

①  $(1+i)^{10}$  in polar form as well as in "a+bi" form  $\rightarrow (\sqrt{2})^{10} e^{i\frac{5\pi}{2}} \rightarrow (\sqrt{2})^{10} i$

②  $\oint_{|z|=1} z^2 \sin\left(\frac{1}{z}\right) dz \rightarrow \left\{ \frac{-i\pi}{3} \right\} a_{-1} 2\pi i$

③ a. First two terms in the Laurent Series of

$\frac{1}{\cos(z)-1}$  where  $z = \varepsilon \ll 1$ ,  $|z| > 0$

$$-\frac{2}{z^2} - \frac{1}{6}$$

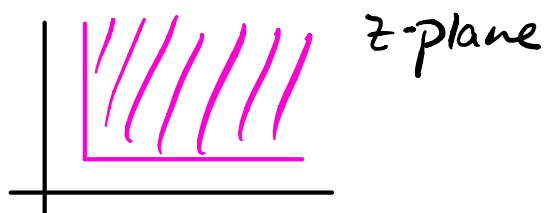
b. How large can our outer annulus radius  $R$  be such that the series is still valid?

$$0 < |z| < R$$

$R = 2\pi$   $\swarrow$  Next Singularity!

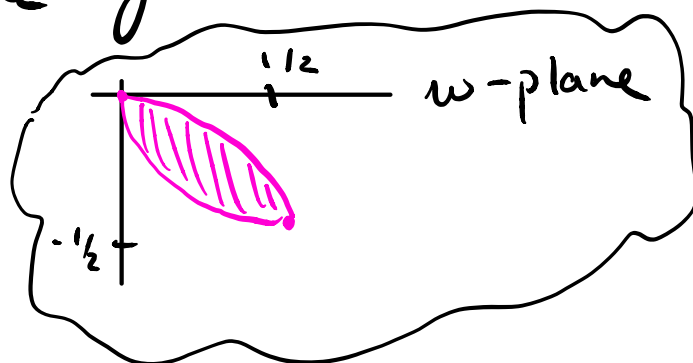
$$e \approx 2.718$$

$$(4) \quad z \begin{cases} 1 < x < \infty \\ 1 < y < \infty \end{cases}$$



Sketch mapping in  $w$ -plane of

$$w = \frac{1}{z}$$



(5) Some Rouché's Theorem question, straightforward application

$$(6) \quad \text{Evaluate } \int_0^{\infty} \frac{x^{\alpha-1}}{x+1} dx$$

What are the limitations on  $\alpha$  such that the integral is finite.

$$\frac{\pi}{\sin(\pi\alpha)}, \quad 0 < \alpha < 1$$