Formal Properties of Stochastic Matrices for Subjective Probabilistic Operators in Kripke Frames

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Abstract

Right stochastic matrices (square matrices where each entry is non-negative and the sum of each row is 1) can be used to represent Kripke frames for a probabilistic operator of modal logic. In this work, we focus on matrices for a subjective probability operator rather than a stochastic process. Each world represents a state of affairs considered by an agent, and M_{ij} is the probability that the agent in state i assigns to state j. This study aims to explore certain formal properties of these matrices, depending on the axioms they satisfy:

- A matrix M represents a Kripke frame that satisfies the introspection axiom in its probabilistic version if and only if it can be arranged as a series of block diagonal submatrices, the other values being 0. Within each block, all rows are identical. This matrix M is thus idempotent.

$$M = \begin{pmatrix} 0.6 & 0.4 & \dots & \dots & 0 & 0 & 0 \\ 0.6 & 0.4 & \dots & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & 0 & 0 & 0 \\ 0 & 0 & \dots & \dots & 0.5 & 0.3 & 0.2 \\ 0 & 0 & \dots & \dots & 0.5 & 0.3 & 0.2 \\ 0 & 0 & \dots & \dots & 0.5 & 0.3 & 0.2 \end{pmatrix}$$

- It is proven that the Kripke frame represented by this matrix is simultaneously transitive, dense, and also Euclidean.

- A matrix satisfies the probabilistic positive introspection axiom if and only if it satisfies the negative introspection axiom. In contrast, these axioms are independent in non-probabilistic epistemic logic.

- If this introspective Kripke frame is also reflexive, it represents an $S5$ frame, and thus each block represents an equivalence class in W.