Week 7

Let V be a vector space over a field $\mathbb{F}$. Let $\mathbf{v} \in V$ and $0 \neq a \in \mathbb{F}$, such that $a\mathbf{v} = \mathbf{0}$. Using the fact that $b\mathbf{0} = \mathbf{0}$ for all $b \in \mathbb{F}$, what is the best way to show that this implies $\mathbf{v} = \mathbf{0}$?

Answers

1. Since $\mathbb{F}$ is a field, the element a has an inverse $a^{-1} \in \mathbb{F} \setminus \{0\}$. Using scalar multiplication we see that $\mathbf{v} = 1\mathbf{v} = (a^{-1}a)\mathbf{v} = a^{-1}(a\mathbf{v}) = a^{-1}\mathbf{0} = \mathbf{0}$.
2. Since $a \neq 0$ we must have $\mathbf{v} = \mathbf{0}$ due to the properties of multiplication.
3. Assume that $\mathbf{v} = \mathbf{0}$. This can only happen if $a\mathbf{v} = \mathbf{0}$ for all $0 \neq a \in \mathbb{F}$.
4. Assume that $v \neq 0$. Since $\mathbb{F}$ is a field, we have that $av = 0 \Rightarrow a = v^{-1}0 = 0$; a contradiction.