

# The role of force field parameter uncertainty in the prediction of the pressure-viscosity coefficient

Richard A. Messerly

*Thermodynamics Research Center, National Institute of Standards and Technology, Boulder, Colorado, 80305*

Michelle C. Anderson

*Thermodynamics Research Center, National Institute of Standards and Technology, Boulder, Colorado, 80305*

S. Mostafa Razavi

*Department of Chemical and Biomolecular Engineering, The University of Akron, Akron, Ohio, 44325-3906*

J. Richard Elliott

*Department of Chemical and Biomolecular Engineering, The University of Akron, Akron, Ohio, 44325-3906*

---

## Abstract

In response to the 10<sup>th</sup> Industrial Fluid Properties Simulation Challenge, we report viscosity ( $\eta$ ) estimates of 2,2,4-trimethylhexane at 293 K for a range of pressures ( $P$ ) from 0.1 MPa to 1000 MPa. The Potoff force field is utilized in this study, as a previous study demonstrated that it provides reliable estimates of  $\eta$  with respect to  $P$ . Whereas most studies report only the uncertainties associated with random fluctuations in the simulation output, we investigate the effect of uncertainties arising from the force field non-bonded and torsional parameters. The pressure-viscosity coefficient as a function of pressure is reported for several different empirical fitting models. Although the uncertainties increase substantially with increasing pressure, cross-validation model selection provides quantitative evidence supporting so-called super-Arrhenius behavior with an inflection point in a  $\log_{10}(\eta)$ - $P$  plot around 200 MPa.

*Keywords:*

---

*Email addresses:* richard.messerly@nist.gov (Richard A. Messerly), michelle.anderson@nist.gov (Michelle C. Anderson), sr87@ziips.uakron.edu (S. Mostafa Razavi), elliot1@uakron.edu (J. Richard Elliott)

## 1. Introduction

The Industrial Fluid Properties Simulation Challenge (IFPSC) is an open international competition aimed at aligning the molecular simulation community, which is primarily academic, with the goals of industrial research. The present work is a submission to the 10<sup>th</sup> Industrial Fluid Properties Simulation Challenge (IFPSC10). The 10<sup>th</sup> challenge is to predict the viscosity ( $\eta$ ) of 2,2,4-trimethylhexane (224TMH) over a wide range of pressures ( $P$ ), specifically, from 0.1 MPa (atmospheric) to 1000 MPa, at a constant temperature ( $T$ ) of 293 K.

The practical application of IFPSC10 is elastohydrodynamic lubrication (EHL), where knowledge of the pressure-viscosity relationship is paramount. The challenge compound was chosen as an ideal lubricating oil candidate for which no published experimental viscosity data are available above ambient pressure. New experimental measurements are performed by Scott Bair of Georgia Tech with a sample of greater than 98 % purity. The estimated experimental uncertainties for  $\eta$ ,  $T$ , and  $P$  are, respectively, 3 %, 0.3 K, and the greater of 1 MPa and 0.4 %.

Classical film thickness formulas rely heavily on the so-called pressure-viscosity coefficient ( $\alpha$ ), which is essentially an Arrhenius-like activation parameter that is obtained from the slope for an  $\log_{10}(\eta)$ - $P$  plot. However, faster-than-exponential, a.k.a. super-Arrhenius, dependence on pressure has been observed through experimental viscometry measurements for nearly a century [1]. This super-Arrhenius trend is typically manifest by an inflection point in the  $\log_{10}(\eta)$ - $P$  plot. While this behavior is common in experimental measurements, we are not aware of any rheological molecular simulation studies that have addressed this topic, as most simply assume an Arrhenius relationship when reporting  $\alpha$  [2, 3, 4]. IFPSC10 is an ideal opportunity to demonstrate whether or not molecular simulation can provide evidence supporting or opposing the existence of

super-Arrhenius behavior.

In a previous study, we investigated the adequacy of four different united-atom (UA) Mie  $\lambda$ -6 (generalized Lennard-Jones, LJ) force fields for predicting the viscosity-pressure trend, namely, the Transferable Potentials for Phase Equilibria (TraPPE-UA [5, 6, 7]), Transferable Anisotropic Mie (TAMie) [8, 9], Potoff [10, 11], and fourth generation anisotropic-united-atom (AUA4) [12, 13]. The comparisons with experimental data were made for saturated liquid viscosity ( $\eta_{\text{liq}}^{\text{sat}}$ ) over a wide temperature range and compressed liquid viscosity ( $\eta_{\text{liq}}^{\text{comp}}$ ) at 293 K from atmospheric pressure to 1000 MPa. The compounds in question were *n*-alkanes ranging in length from ethane to *n*-docosane and branched alkanes ranging in size from 2-methylpropane to 2,2,4-trimethylpentane (224TMP). The 224TMP results at high pressures are especially useful as this compound is a close analogue to the challenge compound and, in contrast with 224TMH, 224TMP has been well studied experimentally.

While TraPPE and AUA4 (LJ 12-6 based potentials) under predict  $\eta_{\text{liq}}^{\text{sat}}$  by greater than 30 % for all compounds studied, TAMie (Mie 14-6) and Potoff (Mie 16-6) predict  $\eta_{\text{liq}}^{\text{sat}}$  within 10 % for most compounds. For  $\eta_{\text{liq}}^{\text{comp}}$ , TAMie is the most reliable at predicting the viscosity-density dependence, while Potoff significantly over estimates  $\eta_{\text{liq}}^{\text{comp}}$  with respect to density. However, since Potoff also over estimates pressure at high densities [14], the viscosity-pressure trend for Potoff is remarkably accurate even at pressures approaching 1000 MPa. In particular, the Potoff force field predicts the viscosity-pressure trend for 2,2,4-trimethylpentane to within 10 % accuracy. For this reason, we implement the Potoff Mie 16-6 force field to predict  $\eta$  and  $\alpha$  for the challenge compound. We should note, however, that our previous study did not provide any definitive evidence that the Potoff force field could predict a super-Arrhenius trend for the compounds studied and, specifically, for 224TMP.

One of the entry guidelines for IFPSC is “an analysis of the uncertainty in the calculated results.” Traditionally, simulation uncertainties are limited to the random fluctuations of simulation output and/or the uncertainty related to data post-processing. This

class of uncertainty is referred to as “numerical uncertainty” (frequently referred to as “statistical uncertainty”). Two other classes of uncertainty, namely, “parameter uncertainty” and “functional form uncertainty” (also referred to as “model uncertainty”) are typically ignored in uncertainty quantification (UQ) due to the increased computational cost. The latter refers to the uncertainty associated with the choice of force field functional form, while the former refers to the uncertainty in the force field parameters for a given force field functional form.

Quantifying the functional form uncertainty is an extremely difficult task, as it often requires performing simulations with numerous force field functional forms. For this reason, we focus on numerical and parameter uncertainties without addressing functional form uncertainties. Specifically, we apply bootstrap re-sampling and Bayesian inference Markov Chain Monte Carlo (MCMC) to quantify numerical and parameter uncertainties, respectively. The chosen functional form is the same as the Potoff force field, namely, a united-atom, fixed bond length, harmonic angular potential, cosine series torsional potential, and a Mie 16-6 non-bonded potential (see Section 2.1 for details). As viscosity is highly sensitive to the non-bonded [15, 16] and torsional [17, 18] potentials, we limit our parameter uncertainty investigation to the non-bonded and torsional parameters.

The outline for the present work is the following. Section 2 explains the force field, simulation methodology, and data analysis. Section 3 presents the simulation results, with an emphasis on uncertainty quantification. Section 4 discusses some important observations and limitations. Section 5 recaps the primary conclusions from this work.

## 2. Methods

### 2.1. Force fields

We utilize the Potoff force field as it provides reliable estimates of the  $\eta$ - $P$  dependence for normal and branched alkanes that are similar to the challenge compound [16]. In addition, we quantify the uncertainty in  $\eta$  that arises from uncertainties in the non-bonded Mie 16-6 and torsional parameters. The parameter uncertainties are obtained using Bayesian

inference Markov Chain Monte Carlo (MCMC). This UQ analysis is performed sequentially. First, we account for only the non-bonded uncertainties (referred to as MCMC-nb). Then, we include both the non-bonded and torsional uncertainties (MCMC-nb-tors). This sequential approach provides insight into which source of uncertainty has a greater impact on  $\eta$ .

The Potoff Mie  $\lambda$ -6 force field utilizes united-atom (UA) sites, where 2,2,4-trimethylhexane is represented with CH<sub>3</sub>, CH<sub>2</sub>, CH, and C UA sites. Neighboring UA sites are separated by a fixed 0.154 nm bond length. Note that we observed in our previous study that the use of flexible bonds can impact  $\eta$  by several percent. Therefore, the choice of fixed bonds was not arbitrary and is a possible source of uncertainty for which we did not rigorously account.

The angular contribution to energy is computed using a harmonic potential:

$$u^{\text{bend}} = \frac{k_\theta}{2} (\theta - \theta_0)^2 \quad (1)$$

where  $u^{\text{bend}}$  is the bending energy,  $\theta$  is the instantaneous bond angle,  $\theta_0$  is the equilibrium bond angle (see Table 1), and  $k_\theta$  is the harmonic force constant with  $k_\theta/k_B = 62500 \text{ K/rad}^2$  for all bonding angles, where  $k_B$  is the Boltzmann constant.

Table 1: Equilibrium bond angles ( $\theta_0$ ) [6, 11]. CH<sub>*i*</sub> and CH<sub>*j*</sub> represent CH<sub>3</sub>, CH<sub>2</sub>, CH, or C sites.

Bending sites	$\theta_0$ (degrees)
CH <sub><i>i</i></sub> -CH <sub>2</sub> -CH <sub><i>j</i></sub>	114.0
CH <sub><i>i</i></sub> -CH-CH <sub><i>j</i></sub>	112.0
CH <sub><i>i</i></sub> -C-CH <sub><i>j</i></sub>	109.5

Dihedral torsional interactions are determined using a modified cosine series:

$$\begin{aligned} u^{\text{tors}} &= c_0 + c_1[1 + \cos \phi] + c_2[1 - \cos 2\phi] + c_3[1 + \cos 3\phi] + A_s \sin^2 \left[ \frac{3}{2}(\phi + 180^\circ) \right] \\ &= (c_0 - A_s) + c_1[1 + \cos \phi] + c_2[1 - \cos 2\phi] + \left( c_3 + \frac{A_s}{2} \right) [1 + \cos 3\phi] \quad (2) \end{aligned}$$

where  $u^{\text{tors}}$  is the torsional energy,  $\phi$  is the dihedral angle,  $c_n$  are the Fourier constants used in the Potoff force field and listed in Table 2, and  $A_s \sin^2 \left[ \frac{3}{2}(\phi + 180^\circ) \right]$  is an additional term proposed by Nieto-Draghi et al. to shift the torsional barrier heights for normal and branched alkanes [17, 13]. Clearly, the non-shifted Potoff torsional potential is obtained only when  $A_s = 0$ . We follow a convention similar to that of the International Union of Pure and Applied Chemistry (IUPAC) such that  $\phi = 180^\circ$  for the *trans* conformation, whereas Nieto-Draghi et al. define the *trans* conformation as  $0^\circ$  or  $360^\circ$  [17, 13], hence the  $\phi + 180^\circ$  term. As  $\sin^2 \left[ \frac{3}{2}(\phi + 180^\circ) \right]$  has a maximum value of 1 at  $0^\circ$ ,  $120^\circ$ ,  $240^\circ$ , and  $360^\circ$ ,  $u^{\text{tors}}$  is shifted by  $A_s$  at these dihedral angles. By contrast, this additional term does not shift  $u^{\text{tors}}$  for dihedral angles of  $60^\circ$ ,  $180^\circ$ , and  $300^\circ$ , which correspond to the equilibrium conformations of *gauche*<sup>-</sup>, *trans*, and *gauche*<sup>+</sup>, respectively.

Table 2: Fourier constants ( $c_n/k_B$ ) and shifting parameter ( $A_s/k_B$ ) in units of K for Potoff force field [6, 11].  $\text{CH}_i$  and  $\text{CH}_j$  represent  $\text{CH}_3$ ,  $\text{CH}_2$ ,  $\text{CH}$ , or C sites.

Torsion sites	$c_0/k_B$	$c_1/k_B$	$c_2/k_B$	$c_3/k_B$	$A_s/k_B$
$\text{CH}_i\text{-CH}_2\text{-CH-CH}_j$	-251.06	428.73	-111.85	441.27	0.0
$\text{CH}_i\text{-CH}_2\text{-C-CH}_j$	0.0	0.0	0.0	461.29	0.0

Nieto-Draghi et al. set  $A_s$  equal to 40% and 15% of the maximum dihedral barrier for the terminal and internal torsions [17, 13]. For example, this corresponds to  $A_s/k_B \approx 1000$  K and  $\approx 375$  K for the  $\text{CH}_3\text{-CH}_2\text{-CH}_2\text{-CH}_2$  and  $\text{CH}_2\text{-CH}_2\text{-CH}_2\text{-CH}_2$  torsional potentials, respectively. The reason why Nieto-Draghi et al. increase the torsional barrier, i.e.,  $A_s > 0$ , is because AUA4 utilizes a Lennard-Jones 12-6 potential and, therefore, under predicts  $\eta$  by approximately 20 % to 40 %. However, despite the relatively large increase in the torsional barriers, the modified torsional model (AUA4m) typically provides only marginal improvement of 5 % to 15 % compared to AUA4 (see Tables 4 and 5 of Reference 13).

As the Potoff Mie 16-6 potential is already quite reliable for predicting viscosity, we would expect significant over prediction of viscosity if we coupled the Potoff Mie 16-6

potential with  $A_s/k_B \gg 0$ . The actual reason we include this additional term, however, is to provide a simple method for quantifying the uncertainty in the torsional potential. Specifically, we assume that  $A_s$  follows a skewed distribution with a mean value near zero and the lower and upper 95 % confidence intervals correspond to -15 % and +40 % of the maximum barrier height for the non-shifted Potoff torsional potential. Thus, unlike Nieto-Draghi et al., we do not assume that the torsional barriers must be increased unilaterally. The rationale for using this skewed distribution is presented in Supporting Information

Figure 1 compares the non-shifted Potoff torsional potential,  $\pm 40$  % shift in barrier heights, and the MCMC torsional potentials which are used for MCMC-nb-tors. The insets also depict the skewed distributions and the randomly sampled MCMC  $A_s$  sets. Note that the challenge compound consists of four  $\text{CH}_i\text{--CH}_2\text{--CH--CH}_j$  torsions and three  $\text{CH}_i\text{--CH}_2\text{--C--CH}_j$  torsions. Also, unlike Nieto-Draghi et al., we make no distinction between internal and terminal torsions.

Non-bonded interactions between sites located in two different molecules or separated by more than three bonds within the same molecule are calculated using a Mie  $\lambda$ -6 potential (of which the traditional Lennard-Jones, LJ, 12-6 is a subclass) [19]:

$$u^{\text{vdw}}(\epsilon, \sigma, \lambda; r) = \left( \frac{\lambda}{\lambda - 6} \right) \left( \frac{\lambda}{6} \right)^{\frac{6}{\lambda - 6}} \epsilon \left[ \left( \frac{\sigma}{r} \right)^\lambda - \left( \frac{\sigma}{r} \right)^6 \right] \quad (3)$$

where  $u^{\text{vdw}}$  is the van der Waals interaction,  $\sigma$  is the distance ( $r$ ) where  $u^{\text{vdw}} = 0$ ,  $-\epsilon$  is the energy of the potential at the minimum (i.e.,  $u^{\text{vdw}} = -\epsilon$  and  $\frac{\partial u^{\text{vdw}}}{\partial r} = 0$  for  $r = r_{\text{min}}$ ), and  $\lambda$  is the repulsive exponent.

The non-bonded Potoff Mie  $\lambda$ -6 force field parameters are provided in Table 3. Note that Potoff reports a “generalized” and “short/long” (S/L) CH and C parameter set. The “generalized” CH and C parameter set is an attempt at a completely transferable force field, while the “short” and “long” parameters are implemented when the number of carbons in the backbone is  $\leq 4$  and  $> 4$ , respectively. The Potoff results presented in this study are obtained with the “long” parameters.

Non-bonded parameters between two different site types (i.e., cross-interactions) are

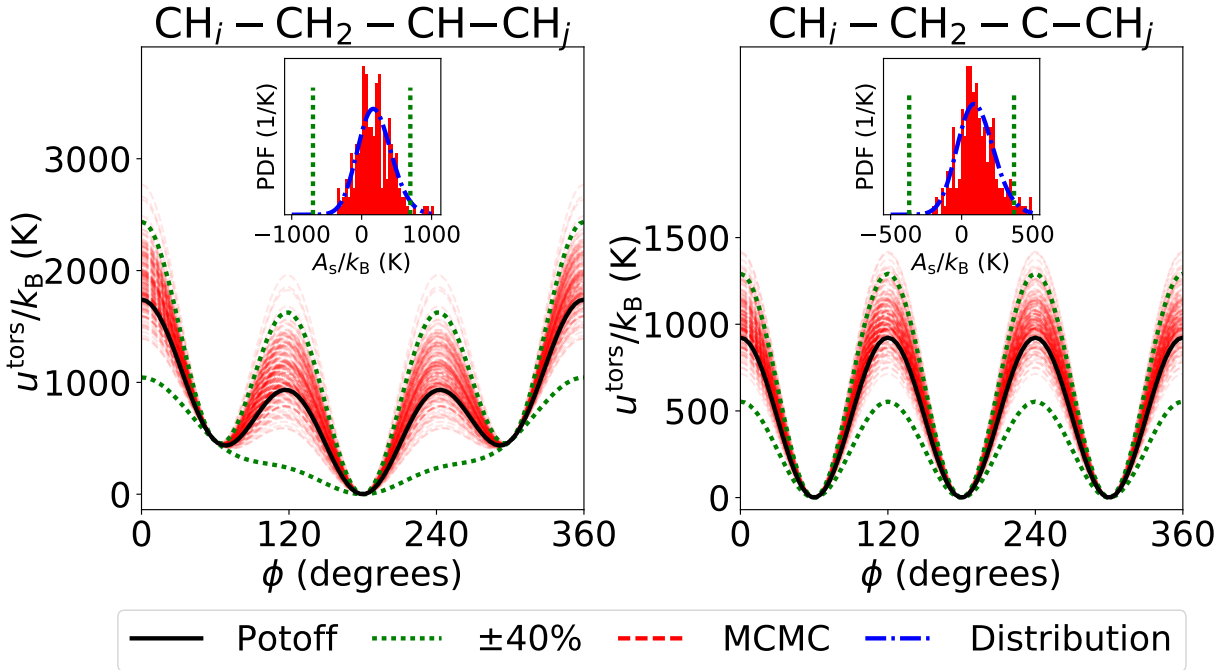


Figure 1: Uncertainty in dihedral potentials. Black line is the Potoff torsional potential. Red lines are the 200 MCMC sampled parameter sets used in this study. Insets show the distribution for  $A_s$ . Both  $u^{\text{tors}}/k_B$  and  $A_s/k_B$  are expressed in units of K.

Table 3: Non-bonded Potoff Mie  $\lambda$ -6 parameters. The “short/long” Potoff CH and C parameters are included in parentheses. \* Generalized C parameters obtained by private communication. Note the discrepancy between the generalized C parameters reported in Table 1 of Reference 11 and the optimal region depicted in Figure 1 of Reference 11.)

	Potoff (S/L)		
United-atom	$\epsilon/k_B$ (K)	$\sigma$ (nm)	$\lambda$
CH <sub>3</sub>	121.25	0.3783	16
CH <sub>2</sub>	61	0.399	16
CH	15 (15/14)	0.46 (0.47/0.47)	16
C	1.05* (1.45/1.2)	0.605* (0.61/0.62)	16



determined using Lorentz-Berthelot combining rules [20] for  $\epsilon$  and  $\sigma$  and an arithmetic mean for the repulsive exponent  $\lambda$  (as recommended in Reference 10):

$$\epsilon_{ij} = \sqrt{\epsilon_{ii}\epsilon_{jj}} \quad (4)$$

$$\sigma_{ij} = \frac{\sigma_{ii} + \sigma_{jj}}{2} \quad (5)$$

$$\lambda_{ij} = \frac{\lambda_{ii} + \lambda_{jj}}{2} \quad (6)$$

where the  $ij$  subscript refers to cross-interactions and the subscripts  $ii$  and  $jj$  refer to same-site interactions.

Figure 2 depicts the MCMC non-bonded parameters for  $\text{CH}_3$ ,  $\text{CH}_2$ ,  $\text{CH}$ , and  $\text{C}$  united-atom sites ( $\epsilon_{\text{CH}_3}$ ,  $\sigma_{\text{CH}_3}$ ,  $\epsilon_{\text{CH}_2}$ ,  $\sigma_{\text{CH}_2}$ ,  $\epsilon_{\text{CH}}$ ,  $\sigma_{\text{CH}}$ ,  $\epsilon_{\text{C}}$ , and  $\sigma_{\text{C}}$ ). Note that  $\lambda_{\text{CH}_3} = \lambda_{\text{CH}_2} = \lambda_{\text{CH}} = \lambda_{\text{C}} = 16$  and parameters are assumed to be transferable, e.g., the  $\text{CH}_2$  MCMC parameters do not depend on the  $\text{CH}_3$  MCMC parameters. The MCMC non-bonded parameters for  $\text{CH}_3$  and  $\text{CH}_2$  sites were reported previously [14]. These parameters were obtained using a likelihood function based on saturated liquid density and saturated vapor pressure data for ethane, propane, *n*-butane, and *n*-octane. By contrast, the MCMC parameters for  $\text{CH}$  and  $\text{C}$  sites were obtained from the scoring function reported by Mick et al. [11] that depends on several vapor-liquid coexistence properties for a diverse set of branched alkanes. Details regarding the generation of MCMC parameter sets from the scoring function are found in Supporting Information. An important observation from Figure 2 is that the uncertainties in the  $\text{CH}$  and  $\text{C}$  non-bonded parameters are considerably larger (on a percent basis) than those for  $\text{CH}_3$  and  $\text{CH}_2$ .

## 2.2. Simulation set-up

Historically, non-equilibrium molecular dynamics (NEMD) has been preferred for highly viscous systems [3, 4]. However, in our recent publication we successfully predicted the viscosity of 2,2,4-trimethylpentane at 293 K and 1000 MPa (the highest pressure required for the challenge) with equilibrium molecular dynamics (EMD). Consistent with our previous study, we use EMD for all pressures in the challenge.

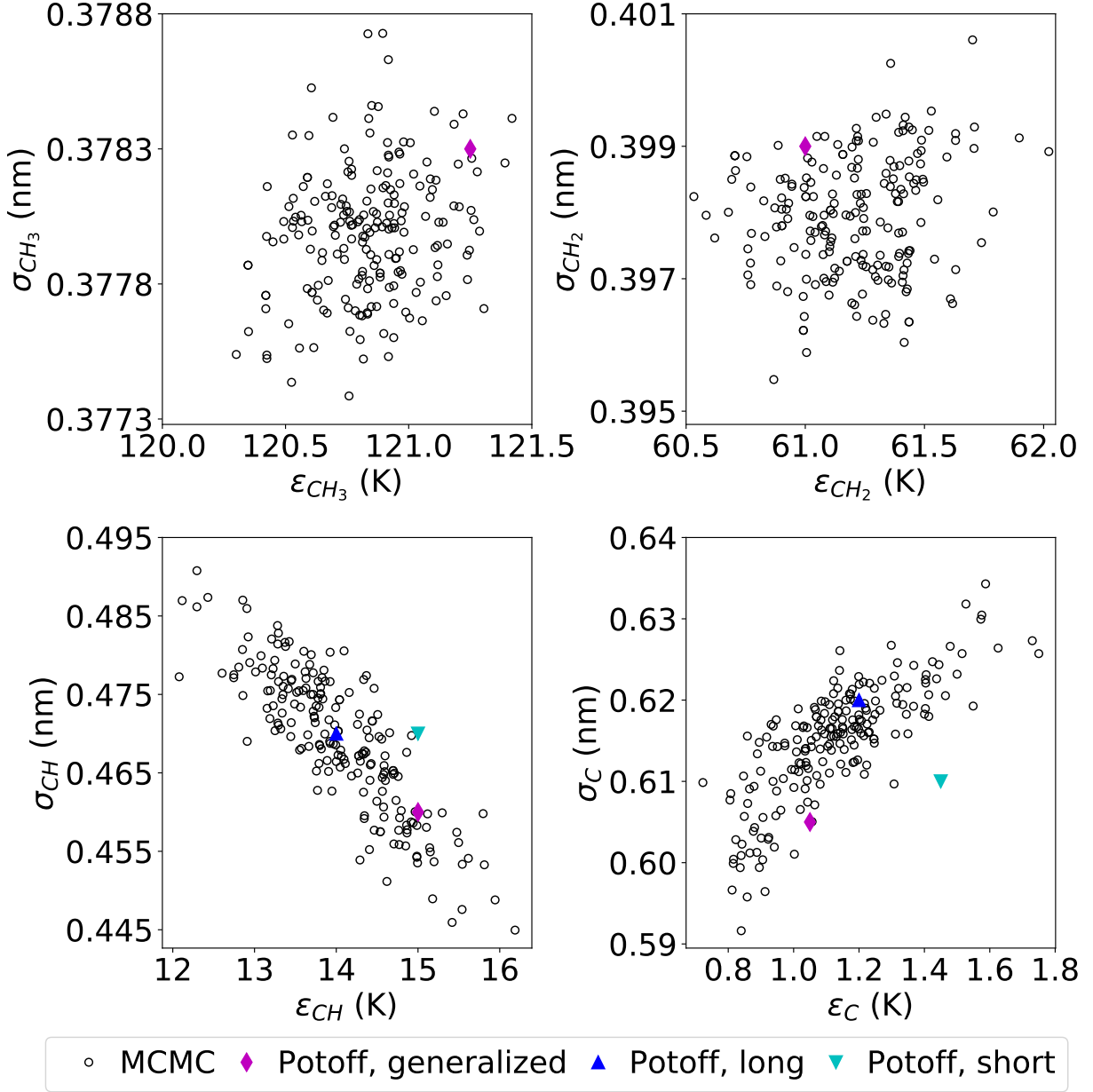


Figure 2: Uncertainty in non-bonded parameters determined with Markov Chain Monte Carlo (MCMC). The Potoff generalized and S/L parameters are also included as a reference [10, 11]. Top left, top right, bottom left, and bottom right panels correspond to  $CH_3$ ,  $CH_2$ ,  $CH$ , and  $C$  parameters, respectively.

Equilibrium molecular dynamics simulations are performed using GROMACS version 2018 with “mixed” (single and double) precision [22]. GROMACS is compiled using GNU 7.3.0, OpenMPI enabled, and GPU support disabled. The simulations are run using

Linux 4.4.0-112-generic x86\_64 on an Intel(R) Xeon(R) CPU E5-2699 v4 @ 2.20GHz machine. Example GROMACS input files (.top, .gro. and .mdp) with corresponding shell and python scripts for preparing, running, and analyzing simulations are provided as Supporting Information.

We utilize the same simulation specifications as our previous study [16]. The general simulation specifications are provided in Table 4. Our previous study demonstrates that, for compounds smaller than *n*-dodecane, the correct system dynamics are obtained using a 2 fs time-step and 1.4 nm non-bonded cut-off distance with analytical tail corrections [23]. Our previous study also shows that finite size effects are negligible for a 400 molecule system.

Table 4: General simulation specifications.

Time-step (fs)	2
Cut-off length (nm)	1.4
Tail-corrections	$U$ and $P$
Constrained bonds	LINCS [24, 25]
LINCS-order	8
Number of molecules	400

We perform a sequence of six simulation stages: energy minimization, *NPT* equilibration, *NPT* production, energy minimization, *NVT* equilibration, and *NVT* production. Table 5 lists the integrators, thermostats, barostats, and simulation time used for each *NPT* and *NVT* equilibration and production stage. These specifications are also the same as our previous study, with the exception of the *NVT* production simulation times, which are state point dependent. The specific production times for the *NVT* production stage are provided in Table 6.

A large number of replicate simulations are required at each state point to improve the precision and to provide more rigorous estimates of uncertainty [26, 27]. We utilize

Table 5: Simulation specifications for equilibration (Equil.) and production (Prod.) stages.

	<i>NPT</i> Equil.	<i>NPT</i> Prod.	<i>NVT</i> Equil.	<i>NVT</i> Prod.
Simulation time (ns)	1	1	1	1 to 32
Integrator	Velocity Verlet	Leap frog	Velocity Verlet	Velocity Verlet
Thermostat	Velocity rescale	Nosé-Hoover	Nosé-Hoover	Nosé-Hoover
Thermostat time-constant (ps)	1.0	1.0	1.0	1.0
Barostat	Berendsen	Parrinello-Rahman	N/A	N/A
Barostat time-constant (ps)	1.0	5.0	N/A	N/A
Barostat compressibility (1/bar)	4.5E-5	4.5E-5	N/A	N/A

Table 6: State point specific production times. Pressure is prescribed only in *NPT* equilibration and production stages.

Pressure (MPa)	<i>NVT</i> Prod. time (ns)
0.1	1
25	1
50	1
100	1
150	1
250	2
400	4
500	8
600	8
700	16
800	16
900	24
1000	32

between 30 and 80 independent replicates when simulating the Potoff force field, while we use 200 replicates for the MCMC parameter sets. To ensure independence between replicates, the entire series of simulation stages are repeated for each replicate where the initial energy minimization stage starts with a different pseudo-random configuration and the initial velocities are randomized for each equilibration stage.

### 2.3. Data analysis

The post-simulation data analysis is identical to that prescribed in our previous study [16]. In brief, we implement the Green-Kubo “time-decomposition” analysis [26, 27]

$$\eta(t) = \frac{V}{k_B T N_{\text{reps}}} \sum_{n=1}^{N_{\text{reps}}} \int_0^t dt' \langle \tau_{\alpha\beta,n}(t') \tau_{\alpha\beta,n}(0) \rangle_{t_0, \alpha\beta} \quad (7)$$

where  $t$  is time,  $V$  is volume,  $N_{\text{reps}}$  is the number of independent replicate simulations,  $\alpha$  and  $\beta$  are  $x, y$ , or  $z$  Cartesian coordinates,  $\tau_{\alpha\beta,n}$  is the  $\alpha$ - $\beta$  off-diagonal stress tensor element for the  $n^{\text{th}}$  replicate, and  $\langle \cdots \rangle_{t_0,\alpha\beta}$  denotes an average over twelve different time origins ( $t_0$ ) and all three unique off-diagonal  $\tau_{\alpha\beta}$  components. For a precise integration of Equation 7,  $\tau_{\alpha\beta,n}$  is recorded every 6 fs (3 time-steps). Equation 7 also averages  $N_{\text{reps}}$  independent replicate simulations, where the Potoff replicate simulations utilize the same force field parameters. By contrast, the MCMC-nb and MCMC-nb-tors replicates utilize different parameter sets (see Figures 2 and 1).

The force field viscosity is obtained by evaluating Equation 7 as  $t \rightarrow \infty$ , i.e., the infinite-time-limit viscosity ( $\eta^\infty$ ). As the long-time tail does not converge, we fit the “running integral” to a double-exponential function

$$\eta(t) = A\alpha\tau_1(1 - \exp(-t/\tau_1)) + A(1 - \alpha)\tau_2(1 - \exp(-t/\tau_2)) \quad (8)$$

where  $A, \alpha, \tau_1$ , and  $\tau_2$  are fitting parameters and  $\eta^\infty = A\alpha\tau_1 + A(1 - \alpha)\tau_2$ . See our previous work for details regarding the fitting procedure [16].

The uncertainty in  $\eta$  is obtained by bootstrap re-sampling and reported at the 95 % confidence level. Specifically, the fitting of Equation 8 is repeated hundreds of times using randomly selected subsets of replicate simulations from which we obtain the distribution of  $\eta^\infty$  estimates. Section SI.III of Supporting Information validates this process for the MCMC non-bonded  $\text{CH}_3$  parameter sets with ethane.

#### 2.4. Pressure-viscosity coefficient

The simulated  $\eta$  with respect to  $P$  values are fit to four different empirical models from which the pressure-viscosity coefficient ( $\alpha$ ) is calculated. As viscosity ranges over several orders of magnitude, the objective function for fitting is the sum-squared-error of  $\log_{10}(\eta)$ . Only the MCMC-nb-tors values are included in the fit, as these simulation results account for more sources of uncertainty than the Potoff and MCMC-nb results. The pressure-viscosity coefficient for each empirical fit is obtained by either analytical or numerical differentiation of  $\log_{10}(\eta)$  with respect to  $P$ .

The first empirical model we implement is the traditional Barus expression [28]

$$\eta = \eta_0 \exp(\alpha P) \quad (9)$$

where  $\eta_0$  and  $\alpha$  are fitting parameters. The second model is the popular Roelands equation [29]

$$\eta = \eta_p \left( \frac{\eta_0}{\eta_p} \right)^{\left( \frac{P_p - P}{P_p} \right)^Z} \quad (10)$$

where  $\eta_0$  and  $Z$  are fitting parameters and  $\eta_p = 6.31 \times 10^{-5}$  Pa-s and  $P_p = -0.196$  GPa. The third model is an alternative form of Equation 10 where  $\eta_p$  and  $P_p$  are additional fitting parameters (rather than fixed constants). This four parameter Roelands equation, which we refer to as “Roelands-Modified,” is more flexible than the standard Roelands model and can predict super-Arrhenius behavior. The fourth model is a hybrid McEwen-Paluch expression [1]

$$\eta = \eta_0 \left( 1 + \frac{a_0}{q} P \right)^q \exp \left( \frac{C_F P}{P_\infty - P} \right) \quad (11)$$

where  $\eta_0$ ,  $a_0$ ,  $q$ ,  $C_F$ , and  $P_\infty$  are fitting parameters. Note that, although the Roelands-Modified equation can predict super-Arrhenius behavior, only the McEwen-Paluch model is capable of representing an inflection point, i.e., a transition from Arrhenius to super-Arrhenius behavior. By contrast, the Barus and Roelands models are only capable of fitting sub-Arrhenius and Arrhenius-like data.

### 3. Results

Table 7 provides tabulated values for viscosity, density, and pressure at the prescribed state points for Potoff, MCMC-nb, and MCMC-nb-tors. These values are also depicted in Figure 3 along with the model fits to the MCMC-nb-tors values and the available experimental viscosity at atmospheric pressure. An inflection point, suggesting super-Arrhenius behavior, is observed in the top panel of Figure 3 around 200 MPa. The bottom panel of Figure 3 presents the percent deviation between simulation results and the McEwen-Paluch empirical model fit. This panel is useful for visually comparing the Potoff, MCMC-nb, and MCMC-nb-tors uncertainties.

Recall that the Potoff results only account for numerical uncertainties, MCMC-nb accounts for numerical and non-bonded parameter uncertainties, and MCMC-nb-tors accounts for numerical, non-bonded and torsional parameter uncertainties. Although somewhat counter intuitive, the reason why the MCMC-nb and MCMC-nb-tors uncertainties are smaller than the Potoff uncertainties is because the latter uses fewer replicate simulations. When the non-bonded and torsional uncertainties are negligible compared to the numerical uncertainties, this increase in  $N_{\text{reps}}$  reduces the numerical uncertainty and, consequently, the overall uncertainty decreases as well. For 224TMH, the  $\text{CH}_3$  parameters contribute to the majority of non-bonded interactions (both between different molecules as well as the 1-5 and 1-6 interactions within the same molecule). Therefore, the relatively small uncertainties assigned to the  $\text{CH}_3$  parameters (recall Figure 2) are likely the cause for the negligible impact of non-bonded uncertainties.

Table 7: Simulation results for Potoff, MCMC-nb, and MCMC-nb-tors.

		Potoff		MCMC-nb		MCMC-nb-tors	
$T$ (K)	$P$ (MPa)	$\rho$ (kg/m <sup>3</sup> )	$\eta$ (Pa-s)	$\rho$ (kg/m <sup>3</sup> )	$\eta$ (Pa-s)	$\rho$ (kg/m <sup>3</sup> )	$\eta$ (Pa-s)

Figure 4 presents the predicted pressure-viscosity coefficient ( $\alpha$ ), as determined by fitting the MCMC-nb-tors results to Equations 9, 10, and 11. The uncertainties in  $\alpha$  are obtained with bootstrap re-sampling for the empirical model fits. Note that the  $\alpha$  magnitudes for all empirical models are reasonable (i.e., similar in magnitude to other lubricants [2, 3, 4]) over the entire range of pressures. However, as expected, the traditional Barus  $\alpha$  value is constant with respect to pressure. By contrast, the Roelands  $\alpha$  value decreases with increasing pressure, while the Roelands-Modified  $\alpha$  value increases with respect to pressure but without a change from negative to positive slope. Only the McEwen-Paluch  $\alpha$  value shows the marked change in slope which corresponds to an inflection point in the  $\log_{10}(\eta)$ - $P$  plot.

Although the hybrid McEwen-Paluch model clearly reproduces the simulation results



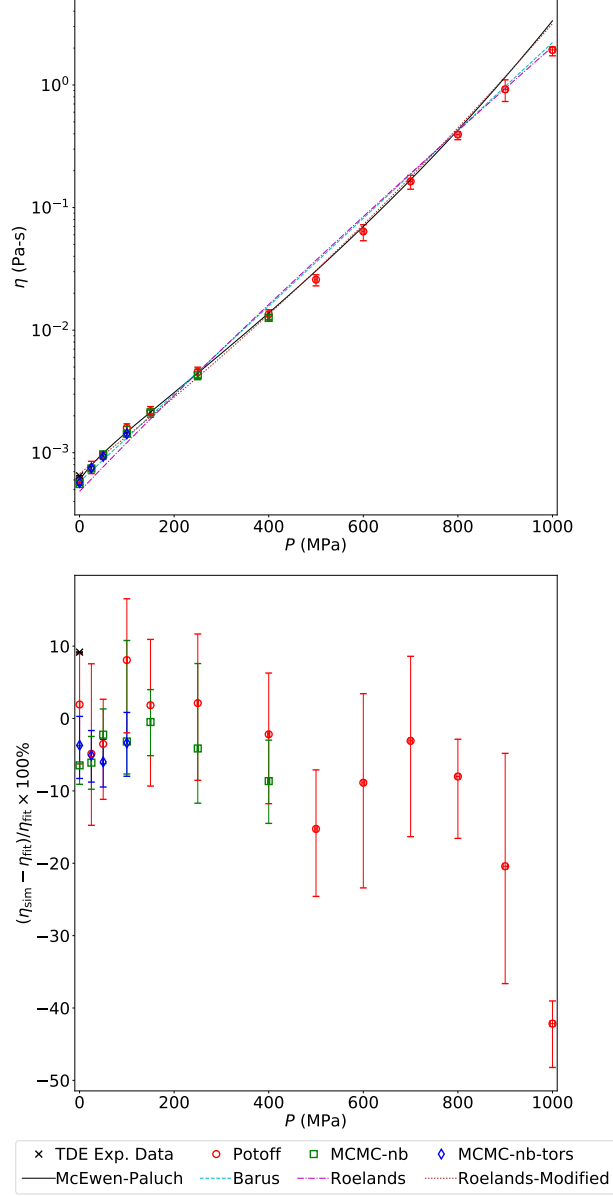


Figure 3: Viscosity-pressure results for Potoff, MCMC-Mie, and MCMC-Mie-tors compared with experimental data. Top panel plots  $\log_{10}(\eta)$ - $P$  where different line colors and styles represent the best fit of Equations 9, 10, and 11 to simulation results. Bottom panel is a percent deviation plot relative to the McEwen-Paluch fit.

with lower deviations than those of the Roelands and Barus models (see Figures 3 and 5), this should be anticipated considering the McEwen-Paluch model has five fitting parameters while the Barus and Roelands models only have two. Note that the four parameter

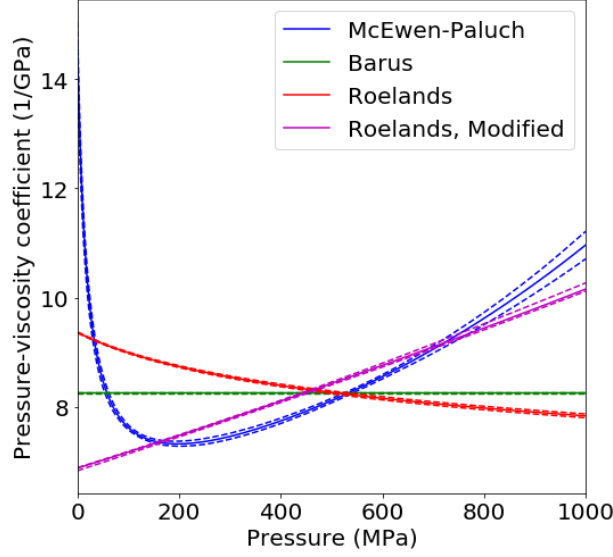


Figure 4: Pressure-viscosity coefficient predicted with empirical model fits (Equations 9, 10, and 11). Dashed lines represent 95 % confidence intervals obtained from bootstrap re-sampling.

Roelands-Modified model also has lower deviations than the Roelands and Barus models. Therefore, it is possible that the McEwen-Paluch model is actually over-fit to our simulation results. To assess this possibility, Figure 5 presents the cross-validation results for each model. Specifically, we implement a Monte Carlo cross-validation scheme where thousands of random sub-samples are selected for the training and testing set. Approximately 70 % of the data are included in the training set while 30 % are excluded as the testing set. Since the mean-squared-error (MSE) for the training set is approximately equal to the MSE for the testing set, we conclude that the McEwen-Paluch model is not over-fit to the data. Therefore, there is statistical evidence that the Mie 16-6 united-atom force field predicts an inflection point followed by super-Arrhenius behavior.

#### 4. Discussion

It is extremely surprising that both the non-bonded and torsional parameter uncertainties are negligible compared to the numerical uncertainties in  $\eta$ . A possible explanation for why the non-bonded parameter uncertainty have a negligible impact on  $\eta$  is that the

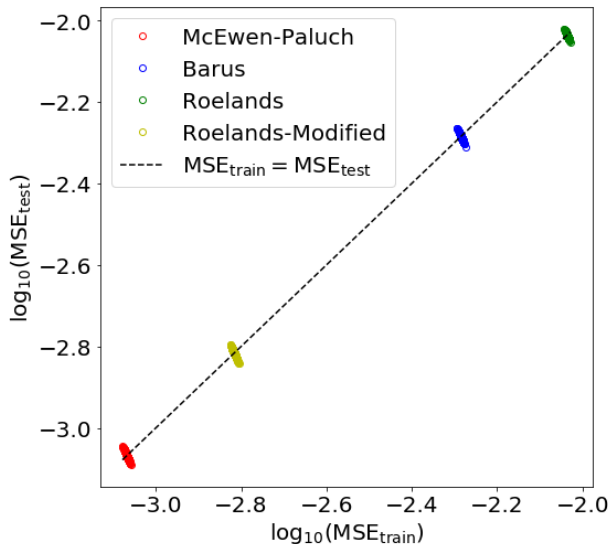


Figure 5: Monte Carlo cross-validation for empirical model fits (Equations 9, 10, and 11).  $\text{MSE}_{\text{train}}$  and  $\text{MSE}_{\text{test}}$  are the mean-squared-error for the “training” and “testing” sets, respectively.

$\text{CH}_3$  uncertainties are considerably smaller than those for  $\text{CH}_2$ ,  $\text{CH}$ , and  $\text{C}$ . As 224TMH consists primarily of  $\text{CH}_3$  sites, the larger uncertainties in  $\text{CH}_2$ ,  $\text{CH}$ , and  $\text{C}$  did not affect the results significantly. By contrast, no clear explanation exists for why the torsional parameter uncertainties did not affect the precision for  $\eta$ . Previous studies suggest that the 15 % to 40 % increase in the torsional barriers should increase the viscosities by a similar amount. However, we did not observe such a strong dependence (or any at all), suggesting that this particular molecule is much less sensitive to the torsional parameters than the long straight-chained alkanes studied in Reference [17].

Although the Potoff force field demonstrates super-Arrhenius behavior, we should caution that this could be an anomaly of the force field. Since the Mie 16-6 potential is known to be overly repulsive at short distances [14, 16], it is possible that this causes the rapid increase in  $\eta$  at high pressures.

Other studies [4] correct for systematic errors in viscosity by normalizing  $\eta$  with respect to an experimental viscosity value at low pressure. This approach would be possible for the challenge compound since a single experimental data point is available at saturation pressure. Although this may result in a more accurate prediction, we prefer not to use

an empirical correction, rather we truly test the force field’s transferability and predictive capabilities.

The slow system dynamics at high pressures requires extremely long simulations and large amounts of replicates. An attractive alternative is the so-called time-temperature superposition method, where simulations are performed at higher temperatures (to enhance the configurational sampling) and the viscosity at 293 K is obtained through extrapolation. Despite some obvious benefits, we are weary of the inordinately large uncertainties that this method can produce (see Figure 11 of Ref. 4). Determining the existence of super-Arrhenius behavior necessitates manageable uncertainties at high pressures. For this reason, we choose the more arduous brute-force approach.

## 5. Conclusions

Previous work demonstrated that the Potoff force field provides reliable viscosities (typically within 10 %) for well-studied *n*-alkane and branched alkanes both at saturation and elevated pressures. For this reason, the Potoff force field was chosen to predict the viscosity-pressure relationship of 2,2,4-trimethylhexane as part of the 10<sup>th</sup> Industrial Fluid Properties Simulation Challenge. In addition, we investigate the parameter uncertainty in the simulation results with Bayesian inference. Specifically, the non-bonded and torsional potentials are varied from run to run according to a Markov Chain in force field parameter space. Surprisingly, the non-bonded and torsional parameter uncertainties are typically negligible compared to the numerical fluctuations in simulation output. Furthermore, we use cross validation model selection to verify the existence of so-called super-Arrhenius behavior at high pressures.

## Supporting Information

Section SI.I provides GROMACS input files. Section SI.II describes how the CH and C non-bonded MCMC parameter sets are obtained. Section SI.III validates the MCMC-nb uncertainty quantification approach. Section ?? presents the average Green-Kubo integrals for each state point.

## Acknowledgments

We are grateful for the internal review provided by BLANK and BLANK of the National Institute of Standards and Technology (NIST). This research was performed while Richard A. Messerly held a National Research Council (NRC) Postdoctoral Research Associateship at NIST and while Michelle C. Anderson held a Summer Undergraduate Research Fellowship (SURF) position at NIST.

Commercial equipment, instruments, or materials are identified only in order to adequately specify certain procedures. In no case does such identification imply recommendation or endorsement by NIST, nor does it imply that the products identified are necessarily the best available for the intended purpose.

Partial contribution of NIST, an agency of the United States government; not subject to copyright in the United States.

## References

- [1] Scott Bair, Laetitia Martinie, and Philippe Vergne. Classical ehl versus quantitative ehl: A perspective part II-super-arrhenius piezoviscosity, an essential component of elastohydrodynamic friction missing from classical ehl. *Tribology Letters*, 63(3):37, 2016.
- [2] Christopher J. Mundy, Michael L. Klein, and J. Ilja Siepmann. Determination of the pressure-viscosity coefficient of decane by molecular simulation. *The Journal of Physical Chemistry*, 100(42):16779–16781, 1996.
- [3] Clare McCabe, Shengting Cui, Peter T. Cummings, Peter A. Gordon, and Roland B. Saeger. Examining the rheology of 9-octylheptadecane to giga-pascal pressures. *The Journal of Chemical Physics*, 114(4):1887–1891, 2001.
- [4] Pinzhi Liu, Hualong Yu, Ning Ren, Frances E. Lockwood, and Q. Jane Wang. Pressure-viscosity coefficient of hydrocarbon base oil through molecular dynamics simulations. *Tribology Letters*, 60(3):34, 2015.

- [5] M. G. Martin and J. I. Siepmann. Transferable potentials for phase equilibria. 1. United-atom description of *n*-alkanes. *The Journal of Physical Chemistry B*, 102(14):2569–2577, 1998.
- [6] Marcus G. Martin and J. Ilja Siepmann. Novel configurational-bias monte carlo method for branched molecules. Transferable Potentials for Phase Equilibria. 2. United-Atom Description of Branched Alkanes. *The Journal of Physical Chemistry B*, 103(21):4508–4517, 1999.
- [7] Mansi S. Shah, J. Ilja Siepmann, and Michael Tsapatsis. Transferable potentials for phase equilibria. Improved united-atom description of ethane and ethylene. *AIChE Journal*, 63(11):5098–5110, 2017.
- [8] Andrea Hemmen and Joachim Gross. Transferable anisotropic united-atom force field based on the Mie potential for phase equilibrium calculations: *n*-alkanes and *n*-olefins. *The Journal of Physical Chemistry B*, 119(35):11695–11707, 2015.
- [9] Dominik Weidler and Joachim Gross. Transferable anisotropic united-atom force field based on the Mie potential for phase equilibria: Aldehydes, ketones, and small cyclic alkanes. *Industrial & Engineering Chemistry Research*, 55(46):12123–12132, 2016.
- [10] J. J. Potoff and D. A. Bernard-Brunel. Mie potentials for phase equilibria calculations: Applications to alkanes and perfluoroalkanes. *The Journal of Physical Chemistry B*, 113(44):14725–14731, 2009.
- [11] Jason R. Mick, Mohammad Soroush Barhaghi, Brock Jackman, Loren Schwiebert, and Jeffrey J. Potoff. Optimized Mie potentials for phase equilibria: Application to branched alkanes. *Journal of Chemical & Engineering Data*, 62(6):1806–1818, 2017.
- [12] Philippe Ungerer, Christele Beauvais, Jerome Delhommelle, Anne Boutin, Bernard Rousseau, and Alain H. Fuchs. Optimization of the anisotropic united atoms intermolecular potential for *n*-alkanes. *The Journal of Chemical Physics*, 112(12):5499–5510, 2000.

- [13] Carlos Nieto-Draghi, Anthony Bocahut, Benoît Creton, Pascal Have, Aziz Ghoufi, Aurélie Wender, , Anne Boutin, Bernard Rousseau, and Laurent Normand. Optimisation of the dynamical behaviour of the anisotropic united atom model of branched alkanes: application to the molecular simulation of fuel gasoline. *Molecular Simulation*, 34(2):211–230, 2008.
- [14] Richard A. Messerly, Michael R. Shirts, and Andrei F. Kazakov. Uncertainty quantification confirms unreliable extrapolation toward high pressures for united-atom Mie  $\lambda$ -6 force field. *Journal of Chemical Physics*, Pending publication, 2018.
- [15] Peter A. Gordon. Development of intermolecular potentials for predicting transport properties of hydrocarbons. *The Journal of Chemical Physics*, 125(1):014504, 2006.
- [16] Richard A. Messerly, Michelle C. Anderson, Mostafa S. Razavi, and J. Richard Elliott. Improvements and limitations of mie  $\lambda$ -6 potential for prediction of saturated and compressed liquid viscosity. *Fluid Phase Equilibria*, Pending publication, 2018.
- [17] Carlos Nieto-Draghi, Philippe Ungerer, and Bernard Rousseau. Optimization of the anisotropic united atoms intermolecular potential for  $n$ -alkanes: Improvement of transport properties. *The Journal of Chemical Physics*, 125(4):044517, 2006.
- [18] Carlos Braga and Karl P. Travis. Computer simulation of the role of torsional flexibility on mass and momentum transport for a series of linear alkanes. *The Journal of Chemical Physics*, 137(6):064116, 2012.
- [19] Carmelo Herdes, Tim S. Totton, and Erich A. Müller. Coarse grained force field for the molecular simulation of natural gases and condensates. *Fluid Phase Equilibria*, 406:91 – 100, 2015.
- [20] M. P. Allen and D. J. Tildesley. *Computer Simulation of Liquids*. Clarendon Press ; Oxford University Press, Oxford England New York, 1987.

- [21] Richard A. Messerly, Thomas A. Knotts IV, and W. Vincent Wilding. Uncertainty quantification and propagation of errors of the Lennard-Jones 12-6 parameters for *n*-alkanes. *The Journal of Chemical Physics*, 146(19):194110, 2017.
- [22] M.J. Abraham, D. van der Spoel, E. Lindahl, B.Hess, and the GROMACS development team. *GROMACS User Manual version 2018*, [www.gromacs.org](http://www.gromacs.org) (2018).
- [23] GROMACS non-bonded tail corrections assume that the long-range contribution from the  $r^{-\lambda}$  term is negligible compared to the  $r^{-6}$  term. By comparing the GROMACS output with other (slower) simulation packages, we verified that the small error introduced with this approximation does not significantly affect our results. For this reason, although it is straightforward to include the  $r^{-\lambda}$  contribution, we did not attempt to modify the GROMACS default tail correction values.
- [24] Berk Hess, Henk Bekker, Herman J. C. Berendsen, and Johannes G. E. M. Fraaije. LINCS: A linear constraint solver for molecular simulations. *Journal of Computational Chemistry*, 18(12):1463–1472, 1998.
- [25] Berk Hess. P-LINCS: A parallel linear constraint solver for molecular simulation. *Journal of Chemical Theory and Computation*, 4(1):116–122, 2008.
- [26] Edward J. Maginn, Richard A. Messerly, Daniel J. Carlson, Daniel R. Roe, and J. Richard Elliott. Best practices for computing transport properties 1. Self-diffusivity and viscosity from equilibrium molecular dynamics v1. *Living Journal of Computational Molecular Science*, Pending publication, 2018.
- [27] Yong Zhang, Akihito Otani, and Edward J. Maginn. Reliable viscosity calculation from equilibrium molecular dynamics simulations: A time decomposition method. *Journal of Chemical Theory and Computation*, 11(8):3537–3546, 2015.
- [28] C. Barus. Isothermals, isopiestic and isometrics relative to viscosity. *American Journal of Science*, 45:87–96, 1893.



- [29] C.J.A. Roelands. *Correlational Aspects of the Viscosity-temperature-pressure Relationships of Lubricating Oils*. PhD thesis, TU Delft, Delft University of Technology, 1966.