$T \in \mathcal{L}(\mathbb{R}^3, \mathbb{R}^3)$

$$T\left(\underbrace{\begin{bmatrix}1\\0\\-1\end{bmatrix}}_{b_1}\right) = \begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$T\left(\underbrace{\begin{bmatrix}0\\1\\-1\end{bmatrix}}_{b_2}\right) = \begin{bmatrix}1\\1\\0\end{bmatrix}$$

$$T\left(\underbrace{\begin{bmatrix}0\\0\\1\end{bmatrix}}_{b_3}\right) = \begin{bmatrix}1\\0\\0\end{bmatrix}$$

$b_1 = e_1 - e_3$

$b_2 = e_2 - e_3$

$b_3 = e_3$

$e_1 = b_1 + e_3 b_3$

$e_2 = b_2 + e_3 b_3$

$e_3 = b_3$

$T(e_1) = T(b_1) + T(b_3)$

$T(e_2) = T(b_2) + T(b_3)$

$T(e_3) = T(b_3)$

$S = e_1, e_2, e_3$

$[T]_{S \leftarrow S}$?

$$T(\vec{b}_1, \vec{b}_2, \vec{b}_3) = \begin{bmatrix}1 & 1 & 1\\1 & 1 & 0\\1 & 0 & 0\end{bmatrix}$$

$$T\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right) = \begin{bmatrix}2\\1\\1\end{bmatrix}$$

$$T\left(\begin{bmatrix}0\\1\\0\end{bmatrix}\right) = \begin{bmatrix}2\\1\\0\end{bmatrix}$$

$$T\left(\begin{bmatrix}0\\0\\1\end{bmatrix}\right) = \begin{bmatrix}1\\0\\0\end{bmatrix}$$

$$[T]_{S \leftarrow S} = \begin{bmatrix}2 & 2 & 1\\1 & 1 & 0\\1 & 0 & 0\end{bmatrix}$$

$$T \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$