1. ECE graduate students participate in an annual volleyball tournament. Suppose 15 students are willing to participate this year.

(a) How many distinct 6-player rosters exist, with players drawn from among the willing participants?
(b) Six of the willing participants are women and nine are men. How many of the rosters you found in (a) consist of exactly two women and four men?
(c) How many rosters satisfy the constraint that they include at least two women?
(d) Suppose one of the graduate students writes a program to generate a roster randomly, so all 6-player rosters are equally probable program outputs. What is the probability that the program's outputted roster includes exactly two women? What is the probability that it includes at least two women?

2. Prior to taking an exam, members of a class of n students leaves their cell phones in a box with the TA. After the exam, the TA redistributes the phones at random, so all assignments of phones to students are equally probable. What is the probability that

(a) every student gets his or her phone back?
(b) the first m students receiving phones from the TA get their own phones back, where $m \leq n$?
(c) each of the first m students receiving a phone from the TA gets a phone belonging to one of the last m students, where $m \leq n$?

Now suppose in addition that every phone deposited in the box has probability p of getting scratched independent of what happens to other phones and independent of who deposits the phone or who receives the phone from the TA after the exam. What is the probability that

(d) the first m students will receive scratched phones from the TA, where $m \leq n$?
(e) exactly m students will receive scratched phones from the TA, where $m \leq n$?

3. An experiment calls for two independent rolls — call them Roll 1 and Roll 2 — of a fair six-sided die. Define random variables X and Y as follows: X is the difference Roll 1 − Roll 2 and $Y = 2^X$.

(a) What are the possible values of X?
(b) What are the possible values of Y?
(c) Find the pmf of X and the pmf of Y.
(d) Find the probability of the event that $|X| \leq 4$.
(e) Find the probability of the event that $Y > 1/32$.
(f) Find $\mathbb{E}(X)$ and $\mathbb{E}(Y)$.

4. Frodo makes n independent flips of a coin that has probability p of coming up heads. Assume n is even, and define the random variable X as the product of number of heads obtained in the n flips with the number of tails obtained.

(a) Find the probability of the event $\{X = 0\}$.
(b) What are the possible values of X?
(c) Find the pmf of X when $n = 4$.

5. Find pmfs for two different discrete random variables X and Y that satisfy $\mathbb{E}(X) = \mathbb{E}(Y) = 0$ and $\mathbb{E}\left(X^2\right) = \mathbb{E}\left(Y^2\right) = 4$. For this problem, you need not talk about Ω, $\mathbb{P}$, etc. — you need only solve for pmfs $p_X(x)$ and $p_Y(y)$.

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① 15 students

(a) How many 6 player rosters?

$$\#(\text{6 player rosters}) = \binom{15}{6}$$

(b) 6 women, 9 men.

Team of exactly 2 woman, 4 men

ways to get 2 women from 6

$$\binom{6}{2} \cdot \binom{9}{4}$$

ways to get 4 men from 9

(c) Rosters that include <u>at least</u> 2 women.

$$\binom{15}{6} - \binom{6}{1}\binom{9}{5} - \binom{6}{0}\binom{9}{6}$$

Total amount − 1 women roster − no women roster

(d) $\mathbb{P}(\text{at least two women}) = \dfrac{\binom{15}{6} - \binom{6}{1}\binom{9}{5} - \binom{6}{0}\binom{9}{6}}{\binom{15}{6}}$

$\mathbb{P}(\text{exactly two women}) = \dfrac{\binom{6}{2}\binom{9}{4}}{\binom{15}{6}}$

② (a) $\mathbb{P}(\text{every student gets phone back}) = \dfrac{1}{n!}$

Why? $\mathbb{P}(\text{First student gets phone back}) = \dfrac{1}{n}$

$\mathbb{P}(\text{Second student gets phone back}) = \dfrac{1}{n-1}$

$\vdots$

$\mathbb{P}(N^{th} \text{ student gets phone back}) = 1$

(b) $\mathbb{P}(\text{first } m \text{ students get phone back}) = \dfrac{(n-m)!}{n!}$

$m \leq n$

Why?

$m=1$

$\mathbb{P}(\text{First student gets phone back}) = \dfrac{n-1}{n} \cdot \dfrac{n-2}{n-1} \cdot \dfrac{n-3}{n-2} \cdots$

$\mathbb{P}(m=2) = \dfrac{n-2}{n} \cdot \dfrac{n-3}{n-1} \cdots$

$\mathbb{P}(m=3) = \dfrac{n-3}{n} \cdot \dfrac{n-4}{n-1} \cdots$

$\vdots$

$\mathbb{P}(m=n) = \dfrac{1}{n} \cdot \dfrac{1}{n-1} \cdot \dfrac{1}{n-2} \cdots$

(c) $\mathbb{P}(\text{first } m \text{ students get phone belonging to one of last } m \text{ students})$

$$\frac{1}{\binom{n}{m}} = \frac{m!\,(n-m)!}{n!}$$

m ways from n to choose

(d) scratched or not ⇒ Bernoulli p.

$$\left.\begin{array}{l} \mathbb{P}(\text{first student has scratched phone}) = p \\ \mathbb{P}(\text{second student " }) = p \\ \quad\vdots \\ \mathbb{P}(m^{th} \text{ " }) = p \end{array}\right\} \text{ independent}$$

So $\mathbb{P}(\text{first } m \text{ students have scratched phone}) = p^m$

(e) m students receive a scratched phone.

$m \leq n$.

So $n-m$ do **NOT** receive a scratched phone.

$$\boxed{\binom{n}{m} p^m (1-p)^{n-m}} \leftarrow \mathbb{P}(\text{exactly } m \text{ students get a scratched phone})$$

how many distributions

③ $X = \text{roll } 1 - \text{roll } 2$

$Y = 2^X$

(a) $X = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$

(b) $Y = \{\frac{1}{32}, \frac{1}{16}, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, 2, 4, 8, 16, 32\}$

(c)

$$P_X(x) = \begin{cases} 1/36, & x = -5 \\ 2/36, & x = -4 \\ 3/36, & x = -3 \\ 4/36, & x = -2 \\ 5/63, & x = -1 \\ 6/36, & x = 0 \\ 5/36, & x = 1 \\ 4/36, & x = 2 \\ 3/36, & x = 3 \\ 2/36, & x = 4 \\ 1/36, & x = 5 \end{cases}$$

$$P_Y(y) = \begin{cases} 1/36, & y = 1/32 \\ 2/36, & y = 1/16 \\ 3/36, & y = 1/8 \\ 4/36, & y = 1/4 \\ 5/63, & y = 1/2 \\ 6/36, & y = 1 \\ 5/36, & y = 2 \\ 4/36, & y = 4 \\ 3/36, & y = 8 \\ 2/36, & y = 16 \\ 1/36, & y = 32 \end{cases}$$

(d) $\mathbb{P}(|X| \leq 4) = 1 - \mathbb{P}(|X| > 4)$

$= 1 - 2/36 = 34/36$

(e) $\mathbb{P}(Y > 1/32) = 1 - \mathbb{P}(Y \leq 1/32)$

$= 1 - 1/36 = 35/36$

(f) $\mathbb{E}(X) = \sum_{x \in X} x p_X(x) = 0$ ← obvious that it cancels

$\mathbb{E}(Y) = \sum_{y \in Y} y p_Y(y) = \frac{1}{32} \cdot \frac{1}{36} + \frac{1}{16} \cdot \frac{2}{36} + \cdots\cdots + \frac{1}{36} \cdot 32 = 3.445\ldots$

you get the point

(4) (a) $\mathbb{P}(\{X=0\})$

X is product of #(heads) #(tails) in n flips

$\mathbb{P}(\{X=0\})$ only occurs if **No** heads **OR** **No** tails

$\Rightarrow$ all heads no tails
OR
all tails no heads

$$\mathbb{P}(\{X=0\}) = p^n + (1-p)^n$$

heads, # tails, some m satisfying this

(b) $X = \{ n(n-m) : 0 \le m \le n \}$

possible values of k evident from product

3 heads 1 tail, 1 head 3 tails

(c)
$$\underbrace{P_X(k)}_{\text{when } n=4} = \begin{cases} p^4 + (1-p)^4, & k=0 \\ \binom{4}{3}p^3(1-p) + \binom{4}{1}p(1-p)^3, & k=3 \\ \binom{4}{2}p^2(1-p)^2 + \binom{4}{2}p^2(1-p)^2, & k=4 \end{cases}$$

⑤ $E(X) = E(Y) = 0$ ← different

could have something like in 3

$E(X^2) = E(Y^2) = 4$

$g(X) = X^2$
$g(Y) = Y^2$

$$\mathbb{E}(X^2) = \sum_{x \in \mathcal{X}} g(x) p_X(x)$$

So need it to sum to 4 squared, cancel out otherwise

$$p_X(x) = \begin{cases} \frac{1}{2}, & x = 2 \\ \frac{1}{2}, & x = -2 \end{cases}$$

Similar for Y,

$$p_Y(y) = \begin{cases} \frac{1}{4}, & x = \sqrt{2} \\ \frac{1}{4}, & x = -\sqrt{2} \\ \frac{1}{4}, & x = 1 \\ \frac{1}{4}, & x = \sqrt{11} \end{cases}$$

work below ↓

Say $x_1 = \sqrt{2}$

$$\frac{1}{4}x_1^2 + \frac{1}{4}x_2^2 + \frac{1}{4}x_3^2 + \frac{1}{4}x_4^2 = 4$$

$x_2 = -\sqrt{2}$

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 = 16$$

$$x_2^2 + x_3^2 + x_4^2 = 14$$

$x_3 = 1$

$$x_3^2 + x_4^2 = 12$$

$$x_4^2 = 11$$

$$x_4 = \sqrt{11}$$