1. (Problem 3.18 in the book) Let X and Y be independent Gaussian random variables with respective means μ_x and μ_y and respective variances σ_x and σ_y. Find the joint pdf $p_{X,Y}(x, y)$ and describe its geometric form.

2. A defective coin-minting machine produces coins whose probability of heads is a random variable P with pdf

$$f_P(p) = \begin{cases} pe^p & \text{when } p \in [0, 1] \\ 0 & \text{otherwise.} \end{cases}$$

Sam takes a coin produced by the machine and tosses it repeatedly and independently.

(a) Find the probability that Sam tosses a head on any given toss.
(b) Find the conditional pdf of P given that Sam tosses a head on his first toss.
(c) Find the conditional probability that Sam tosses a head on his second toss given that he tossed a head on his first toss.

3. (Problem 3.35 in the book) Let X and Y be independent continuous random variables with respective pdfs $f_X(x)$ and $f_Y(y)$ and let $Z = X + Y$.

(a) Show that $f_{Z|X}(z \mid x) = f_Y(z - x)$.
(b) Find the conditional pdf of X given that $Z = z$ when X and Y are both exponentially distributed with rate parameter λ.
(c) Repeat part (b) when X and Y are Gaussian with zero means and respective variances σ_x^2 and σ_y^2.

4. (Problem 4.14 in the book) We model the lifetimes of two light bulbs as independent exponential random variables X and Y with respective rate parameters λ and ζ. The time at which the first light-bulb burnout occurs is

$$Z = \min\{X, Y\} .$$

Show that Z is exponential with rate parameter $\lambda + \zeta$.