AFFINE PAVING OF PARTIAL FLAG QUIVER VARIETY

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Abstract. In this article, we establish an affine paving for partial flag quiver varieties when the quiver is of Dynkin type. By copying results in [1, section 6] word by word, the same problem for affine quiver reduced to the case where the representation is regular quasi-simple. The idea of the proof mainly comes from [1], and the result is a natural continuation of [2].

Contents

1. Introduction

Let Q be a quiver of Dynkin or affine type(without loops), $X \in \mathrm{Rep}(Q)$ be an quiver representation.[1] We are interested in three objects related to $X \in \mathrm{Rep}(Q)$:

quiver Grassmannian		$\mathrm{Gr}^{\mathrm{KQ}}(X) \colon = \{M_1 \mid M_1 \subseteq X\}$
partial flag variety	$d \geqslant 1$	$\mathrm{Flag}_{\mathbf{d}}(X) \colon = \{0 \subseteq M_1 \subseteq \cdots M_d \subseteq X\}$
strict partial flag variety	$d \geqslant 2$	$\mathrm{Flag}_{\mathbf{d},\mathrm{str}}(X) \colon = \{0 \subseteq M_1 \subseteq \cdots M_d \subseteq X \mid x.M_{i+1} \subseteq M_i\}$ [2]

It's easy to see that $\mathrm{Flag}_{\mathbf{1}}(X) = \mathrm{Gr}^{\mathrm{KQ}}(X)$. These geometrical objects can be divided into different pieces according to the dimension vectors of $M_1, \ldots, M_d$, and each piece have its own natural (complex/Zarisky) topology. It was proved in [1] that $\mathrm{Gr}^{\mathrm{KQ}}(X)$ have an affine paving, and in [2] that $\mathrm{Flag}_{\mathbf{d}}(X)$ have the same property when Q is Dynkin quiver of type A/E. Here we go one step further, the results are concluded in the ???.

???Here is one table

The idea of proof is very simple: first, we view the partial flag quiver variety as the quiver Grassmannian of the more complicated quiver; then we establish the decomposition so that one may solve the problem by induction; finally we set a special way of decomposition for

[1] We fix the base field $K = \mathbb{C}$ for convinience.

[2] for any $x \in Q_1, i \in \{2, \ldots, d\}$.

each indecomposable module so that we can avoid meeting the bad decomposition. These contents are in Section ???, accordingly.

2. Preliminary Facts

2.1. **Ext-vanishing properties.**

3. Main Theorem

4. Application: Dynkin Case

5. Application: Affine Case

References

[1] Giovanni Cerulli Irelli, Francesco Esposito, Hans Franzen, and Markus Reineke. Cell decompositions and algebraicity of cohomology for quiver grassmannians, 2019.

[2] Ruslan Maksimau. Flag versions of quiver grassmannians for dynkin quivers have no odd cohomology over $\mathbb{Z}$, 2019.

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