

# Link Analysis

## Lecture 7



# Outline

## 1. Let's Build a Search Engine :)

- The Model
- Problem #1: Fast Text Search
- Problem #2: Ranking Documents
- Enter the  ers

## 2. Bringing Order to the Web

- Voting with Your Links
  - Two Equivalent Views
- Problem Representation
  - Spiders Everywhere!
- Return of the Spammer (Farms)

## 3. Related Approaches

- Topic-Specific PageRank
- SimRank
- HITS: Hubs and Authorities



# What Makes a Search Engine?

- Inputs
  - The Web (set of webpages)
    - Text, images, downloads
    - Links to other pages
  - User query
    - Simplest: set of words
- Outputs
  - Ranked list of the “best” documents related to the query
  - Desired properties
    - Fast & scalable
    - Relevant results
    - Expressive queries
    - Up-to-date



# Problem #1

northeastern university

All Maps News Images Videos More Settings Tools

About 14,800,000 results (0.49 seconds)

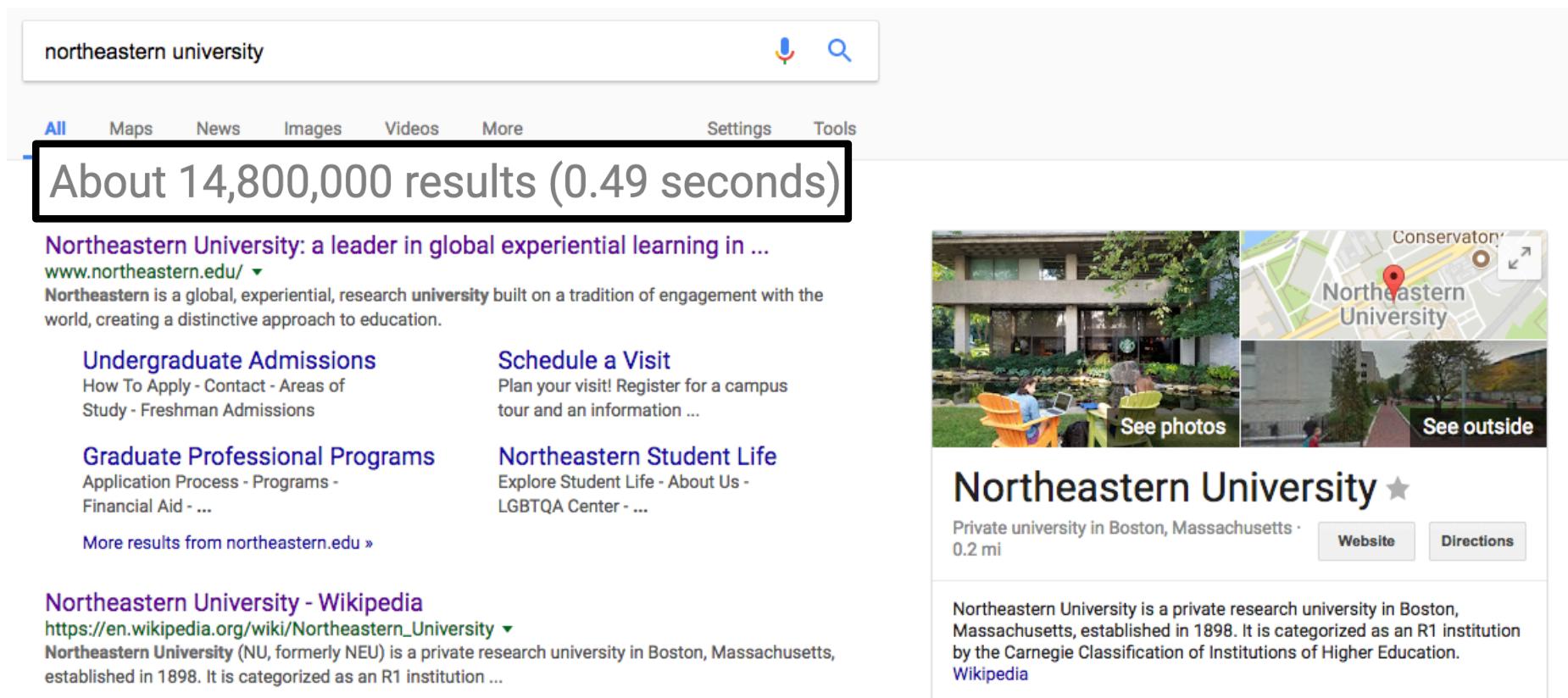
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[www.northeastern.edu/](http://www.northeastern.edu/) ▾  
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[More results from northeastern.edu »](#)

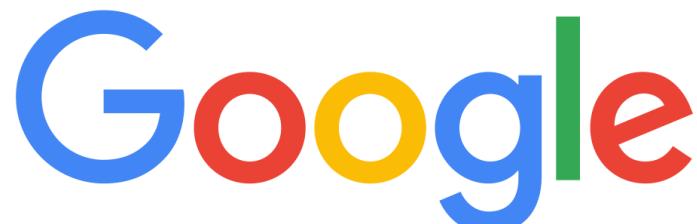
[Northeastern University - Wikipedia](#)  
[https://en.wikipedia.org/wiki/Northeastern\\_University](https://en.wikipedia.org/wiki/Northeastern_University) ▾  
Northeastern University (NU, formerly NEU) is a private research university in Boston, Massachusetts, established in 1898. It is categorized as an R1 institution ...



The image shows a Google search results page for the query "northeastern university". The top result is a link to the official website of Northeastern University. Below it are links for undergraduate admissions, graduate professional programs, and more results from the university's website. At the bottom is a link to the university's entry on Wikipedia. To the right of the search results, there is a sidebar featuring a photograph of a building, a map of the university's location in Boston, and a "See photos" button. Below this is a summary box for Northeastern University, which includes its status as a private university in Boston, Massachusetts, its distance from the user (0.2 mi), and buttons for "Website" and "Directions".



# Fast & Scalable: # of Documents



**> 130 Trillion Pages**



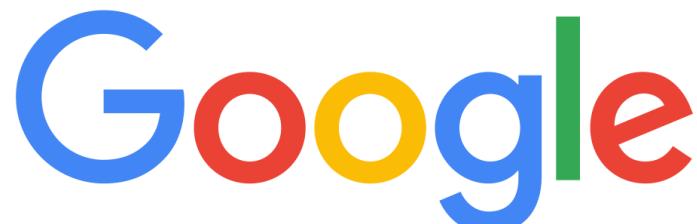
**~ 2 Billion Users**



**~ 400 Million Products**



# Fast & Scalable: Queries/sec



> 63K



~ 6K



# Linear-Time Google

- Assume simple query:

northeastern university

~10.4M blu ray  
~ 7.75 miles  
~ 15 x Burj Khalifa

- Single repo of all pages
  - $130T * 250 \text{ w/page} * 8 \text{ bytes/w} \sim 260 \text{ PB}$
- Require 1s response time
  - $8M * 3\text{GHz} 64\text{-bit CPU} (\text{assume 1 cycle/w})$
  - $(8M * 63K) \text{ CPUs} * 35W/\text{CPU} * \$0.14/\text{kWH}$   
 $\sim \$0.7M/\text{sec}$

~67 CPU/person

~ \\$21.6T/year  
~ 1.25 x US GDP



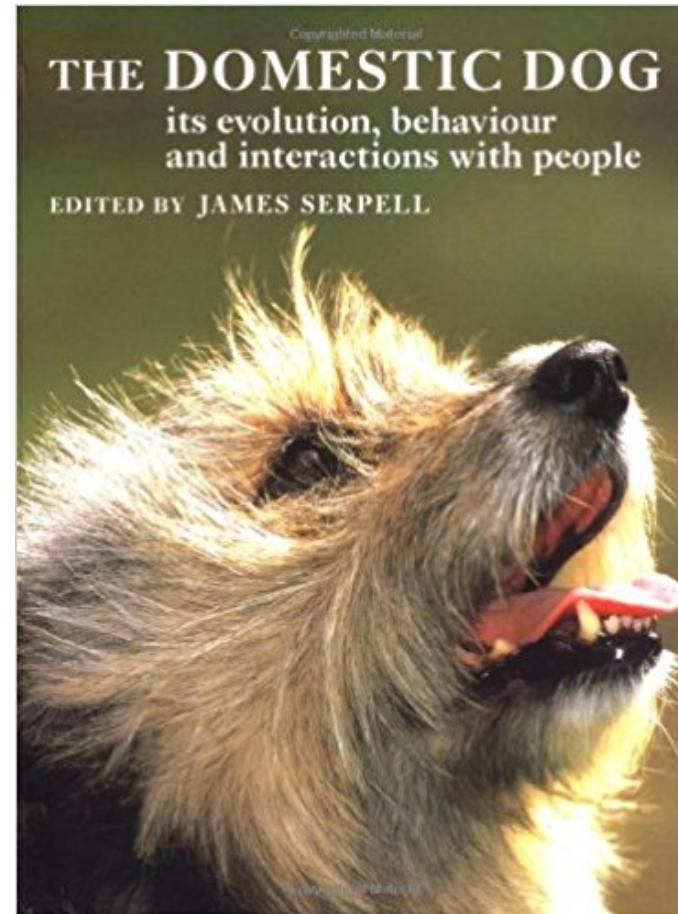
# Enter: The Inverted Index

## Physical Book

Find all pages that contain  
the word “Husky”

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Link Analysis

# In the Beginning...



# In the Beginning...

 Serious Sports Fans Only \$1,000,000 in Cash and Prizes!  
For serious sports fans only! Play Fantasy Football!



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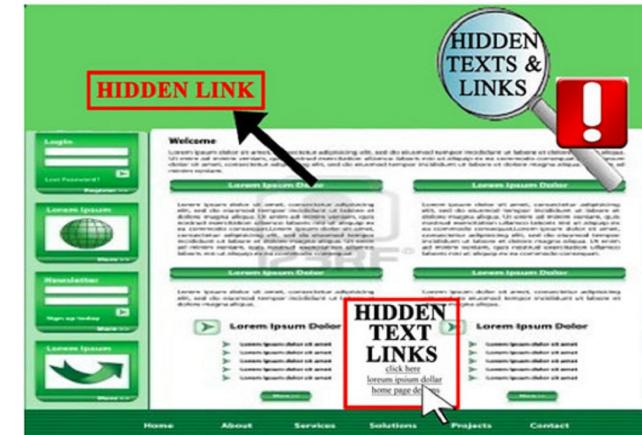
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Link Analysis

# Initial Approaches

- Human-curated (e.g. Yahoo)
  - Hand-written descriptions
  - Wait time for inclusion
- Text-search (e.g. Lycos)
  - Prone to term spam
- **Core Question:** how to automatically rank pages (i.e. efficiently) in a quality way that is resistant to term spam?
  - And, at least early on, in an **unsupervised** fashion (i.e. given only the contents of the pages + structure of the web)



# A Humble Academic Paper

## *L. Page, S. Brin, R. Motwani, T. Winograd*

### The PageRank Citation Ranking: Bringing Order to the Web

January 29, 1998

#### Abstract

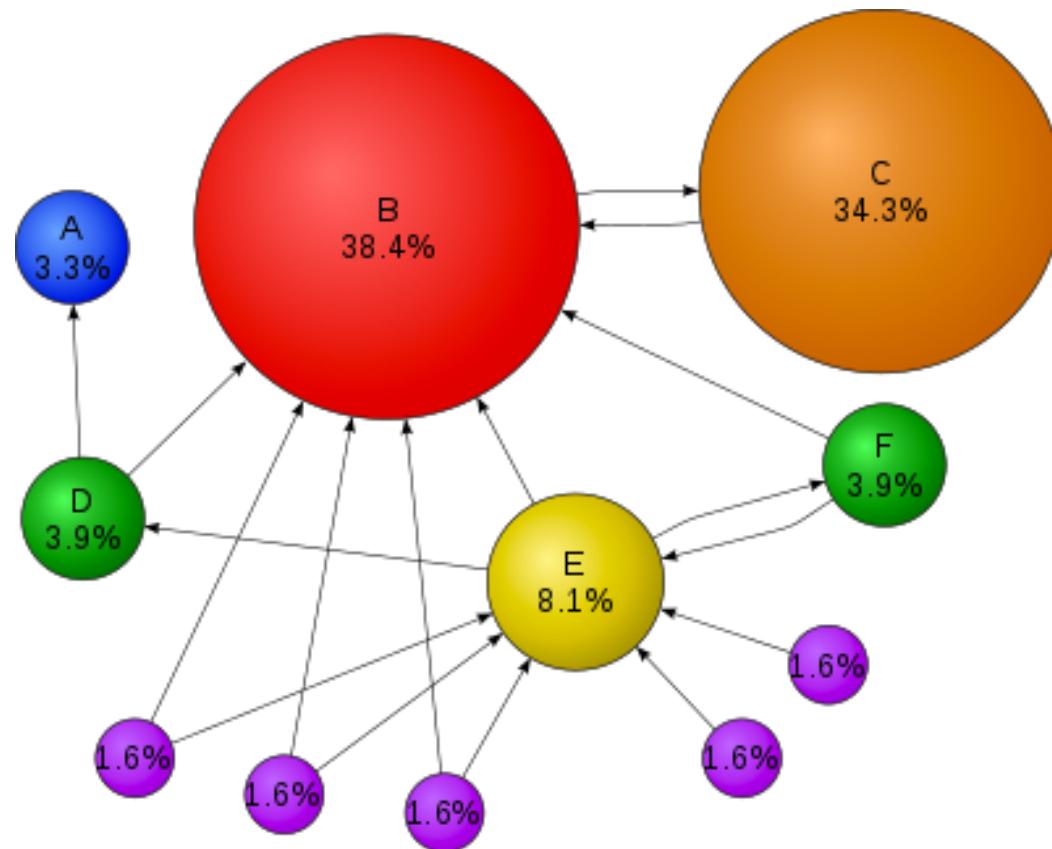
The importance of a Web page is an inherently subjective matter, which depends on the readers interests, knowledge and attitudes. But there is still much that can be said objectively about the relative importance of Web pages. This paper describes PageRank, a method for rating Web pages objectively and mechanically, effectively measuring the human interest and attention devoted to them.

We compare PageRank to an idealized random Web surfer. We show how to efficiently compute PageRank for large numbers of pages. And, we show how to apply PageRank to search and to user navigation.



# Basic Assumptions

- Pages with more **inbound links** are more important
- Inbound links from important pages carry more weight



# The PageRank Model

- Each out-link is an implicit conveyance of authority to the target page
  - Equal amount per link
- So the PageRank of node i,  $P(i)$ , equals the sum of each incoming link's PageRank proportion

$$P(i) = \sum_{(j,i) \in E} \frac{P(j)}{O_j}$$

$O_j$  : number of out-links of page j

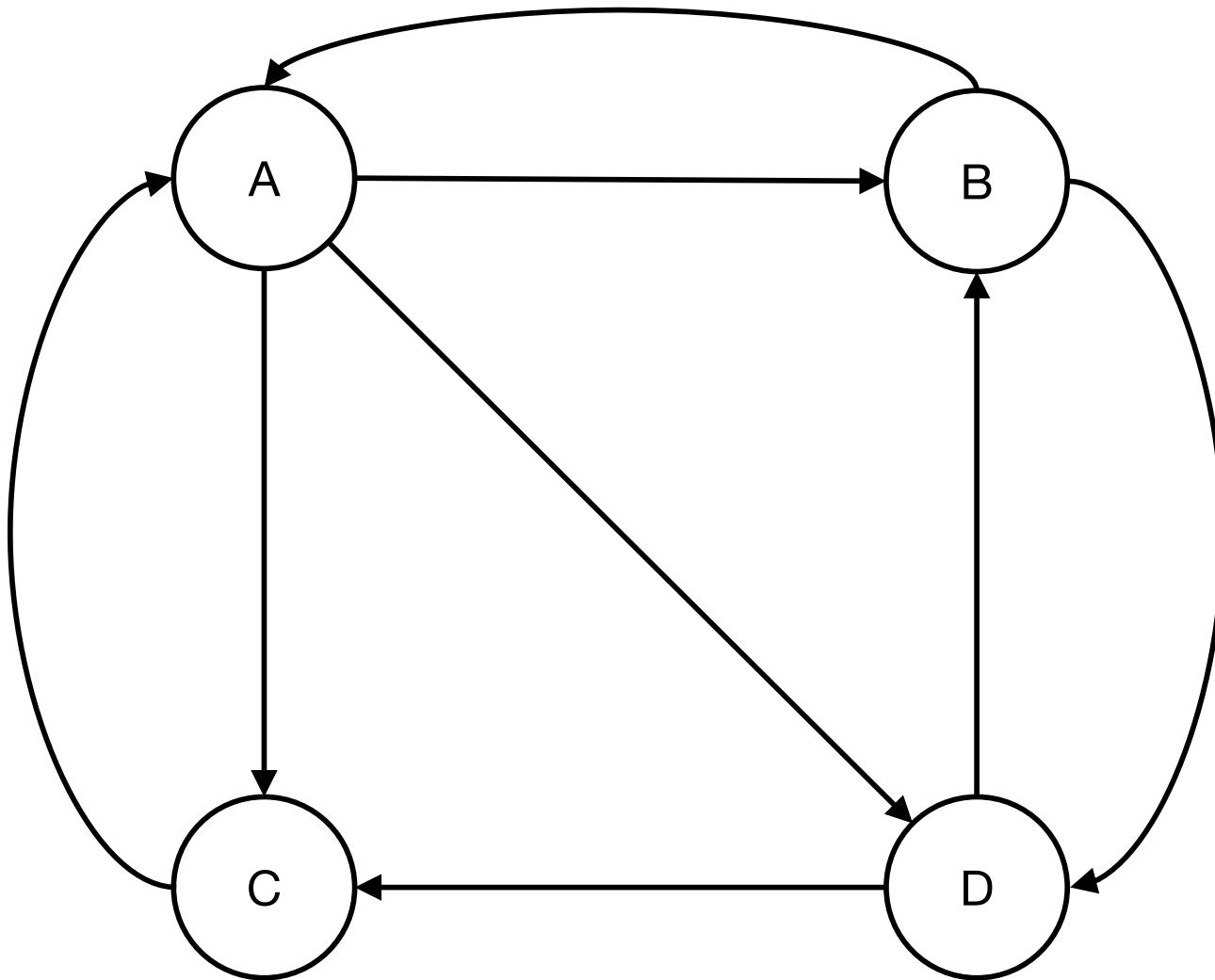


# An Equivalent View of PageRank

- At time  $t$  a surfer is on some page
  - Start uniformly at random
- At time  $t+1$  the surfer follows a link to a new page at random
  - A *Markov Chain*
- Define PageRank where the surfer is likely to be after a long time



# The Model as a Directed Graph

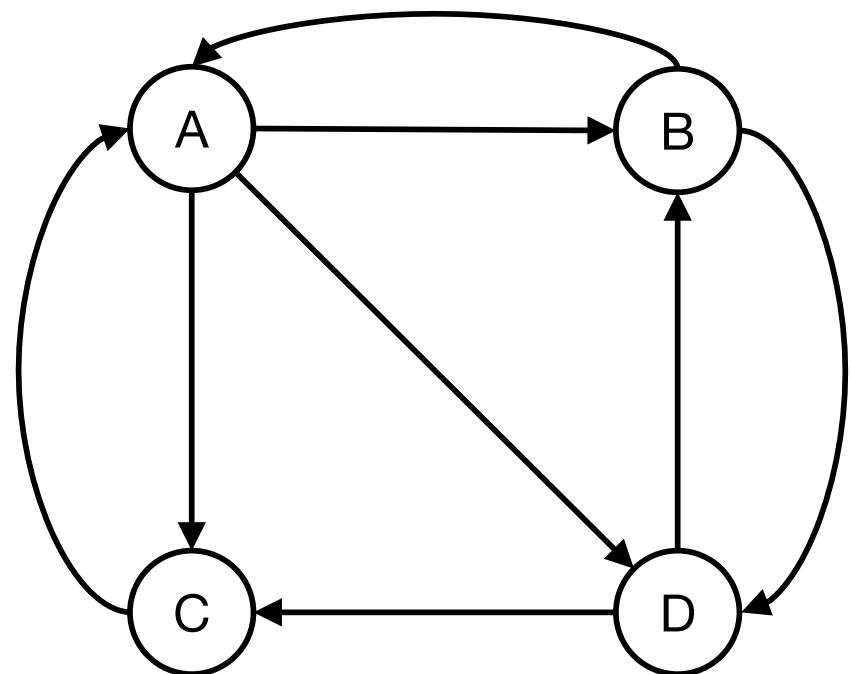


# Random Surfer via Transition Matrix

- Weight each edge equally...

$P(X|A)$   
*Starting at A, probability of arriving at node X*

$$\begin{bmatrix} 0 & 1/2 & 1 & 0 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 1/2 & 0 & 0 \end{bmatrix}$$



$P(A|X)$   
*Starting at X, “probability” of arriving at node D*



# Checkup

- Assume a surfer has an equal probability of starting at any site
- What is the probability of arriving at each of the four sites at the next time step given the transition matrix?

$$\begin{bmatrix} 0 & 1/2 & 1 & 0 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 1/2 & 0 & 0 \end{bmatrix}$$



# Answer

$$\begin{bmatrix} 0 & 1/2 & 1 & 0 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 1/2 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1/4 \\ 1/4 \\ 1/4 \\ 1/4 \end{bmatrix} = \begin{bmatrix} 9/24 \\ 5/24 \\ 5/24 \\ 5/24 \end{bmatrix}$$



# A Few More Waves...

$$\begin{bmatrix} 1/4 \\ 1/4 \\ 1/4 \\ 1/4 \end{bmatrix} \begin{bmatrix} 9/24 \\ 5/24 \\ 5/24 \\ 5/24 \end{bmatrix} \begin{bmatrix} 15/48 \\ 11/48 \\ 11/48 \\ 11/48 \end{bmatrix} \begin{bmatrix} 11/32 \\ 7/32 \\ 7/32 \\ 7/32 \end{bmatrix} \dots \begin{bmatrix} 3/9 \\ 2/9 \\ 2/9 \\ 2/9 \end{bmatrix}$$



# An Aside... Was This Surprising?

0	1/2	1	0
1/3	0	0	1/2
1/3	0	0	1/2
1/3	1/2	0	0

Same probability of arriving at B/C/D – call each X



# An Aside... Was This Surprising?

	0	1/2	1	0
	1/3	0	0	1/2
	1/3	0	0	1/2
	1/3	1/2	0	0

$$(1/2)X + X$$

Same probability of arriving at B/C/D – call each X



# An Aside... Was This Surprising?

	0	1/2	1	0
	1/3	0	0	1/2
	1/3	0	0	1/2
	1/3	1/2	0	0

$$(1/2)X + X$$

Same probability of arriving at B/C/D – call each X

$$X + X + X + (3/2)X = 1$$

$$\frac{9}{2}X = 1$$

$$X = \frac{2}{9} \quad \frac{3}{2} \cdot X = \frac{3}{2} \cdot \frac{2}{9} = \frac{3}{9}$$



# Checkup

$$\begin{bmatrix} 0 & 1/2 & 1 & 0 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 1/2 & 0 & 0 \end{bmatrix} \begin{bmatrix} 3/9 \\ 2/9 \\ 2/9 \\ 2/9 \end{bmatrix} =$$



# Answer

$$\begin{bmatrix} 0 & 1/2 & 1 & 0 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 1/2 & 0 & 0 \end{bmatrix} \begin{bmatrix} 3/9 \\ 2/9 \\ 2/9 \\ 2/9 \end{bmatrix} = \begin{bmatrix} 3/9 \\ 2/9 \\ 2/9 \\ 2/9 \end{bmatrix}$$

$$MP = P$$

$$MP = \lambda P$$

$$\lambda = ?$$



# What Just Happened!!??

- If certain conditions hold, 1 is the largest eigenvalue and the PageRank vector is the principal eigenvector of the transition matrix
  - The conditions held for our example, but not in the general case for the web (yet!)
- We intuitively used a method called **power iteration** to compute  $P$ 
  - Useful particularly when the matrix in question is large & sparse, as the method doesn't require any decomposition
  - Convergence: typically when the residual (norm of the difference between  $P$  vs  $P'$ ) is below a threshold; may require many iterations, typically few are good enough for the web (according to Google)

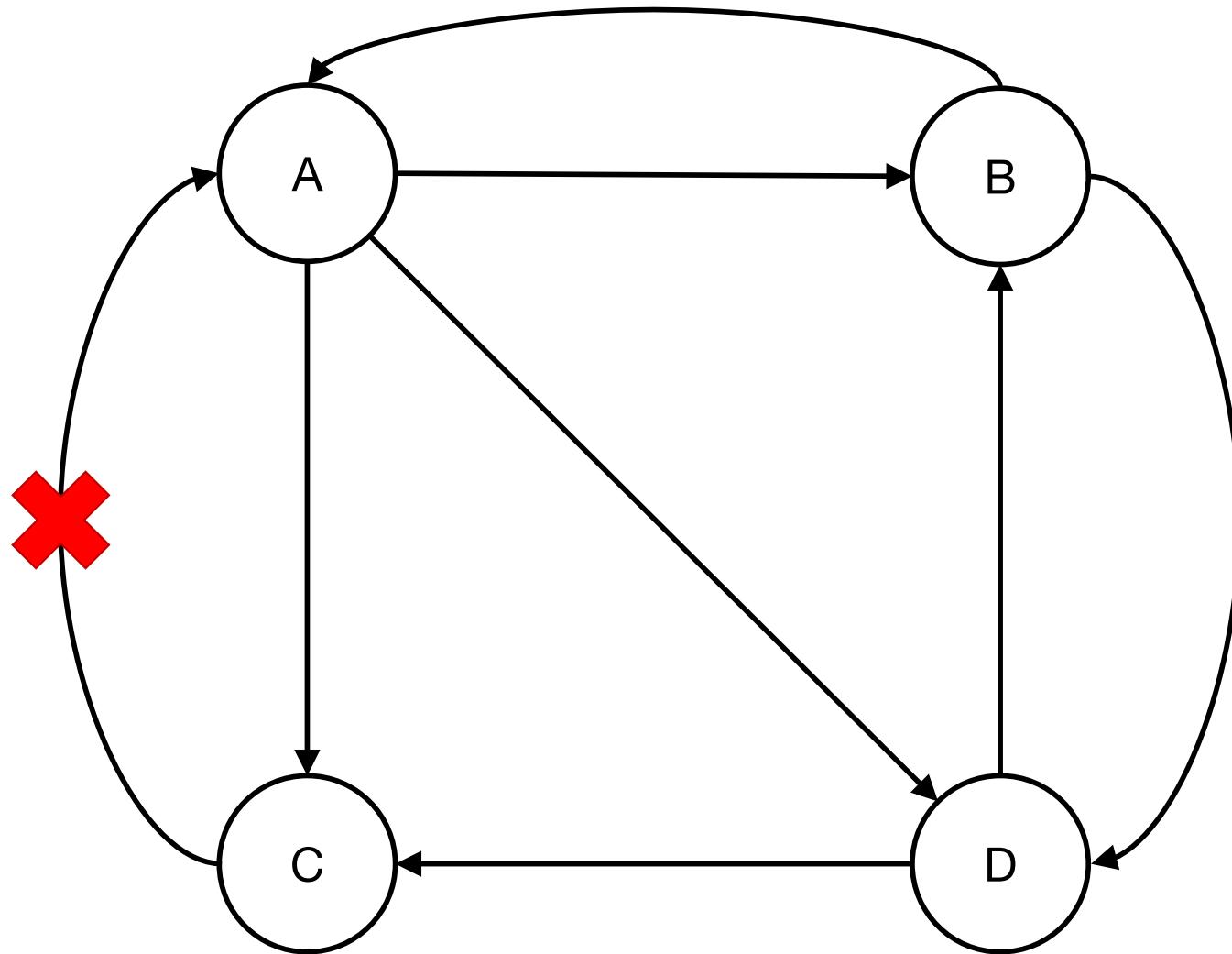


# So What are these Conditions...?

- Stochastic: columns sum to 1
  - Done... right?
- Strongly connected: possible to get from any node to any other node
  - Might be very unlikely, but must be possible!

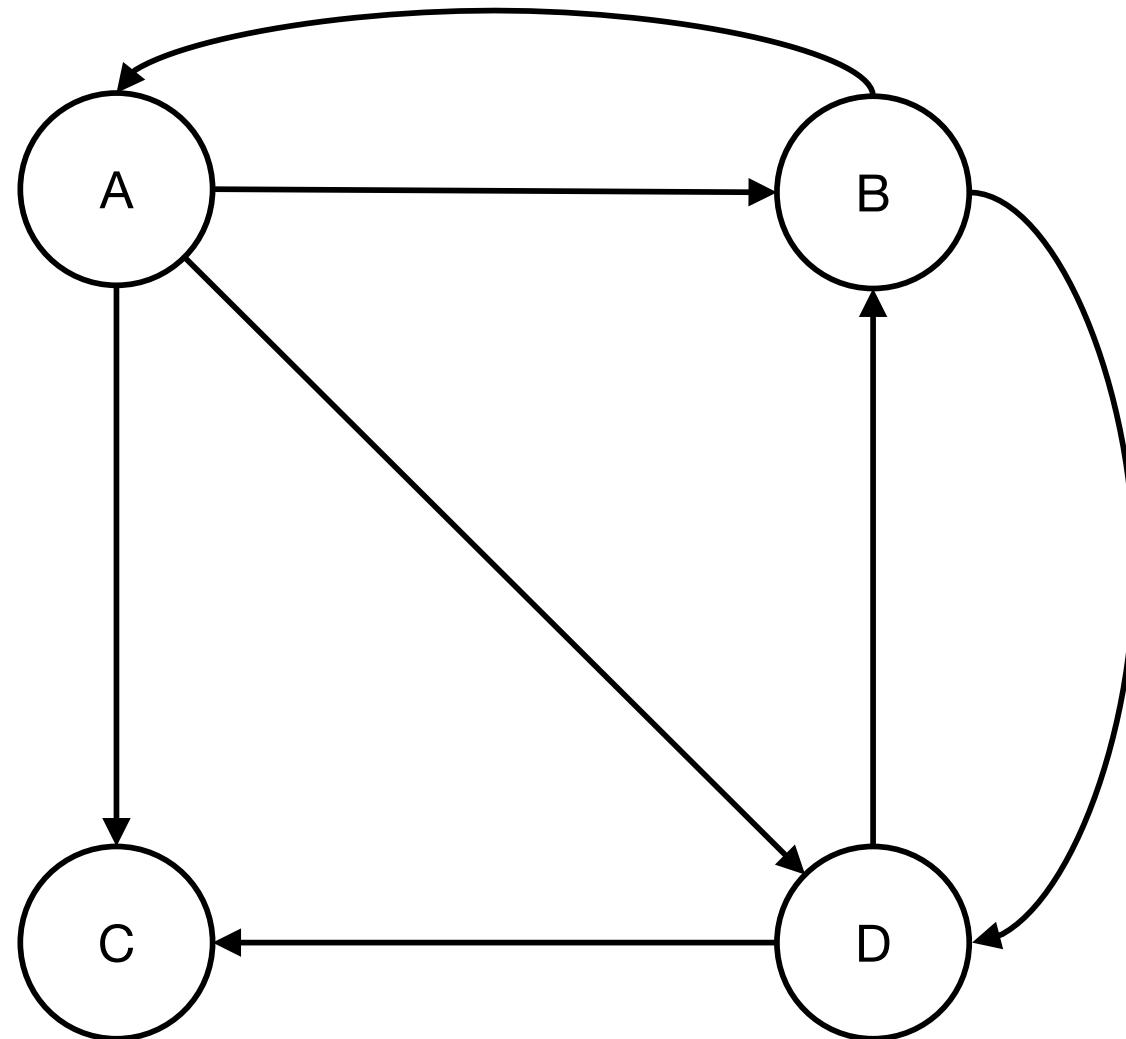


# What Would Happen If...



# What Would Happen If...

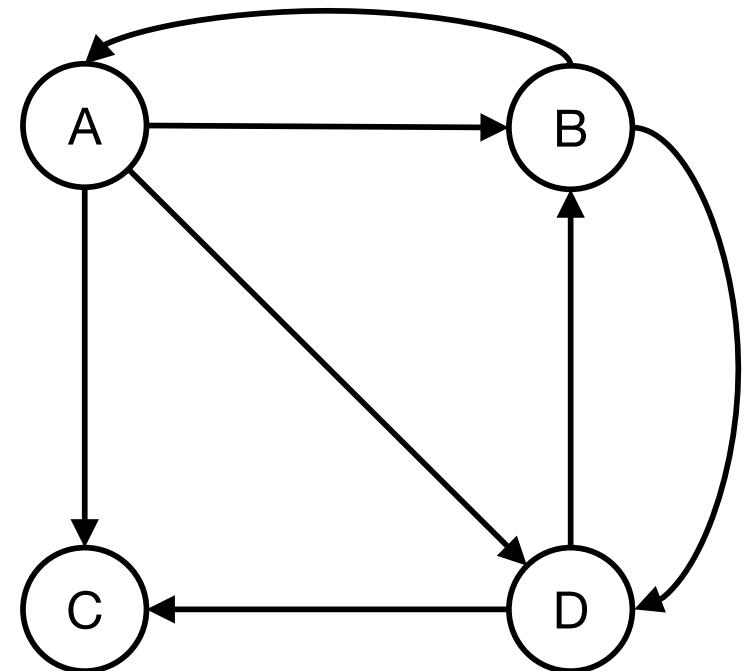
C is termed a  
**dead end**



# New Transition Matrix

- Weight each edge  
equally...

$$\begin{bmatrix} 0 & 1/2 & 0 & 0 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 1/2 & 0 & 0 \end{bmatrix}$$



Now **substochastic**  
(columns sum to at most 1)



# Checkup

- Assume a surfer has an equal probability of starting at any site
- What is the probability of arriving at each of the four sites at the next time step given the transition matrix?

$$\begin{bmatrix} 0 & 1/2 & 0 & 0 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 1/2 & 0 & 0 \end{bmatrix}$$



# Answer

$$\begin{bmatrix} 0 & 1/2 & 0 & 0 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 1/2 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1/4 \\ 1/4 \\ 1/4 \\ 1/4 \end{bmatrix} = \begin{bmatrix} 3/24 \\ 5/24 \\ 5/24 \\ 5/24 \end{bmatrix}$$

25% loss!



# A Few More Waves...

$$\begin{bmatrix} 1/4 \\ 1/4 \\ 1/4 \\ 1/4 \end{bmatrix} \begin{bmatrix} 3/24 \\ 5/24 \\ 5/24 \\ 5/24 \end{bmatrix} \begin{bmatrix} 5/48 \\ 7/48 \\ 7/48 \\ 7/48 \end{bmatrix} \begin{bmatrix} 21/288 \\ 31/288 \\ 31/288 \\ 31/288 \end{bmatrix} \dots \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

1

0.75

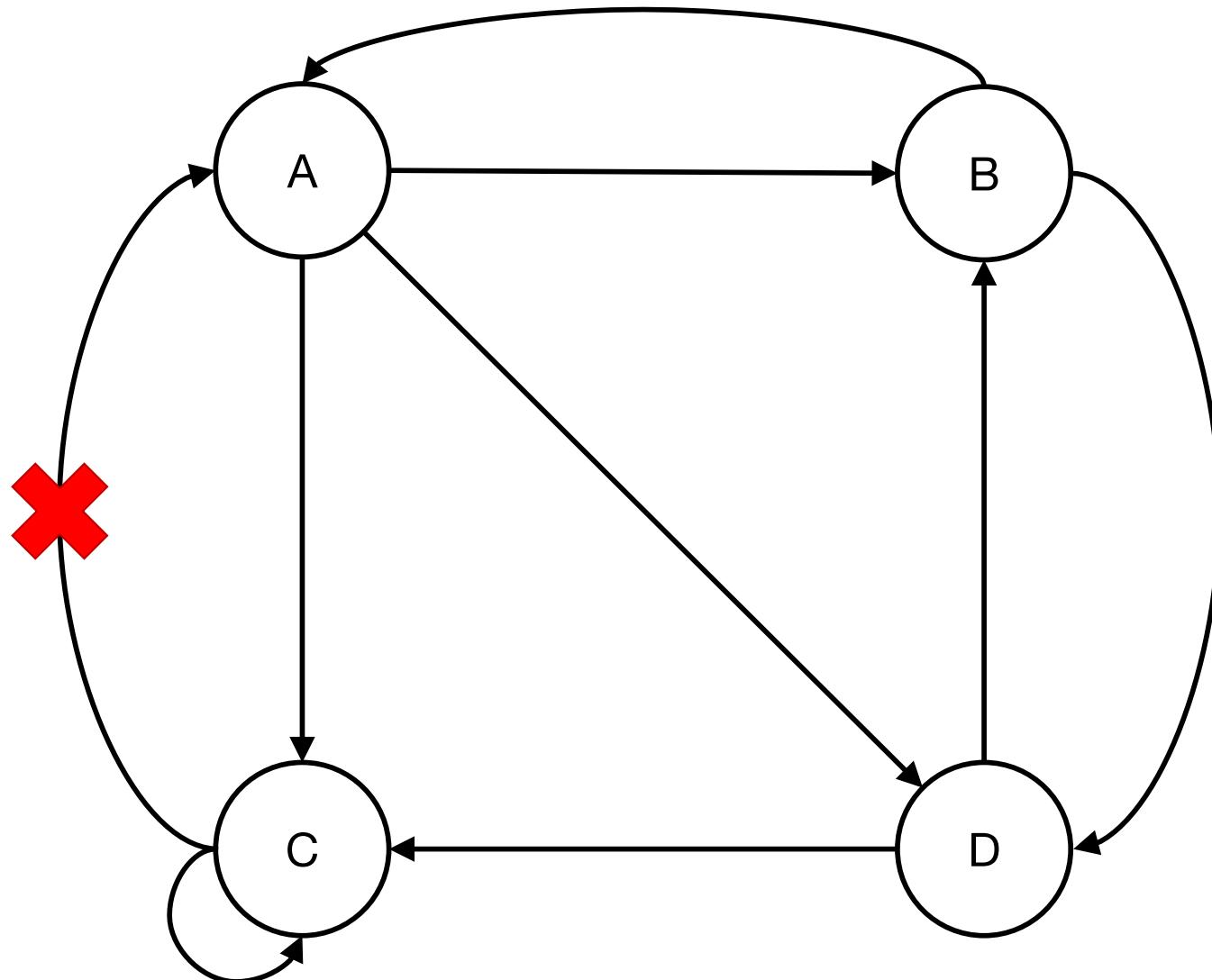
0.54

0.29

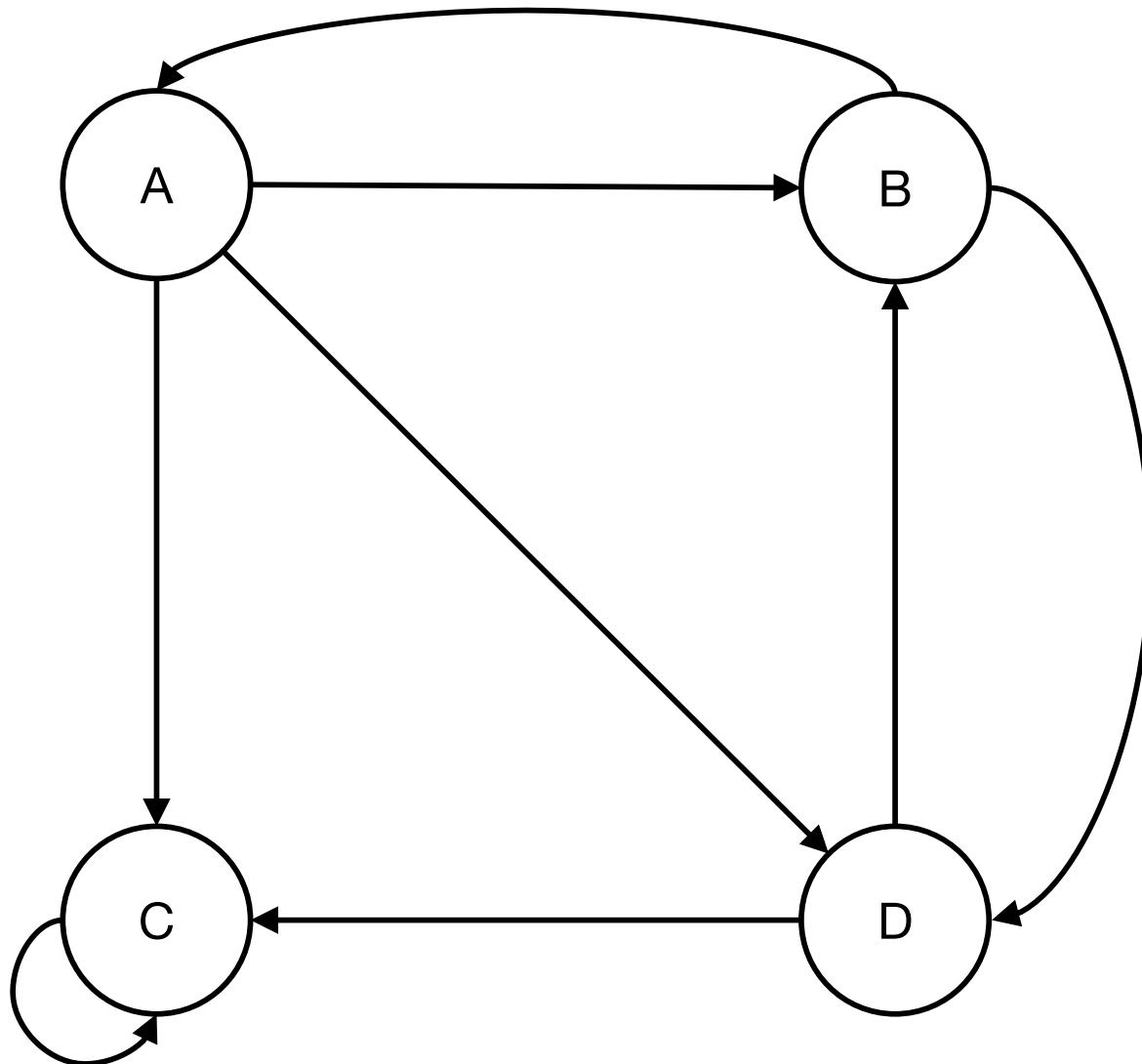
0



# What Would Happen If...



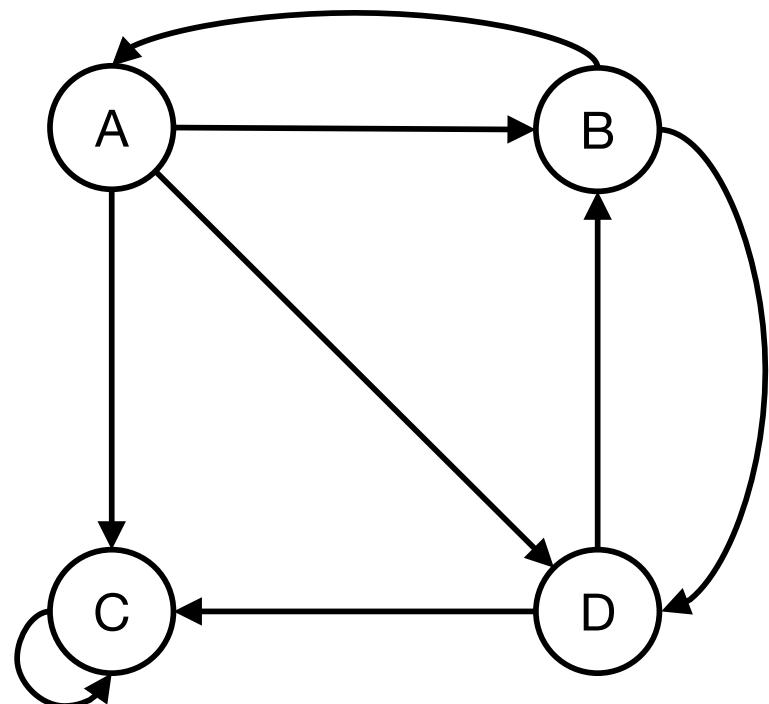
# What Would Happen If...



# Sticky Transition Matrix

- Weight each edge  
equally...

$$\begin{bmatrix} 0 & 1/2 & 0 & 0 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 0 & 1 & 1/2 \\ 1/3 & 1/2 & 0 & 0 \end{bmatrix}$$



Stochastic – yes!  
But strongly connected?



# Checkup

- Assume a surfer has an equal probability of starting at any site
- What is the probability of arriving at each of the four sites at the next time step given the transition matrix?

$$\begin{bmatrix} 0 & 1/2 & 0 & 0 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 0 & 1 & 1/2 \\ 1/3 & 1/2 & 0 & 0 \end{bmatrix}$$



# Answer

$$\begin{bmatrix} 0 & 1/2 & 0 & 0 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 0 & 1 & 1/2 \\ 1/3 & 1/2 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1/4 \\ 1/4 \\ 1/4 \\ 1/4 \end{bmatrix} = \begin{bmatrix} 3/24 \\ 5/24 \\ 11/24 \\ 5/24 \end{bmatrix}$$



# A Few More Waves...

$$\begin{bmatrix} 1/4 \\ 1/4 \\ 1/4 \\ 1/4 \end{bmatrix} \begin{bmatrix} 3/24 \\ 5/24 \\ 11/24 \\ 5/24 \end{bmatrix} \begin{bmatrix} 5/48 \\ 7/48 \\ 29/48 \\ 7/48 \end{bmatrix} \begin{bmatrix} 21/288 \\ 31/288 \\ 205/288 \\ 31/288 \end{bmatrix} \dots \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$



# Solution: Damping/Taxation

- Allow a relatively small probability of hopping from any page to any other page (teleporting!)
  - Typical: 10-15%
- Ensures the requirements of the model



# PageRank Model

## Simplified

$$MP = P$$

## Damped

$$((1 - d) \frac{E}{n} + dM)P = P$$

$$(1 - d) \frac{\mathbf{e}}{n} + dMP = P$$

n: number of nodes

E:  $\mathbf{e}\mathbf{e}^T$

- e: n-length column of 1's

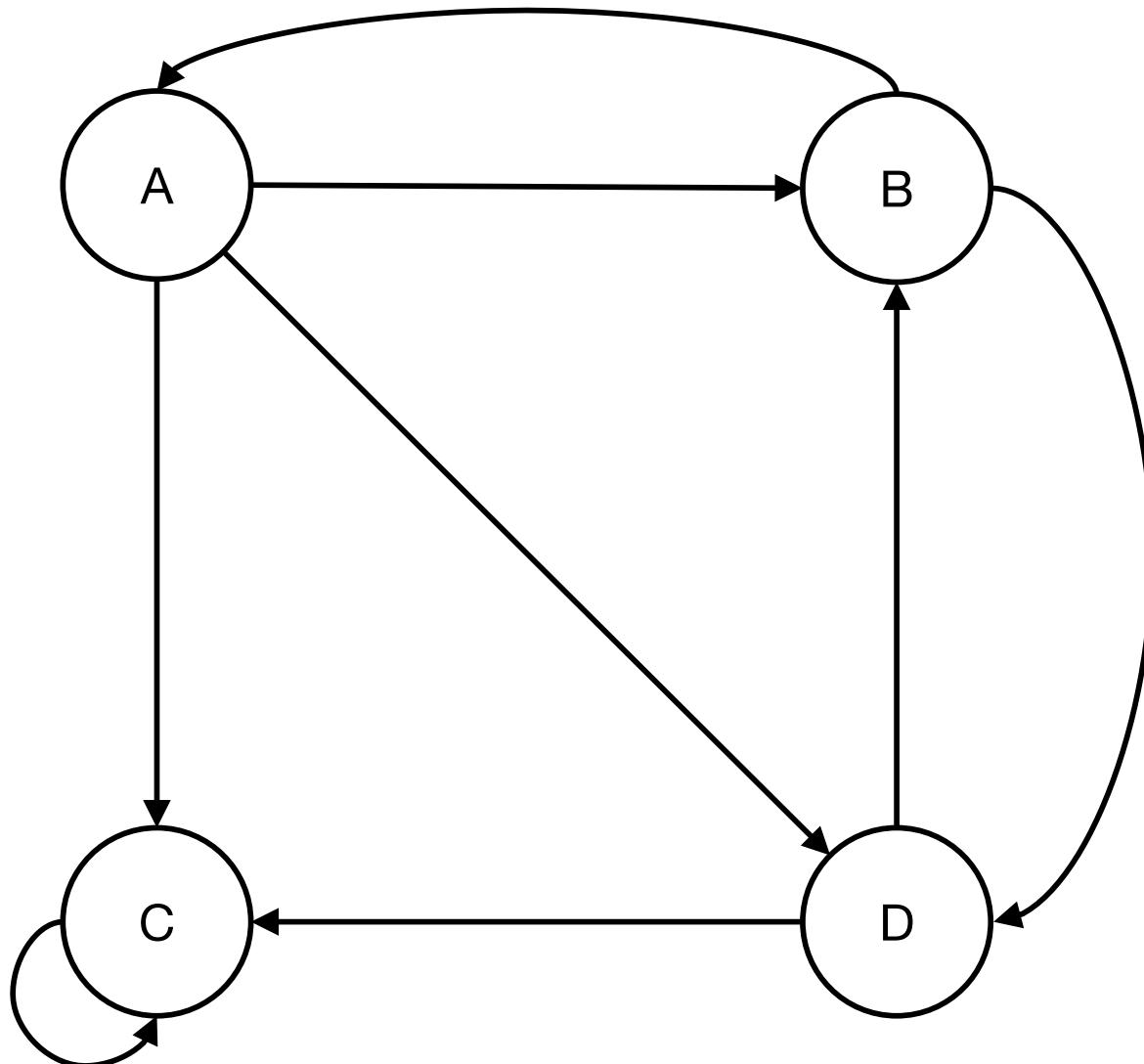
Think back to Naïve Bayes

Anything seem *smoother*?

Think about extreme values of d...



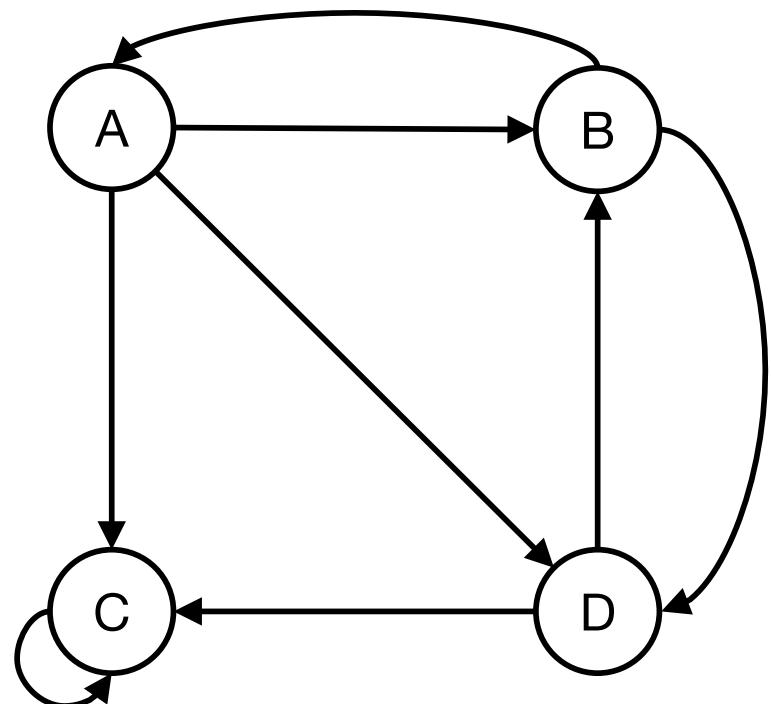
# Recall the Spider Trap



# New Model: $d=0.8$

- Weight each edge equally...

$$\begin{bmatrix} 0 & 2/5 & 0 & 0 \\ 4/15 & 0 & 0 & 2/5 \\ 4/15 & 0 & 4/5 & 2/5 \\ 4/15 & 2/5 & 0 & 0 \end{bmatrix}$$



# Checkup

- Assume a surfer has an equal probability of starting at any site
- What is the probability of arriving at each of the four sites at the next time step given the transition and damping matrices?

$$\begin{bmatrix} 0 & 2/5 & 0 & 0 \\ 4/15 & 0 & 0 & 2/5 \\ 4/15 & 0 & 4/5 & 2/5 \\ 4/15 & 2/5 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1/20 \\ 1/20 \\ 1/20 \\ 1/20 \end{bmatrix}$$



# Answer

$$\begin{bmatrix} 0 & 2/5 & 0 & 0 \\ 4/15 & 0 & 0 & 2/5 \\ 4/15 & 0 & 4/5 & 2/5 \\ 4/15 & 2/5 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1/4 \\ 1/4 \\ 1/4 \\ 1/4 \end{bmatrix} + \begin{bmatrix} 1/20 \\ 1/20 \\ 1/20 \\ 1/20 \end{bmatrix} = \begin{bmatrix} 9/60 \\ 13/60 \\ 25/60 \\ 13/60 \end{bmatrix}$$



# A Few More Waves...

$$\begin{bmatrix} 1/4 \\ 1/4 \\ 1/4 \\ 1/4 \end{bmatrix} \quad \begin{bmatrix} 9/60 \\ 13/60 \\ 25/60 \\ 13/60 \end{bmatrix} \quad \begin{bmatrix} 41/300 \\ 53/300 \\ 153/300 \\ 53/300 \end{bmatrix} \quad \dots \quad \begin{bmatrix} 15/148 \\ 19/148 \\ 95/148 \\ 19/148 \end{bmatrix}$$

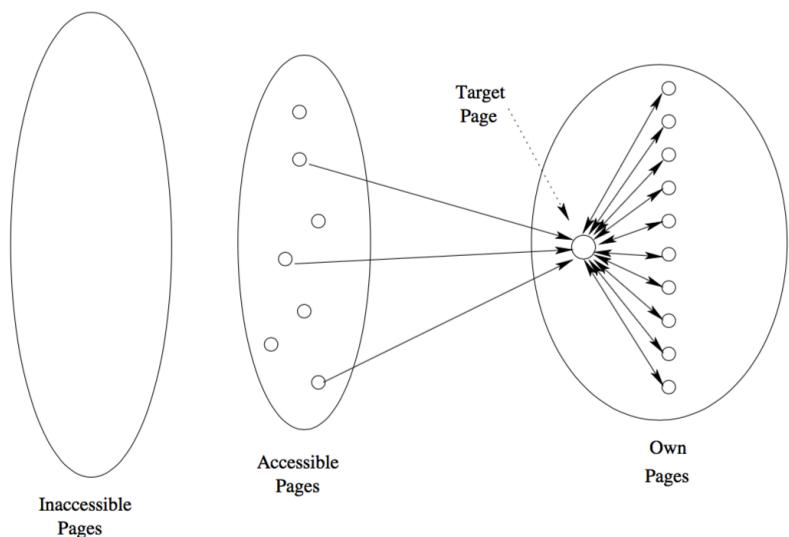
$$\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Compare to...



# PageRank in Practice

- Efficient implementation via MapReduce
  - See LRU
- Still have to deal with Spam Farms
  - See LRU



# Some Related Approaches

- Topic-Sensitive PageRank
- SimRank
- HITS

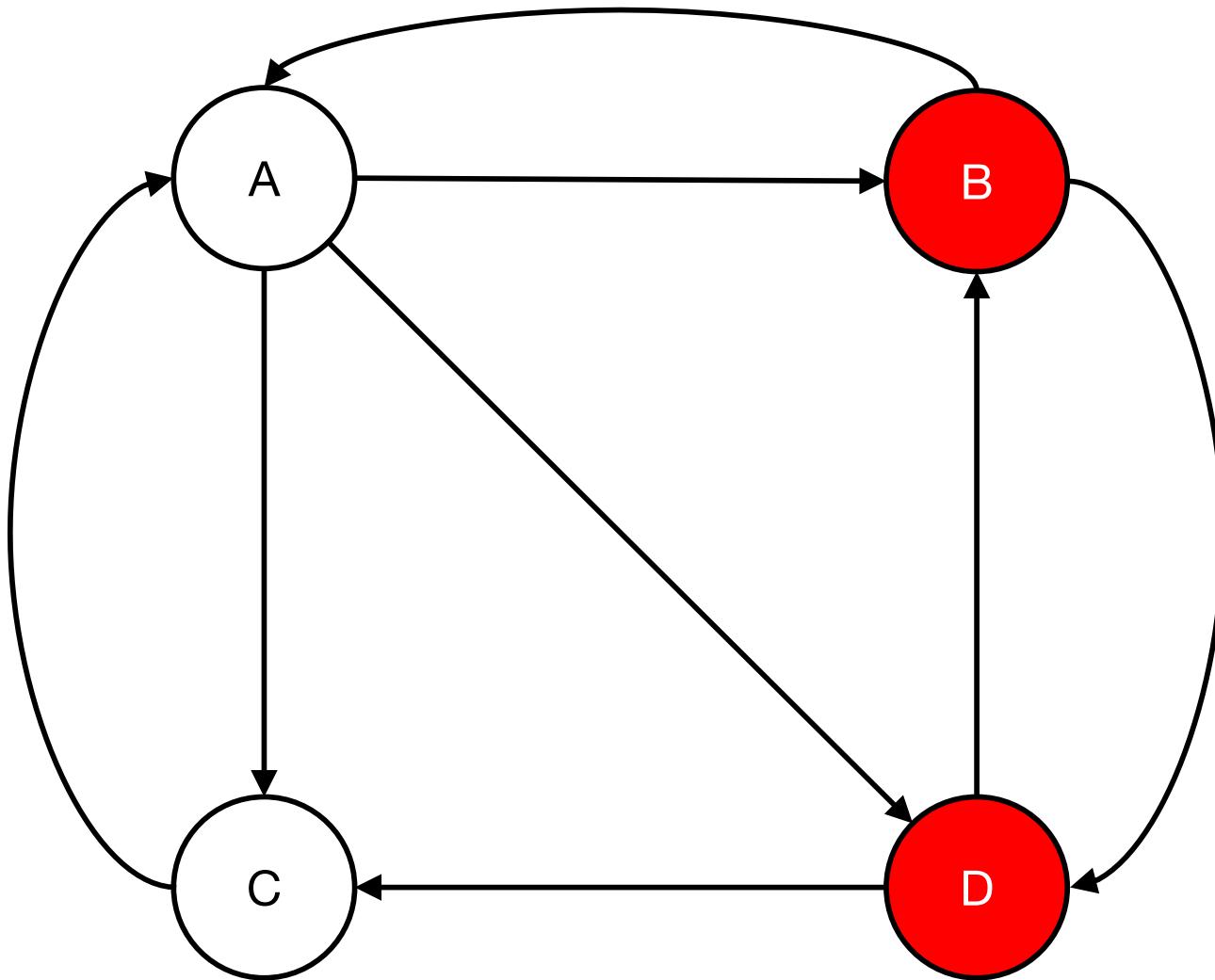


# Biased Random Walks

- Suppose we wish to create a ranking for “sports” (or some other topic)
- We can modify PageRank via a **teleport set** (representative topic-related pages)
  - Could look to a known directory/authority (e.g. <http://dmoztools.net>)
- Start randomly from within this set & only include them within the damping options



# Creating a **RED** PageRank



# Checkup

- Assume a surfer has an equal probability of starting at any RED site (B, D)

$$\begin{bmatrix} 0 & 2/5 & 4/5 & 0 \\ 4/15 & 0 & 0 & 2/5 \\ 4/15 & 0 & 0 & 2/5 \\ 4/15 & 2/5 & 0 & 0 \end{bmatrix}$$

- What is the probability of arriving at each of the four sites at the next time step given the transition and damping matrices?

$$\begin{bmatrix} 0 \\ 1/10 \\ 0 \\ 1/10 \end{bmatrix}$$

(1 - 0.8) / 2 nodes



# Answer

$$\begin{bmatrix} 0 & 2/5 & 4/5 & 0 \\ 4/15 & 0 & 0 & 2/5 \\ 4/15 & 0 & 0 & 2/5 \\ 4/15 & 2/5 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0/2 \\ 1/2 \\ 0/2 \\ 1/2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/10 \\ 0 \\ 1/10 \end{bmatrix} = \begin{bmatrix} 2/10 \\ 3/10 \\ 2/10 \\ 3/10 \end{bmatrix}$$



# Application Steps

1. Choose a topic set
2. Choose a teleport set for each topic
  - Solve for PageRank vector
3. For a user/query, choose the topics that are most relevant
  - Hard task in-and-of itself
4. Weight results via combined PageRank vectors



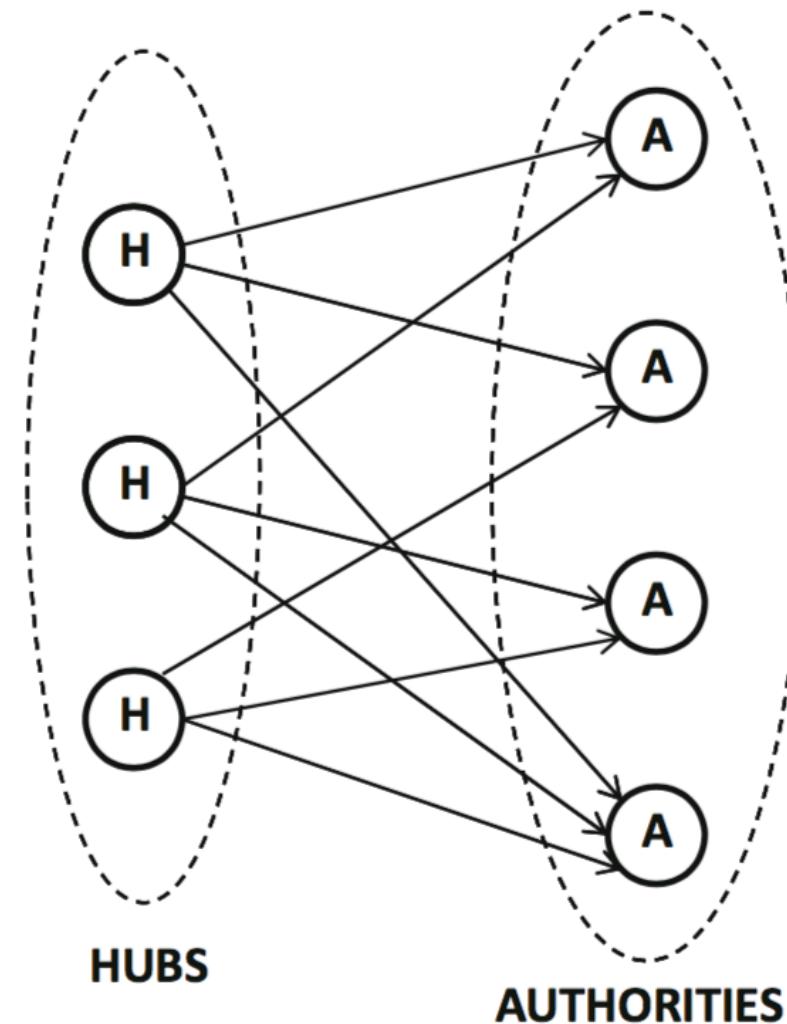
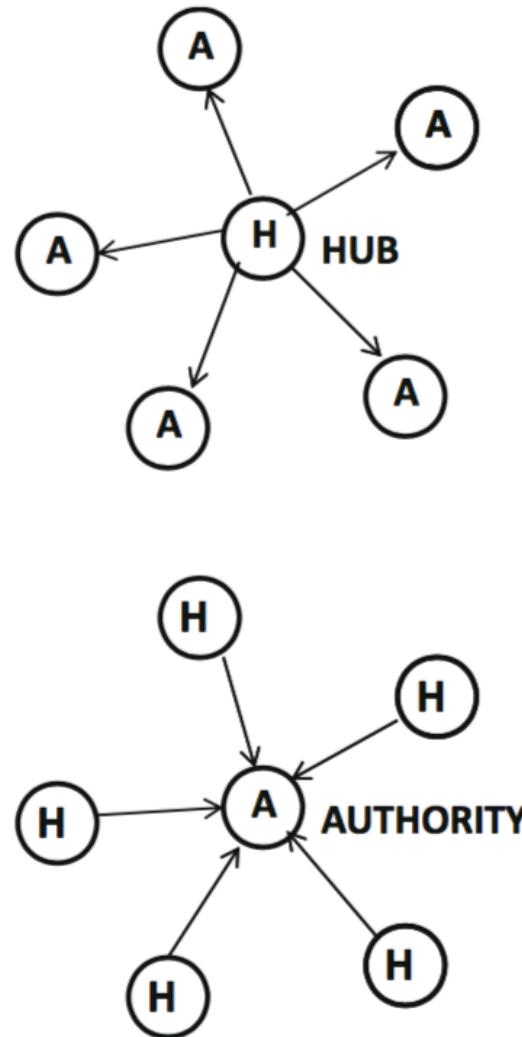
# Structural Similarity Between Nodes

- One possibility: Topic-Sensitive PageRank with teleport set of 1 (node in question)
  - Provides an *asymmetric* ranking of nodes that are structurally close
- SimRank( $i, j$ )
  - Basic idea: what would the expected distance be if two random surfers walked from nodes  $i/j$

$$\text{SimRank}(i, j) = \frac{C}{|\text{In}(i)| \cdot |\text{In}(j)|} \sum_{p \in \text{In}(i)} \sum_{q \in \text{In}(j)} \text{SimRank}(p, q)$$



# Hubs and Authorities



# Hypertext Induced Topic Search

1. Collect top- $r$  (e.g. 200) most relevant results to a query (e.g. via Google), R
2. Produce base set S as nodes that are in/out-links of nodes in R, edges (A) within S
  - Commonly need to limit size
3.  $h^{0(i)} = a^{0(i)} = 1/\sqrt{|S|}$
4. Iterate

for each  $i \in S$  set  $a^{t+1}(i) \leftarrow \sum_{j:(j,i) \in A} h^t(j);$

for each  $i \in S$  set  $h^{t+1}(i) \leftarrow \sum_{j:(i,j) \in A} a^{t+1}(j);$

Normalize  $L_2$ -norm of each of hub and authority vectors to 1;



# HITS

- Because it is query dependent, runs at query time, not indexing time
- Produces two scores per document
  - And only looks at a subset relevant to the query
- Not commonly used by search engines
  - Might be used by Ask
- Not affected by dead ends/spider traps!
  - So no need for damping

