Consider the function $T : \mathbb{R}^2 \to \mathbb{R}^3$, defined by

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x+y \\ x+1 \\ 3y \end{bmatrix}$$

Which is the best way to show that T does not send the zero vector to the zero vector?

Answers

1. $T(0) = [0, 1, 0]^T$
2. $T(\begin{bmatrix} 0 \\ 0 \end{bmatrix}) = [0+0, 0+1, 0]^T = [0, 1, 0]^T \neq [0, 0, 0]^T$
3. $T(\begin{bmatrix} x \\ y \end{bmatrix}) = [0, 0, 0]^T \Leftrightarrow x + y = 0$, $x + 1 = 0$ and $3y = 0$, but $\nexists x, y \in \mathbb{R}$ such that all three equations hold.
4. Every linear map sends the zero vector to the zero vector.