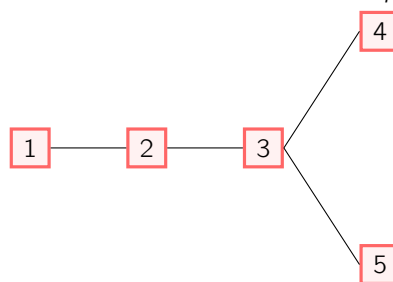


### Probability and Random Processes

- (2.1) A bias coin has the probability  $2/3$  of turning up heads. The coin is thrown 4 times.
- What is the probability that the total number of heads shown is 3?
  - Suppose that we know that outcome of the first throw is a head. Find the probability that the total number of heads shown is 3.
  - If we know that the total number of heads shown is 3, find the probability that the outcome of the first throw was a head.
- (2.2) Suppose that 15 percent of the messages arriving at a mailbox are spam and that 20 percent of spam messages arriving there contain the word "winner". Suppose also that the probability that the word "winner" appears in a non-spam message is 5 percent.
- What percentage of the received emails contain the word "winner"?
  - Suppose that a message is tagged as spam based on containing the word "winner". Find the probability that the message is indeed a spam.
- (2.3) Suppose  $M$  is an integer randomly chosen from the set  $\{1, 2, \dots, 10\}$ . Once  $M$  is chosen, the integer  $N$  is chosen from the set  $\{1, 2, \dots, M\}$ . For instance if it turns out that  $M = 7$ , then  $N$  can take one of the values  $1, \dots, 7$ , each with probability  $1/7$ .
- Find the probability that  $N = 7$ .
  - Find the probability of the event  $M = N$ .
- (2.4) Suppose  $1 \leq i \leq 5$ . A mouse starts at station  $i$  of the network below. At each junction, she moves to one of the adjacent stations with equal probability. For instance, once at 1, she can move to 0 or 2 with probability  $1/2$ , or from 0 she can move to 1 with probability 1. She will stop when she arrived at one of the stations 4 or 5. Let  $p_i$  denote the probability that the mouse eventually ends up at the station 4. Find a formula for  $p_i$  for  $0 \leq i \leq 5$ .



- (2.5) We say that an event  $A$  attracts  $B$  if  $\mathbb{P}[B|A] > \mathbb{P}[B]$ .
- Show that if  $A$  attracts  $B$  then  $B$  also attracts  $A$ .
  - Suppose  $A$  attracts  $B$  and  $B$  attracts  $C$ . Does this imply that  $A$  attracts  $C$ ?