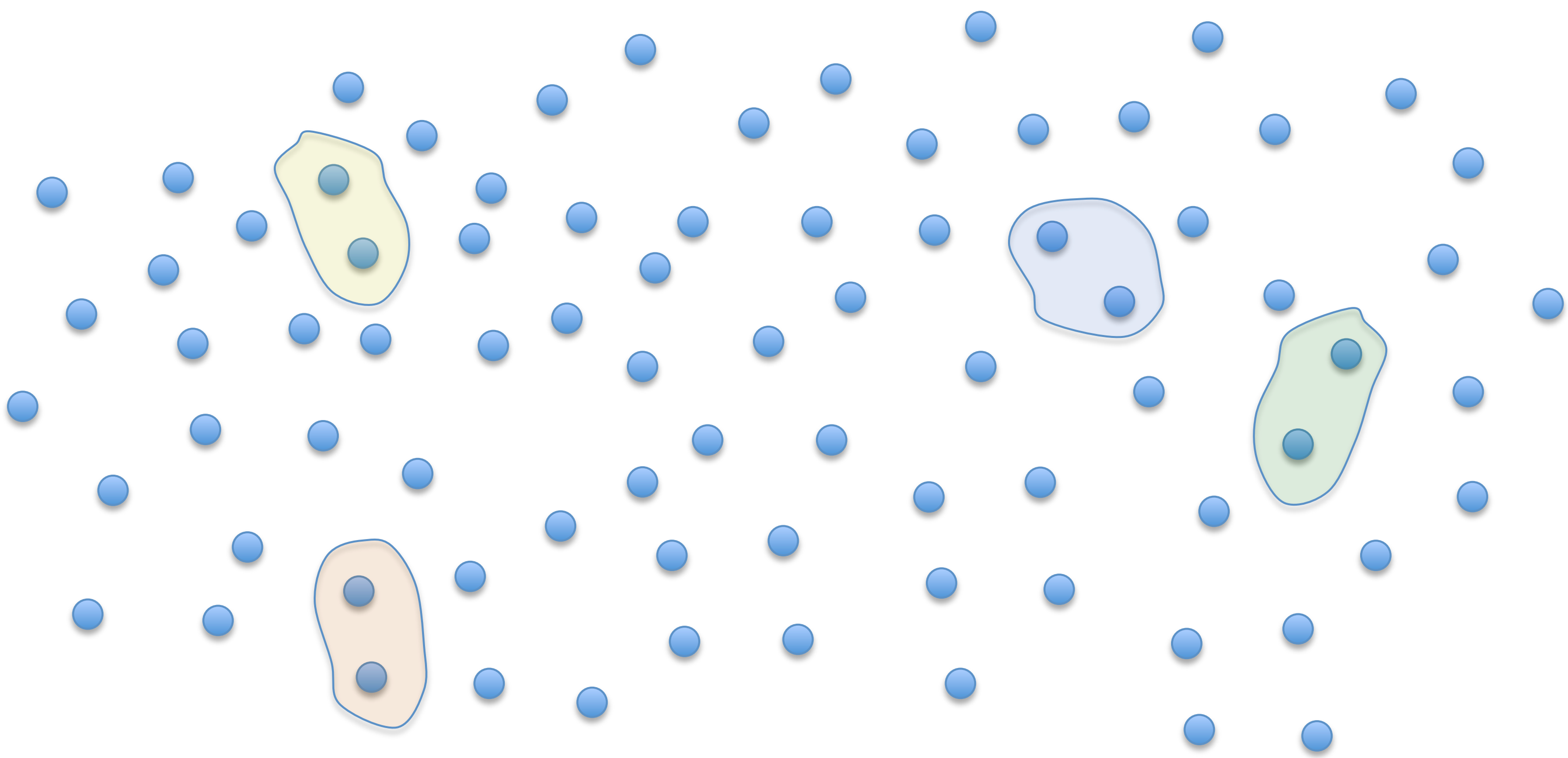

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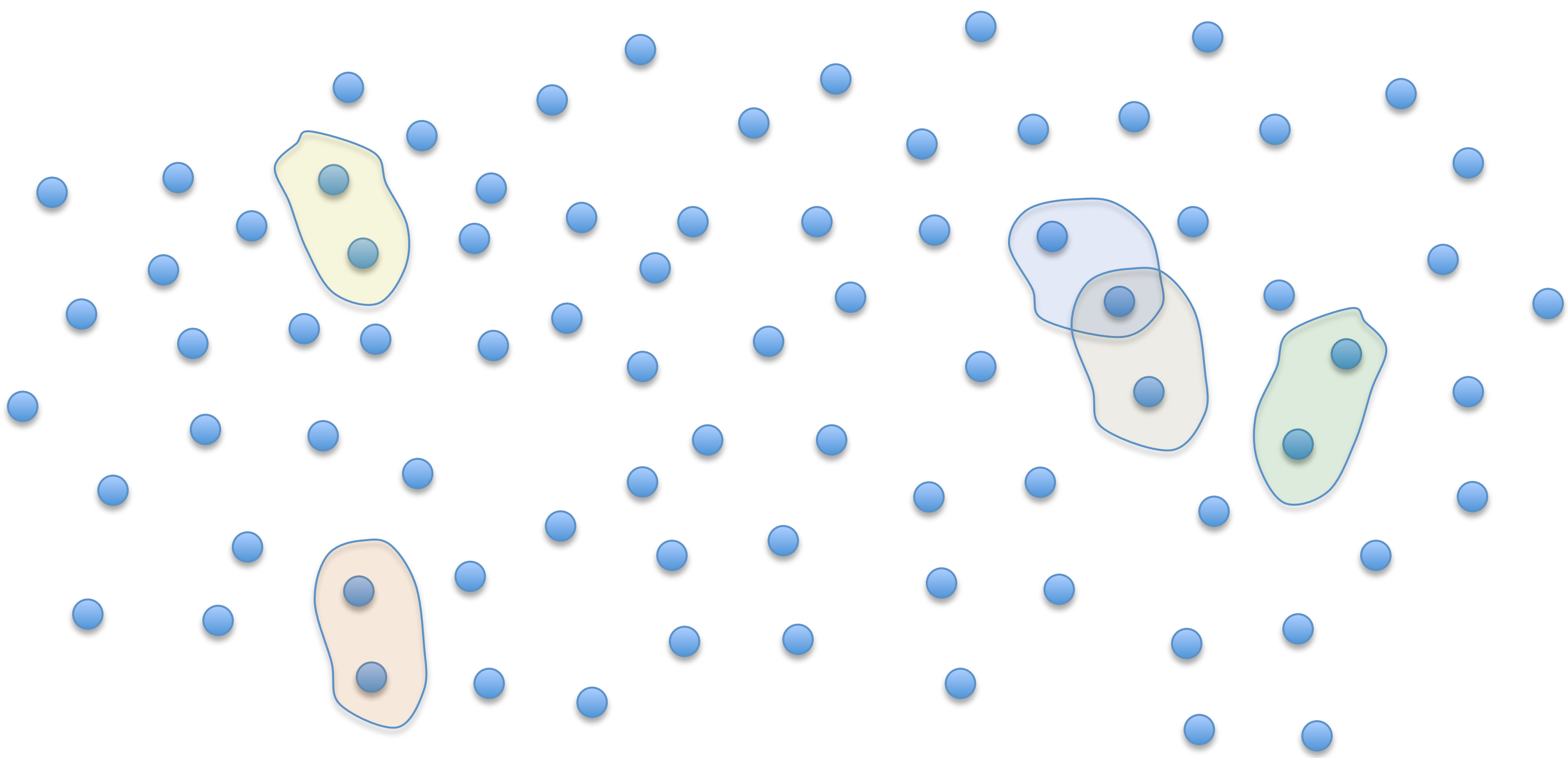

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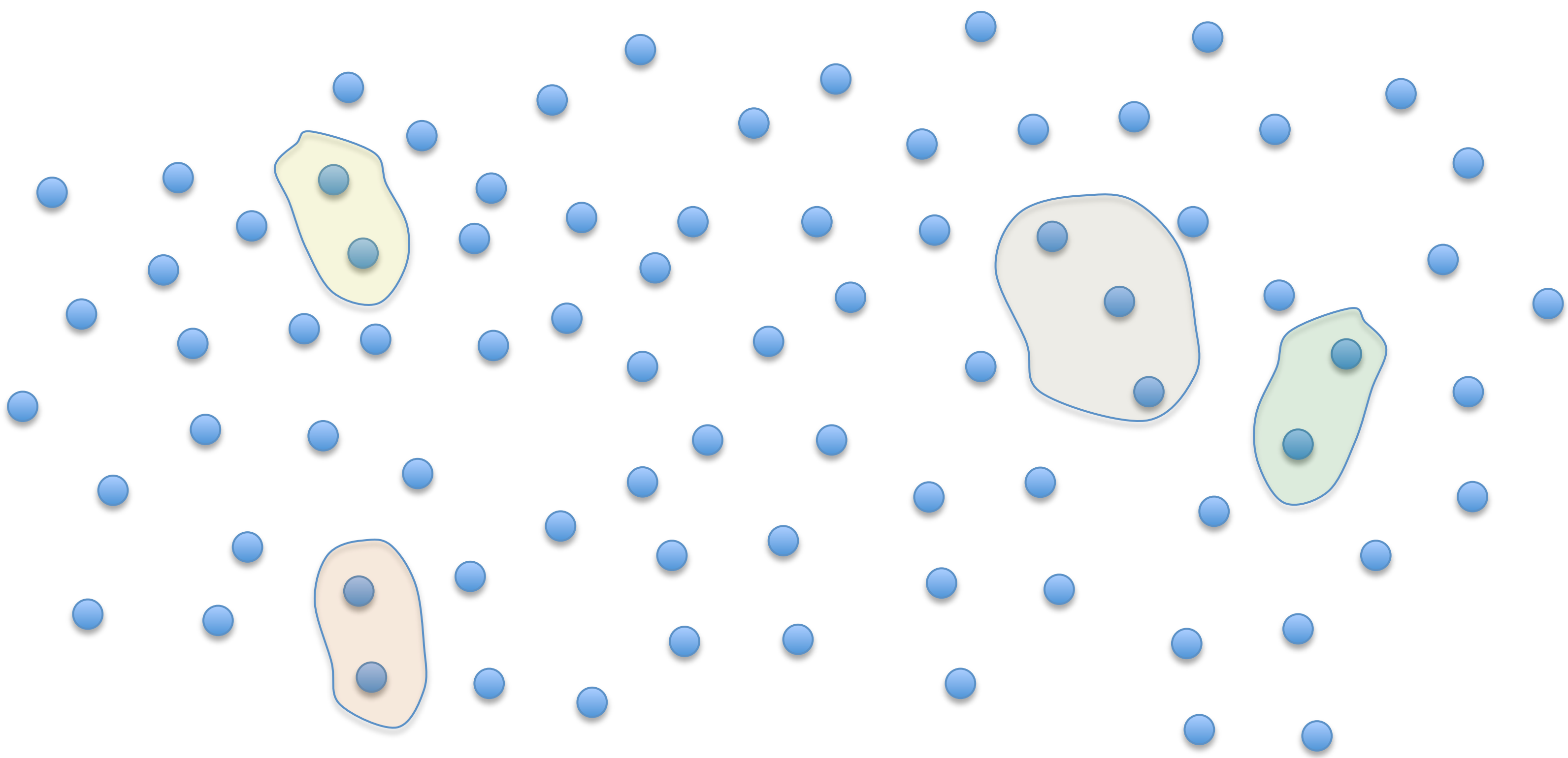

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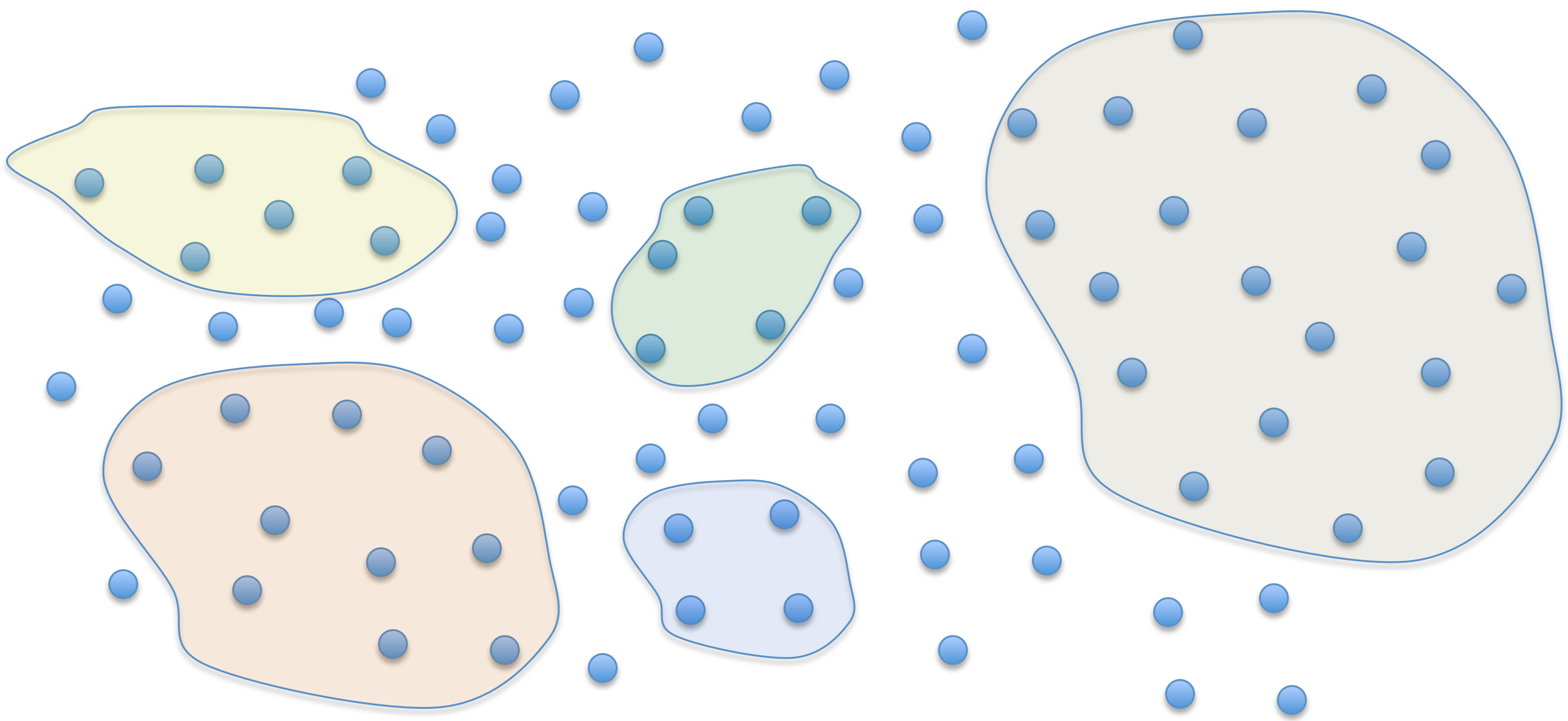
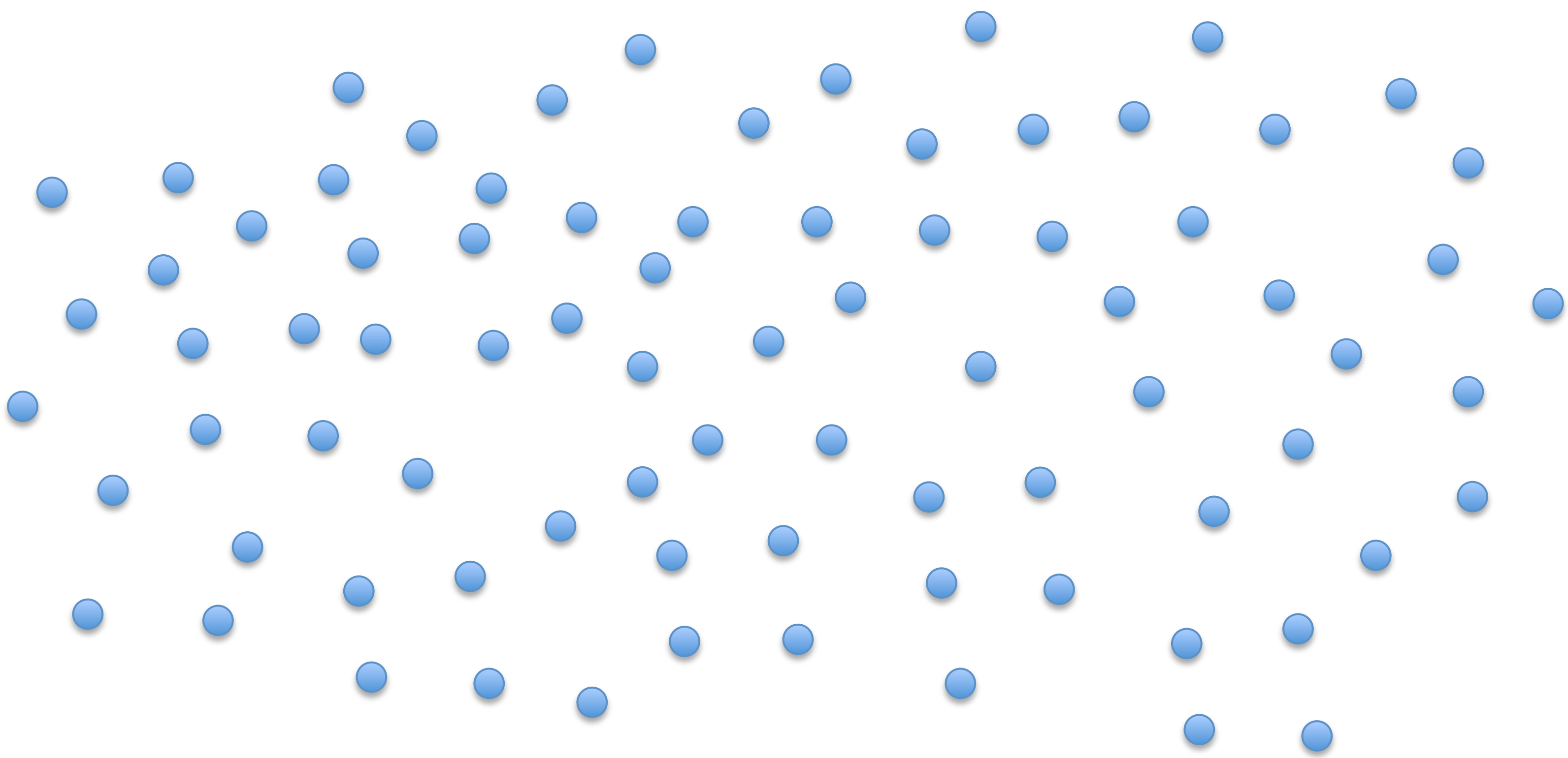


Finding Communities using Betweenness

Method 2:

- Start from all existing edges. The graph may look like one big component.
- Keep removing edges starting from highest betweenness
- Gradually split large components to arrive at communities



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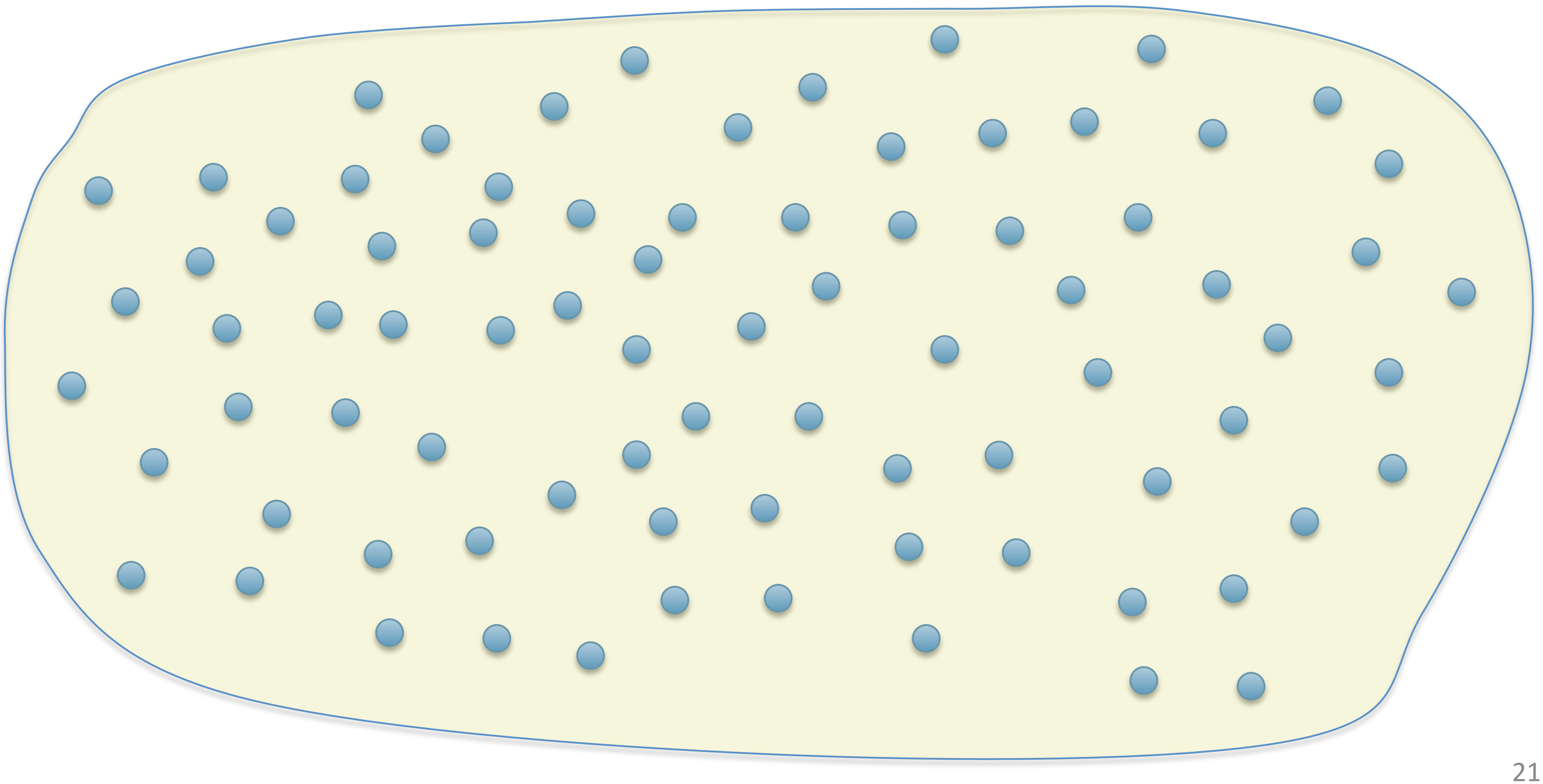

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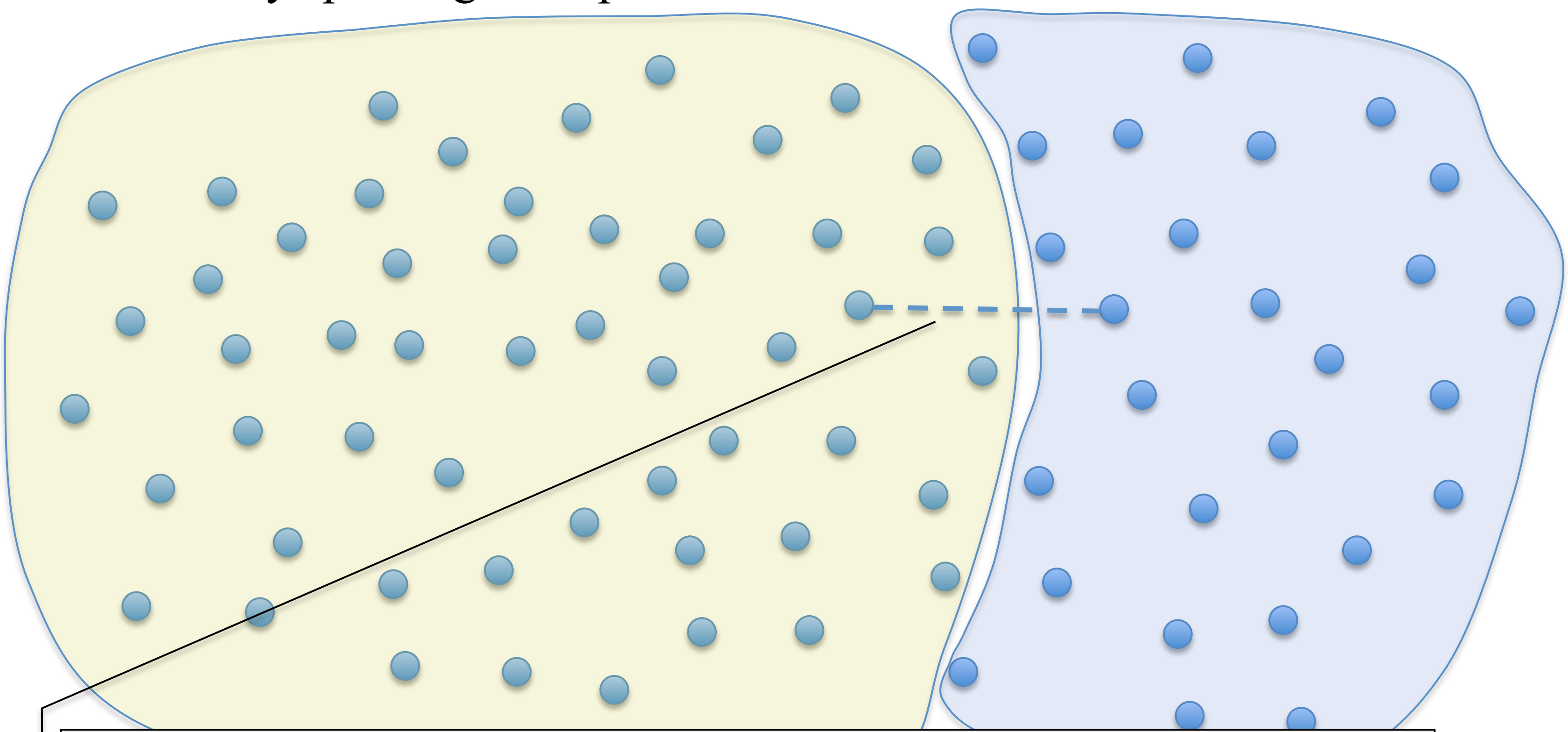


At some point, removing the edge with highest betweenness would split the graph into separate components

Finding Communities using Betweenness

- For a fixed threshold of betweenness, both methods would ultimately produce the same clustering
- However, a suitable threshold is not known beforehand
- Method 1 vs Method 2
  - Method 2 is likely to take less number of operations. Why?
  - Inter-community edges are less than intra-community edges

Triangles in Social Network Graph

- Number of triangles in a social network graph is expected to be much larger than a random graph with the same size
  - The locality property
- Counting the number of triangles
  - How much the graph looks like a social network
  - Age of community
    - A new community forms
    - Members bring in their *like minded* friends
    - Such new members are expected to eventually connect to other members directly

Triangle Counting Algorithm

Graph (V, E); $|V| = n$, $|E| = m$

- Step 1: Compute degree of each node
  - Examine each edge
  - Add degree 1 to each of the two nodes
  - Takes $O(m)$ time
- Step 2: A hash table $(v_i,v_j) \rightarrow 1$
  - So that, given two nodes, we can determine if they have an edge between them
  - Construction takes $O(m)$ time
  - Each query $\sim_{\text{expected}}$ $O(1)$ time, with a proper hash function
- Step 3: An index $v \rightarrow$ list of nodes adjacent to v
  - Construction takes $O(m)$ time, querying takes $O(1)$ time

Counting Heavy Hitter Triangles

- *Heavy hitter* node: a node with degree $\geq \sqrt{m}$
- Note: there are at most $2\sqrt{m}$ heavy hitter nodes
  - More than $2\sqrt{m}$ nodes → total degree $> 2m$ (but $|E| = m$)
- Heavy hitter triangle: triangle with all 3 heavy hitter nodes
- Number of possible heavy hitter triangles: at most ${}^{2\sqrt{m}}C_3 \sim O(m^{3/2})$
- For each possible triangle, use hash table (step 2) to check if all three edges exist
- Takes $O(m^{3/2})$ time

Counting other Triangles

- Consider an ordering of nodes $v_i << v_j$ if
  - Either degree(v_i) < degree(v_j), and
  - If degree(v_i) = degree(v_j) then $i < j$
- For each edge (v_i,v_j)
  - If both nodes are heavy hitters, skip (already done)
  - Suppose v_i is not a heavy hitter
  - Find nodes $w_1, w_2, \ldots, w_k$ which are adjacent to v_i (using node → adjacent nodes index, step 3) [Takes $O(k)$ time]
  - For each w_l , $l = 1, \ldots , k$ check if edge $v_j w_l$ exist, in $O(1)$ time, total $O(k)$ time
  - Count the triangle $\{v_i\, v_j\, w_l\}$ if and only if
    - Edge $v_j w_l$ exists
    - Also $v_i << w_l$
  - Total time for each edge (v_i,v_j) is $O(\sqrt{m})$
  - There are m edges, total time is $O(m^{3/2})$ time

Optimality

<u>Worst case scenario</u>

- If G is a complete graph
- Number of triangles = ${}^{m}C_3 \sim O(m^{3/2})$
- Cannot even enumerate all triangles in less than $O(m^{3/2})$
- Hence it is the lower bound for computing all triangles

<u>If G is sparse</u>

- Consider a complete graph G' with n nodes, m edges
- Note that $m = {}^{n}C_2 = O(n^2)$
- Construct G from G' by adding a chain of length n^2
- The number of triangles remain the same, $O(m^{3/2})$
- The number of edges remain of the same order $O(m)$
- G is quite sparse, lowering edge to node ratio
- Still cannot compute the triangles in less than $O(m^{3/2})$ time

Directed Graphs in (Social) Networks

- Set of nodes V and directed edges (arcs) $u \rightarrow v$
- The web: pages link to other pages
- Persons made calls to other persons
- Twitter, Google+: people follow other people
- All undirected graphs can be considered as directed
  - Think of each edge as bidirectional

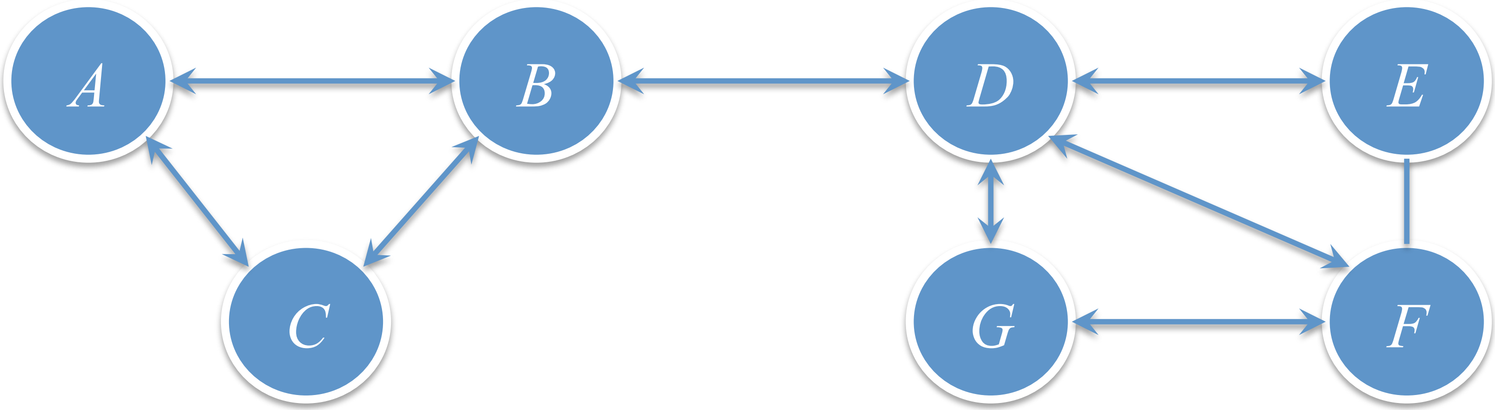
Paths and Neighborhoods

- Path of length k: a sequence of nodes $v_0, v_1, \ldots, v_k$ from v_0 to v_k so that $v_i \rightarrow v_{i+1}$ is an arc for $i = 0, \ldots, k-1$

- Neighborhood $N(v,d)$ of radius d for a node v: set of all nodes w such that there is a path from v to w of length $\leq d$
- For a set of nodes V, $N(V,d) := \{w \mid$ there is a path of length $\leq d$ from some v in V to $w\}$
- Neighborhood profile of a node v: sequence of sizes of its neighborhoods of radius $d = 1, 2, \ldots$; that is

$$|N(v,1)|, |N(v,2)|, |N(v,3)|, \ldots$$

Neighborhood Profile



Neighborhood profile of B

$N(\text{“}B\text{”},1) = 4$

$N(\text{“}B\text{”},2) = 7$

Neighborhood profile of A

$N(\text{“}A\text{”},1) = 3$

$N(\text{“}A\text{”},2) = 4$

$N(\text{“}A\text{”},3) = 7$

Diameter of a Graph

- Diameter of a graph $G(V,E)$: the smallest integer d such that for any two nodes v, w in V, there is a path of length at most d from v to w
  - Only makes sense for *strongly connected* graphs
  - Can reach any node from any node
- The web graph: not strongly connected
  - But there is a large strongly connected component
- The six degrees of separation conjecture
  - The diameter of the graph of the people in the world is six

Diameter and Neighborhood Profile

- Neighborhood profile of a node v

  $|N(v,1)|, |N(v,2)|, |N(v,3)|, \ldots \ldots$ $|V| = N(v,k)$ for some k
- Denote this k as $d(v)$
- If G is a complete graph, $d(v) = 1$
- Diameter of G is $\max_v\{d(v)\}$

Reference

- *Mining of Massive Datasets*, by Leskovec, Rajaraman and Ullman, Chapter 10