ECE 3100 **RECITATION PROBLEMS IV** **Spring 2019**

1. We assign Frodo, Sam, Gandalf, Legolas, and eight miscellaneous hobbits to four teams of three each — call them Team 1, Team 2, Team 3, and Team 4 — by handing out slips of paper drawn uniformly at random without replacement from a box, just as we did in lecture. What is the probability that one of the teams turns out to be a "power team" consisting of three out of the four main characters? (Thanks to Kenneth Fang for raising this question.)

2. (Problem 1.55 in the book) Eight rooks are placed randomly on distinct squares of an (8×8) chessboard, with all possible placements equally probable. Find the probability that all the rooks are safe from each other in the sense that no row or column contains more than one rook.

3. (Problem 1.59 in the book) A used-car lot contains 100 cars, k of which are lemons. You select m cars uniformly at random and take them out for a test drive. Find, as a function of n, m, and k, the probability that n of the cars tested are lemons.

4. (Problem 2.6 from the book) The Red Wings and Maple Leafs are set to face each other in the playoffs. The Leafs win any given game between the two teams with probability p, independent of other games. The general mamnagers need to agree on whether to play a single winner-take-all game or a best-of-3 series. What would the Leafs prefer if $p > 1/2$? Answer the same question if the choice is between a best-of-3 series and a best-of-5 series.

5. (Problem 2.14 in the book) Let the discrete random variable X take each integer value k satisfying $0 \le k \le 9$ with probability $1/10$.

(a) Write a formula for p_X, the pmf of X.
(b) Find a formula for p_Y, the pmf of the random variable $Y = X (\mathrm{mod}\ 3)$.
(c) Find a formula for p_Z, the pmf of the random variable $Z = 5(\mathrm{mod}\ X + 1)$.]