Week 6

Consider the symmetric group S_3. Which is the best way to show that the subgroup $H = \langle (12) \rangle = \{e, (12)\}$ is not normal?

Answers

1. The subgroup **is** normal since $|\langle (12) \rangle| = 2$ and every subgroup of index 2 is normal.
2. Take $(123) \in S_3$, and notice that $(123)H = \{(123), (13)\}$, and $H(123) = \{(123), (23)\}$. Thus $(123)H \neq H(123)$ and so H is not normal.
3. Let $d \in S_3$. Then $dH = \{d, d(12)\}$ and $Hd = \{d, (12)d\}$. But $d(12) \neq (12)d$ so the statement doesn't hold.
4. Assume H is not normal. Then $aH \neq Ha$ for some $a \in S_3$. Picking such an $a \in S_3$ shows the claim.