

Which of the following is the best way to show that

$$\{\lambda I_n \mid \lambda \in \mathbb{C} \setminus \{0\}\} \subseteq Z(GL_n(\mathbb{C}))?$$

Answers

- ① A direct computation shows that if $ZA = AZ$ for all $A \in GL_n(\mathbb{C})$ then we must have $A = \lambda I_n$. Hence the assertion holds.
- ② For all $A \in GL_n(\mathbb{C})$ and all $\lambda \in \mathbb{C} \setminus \{0\}$ we have $A(\lambda I_n) = \lambda A = (\lambda I_n)A$. Hence $\lambda I_n \in Z(GL_n(\mathbb{C}))$, and the assertion holds.
- ③ We have $I_n \in Z(GL(n, \mathbb{C}))$ and thus $\lambda I_n \in Z(GL(n, \mathbb{C}))$ for all $\lambda \in \mathbb{C} \setminus \{0\}$.
- ④ For all $A \in GL_n(\mathbb{C})$ we have $\lambda A = A\lambda$ for all $\lambda \in \mathbb{C} \setminus \{0\}$, and $I_n A = A I_n$. Therefore, $\lambda, I_n \in Z(GL_n(\mathbb{C}))$. Since the $Z(GL_n(\mathbb{C}))$ is a subgroup of $GL_n(\mathbb{C})$, it follows that $\lambda I_n \in Z(GL_n(\mathbb{C}))$.