Logical Structure of Complex Analysis

Analytic function — $f'(z)$ exists independent of the direction "$\Delta z \to 0$"

Harmonic Functions
(Riemann)

Cauchy-Riemann Equations

$\downarrow$

u, v satisfy Laplace's equation

$$\left.\begin{aligned}\nabla^2 u &= 0\\ \nabla^2 v &= 0\end{aligned}\right\} \Rightarrow \text{harmonic}$$

describe steady state phenomena

$\downarrow$

Conformal Mapping

$\rightarrow$ Riemann Surfaces $\rightarrow$ Topology | Geometry

$\rightarrow$ Mathematical Physics

Integration

if analytic

$$\int_\gamma f(z)\, dz$$

$\downarrow$

done by parametrizing γ w/ real variable

$\downarrow$

$$\oint_\gamma f(z)\, dz = 0 \quad \text{if}$$

f has a single-valued anti-derivative

from fund thm of calculus (real vars)

$\downarrow$

$$\oint_\gamma (z-a)^n\, dz = \begin{cases} 0, & n \neq -1 \\ 2\pi i, & n = -1 \end{cases}$$

Cauchy's Theorem

$$\oint_\gamma f(z)\,dz = 0, \text{ for } f_{\text{analytic}}$$

f can be represented near a in this form

$$f(z) = f(a) + f'(a)(z-a) + \varepsilon(z)(z-a)$$

where $\varepsilon(z) \to 0$ as $z \to a$

Cauchy's Integral Formula

$$f(a) = \frac{1}{2\pi i}\oint_\gamma \frac{f(a)}{z-a}\,dz$$

MVT for analytic functions
Liousville's Theorem
Fundamental Theorem of Algebra

Taylor/ Laurent Series

Contour Integrals

Residue Theory, ML Estimate

Winding Numbers,
Argument Principle,
Rouche's Theorem

Control Theory

asymptotic series, and expansions / perturbation methods

Special functions { Bessel, Gamma, etc