Week 10

Let A and B be $n \times n$ matrices over a field $\mathbb{F}$. Which is the best way to show that similarity of matrices is a reflexive relation?

Answers

1. Let $A \in M_{n,n}(\mathbb{F})$. Then $A = I_n^{-1} A I_n$, where I_n denotes the identity matrix. Hence A is similar to itself, and thus similarity of matrices is reflexive.
2. If A is similar to itself, then there exists an $S \in GL_n(\mathbb{F})$ such that $A = S^{-1}AS$. Therefore, similarity is reflexive.
3. Let $A \in M_{n,n}(\mathbb{F})$. Since $A = A$, reflexivity holds.
4. Let $A, B \in M_{n,n}(\mathbb{F})$ such that A is similar to B. Then $\exists S \in GL_n(\mathbb{F})$ such that $S^{-1}AS = B \Rightarrow (S^{-1})^{-1}B(S^{-1}) = A$ and $S^{-1} \in GL_n(\mathbb{F})$. Hence B is similar to A and reflexivity holds.