① $IF = IF_5 = Z_5$

(a) Multiplicative inverse of 1, 2, 3, 4 of IF

$1 * 4 \% 5 = 1$

$2 * 3 \% 5 = 1$

For other values in IF, the multiplicative inverse doesn't lie in IF.

Thus,

4, 4

1, 1

2, 3

(b)

$5 \notin IF_5$

$IF_5 = Z_5 = \{0, 1, 2, 3, 4\}$

(a) $\begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix} + \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

(b) $\begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix} + \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$

(c) $4\begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$

(d) $3 \cdot \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ 9 \\ 12 \end{pmatrix}$

(c) $\begin{pmatrix} a \\ b \\ c \end{pmatrix} \in \mathbb{F}^3$

$$a\begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix} + b\begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix} + c\begin{pmatrix} 3 \\ 4 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{bmatrix} 0 & 3 & 1 \\ 4 & 0 & 3 \\ 3 & 4 & 4 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$