

# Logical Structure of Complex Analysis

**Analytic function** —  $f'(z)$  exists independent of the direction  
" $\Delta z \rightarrow 0$ "

Harmonic Functions  
(Riemann)

Cauchy-Riemann Equations

$u, v$  satisfy Laplace's equation

$$\nabla^2 u = 0$$

$$\nabla^2 v = 0$$

} harmonic

describe steady state phenomena

Conformal Mapping

→ Riemann Surfaces

→ Topology / Geometry

→ Mathematical Physics

Integration

$$\int_{\gamma} f(z) dz$$

done by parametrizing  
 $\gamma$  w/ real variable

$$\oint_{\gamma} f(z) dz = 0 \text{ if}$$

$f$  has a single-valued  
anti-derivative

from fund thm of calculus  
(real vars)

$$\oint_{\gamma} (z-a)^n dz = \begin{cases} 0, n \neq -1 \\ 2\pi i, n = -1 \end{cases}$$

# Cauchy's Theorem

$$\oint_{\gamma} f(z) dz = 0, \text{ for } f \text{ analytic}$$

$f$  can be represented near  $a$  in this form

$$f(z) = f(a) + f'(a)(z-a) + \varepsilon(z)(z-a)$$

where  $\varepsilon(z) \rightarrow 0$  as  $z \rightarrow a$

Cauchy's Integral Formula

$$f(a) = \frac{1}{2\pi i} \oint_{\gamma} \frac{f(z)}{z-a} dz$$

use for analytic functions  
Liouville's Theorem

Fundamental Theorem of Algebra

Taylor/Laurent Series

Contour Integrals

→ Residue Theory, MZ Estimate



Winding Numbers,  
Argument Principle,  
Rouche's Theorem



Control  
Theory

asymptotic series,  
and expansions / perturbation  
methods



Special functions { Bessel, Gamma,  
etc