(3) $I = \langle x^2+1 \rangle \subset \mathbb{R}[x]$ is an ideal, defined to be the subspace $\{p(x)(x^2+1) \mid p(x) \in \mathbb{R}[x]\}$ of all polynomial multiples of x^2+1.

(a)

$$\frac{h(x)}{f(x)} = q(x) + \frac{r(x)}{f(x)} \qquad \deg r < \deg f$$

(b) $R_f : \mathbb{R}[x] \to \mathbb{R}[x]$

$$\frac{h(x)}{f(x)} \mapsto r(x)$$

$$R_f(a h_1(x) + b h_2(x)) = a r_1(x) + b r_2(x)$$
$$= a R_f(h_1(x)) + b R_f(h_2(x))$$

(c) Fix $f(x) = x^2+1$

Find ker, im of R_f.

$$\ker R_f = \left\{ \frac{h(x)}{f(x)} \;\middle|\; R_f\left(\frac{h(x)}{f(x)}\right) = 0 \right\}$$

$$= I \qquad \frac{p(x)\, x^2+1}{x^2+1} = p(x) + \frac{\overset{r(x)}{0}}{}$$

$$\operatorname{im} R_f = \{ p(x) \mid p(x) \neq a(x^2+1) \ \forall a \in \mathbb{R} \}$$

7.

$\mathbb{R}[x]/I$ $\qquad R_f : \mathbb{R}[x] \to \mathbb{R}[x]$

$\ker R_f$ $\qquad \tilde{R}_f : \mathbb{R}[x]/I \to$

$v \in \operatorname{im}(T)$

$Tw = v$ for some $w \in V$

$2Tw = 2v$

$T^2 w = Tv$ $\qquad \boxed{Tv = 2v}$