(1)

$$\sin z = \cos z$$

$$\frac{e^{iz} - e^{-iz}}{\cancel{2}i} = \frac{e^{iz} + e^{-iz}}{\cancel{2}}$$

$$-i(e^{iz} - e^{-iz}) = e^{iz} + e^{-iz}$$

$$i(e^{-iz} - e^{iz}) = e^{iz} + e^{-iz}$$

$$e^{iz} + ie^{iz} = ie^{-iz} - e^{-iz}$$

$$e^{iz}(1+i) = e^{-iz}(i-1)$$

$$i - i^2 - 1 + i$$

$$\frac{e^{iz}}{e^{-iz}} = \frac{i-1}{(1+i)}\frac{(1-i)}{(1-i)} = \frac{i+1-1+i}{2}$$

$$e^{i2z} = i = e^{i\left(\frac{\pi}{2} + 2\pi k\right)}$$

$$i2z = \frac{\pi}{2}i + 2\pi i k, \quad k \in \mathbb{Z}$$

$$z = \frac{\pi}{4} + \pi k, \quad k \in \mathbb{Z}$$

(2) $\phi(z) = A \ln|z| + B$ $\qquad \nabla^2 \phi = 0$ for $|z| > 3$

$0 = A \ln(3) + B$

$B = -A \ln(3)$

$$\phi(z) = A\left(\ln|z| - \ln(3)\right)$$

$$= A\left(\ln\left(\sqrt{x^2+y^2}\right) - \ln(3)\right)$$

③ $$\int_{|z|=4} \frac{z}{(z+5)(z-1)}\,dz$$

$$f(z) = z/z+5$$

$$\int_{|z|=4} \frac{f(z)}{z-1}\,dz = f(1)\,2\pi i$$

$$= \frac{2\pi i}{6} = \frac{\pi i}{3}$$

4)

$$\sum_{k=0}^{\infty} k z^k$$

HMM

$$\frac{d}{dz}\left(z^k\right) = k z^{k-1}$$

$$z \frac{d}{dz}\left[z^k\right] = k z^k$$

Realize $\frac{1}{1-z} = \sum_{k=0}^{\infty} z^k$

$$z \frac{d}{dz}\left[\frac{1}{1-z}\right] = \boxed{\frac{z}{(1-z)^2}}$$

So,

$$\frac{z}{(1-z)^2} = \sum_{k=0}^{\infty} k z^k$$

5)

D: $|z| < 1$ (open unit-disk)

1) f is analytic IN D
2) f is continuous ON D and its boundary
3) $f(0) = i$
4) $|f(z)| \leq 1 \quad \forall z \in D$

All f that satisfy this condition?

$f(z) = i$ ← analytic in D

continuos on D

$f(0) = i$

$|f(z)| = 1 \quad \forall z \in D$

$f(z) = ie^{iz}$ $|z| < 1$

analytic in D

continuos on D

$f(0) = i$

$|f(z)| = 1 \quad \forall z \in D$