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Due: September 25, 2022
Assignment 1

Probability and Random Processes

In each one of the problems below, if you are asked to compute a probability, first identify the sample space and the event in question explicitly.

(**1.1**) A fairs die is thrown twice.
(a) What is the sample space?
(b) Determine the probability p_k that the sum of the numbers shown is k. Plot p_k as a function of k.
(c) What is the probability that the absolute value of the difference between outcomes of the first and the second throw is less than 3?

(**1.2**) A fair die is rolled three times. We say that a match has occurred if the outcome of the first throw is 1, or the outcome of the second throw is 2, or the outcome of the third throw is 3. Find the probability of the event that a match occurs.

(**1.3**) An ordinary deck of playing cards (containing 52 standard cards, 13 of each suit) is randomly divided into two parts, each containing at least one card.
(a) What is the probability that each part contains at least one ace.
(b) Find the probability that each part contains exactly two aces.

(**1.4**) Suppose A and B are two events. Let S be the event that A or B occur, but not both. Show that

$$\mathbb{P}[S] = \mathbb{P}[A] + \mathbb{P}[B] - 2\,\mathbb{P}[A \cap B]\,.$$

Hint: Draw a Venn diagram and use it to describe S as Boolean combination of the given events.

(**1.5**) Alice and Bob are supposed to meet in the cafeteria. Alice arrives at a random time between noon and 1pm, and wait for 15 minutes upon her arrival and then leaves. Bob also also arrives at a random time between noon and 1 pm, but waits up to 20 minutes and then leaves.
(a) What is the probability that Bob arrives before 12:20?
(b) What is the probability that Alice and Bob meet?
(c) If Bob arrives later than Alice, what is the probability that they meet?
(d) Suppose that Alice and Bob have managed to meet. What is the probability that Bob has arrived before 12:20?