1. (Problem 3.1 in the book) Let X be a random variable uniformly distributed on $[0, 1]$. Define the random variable Y by

$$Y = \begin{cases} 1 & \text{when } x \le 1/3 \\ 2 & \text{when } x > 1/3 \; . \end{cases}$$

Find $\mathbb{E}(Y)$ first first deriving Y's pmf and then by the Expected Value Rule.

2. (Problem 3.2 in the book) Let X have pdf

$$f_X(x) = \frac{\lambda}{2} e^{-|\lambda| x} \quad \text{for all } \; x \in \mathbb{R} \; ,$$

where λ is a positive real number.

(a) Verify that $f_X(x)$ satisfies the normalization condition.
(b) Find $\mathbb{E}(x)$.
(c) Find $\mathrm{Var}(X)$.

3. (Problem 3.7 in the book) Legolas shoots an arrow at a circular target of radius r and is equally likely to hit any point on the target. Let X be the distance of Legolas's hit from the center.

(a) Find X's cdf $F_X(x)$.
(b) Find X's pdf, mean, and variance.
(c) The target has an inner ring of radius t. Legolas wins a prize of value $Y = 1/(X + 1)$ when $X \le t$ and wins nothing when $X > t$. Find the cdf of Y. Is Y a continuous random variable?

4. (Problem 3.8 in the book) Y and X are continuous random variables with respective pdfs $f_Y(y)$ and $f_Z(z)$. X is a random variable that equals Y with probability p and Z with probability $1 - p$.

(a) Find the cdf of X and from that derive the formula

$$f_X(x) = p f_y(x) + (1 - p) f_Z(x) \; .$$

(b) Find the cdf of the random variable X with pdf

$$f_X(x) = \begin{cases} p\lambda e^{\lambda x} & \text{when } x < 0 \\ (1 - p)\lambda e^{-\lambda x} & \text{when } x \ge 0 \; , \end{cases}$$

where $\lambda > 0$ and $p \in (0, 1)$.