以下所有内容均为个人翻译，非官方翻译版本（暂无），仅供参考。

S．T．YAU College Student Mathematics Contests

Algebra, Number Theory and Combinatorics (second draft)

代数、数论和组合（修改稿）

Linear Algebra

Abstract vector spaces; subspaces; dimension; matrices and linear transformations; matrix algebras and groups; determinants and traces; eigenvectors and eigenvalues, characteristic and minimal polynomials; diagonalization and triangularization of operators; invariant subspaces and canonical forms; inner products and orthogonal bases; reduction of quadratic forms; hermitian and unitary operators, bilinear forms; dual spaces; adjoints. tensor products and tensor algebras;

线性代数

抽象向量空间；子空间；维度；矩阵和线性变换；矩阵代数和群；行列式和迹；特征向量和特征值；特征多项式和最小多项式；对角化和三角化；不变子空间和典范形式；内积和正交基；二次型的化简；埃尔米特算子和酉算子；双线性型；对偶空间；伴随矩阵，张量积和张量代数；

Integers and polynomials

Integers, Euclidean algorithm, unique decomposition; congruence and the Chinese Remainder theorem; Quadratic reciprocity ; Indeterminate Equations. Polynomials, Euclidean algorithm, uniqueness decomposition, zeros; The fundamental theorem of algebra; Polynomials of integer coefficients, the Gauss lemma and the Eisenstein criterion; Polynomials of several variables, homogenous and symmetric polynomials, the fundamental theorem of symmetric polynomials.

整数和多项式

整数，欧几里得算法，唯一分解；同余和中国剩余定理；二次互反性；不定方程. 多项式，欧几里得算法，唯一分解，零点；代数基本定理；整系数多项式，高斯引理和爱森斯坦判别法；多元多项式，齐次多项式和对称多项式；对称多项式基本定理.

Group

Groups and homomorphisms, Sylow theorem, finitely generated abelian groups.
Examples: permutation groups, cyclic groups, dihedral groups, matrix groups, simple groups, Jordan-Holder theorem, linear groups (GL(n, F) and its subgroups), p-groups, solvable and nilpotent groups, group extensions, semi-direct products, free groups, amalgamated products and group presentations.

群

群和同态，西罗定理，有限生成阿贝尔群.例子：置换群、循环群、二面体群、矩阵群、单群、若尔当-赫尔德定理、线性群（GL（n，F）及其子群）、p群、可解群和幂零群，群扩张，半直积，自由群，融合积和群表示.

Ring

Basic properties of rings, units, ideals, homomorphisms, quotient rings, prime and maximal ideals,

fields of fractions, Euclidean domains, principal ideal domains and unique factorization domains, polynomial and power series rings, Chinese Remainder Theorem, local rings and localization, Nakayama's lemma, chain conditions and Noetherian rings, Hilbert basis theorem, Artin rings, integral ring extensions, Nullstellensatz, Dedekind domains,algebraic sets, Spec(A).

环

环的基本性质，单位元，理想，同态，商环，素理想和最大理想，分式域，欧几里得整环，主理想整环和唯一因子分解整环（高斯整环），多项式环和幂级数环，中国剩余定理，局部环和局部化，中山正引理，链式条件和诺特环，希尔伯特基本定理，阿延环，整环扩张，零点定理（德文），戴德金整环，代数集，Spec（A）

Module

Modules and algebra Free and projective; tensor products; irreducible modules and Schur's lemma; semisimple, simple and primitive rings; density and Wederburn theorems; the structure of finitely generated modules over principal ideal domains, with application to abelian groups and canonical forms; categories and functors; complexes, injective modues, cohomology; Tor and Ext.

模

模和代数，自由和射影；张量积；不可约模和舒尔引理；半单环、单环和本原环；稠密性和韦德伯恩定理；主理想整环上有限生成模的结构及其在阿贝尔群和典范形式上的应用；范畴和函子；复内射模，上同调；挠积和Ext.

Field

Field extensions, algebraic extensions, transcendence bases; cyclic and cyclotomic extensions; solvability of polynomial equations; finite fields; separable and inseparable extensions; Galois theory, norms and traces,Galois theory of number fields, transcendence degree, function fields.

域

域扩张，代数扩张，超越基；循环扩张和分圆扩张；多项式方程和可解性；有限域；可分扩张和不可分扩张；伽罗华理论，范数和迹，数域上的伽罗华理论超越次数，函数域.

Group representation

Irreducible representations, Schur's lemma, characters, Schur orthogonality, character tables, semisimple group rings, induced representations, Frobenius reciprocity, tensor products, symmetric and exterior powers, complex, real, and rational representations.

群表示

不可约表示，舒尔引理，舒尔规范正交性，特征标表，半单群环，诱导表示，弗罗贝尼乌斯互反，张量积，对称幂和外幂，复表示、实表示和有理表示.

Lie Algebra

Basic concepts, semisimple Lie algebras, root systems, isomorphism and conjugacy theorems, representation theory.

李代数

基本概念，半单李代数，根系，同构定理和共轭定理，表示论.

Combinatorics (TBA)

组合学（TBA）

References:

Strang, Linear algebra, Academic Press.

I.M. Gelfand, Linear Algebra
《整数与多项式》冯克勤余红兵著高等教育出版社
Jacobson, Nathan Basic algebra. I. Second edition. W. H. Freeman and Company, New York, 1985. xviii+499 pp.
Jacobson, Nathan Basic algebra. II. Second edition. W. H. Freeman and Company, New York, 1989. xviii+686 pp.
S. Lang, Algebra, Addison-Wesley
冯克勤，李尚志，查建国，章璞，《近世代数引论》
刘绍学，《近世代数基础》
J. P. Serre, Linear representations of finite groups
J. P. Serre: Complex semisimple Lie algebra and their representations
J. Humphreys: Introduction to Lie algebra and representation theory, GTM 009.
W. Fulton, Representation theory, a First Course, GTM 129.

Analysis and differential equations (second draft)

分析和微分方程（修改稿）

Calculus and mathematical analysis

Derivatives, chain rule; maxima and minima, Lagrange multipliers; line and surface integrals of scalar and vector functions; Gauss', Green's and Stokes' theorems. Sequences and series, Cauchy sequences, uniform convergence and its relation to derivatives and integrals; power series, radius of convergence, convergence of improper integrals. Inverse and implicit function theorems and applications; the derivative as a linear map; existence and uniqueness theorems for solutions of ordinary differential equations, explicit solutions of simple equations.; elementary Fourier series.

微积分和数学分析

导数，链式法则；极大值和极小值，拉格朗日乘数法；标量函数与向量函数的曲线积分与曲面积分；高斯公式、格林公式和斯托克斯公式.数列和级数，柯西列，一致收敛及其与可微和可积的关系；幂级数，收敛半径，反常积分和收敛性.反函数定理、隐函数定理及其应用；看作线性映射的求导；常微分方程解的存在唯一性定理，简单方程的隐函数解；傅里叶级数.

Complex analysis

Analytic function, Cauchy's Integral Formula and Residues, Power Series Expansions, Entire Function, Normal Families, The Riemann Mapping Theorem, Harmonic Function, The Dirichlet Problem Simply Periodic Function and Elliptic Functions, The Weierstrass Theory Analytic Continuation, Algebraic Functions, Picard's Theorem

复分析

解析函数，柯西积分公式和留数，幂级数展开，整函数，正规簇，黎曼映射定理，调和函数，狄利克雷问题，简单周期函数和双曲函数，维尔斯特拉斯定理，解析延拓，代数函数，皮卡定理.

Point set topology of Rn

Countable and uncountable sets, the axiom of choice, Zorn's lemma. Metric spaces.Completeness; separability; compactness; Baire category; uniform continuity; connectedness; continuous mappings of compact spaces. Functions on topological spaces. Equicontinuity and Ascoli's theorem; the Stone-Weierstrass theorem; topologies on

function spaces; compactness in function spaces.

Rn上的点集拓扑学

可数集与不可数集，选择公理，佐恩引理.度量空间.完全性（完备性）；分离性；紧性；贝尔范畴；一致连续性；连通性；紧空间的连续映射.拓扑空间里的函数.等度连续性和阿斯科利定理；斯通-维尔斯特拉斯定理；函数空间的拓扑；函数空间的紧性.

Measure and integration

Measures; Borel sets and contor sets; Lebesgue measures; distributions; product measures. Measurable functions. approximation by simple functions; convergence in measure; Construction and properties of the integral; convergence theorems; Radon-Nykodym theorem; Fubini's theorem; mean convergence. Monotone functions; functions of bounded variation and Borel measures; Absolute continuity, convex functions; semicontinuity.

测度和积分

测度；博雷尔集和康托集；勒贝格测度；广义函数；积测度.可测函数.简单函数的逼近；依测度收敛；积分的构造与性质；收敛定理；拉东-Nykodym定理；富比尼定理；平均收敛.单调函数；有界变差函数和博雷尔测度；绝对连续性，反函数；半连续性.

Banach and Hilbert spaces

Lp spaces; C(X); completeness and the Riesz-Fischer theorem; orthonormal bases; linear functionals; Riesz representation theorem; linear transformations and dual spaces; interpolation of linear operators; Hahn-Banach theorem; open mapping theorem; uniform boundedness (or Banach-Steinhaus) theorem; closed graph theorem. Basic properties of compact operators, Riesz- Fredholm theory, spectrum of compact operators. Basic properties of Fourier series and the Fourier transform; Poission summation formula; convolution.

巴拿赫空间和希尔伯特空间

Lp空间；C(X)；完备性和里斯-菲舍尔定理；规范正交基（幺正基）；线性泛函；里斯表示定理；线性变换和对偶空间；线性算子的插值；哈恩-巴拿赫定理；开映射定理；一致有界性定理（巴拿赫-施坦豪斯定理）；闭图象定理.紧算子（全连续算子）的基本性质，里斯-弗雷德霍姆定理，紧算子的谱.傅里叶级数和傅里叶变换的基本性质；泊松求和公式；卷积.

Basic partial differential equations

First order partial differential equations, linear and quasi-linear PDE, Wave equations: initial condition and boundary condition, well-poseness, Sturn-Liouville eigen-value problem, energy functional method, uniqueness and stability of solutions Heat equations: initial conditions, maximal principle and uniqueness and stability, Potential equations: Green functions and existence of solutions of Dirichlet problem, harmonic functions, Hopf’s maximal principle and existence of solutions of Neumann’s problem, weak solutions, eigen-value problem of the Laplace operator, Generalized functions and fundamental solutions of PDE

偏微分方程基础

一阶偏微分方程，线性偏微分方程和拟线性偏微分方程；初始条件和边界条件.，适应性，斯图姆-刘维尔特征值问题，能量泛函方法，解的唯一性和稳定性，热传导方程；初始条件，最大值原理和唯一性、稳定性，位势方程；格林函数和狄利克雷问题的解的存在性，调和函数，霍普夫最大值原理和诺依曼问题的解的存在性，弱解，

拉普拉斯算子的特征值问题，广义函数和偏微分方程的基础解系.

References:

Rudin, Principles of mathematical analysis, McGraw-Hill.

Courant, Richard; John, Fritz Introduction to calculus and analysis. Vol. I. Reprint of the 1989 edition.Classics in Mathematics. Springer-Verlag, Berlin, 1999.

Courant, Richard; John, Fritz Introduction to calculus and analysis. Vol. II. With the assistance of Albert A. Blank and Alan Solomon. Reprint of the 1974 edition. Springer-Verlag, New York, 1989.

V. I. Arnold, Ordinary Differential Equations, Springer-Verlag, Berlin, 2006.

Valerian Ahlfors, An Introduction to the Theory of Analytic Functions of One Complex Variable

K. Kodaira, Complex Analysis

Rudin, Real and complex analysis

龚升，简明复分析

Royden, Real Analysis, except chapters 8, 13, 15.

E.M. Stein and R. Shakarchi; Real Analysis: Measure Theory, Integration, and Hilbert Spaces, Princeton University Press, 2005

周民强，实变函数论，北京大学出版社, 2001

夏道行等，《实变函数论与泛函分析》，人民教育出版社.

Peter D. Lax, Functional Analysis, Wiley-Interscience, 2002.

《Basic Partial Differential Equations》, D. Bleecker, G. Csordas 著，李俊杰译，高等教育出版社，2008.

《数学物理方法》，柯朗、希尔伯特著。

Computational Mathematics, Applied Mathematics, Probability and Statistics (the second draft)

计算数学、应用数学、概率与统计（修改稿）

Computational Mathematics

计算数学

Interpolation and approximation

Polynomial interpolation and least square approximation; trigonometric interpolation and approximation, fast Fourier transform; approximations by rational functions; splines.

插值与逼近

多项式插值和最小二乘法逼近；三角插值和逼近，快速傅里叶变换；有理函数逼近；样条.

Nonlinear equation solvers

Convergence of iterative methods (bisection, secant method, Newton method, other iterative methods) for both scalar equations and systems; finding roots of polynomials.

非线性方程解法

解标量方程及标量方程组的迭代法（二分法，割线法，牛顿法，其它迭代法）的收敛性；求多项式的根.

Linear systems and eigenvalue problems

Direct solvers (Gauss elimination, LU decomposition, pivoting, operation count, banded matrices, round-off error accumulation); iterative solvers (Jacobi, Gauss-Seidel, successive over-relaxation, conjugate gradient method, multi-grid method, Krylov methods); numerical solutions for eigenvalues and eigenvectors.

线性系统和本征值问题

直接解法（高斯消元法，LU分解，基更换，带状矩阵，舍入误差的积累）；迭代方法（雅可比迭代法，高斯-塞德尔迭代法，超松弛迭代法，共轭梯度法，多重网格法，克雷洛夫方法）；特征值和特征向量的数值解.

Numerical solutions of ordinary differential equations

One step methods (Taylor series method and Runge-Kutta method); stability, accuracy and convergence; absolute stability, long time behavior; multi-step methods.

常微分方程的数值解

单步法（泰勒级数方法和龙格-库塔法）；稳定性、精度和收敛性；绝对稳定性，长期性态；多步法.

Numerical solutions of partial differential equations

Finite difference method; stability, accuracy and convergence, Lax equivalence theorem; finite element method, boundary value problems.

偏微分方程的数值解

（有限）差分法；稳定性、精度和收敛性，拉克斯等价定理；有限元方法，边值问题.

References:

[1] C. de Boor and S.D. Conte, Elementary Numerical Analysis, an algorithmic approach, McGraw-Hill, 2000.

[2] G.H. Golub and C.F. van Loan, Matrix Computations, third edition, Johns Hopkins University Press, 1996.

[3] E. Hairer, P. Syvert and G. Wanner, Solving Ordinary Differential Equations, Springer, 1993.

[4] B. Gustafsson, H.-O. Kreiss and J. Oliger, Time Dependent Problems and Difference Methods, John Wiley Sons, 1995.

[5] G. Strang and G. Fix, An Analysis of the Finite Element Method, second edition, Wellesley-Cambridge Press, 2008.

Applied Mathematics

ODE with constant coefficients; Nonlinear ODE: critical points, phase space & stability analysis; Hamiltonian, gradient, conservative ODE's.

Calculus of Variations: Euler-Lagrange Equations; Boundary Conditions, parametric formulation;

optimal control and Hamiltonian, Pontryagin maximum principle.

First order partial differential equations (PDE) and method of characteristics; Heat, wave, and Laplace's equation; Separation of variables and eigen-function expansions; Stationary phase method; Homogenization method for elliptic and linear hyperbolic PDEs; Homogenization and front propagation of Hamilton-Jacobi equations; Geometric optics for dispersive wave equations.

应用数学

常系数常微分方程；非线性常微分方程：临界点，相空间和稳定性分析；
变分法：欧拉-拉格朗日方程；边界条件，参数公式；最优控制和哈密顿算子，庞特里亚金极大原理.
一阶偏微分方程和特征曲线法；热传导方程，波动方程和拉普拉斯方程；分离变量法和本征函数展开；平稳相位法；椭圆型偏微分方程和线性双曲型偏微分方程的齐次化方法；哈密顿-雅可比方程的齐次化和波前传播；色散波方程的几何光学.

References:

W.D. Boyce and R.C. DiPrima, Elementary Differential Equations, Wiley, 2009
F.Y.M. Wan, Introduction to Calculus of Variations and Its Applications, Chapman & Hall, 1995
G. Whitham, "Linear and Nonlinear Waves", John-Wiley and Sons, 1974.
J. Keener, "Principles of Applied Mathematics", Addison-Wesley, 1988.
A. Benssousan, P-L Lions, G. Papanicolaou, "Asymptotic Analysis for Periodic Structures", North-Holland Publishing Co, 1978.
V. Jikov, S. Kozlov, O. Oleinik, "Homogenization of differential operators and integral functions", Springer, 1994.
J. Xin, "An Introduction to Fronts in Random Media", Surveys and Tutorials in Applied Math Sciences, No. 5, Springer, 2009.

Probability

Random Variables; Conditional Probability and Conditional Expectation; Markov Chains; The Exponential Distribution and the Poisson Process; Continuous-Time Markov Chains; Renewal Theory and Its Applications; Queueing Theory; Reliability Theory; Brownian Motion and Stationary Processes; Simulation.

概率论

随机变量；条件概率和条件期望；马尔可夫链；指数分布和泊松过程；连续时间马尔可夫链；更新定理及其应用；排队论；可靠性理论；布朗运动和平稳过程；模拟仿真.

Reference: Sheldon M. Ross, Introduction to Probability Models

Statistics

数理统计

Distribution Theory and Basic Statistics

Families of continuous distributions: Chi-sq, t, F, gamma, beta; Families of discrete distributions: Multinomial, Poisson, negative binomial; Basic statistics: Mean, median, quantiles, order statistics

分布理论与初等统计

连续分布族：χ^2, t, F, γ, β (分布)；离散分布族：多项式、泊松、负二项（分布）；初等数理统计：平均值、中位数、数量、顺序统计量.

Likelihood principle

Likelihood function, parametric inference, sufficiency, factorization theorem, ancillary statistic, conditional likelihood, marginal likelihood.

似然原理

似然函数，参数推断，充分性，因子分解定理，从属统计量，条件似然，边缘似然.

Testing

Neyman-Pearson paradigm, null and alternative hypotheses, simple and composite hypotheses, type I and type II errors, power, most powerful test, likelihood ratio test, Neyman-Pearson Theorem, monotone likelihood ratio, uniformly most powerful test, generalized likelihood ratio test.

检验

奈曼-皮尔逊范例，原假设和被选假设，简单假设和复合假设，I型误差和II型误差，功效，最大功效检验，似然比检验，奈曼-皮尔逊定理，单调似然化，一直最大功率检验，广义似然化检验.

Estimation

Parameter estimation, method of moments, maximum likelihood estimation, unbiasedness, quadratic and other criterion functions, Rao-Blackwell Theorem, Fisher information, Cramer-Rao bound, confidence interval, duality between confidence interval and hypothesis testing.

估计

素数估计，矩量法，最大似然估计，无偏性，二次判据函数及其它判据函数，拉奥-布莱克韦尔定理，费希尔信息，克拉姆-拉奥边界，置信区间，置信区间与假设检验间的对偶性.

Bayesian Statistics

Prior, posterior, conjugate priors, Bayesian loss

贝叶斯统计

先验，后验，先验共轭，贝叶斯损失

Nonparametric statistics

Permutation test, permutation distribution, rank statistics, Wilcoxon-Mann-Whitney test, log-rank test, Kolmogorov-Smirnov-type tests.

非参数统计

置换检验，置换分布，秩统计量，威尔科克森-曼恩-惠特尼检验，对数秩检验，柯尔莫哥洛夫-斯米尔诺夫检验.

Regression

Linear regression, least squares method, Gauss-Markov Theorem, logistic regression, maximum likelihood

回归

线性回归，最小二乘法，高斯-马尔可夫定理，逻辑斯谛回归，最大似然.

Large sample theory

Consistency, asymptotic normality, chi-sq approximation to likelihood ratio statistic, large-sample based confidence interval, asymptotic properties of empirical distribution.

大样本理论

相容性，渐进正态性，似然比统计量的 χ^2 逼近，大样本基本置信区间，经验分布的渐近性质.

References

Casella, G. and Berger, R.L. (2002). Statistical Inference (2nd Ed.) Duxbury Press.

茆诗松，程依明，濮晓龙，概率论与数理统计教程（第二版），高等教育出版社，2008.

陈家鼎，孙山泽，李东风，刘力平，数理统计学讲义，高等教育出版社，2006.

郑明，陈子毅，汪嘉冈，数理统计讲义，复旦大学出版社，2006.

陈希孺，倪国熙，数理统计学教程，中国科学技术大学出版社，2009.

Geometry and Topology (the second draft)

几何与拓扑（修改稿）

Space curves and surfaces

Curves and Parametrization, Regular Surfaces; Inverse Images of Regular Values. Gauss Map and Fundamental Properties; Isometries; Conformal Maps; Rigidity of the Sphere.

空间曲线和曲面

曲线和参数化，正则曲面；正则值的逆象.高斯映射及其基本性质；等矩；共形映射；球的刚性.

Topological space

Space, maps, compactness and connectedness, quotients; Paths and Homotopy. The Fundamental Group of the Circle. Induced Homomorphisms. Free Products of Groups. The van Kampen Theorem. Covering Spaces and Lifting Properties; Simplex and complexes. Triangulations. Surfaces and its classification.

拓扑空间

空间，映射，紧性与连通性，商；路径和同伦.院的基本群.有道同态.群的自由积.范坎彭定理.覆叠空间和提升性质；单纯形和复形.三角剖分.曲面及其分类.

Differential Manifolds

Differentiable Manifolds and Submanifolds, Differentiable Functions and Mappings; The Tangent Space, Vector Field and Covector Fields. Tensors and Tensor Fields and differential forms. The Riemannian Metrics as examples, Orientation and Volume Element; Exterior Differentiation and Frobenius's Theorem; Integration on manifolds, Manifolds with Boundary and Stokes' Theorem

微分流形

微分流形和子流形，可微函数和可微映射；切空间，向量场和余向量场.张量、张量场和微分形式.黎曼度量及举例，定向和体积元，外微分和弗罗贝尼乌斯定理；流形上的积分，有界流形和斯托克斯定理.

Homology and cohomology

Simplicial and Singular Homology. Homotopy Invariance. Exact Sequences and Excision. Degree. Cellular Homology. Mayer-Vietoris Sequences. Homology with Coefficients. The Universal Coefficient Theorem. Cohomology of Spaces. The Cohomology Ring. A Kunneth Formula. Spaces with Polynomial Cohomology. Orientations and Homology. Cup Product and Duality.

同调和上同调

单纯同调和奇异同调.同伦不变性.正合序列和分割.阶.胞腔同调.迈尔-菲托里斯序列.具有系数的同调.上同调环.屈内特公式.具有多项式上同调的空间.定向和同调.上积和对偶.

Riemannian Manifolds

Differentiation and connection, Constant Vector Fields and Parallel Displacement， Riemann Curvatures and the Equations of Structure， Manifolds of Constant Curvature, Spaces of Positive Curvature, Spaces of Zero Curvature, Spaces of Constant Negative Curvature

黎曼流形

微分和联络，常向量场和平行位移，黎曼曲率和结构方程，常曲率的流形，正曲率空间，零曲率空间，常负曲率空间.

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