

1. (Extension of Problem 2.14 in the book) Let the random variable  $X$  take each integer value  $k$  satisfying  $0 \leq k \leq 9$  with probability  $1/10$ .

- (a) Find the expected value of the random variable  $Y = X \bmod 3$ .
- (b) Find the expected value of the random variable  $Y = 5 \bmod (X + 1)$ .

2. (Problem 2.16 in the book) Let  $X$  be a random variable with pmf

$$p_X(x) = \begin{cases} x^2/a & \text{when } x = -3, -2, -1, 0, 1, 2, 3 \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Find  $a$  and  $\mathbb{E}(X)$ .
- (b) What is the pmf of the random variable  $Z = (X - \mathbb{E}(X))^2$ ?
- (c) Find the variance of  $X$  using the result of part (c).
- (d) Find the variance of  $X$  using the Expected Value Rule

$$\mathbb{E}(g(X)) = \sum_x g(x)p_X(x) .$$

3. (Problem 2.24 in the book) A stock market trader buys 100 shares of stock  $A$  and 200 shares of stock  $B$  at the beginning of a certain time period. Let  $X$  and  $Y$  be the price changes respectively of stocks  $A$  and  $B$  from the beginning to the end of that time period, and assume that the joint pmf  $p_{X,Y}(x,y)$  of  $X$  and  $Y$  is uniform of the set of integers  $x$  and  $y$  satisfying  $-2 \leq x \leq 4$  and  $-1 \leq y - x \leq 1$ .

- (a) Find the marginal pmfs and expected values of  $X$  and  $Y$ .
- (b) Find the mean of the trader's profit if she sells the stocks at the end of the time period.

4. Three tests consist of three questions each, with each question worth 1 point. Frodo can answer each question on the easy test independently with probability 0.9, on the medium test with probability 0.7, and on the hard test with probability 0.5. Frodo chooses one test uniformly at random and answers the questions on that test. Find the pmf of his score  $X$ .