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Due: November 2, 2022
Assignment 4

Probability and Random Processes

(4.1) Consider a discrete random variable X with the probability mass function given by

$$p_X(x) = \begin{cases} k \cdot 2^x & \text{if } x = -1, 0, 1, 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) Compute the value of k.
(b) Find $\mathbb{P}[X \text{ is even}]$.
(c) Determine $\mathbb{E}[X]$.

(4.2) Suppose X is a discrete random variable with $\mathbb{E}[X] = 5$ and $\text{Var}[X] = 15$. Find the values of $\mathbb{E}\left[X^2\right]$, $\mathbb{E}[2 - X]$, $\text{Var}[3X + 1]$.

(4.3) Let X be a Poisson random variable with parameter λ. Show that $\mathbb{E}[X] = \lambda$.

(4.4) The number of items arriving daily in the lost-and-found section of Bremen train station has a Poisson distribution with parameter $\lambda = 4$.
(a) Find the probability of the event that on a given day no items arrive.
(b) Knowing that at least one item has arrived today, find the probability that there is at least one more item arriving today?
(c) Let W denote the number of items arriving from January 1st to January 15th. What is $\mathbb{E}[W]$?

(4.5) There are 5 people in an elevators. An elevator goes up a building with 10 floors and stops at each floor where at least one person wants to get off. If X denote the number of stops, find $\mathbb{E}[X]$.

Hint: For each $1 \leq i \leq n$, set X_i to denote the Bernoulli random variable defined by $X_i = 1$ when the elevator stops at the i-th floor. Use $X = X_1 + \cdots + X_{10}$.