Jacobs University Bremen
Keivan Mallahi-Karai

Due: October 21, 2022
Assignment 3

Probability and Random Processes

(3.1) Suppose X is a discrete random variable taking values $x \in \{1, 2, 3, 4, 5\}$ such that

$$\mathbb{P}[X = x] = kx^2$$

for some k.

(a) Find the value of k
(b) Determine $\mathbb{P}[X > 2]$.

(3.2) Let X be a continuous random variable with the density function

$$f_X(x) = \begin{cases} 2x & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Determine the distribution function $F_X(t)$.
(b) Let $Y = X^2$. Find the distribution and density function of Y.

(3.3) A fair die is thrown twice with outcomes X_1, X_2. Let $Y = \max(X_1, X_2)$. Find the probability mass function of X.

(3.4) Let X be a continuous random variable with the density function

$$f_X(x) = \begin{cases} \lambda x^2 & \text{if } -2 < x < 2 \\ & \text{otherwise} \end{cases}$$

(a) Determine the value of λ.
(b) Find $\mathbb{P}[X > 1]$.

(3.5) Let X and Y be Bernoulli random variables with parameters p and q, respectively. Show that $Z := XY$ is a Bernoulli random variable with a parameter r with $p + q - 1 \leq r \leq \min(p, q)$.