

MODULI IN ALGEBRAIC GEOMETRY

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ABSTRACT. In this personal survey, we conclude the definitions of moduli functors in the algebraic geometry. Most of the results are in the black box, so it's very possible that they're wrong. And also I'm not responsible for the completeness of the whole theory. However, I'm still happy to improve this document, and make it better over time.

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1. GOAL AND RELATED CONCEPTS

The personal survey is motivated by the three courses in Bonn: “the moduli space of curves”, “moduli of elliptic curves” and “moduli of vector bundles”. I want to construct my personal understanding on the moduli, and find out the details I missed in the courses.

“Some mathematicians are birds, others are frogs.” This document is devoted to those “birds” from a “frog” who gets stuck in the mud.

1.1. representable functor.

1.2. **coarse moduli space.** We need to define the fine moduli space, coarse moduli space and some related concepts.

1.3. **stack.** We need to define stack, algebraic stack, Deligne-Munford stack and some related concepts.

1.4. goal.

2. BASIC OBJECT

In this section, we present some algebraic geometric objects which can be viewed as moduli.

Here is a picture showing the relationships of these objects:

???

2.1. projective space.

2.2. **Grassmannian.** It's well-written in [2, 16.7].

2.3. **flag variety, partial flag variety.**

2.4. **Hilbert scheme.**

2.5. **Quot scheme.**

2.6. **Misc.** The representable functor is also used to construct the fibered product of schemes, see [2, 9.1.6-7] for more details.

3. MODULI OF CURVE

The content of this section is already well written in the course “the moduli space of curves”. This section is just for the completeness of the survey.

4. MODULI OF ELLIPTIC CURVE

The elliptic curve theory is especially rich compared to the other curves. That's why we'd like to put it a special section.

4.1. **differential.**

4.2. **level structure.**

4.3. **complex case.** In this subsection, we will show that how the moduli is connected to the modular curve $\mathcal{H}/\mathrm{SL}_2(\mathbb{Z})$.

5. MODULI OF VECTOR BUNDLE

Here we refer to [1]. It's not easy to read, but I don't know the other better reference.

REFERENCES

- [1] Daniel Huybrechts and Manfred Lehn. *The geometry of moduli spaces of sheaves*. Cambridge University Press, 2010.
- [2] Ravi Vakil. MATH 216: Foundations of Algebraic Geometry.

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