Probability and Random Processes

(7.1) Show that for a non-negative random variable X with mean $\mathbb{E}[X] = \mu$, we have

$$\mathbb{P}[X \geq 2\mu] \leq \frac{1}{2}.$$

Give an example of a non-negative random variable X with mean $\mathbb{E}[X] = \mu > 0$ such that

$$\mathbb{P}[X \geq 2\mu] = \frac{1}{2}.$$

(7.2) Use the Central Limit theorem to compute the approximate value of the probability that the average of 1200 points chosen randomly according to the uniform distribution from the interval $(0, 1)$ is within 0.01 of the midpoint of the interval? The answer can be given in terms of the distribution function F of a standard normal random variable.

(7.3) Suppose X has geometric distribution with parameter p. Show that the moment generating function of X is given by

$$M_X(t) = \frac{pe^t}{1 - (1-p)e^t}.$$

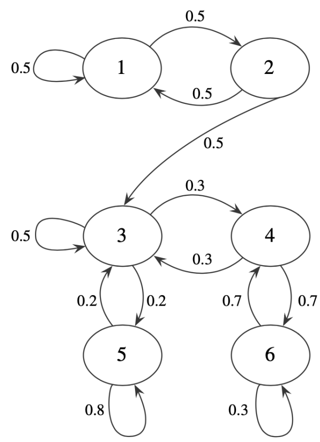
(7.4) Suppose X is a random variable whose moment generating function is given by

$$M_X(t) = \frac{1}{4}e^{2t} + \frac{1}{3}e^{-t} + \frac{5}{12}.$$

Find the probability $\mathbb{P}[|X| \leq 1]$.

Hint: Try to guess a candidate for the random variable X and then use the uniqueness theorem.

(7.5) Consider the following Markov chain on the state space



(a) Compute the transition matrix of this Markov chain.

(b) Compute the probability $p_{12}^{(2)}$.

(c) Determine the transient and absorbing states and compute absorbing probabilities.