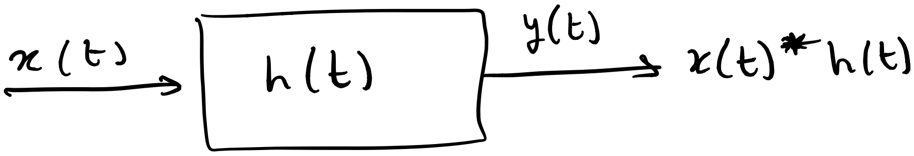
Small Review

ECE 2200:

$$x(t) = a\cos(w_c t + \pi/3)$$



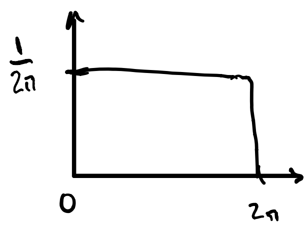
$\mathbf{X}$ $\quad$ $\mathbf{H}$ $\quad$ $\mathbf{Y} = \mathbf{X}(\omega)\mathbf{H}(\omega)$

ECE 3100

Random variables

$$\Theta \sim U(0, 2\pi]$$

pdf



} for continuous

Combined ???

$$X(t) = a\cos(w_c t + \Theta)$$

phase shift is randomly distributed

Review of Basic Probability

Probability Space

Example of Random Experiments

- R ① flip a fair coin twice
- R ② throw a die
- R ③ take ECE4110 until you pass
- R ④ throw a dart to unit disk

Possible OUTCOMES

R1: HT, HH, TH, TT

R2: 1, 2, 3, 4, 5, 6

R3: Pass, Fail

R4: $\{x^2+y^2<1 : -1<x<1, -1<y<1\}$

Events : a subset of outcomes

Probability Space

$$(\Omega, \mathcal{F}, Pr)$$

probability measure on Ω

Sample space

collection of events I want to assign probabilities to

level of resolution we observe our random experiment at

All events $\subset \Omega$

Ω

$\mathcal{F}$ example: For R2, even or odd?

$$\mathcal{F} = \{\emptyset, \Omega, \{1,3,5\}, \{2,4,6\}\}$$

Axioms

$\boxed{\mathcal{F}}$

(i) $\Omega \in \mathcal{F}$

(ii) If $A \in \mathcal{F}$, $A^c \in \mathcal{F}$

(iii) If $A_i \in \mathcal{F}$, $\bigcup_i A_i \in \mathcal{F}$

countable intersections also there!

$A \cap B = (A^c \cup B^c)^c$!

Only require a countable union to belong to $\mathcal{F}$

$\boxed{Pr}$ $\mathcal{F} \longrightarrow [0,1]$

(i) $0 \leq Pr(A) \leq 1$

if one event occurs the other events do not

(ii) $Pr(\Omega) = 1$

(iii) If $A_1, A_2, \ldots,$ is a sequence of mutually exclusive events then

$$Pr\left(\bigcup_i A_i\right) = \sum_i Pr(A_i)$$

Properties that Follow

(i) $Pr(A^c) = 1 - Pr(A)$

Proof: $Pr(A \cup A^c) = Pr(A) + Pr(A^c)$

$Pr(\Omega) = 1 = Pr(A) + Pr(A^c)$

$Pr(A^c) = 1 - Pr(A)$

(ii) $Pr(\phi) = 0$, $\phi = \Omega^c$

(iii) If $A \subseteq B$ then $Pr(A) \leq Pr(B)$

Proof: $B = A \cup (A^c \cap B)$

$Pr(B) = Pr(A) + Pr(A^c \cap B) \geq Pr(A)$

(iv) $Pr(A \cup B) = Pr(A) - Pr(B) - Pr(A \cap B)$

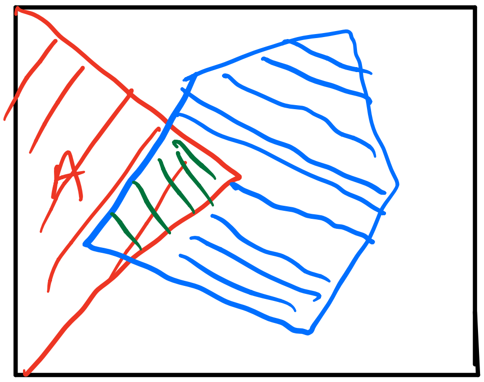
Union Bound

$$Pr\left(\bigcup_i A_i\right) \leq \sum_{i=1}^{n} Pr(A_i)$$

Conditional Probability

If A and B are two events, $Pr(B) \neq 0$, then

$$Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)}$$



"B becomes the new universe, $B = \Omega$"

Paradox of Two Children

I have two children.

(i) one of them is a boy

(ii) one of them is a boy who was born on Wednesday

$Pr($both are boys$)$?

(i) $$Pr(BB \mid (B,B),(B,G)) = \frac{Pr(BB)}{Pr(BB, BG)} = \frac{Pr(BB)}{Pr(BB) + Pr(BG)} = \frac{1/4}{\frac{1}{4} + \frac{2}{4}} = 1/3$$

(ii) B^* = boy born Wednesday, B = boy born NOT on Wednesday

$$Pr(B^*B^* \cup B^*B \mid B^*B^*, B^*B, B^*G) = \frac{Pr(B^*B^*) + Pr(B^*B)}{Pr(B^*B^*) + Pr(B^*B) + Pr(B^*G)}$$

$$= \frac{\frac{1}{14}\cdot\frac{1}{14} \cdot 2\frac{1}{14}\cdot\frac{6}{14}}{\left(\frac{1}{14}\right)^2 + \frac{12}{14^2} + 2\left(\frac{1}{14}\cdot\frac{1}{2}\right)}$$

$$= \frac{13}{27}$$

WOAH