1. (Problem 2.38 in the book) Alexa passes through four traffic lights on the way to work, and each light is equally likely to be green or red, independent of the others.

(a) Find the pmf, the mean, and the variance of the number of red lights Alexa encounters.

(b) Suppose each red light delays Alexa by exactly two minutes. What is the variance of her commuting time?

2. (Problem 2.44 in the book) Let X and Y be discrete random variables with joint pmf $p_{X,Y}(x, y)$ and let g and h be functions respectively of X and Y. Show that if X and Y are independent, then so are $g(X)$ and $h(Y)$.

3. (A classic due to Thomas Cover) Exactly one of six bottles of wine has gone bad and therefore tastes terrible. Ler p_i be the probability that Bottle i is bad. It turns out that p_1, p_2, $\ldots$, p_6 are respectively 8/23, 6/23, 4/23, 2/23, 2/23, and 1/23. You want to determine the bad wine by tasting. Suppose you taste the wines one at a time. Choose the order of tastings to minimize the expected number of tastings required to determine the bad bottle. Remember, if the first five wines pass the test, you need not taste the last.

(a) What is the expected number of tastings required?

(b) Which bottle should you taste first?

Now you get smart. For the first sample, you mix some of the wines in a fresh glass and sample the mixture. You repeat this process, mixing and tasting, stopping once you've determined the bad bottle.

(c) What is the minimum expected number of tastings required to determine the bad wine?

(d) What mixture should you taste first?

4. Let X be a continuous random variable defined on a sample space Ω equipped with a probability law $\mathbb{P}$. Let $f_X(x)$ be the probability density function of X. Show that the following statement can't hold: there exists a constant α such that for every interval $[a, b] \subset \mathbb{R}$

$$\mathbb{P}(\{X \in [a, b]\}) = \alpha(b - a) .$$