ECE/ME 851 – Fall 2022
Homework # 7
Due Thursday, November 17, 2022

1. For the system $\dot{x} = Ax + Bu$, where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & -1 & -3 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

determine F so that the eigenvalues of $A + BF$ are at -1, -2 and $-1 \pm 2j$. Use the following methods to choose F.

   (a) Transform (A, B) into a controller form.

   (b) Take $u_2 = 0$ and transform (A, b_1) into a controller form.

   (c) Take $u_1 = 0$ and transform (A, b_2) into a controller form.

   (d) Assign the closed-loop eigenvectors such that the first component of the eigenvectors associated with $-1 \pm 2j$ is zero.

   (e) Assign the closed-loop eigenvectors such that the first component of the eigenvector associated with -1 is zero.

   (f) Let $F = g \cdot f$ where $g \in R^2$ and $f^T \in R^4$ are vectors to be selected. Choose g such that (A, Bg) is controllable. Then f can be determined by solving the eigenvalue placement problem for the single input system (A, Bg). (See Problem 9.2 of the textbook).

   (g) Use the "place" command of Matlab.

   At the end of each step, check the eigenvalues of $(A + BF)$. You can use MATLAB in all parts of this problem.

2. Consider the system

$$\dot{x} = Ax + Bu$$

   where

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

   (a) Find the feedback gain F to assign the closed-loop eigenvalues at $(-1 \pm j\sqrt{3})\omega$.

   (b) For $x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $u = Fx$, plot x_1, x_2 and u for $\omega = 1$ and $\omega = 10$. Comment on the results.

   (c) Using simulation, find the largest ω for which $|u(t)| \leq 3$.