1. All events named or described in this problem are subsets of a given sample space Ω. Express each described event in terms of unions, intersections, and complements with respect to Ω of the named events. Draw a Venn diagram covering each case.

(a) At least two of events A, B, and C occur.
(b) Events A and C occur, but neither B nor D does.
(c) Exactly one of the events A, B, or C occurs.
(d) Either event A occurs or, if not, event B also does not occur.
(e) At most one of the events A, B, or C occurs.

2. A spinner such as the kind used in games like Twister is colored as in the accompanying diagram. In terms of the labels around the outside, red covers the wedge $0 \leq \theta < \pi/2$, green the wedge $\pi/2 \leq \theta < \pi$, yellow the wedge $\pi \leq \theta < 3\pi/2$, and blue the wedge $3\pi/2 \leq \theta < 2\pi$. In a certain game of chance, what's relevant about the spinner is what color it lands on after being spun. Assume that the spinner is equally likely to land pointing in any direction $0 \leq \theta < 2\pi$.

(a) Construct a finite sample space Ω along with probability law $\mathbb{P}$ on Ω that suffice to model this situation. In terms of your Ω and $\mathbb{P}$, what is the event "spinner lands on blue"? What is the event B that "spinner lands on neither yellow nor red" and what is $\mathbb{P}(B)$?
(b) Repeat part (a) with the word "finite" replaced by "infinite."

3. Given that 23% of Cornell engineering students like Radiohead, 29% like Ed Sheeran, and 61% like neither, what is the probability that a Cornell engineering student drawn at random likes

(a) both Radiohead and Ed Sheeran?
(b) either Radiohead or Ed Sheeran?

4. Let A and B be disjoint events in a sample space Ω, and let $\mathbb{P}$ be a probability law on Ω satisfying $\mathbb{P}(A) = 0.35$ and $\mathbb{P}(B) = 0.55$. Find the probability that

(a) both A and B occur.
(b) A doesn't occur but B does.
(c) neither A nor B occurs.

5. This problem alludes to conditional probabilities. Although we haven't covered those in section yet, all you need to know here is that for any events A and B with $\mathbb{P}(B) > 0$,

$$\mathbb{P}(A \mid B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)} \ .$$

When rolling a twelve-sided die with faces $k = 0, 1, 2, \ldots, 11$, the probability that the die lands with face k on the bottom turns out to equal $\alpha(k+1)$ for some number α.

(a) Find α. You might want to use the identity $\sum_{m=1}^{n}(m) = n(n+1)/2$.
(b) Let B be the event that an odd number k lands on the bottom, and let A be the event that 3, 4, or 7 lands on the bottom. Find $\mathbb{P}(A \mid B)$.
(c) With A and B as in part (c), find $\mathbb{P}(B \mid A)$.

6. A box contains three crayons, one of each color cyan, magenta, and carnelian. An experiment consists of drawing one crayon from the box, returning it to the box, and drawing a second crayon. The outcome of the experiment is the result of the two draws. Assume that when drawing a crayon from the box, any crayon in the box is equally likely to be drawn.

(a) Construct a sample space Ω and probability law $\mathbb{P}$ that suffice to model this experiment. For each $s \in \Omega$, find $\mathbb{P}(\{s\})$.

(b) Change the experiment as follows: return the crayon from the first draw to the box if and only if it's cyan. Repeat part (a) for this new experiment.

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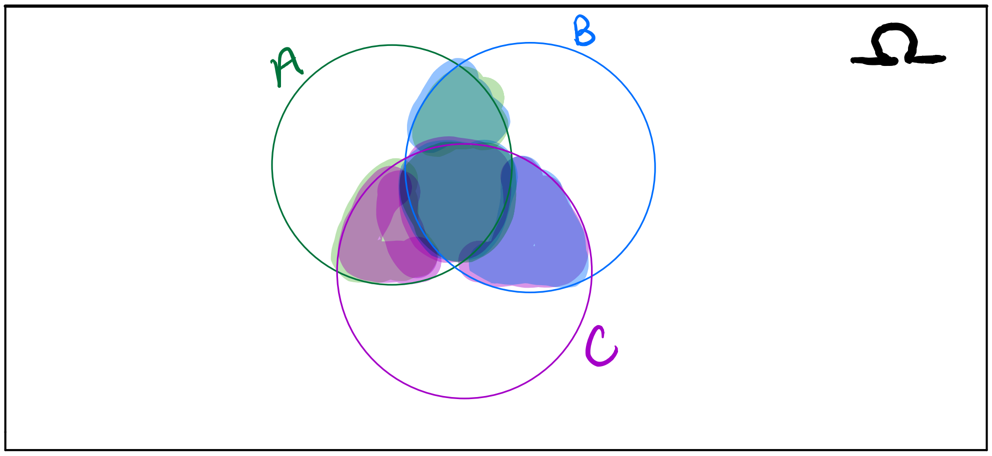
HW1 ECE3100

2/6/19

① All events $\subset \Omega$

(a) At least two of A, B, C occurs?

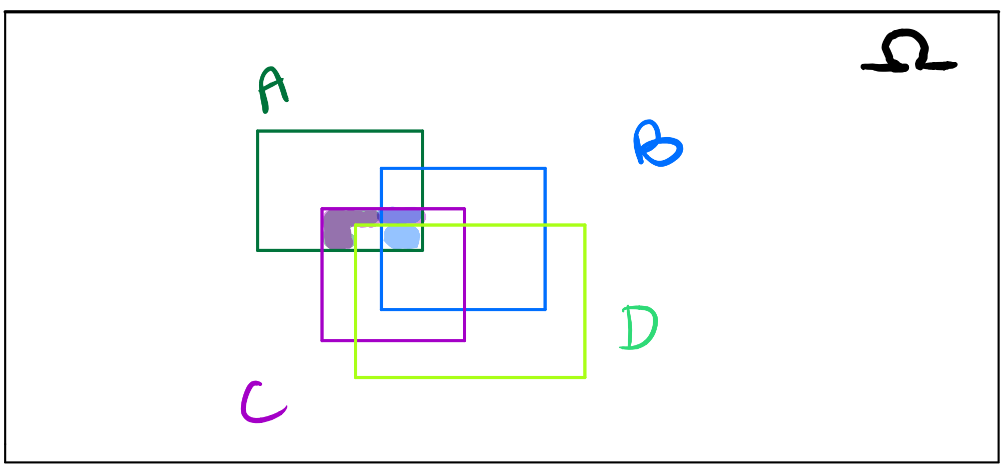
At least two occur $\Rightarrow (A \cap B) \cup (A \cap C) \cup (B \cap C) \cup (A \cap B \cap C)$



(b) Events A, C occur, but neither B nor D does.

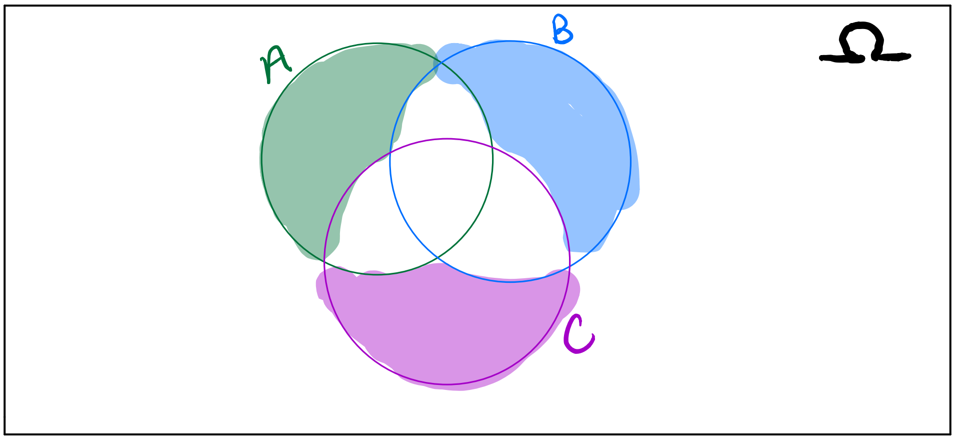
A occurs, C occurs $\Rightarrow (A \cap C)$ occurs

So, $(A \cap C) \cap (B^c \cap D^c)$



(c) Exactly one of the events A or B or C occurs.

$$[A\cap(B'\cap C')]\cup[B\cap(A'\cap C')]\cup[C\cap(A'\cap B')]$$



(d) Either event A occurs, or, if not, B also does not occur.

$$A \cup (A^c \cap B^c)$$

A occurs

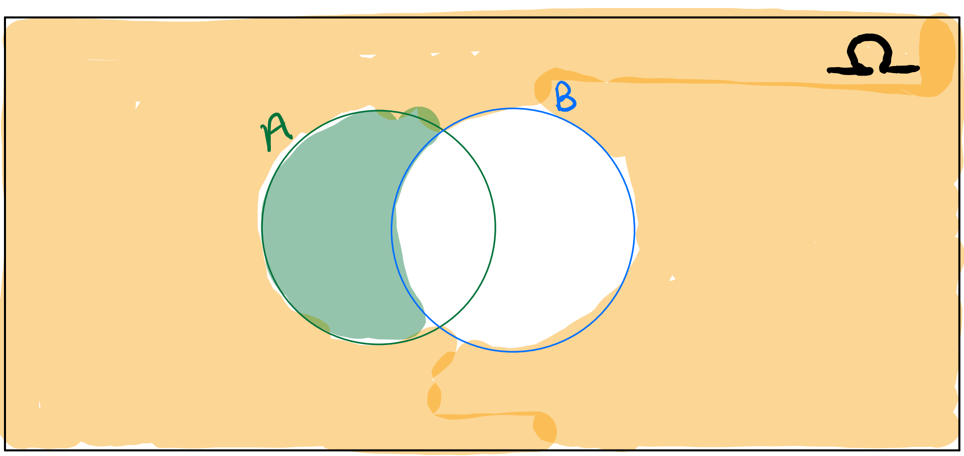
or

and

A does NOT

B does NOT occur

occur



(e) **At most** one of the events A, B, C occurs.

$$[A \cap (B' \cap C')] \cup [B \cap (A' \cap C')] \cup [C \cap (A' \cap B')] \cup [A' \cap B' \cap C']$$

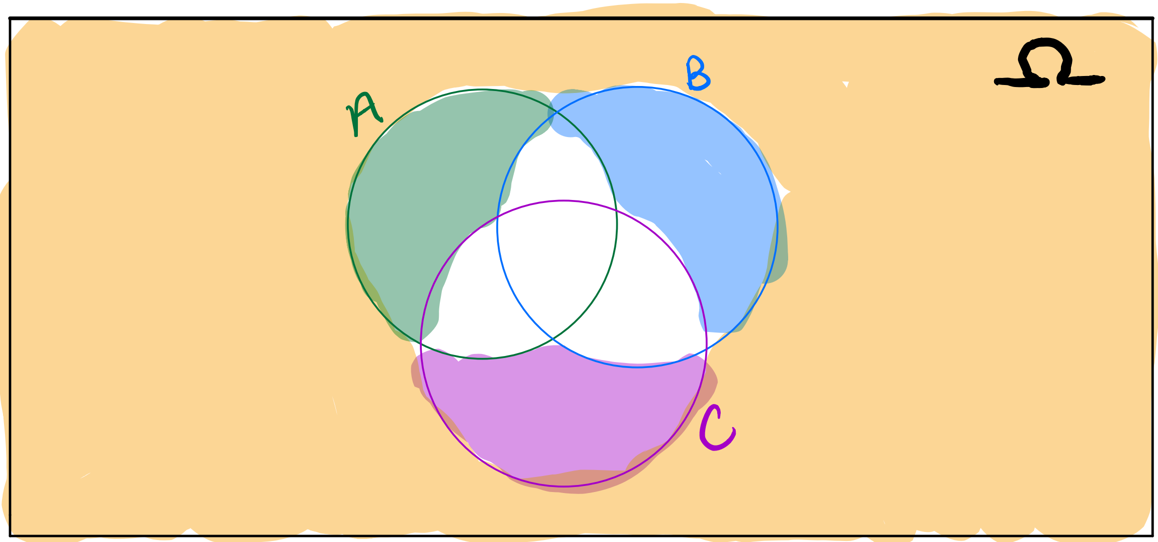
or — or — or

A occurs. B, C do NOT occur

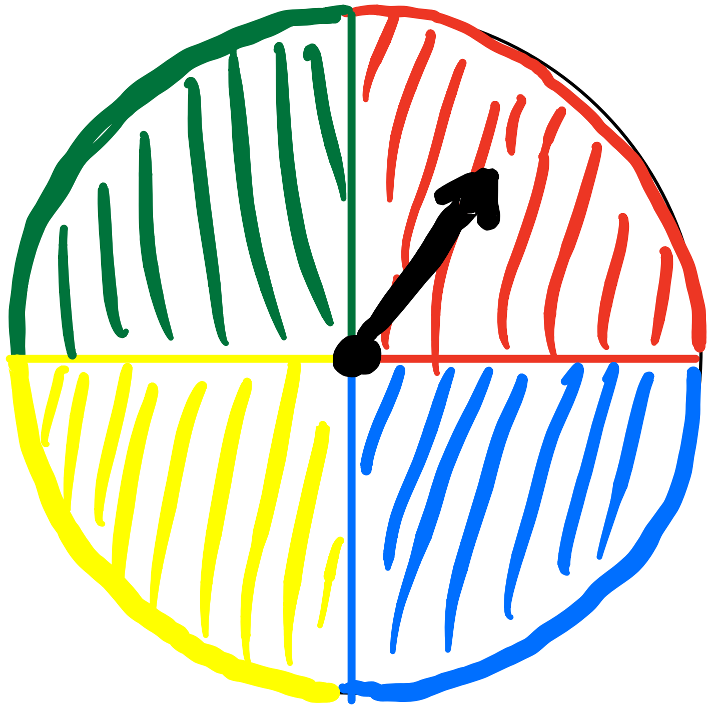
B occurs, A, C do NOT occur

C occurs. A, B do NOT occur

NONE of them occurs!



②



Spinner is equally likely to land pointing in any direction $0 \le \theta < 2\pi$.

a) Let

$$\Omega = \{\text{red}, \text{green}, \text{yellow}, \text{blue}\}$$

$\mathbb{P}$("spinner lands on any given color") = $\frac{1}{4}$

$\mathbb{P}$("spinner lands on blue) = $\frac{1}{4}$

B = "spinner lands on neither yellow nor red"

B^c = "spinner lands on green or blue"

$\mathbb{P}(B^c) = \frac{1}{2} \Rightarrow \boxed{\mathbb{P}(B) = 1 - \frac{1}{2} = \frac{1}{2}}$

b) Let

infinite θ values in $[0, 2\pi)$

$$\Omega = \{\theta : 0 \le \theta < 2\pi\}$$

$$\mathbb{P}(\text{"spinner lands on any color"}) = \frac{\text{Range each colors } \theta \text{ values occupy}}{\text{Total range } \theta \text{ values}}$$

$$\mathbb{P}(\text{"spinner lands on blue"}) = \frac{\frac{\pi}{2} - 0}{2\pi} = \frac{\pi}{4\pi} = \frac{1}{4}$$

B = "spinner lands on neither yellow nor red"

B^c = "spinner lands on green or blue"

$$\mathbb{P}(B^c) = \frac{\overbrace{\left(\pi - \frac{\pi}{2}\right)}^{\text{Range of green}} + \overbrace{\left(2\pi - \frac{3\pi}{2}\right)}^{\text{Range of blue}}}{2\pi} = \frac{\pi}{2\pi} = \frac{1}{2}$$

$$\mathbb{P}(B) = 1 - \mathbb{P}(B^c) = \frac{1}{2}$$

3. • 23% of Cornell engineering students like radiohead.
   • 29% like Ed Sheeran
   • 61% like neither.

Cornell student drawn at random. - Say a male

a) Probability he likes both radiohead and Ed Sheeran?

Let A = event student likes Radio head

B = event student likes Ed Sheeran

Want

$$\mathbb{P}(A \cap B)$$

Know that

$$\mathbb{P}(A) = 0.23$$

$$\mathbb{P}(B) = 0.29$$

$$\mathbb{P}(A^c \cap B^c) = 0.61$$

$$\mathbb{P}(A \cap B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cup B)$$

Note:

$$\mathbb{P}(A \cup B) = \mathbb{P}((A^c \cap B^c)^c) = 1 - \mathbb{P}((A^c \cap B^c))$$

$$= 1 - 0.61$$

$$= 0.39$$

So,

$$\boxed{\begin{aligned}\mathbb{P}(A \cap B) &= 0.23 + 0.29 - 0.39 \\ &= 0.13\end{aligned}}$$

(b)

$$\boxed{\begin{aligned}\mathbb{P}(A \cup B) &= \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B) \\ &= 0.23 + 0.29 - 0.13 \\ &= 0.39\end{aligned}}$$

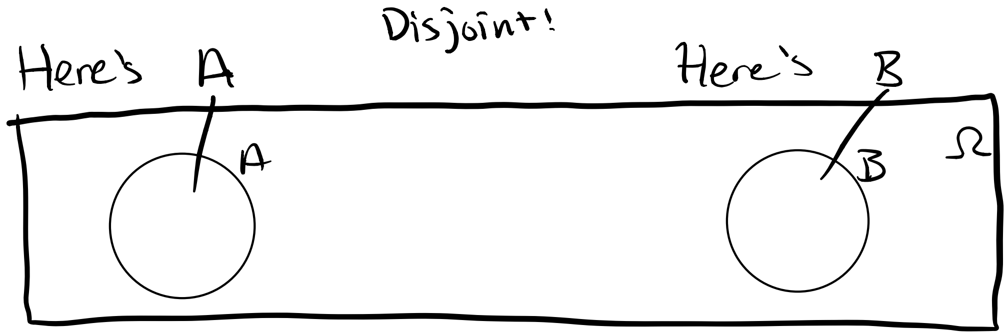
④ A, B are **disjoint** events in sample space Ω. Let $\mathbb{P}$ be a probability law on Ω satisfying

$$\mathbb{P}(A) = 0.35$$
$$\mathbb{P}(B) = 0.55$$

Probability that

(a) both A, B occur?

$\mathbb{P}(A \cap B) = 0$! Why?



(b) A doesn't occur but B does

$$\mathbb{P}(\underbrace{A^c}_{\text{everything outside } A} \cap \underbrace{B}_{B}) = \underbrace{\mathbb{P}(B)}_{\text{see diagram above}} = 0.55$$

(c) Neither A NOR B occurs.

$$\mathbb{P}(\underbrace{A^c \cap B^c}_{\text{everything outside A AND B.}}) = \mathbb{P}((A \cup B)^c) = 1 - \mathbb{P}(A \cup B) = 1 - \boxed{\mathbb{P}(A) + \mathbb{P}(B)}$$
$$\boxed{= 1 - 0.9 = 0.1}$$

⑤ For any events A & B w/ $\mathbb{P}(B)>0$,

$$\mathbb{P}(A|B)=\frac{\mathbb{P}(A\cap B)}{\mathbb{P}(B)}$$

When rolling a 12-sided die with faces $k=0,1,2,\ldots,11$, the probability that the die lands on face k on the bottom turns out to equal

$$\alpha(k+1)$$

for some number α.

(a) Find α.

If each face k has probability $\alpha(k+1)$ Then

$$\sum_k \alpha(k+1) = 1$$

$$\alpha = \frac{1}{\sum_{k=0}^{11}(k+1)} = \frac{1}{\sum_{k=1}^{12} k} = \frac{1}{\frac{12}{2}(13)} = \frac{1}{78}$$

So $\mathbb{P}(\text{land on face } k) = \frac{1}{78}(k+1)$

(b) B is the event an odd number k lands on bottom.

A is the event 3, 4, or 7 lands on the bottom.

Find $\mathbb{P}(A|B)$

$$\Omega = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$B = \{1, 3, 5, 7, 9, 11\}$$

$$A = \{3, 4, 7\}$$

A and B share two possibilities

$$\mathbb{P}(k=3) = \frac{4}{78}, \quad \mathbb{P}(k=7) = \frac{8}{78}, \quad \mathbb{P}(k=4) = \frac{5}{78}$$

$$\mathbb{P}(k = \text{odd}) = \frac{2}{78} + \frac{4}{78} + \frac{6}{78} + \frac{8}{78} + \frac{10}{78} + \frac{12}{78} = \frac{42}{78}$$

$$\mathbb{P}(A|B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)} = \frac{\mathbb{P}(k=3) + \mathbb{P}(k=7)}{\mathbb{P}(k = \text{odd})} = \frac{12/78}{42/78} = \boxed{\frac{6}{21}}$$

(c) $$\mathbb{P}(B|A) = \frac{\mathbb{P}(k=3) + \mathbb{P}(k=7)}{\mathbb{P}(k = 3, 4, \text{or } 7)} = \frac{12/78}{17/78} = \boxed{\frac{12}{17}}$$

⑥ A box has three crayons $\begin{cases} \text{Cyan} \\ \text{Magenta} \\ \text{Carnelian} \end{cases}$

- An experiment consists of drawing one crayon from the box, returning it to the box, and drawing a second crayon.

- The outcome of the experiment is the result of two draws.

- Any crayon is equally likely to be drawn.

(a) Let

first draw

second draw

$\Omega = \{CC, CM, CC, MM, MC, MC, CC, CC, CM\}$

Where each coupling of letters denotes drawing those two colors in that order. The probability of each coupling of letters is $\frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$.

A uniform probability law would be best here.

For each $s \in \Omega$, $\mathbb{P}(\{s\}) = \frac{1}{n} = \frac{1}{9}$

(b) Now we only return the first crayon if it is cyan.

let $\Omega = \{CC, CM, CC, MC, MC, CC, CM\}$ with indices 0 1 2 3 4 5 6 — k indices

Where each coupling of letters denotes drawing those two colors in that order. The probability of the coupling of letters when cyan is drawn first is $\frac{1}{3} \cdot \frac{1}{3} = 1/9$. If cyan is not drawn first, then the probability of that coupling is $\frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$.

So, assigning an index k, $0 \leq k \leq 6$ to each event in Ω above so

$s = \Omega_k$ = event at k^{th} index in Ω

$$\mathbb{P}(\{s\}) = \begin{cases} \frac{1}{9}, & 0 \leq k \leq 2 \\ \frac{1}{6}, & 3 \leq k \leq 6 \end{cases}$$