## Set Theory Notations

$$\begin{aligned}
\text{Universal Set} &: \Omega \\
\text{Empty Set} &: \emptyset \\
\text{Set} &: S = \{x \in \Omega \mid x \in S\} \\
\text{Complement of Set} &: S^C = \{x \in \Omega \mid x \notin S\} \\
\text{Union} &: A \cup B = \{x \mid x \in A \text{ or } x \in B\} \\
\text{Intersection} &: A \cap B = \{x \mid x \in A \text{ and } x \in B\}
\end{aligned}$$

## Algebra of Sets

Let $A$, $B$, and $C$ be subsets of $\Omega$

- **Complement Laws:**

$$\begin{aligned} A \cup A^C &= \Omega \\ A \cap A^C &= \emptyset \end{aligned}$$

- **Identity Laws:**

$$\begin{aligned} A \cup \emptyset &= A \\ A \cap \Omega &= A \end{aligned}$$

- **Commutative Laws:**

$$\begin{aligned} A \cup B &= B \cup A \\ A \cap B &= B \cap A \end{aligned}$$

- **Associative Laws:**

$$\begin{aligned} (A \cup B) \cup C &= A \cup (B \cup C) \\ (A \cap B) \cap C &= A \cap (B \cap C) \end{aligned}$$

- **Distributive Laws:**

$$\begin{aligned} A \cup (B \cap C) &= (A \cup B) \cap (A \cup C) \\ A \cap (B \cup C) &= (A \cap B) \cup (A \cap C) \end{aligned}$$

- **De Morgan's Laws:**

$$( \cup_{i=1}^{n} A_i )^C = \cap_{i=1}^{n} A_i^C$$
$$( \cap_{i=1}^{n} A_i )^C = \cup_{i=1}^{n} A_i^C$$

  - Outline of Proof:
    If we are able to show that $( \cup_{i=1}^{n} A_i )^C \subset \cap_{i=1}^{n} A_i^C$ is true and $\cap_{i=1}^{n} A_i^C \subset ( \cup_{i=1}^{n} A_i )^C$ is also true, then we can conclude that $( \cup_{i=1}^{n} A_i )^C = \cap_{i=1}^{n} A_i^C$.

$$\begin{aligned} \text{Let } x \in ( \cup_{i=1}^{n} A_i )^C &\Rightarrow x \notin ( \cup_{i=1}^{n} A_i ) \\ &\Rightarrow x \notin A_1, \ x \notin A_2, \ ... \ x \notin A_n \\ &\Rightarrow x \in A_1, \ x \in A_2, \ ... \ x \in A_n \\ &\Rightarrow x \in \cap_{i=1}^{n} A_i^C \\ &\Rightarrow ( \cup_{i=1}^{n} A_i )^C \subset \cap_{i=1}^{n} A_i^C \end{aligned}$$

Similar arguments can be made to show that $\cap_{i=1}^{n} A_i^C \subset ( \cup_{i=1}^{n} A_i )^C$