"Network Sensor of Human Beings"

A random variable assigns to each outcome $\omega \in \Omega$ of a probabilistic experiment, a real number $X(\omega) \in \mathbb{R}$.

By convention $Pr(X(\omega) = \infty) = Pr(X(\omega) = -\infty) = 0$

Characterization of a Random Variable

① Cumulative Distribution Function

Let $F_X(x)$ denote the cumulative distribution function

$$F_X(x) = Pr(X(\omega) \leq x) \quad \forall x \in \mathbb{R}$$

(i) $F_X(-\infty) = 0, \quad F_X(\infty) = 1$

(ii) $Pr(X \in (a, b]) = Pr(a < X \leq b) = F_X(b) - F_X(a)$

(iii) $F_X(x)$ is non-decreasing in x

② Probability Mass Function

For a discrete-valued rv $X(\omega) \in \{x_1, \ldots, x_M\}$

$$f_X(x_i) = Pr(X = x_i), \quad i = 1, \ldots, M$$

$$\sum_{i=1}^{M} f_X(x_i) = 1$$

(3) Probability Density Function

For continuous valued rv $X(\omega) \in \mathbb{R}$. If $F_X(x)$ is differentiable

$$f_X(x) = \frac{dF_X(x)}{dx} \quad \text{equivalently} \quad F_X(x) = \int_{-\infty}^{x} f_X(s)\,ds$$

Remarks

(i) $F_X(-\infty) = 0$, $F_X(\infty) = 1$

(ii) $Pr(X \in (a,b]) = Pr(a < X \leq b) = F_X(b) - F_X(a)$

(iii) $F_X(x)$ is non-decreasing in x

(iv) For continuous-valued rv $Pr(X = a) = 0 \quad \forall a \in \mathbb{R}$

Statistics of Random Variables

1. Expected Value

$$\mathbb{E}[X] = \int_{-\infty}^{+\infty} f_X(x) \cdot x \, dx$$

Recall: $\mathbb{E}[X]$ is deterministic

Properties

(i) For any function g

$$\mathbb{E}[g(X)] = \int_{-\infty}^{+\infty} g(x) f_X(x)\, dx$$

(ii) $\mathbb{E}[aX + bY] = a\mathbb{E}[X] + b\,\mathbb{E}[Y]$

Note: in general $\mathbb{E}[g(x)] \neq g(\mathbb{E}[x])$

ONLY true if g is linear

2. Variance

If $X \in \mathbb{R}$ is a rv with $\mathbb{E}[X] = \mu$ then

$$\text{var}(X) = \mathbb{E}[(X-\mu)^2] = \left.\int_{-\infty}^{+\infty} (X-\mu)^2 f_X(x)\, dx \right\} \text{always} \geq 0$$

Standard Deviation $= \sqrt{\text{var}(X)}$