

1. (Problem 3.13 in the book) We model a city's temperature as a Gaussian random variable with mean and standard deviation both equal to 10 degrees Celsius. What is the probability that the temperature at a randomly chosen time will be less than or equal to 59 degrees Fahrenheit?

2. (Problem 3.15 in the book) The coordinates  $(X, Y)$  of a point are distributed uniformly over the semicircle

$$\{(x, y) : x^2 + y^2 \leq r^2 \text{ and } y \geq 0\} ,$$

where  $r > 0$  is given.

- (a) Find the joint pdf  $f_{X,Y}(x, y)$ .
- (b) Find the marginal pdf  $f_Y(y)$  and use it to find  $\mathbb{E}(Y)$ .
- (c) Check your answer for  $\mathbb{E}(Y)$  by computing it directly without using the marginal pdf of  $Y$ .

3. (Problem 3.18 in the book) Let  $X$  be a random variable with pdf

$$f_X(x) = \begin{cases} x/4 & \text{when } 1 \leq x \leq 3 \\ 0 & \text{otherwise,} \end{cases}$$

and let  $A$  be the event  $\{X \geq 2\}$ .

- (a) Find  $\mathbb{E}(X)$ ,  $\mathbb{P}(A)$ ,  $f_{X|A}(x)$ , and  $\mathbb{E}(X | A)$ .
- (b) Find  $\mathbb{E}(Y)$  and  $\text{Var}(Y)$  when  $Y = X^2$ .

4. We start with a stick of length  $h$ . We break it at a point chosen uniformly and keep the piece, of length  $Y$ , that contains the left end of the stick. We repeat the process on the piece we're left with. Let  $X$  be the length of the piece containing the left end after the second break.

- (a) Find the joint pdf of  $X$  and  $Y$ .
- (b) Find the marginal pdf of  $X$ .
- (c) Use the marginal pdf of  $X$  to evaluate  $\mathbb{E}(X)$ .
- (d) Evaluate  $\mathbb{E}(X)$  by exploiting the relation  $X = Y(X/Y)$ .