You should know:

$\mathbb{R}^n$, vectors, span, linear dependence, bases, dim

$T: \mathbb{R}^n \to \mathbb{R}^m$ (linear transformations), Gaussian elimination

kernel / image / null-space / range, eigenvalues, symmetric matrices

We will cover:

- fields (scalars) : $\mathbb{R}$, $\mathbb{C}$, finite fields
- vector spaces
- linear transformations
- dim, bases of vector spaces
- eigenvalues + eigenvectors w/o determinants
- polynomials
- inner product spaces $\left\{ \begin{array}{l} \text{least squares} \\ \text{Spectral theorem} \\ \text{singular value decompositions} \end{array} \right.$
- Structure of linear operators (square matrices)

  Jordan canonical form

  Cayley Hamilton Theorem

Fields

Let

$$A = \begin{bmatrix} 1 & 3 & 5 \\ -2 & -8 & 2 \end{bmatrix}$$

row reduced echelon form

$$\mathrm{rref}(A) = \ ?$$

$$\begin{bmatrix} 1 & 3 & 5 \\ -2 & -8 & 2 \end{bmatrix} \xrightarrow{R_2 \mathrel{+}= 2R_1} \begin{bmatrix} 1 & 3 & 5 \\ 0 & -2 & 12 \end{bmatrix} \xrightarrow{R_2 /= -2} \begin{bmatrix} 1 & 3 & 5 \\ 0 & 1 & -6 \end{bmatrix} \xrightarrow{R_1 -= 3R_2} \begin{bmatrix} 1 & 0 & 23 \\ 0 & 1 & -6 \end{bmatrix}$$

what <u>scalar</u> operations did we use?

Addition $(+)$

Multiplication $(\cdot)$

Division $(/)$

Subtraction $(-)$

also identify 0, 1 ?

Definition (appendix C of FIS): A field is a set $\mathbb{F}$ equipped with the following

ⓐ Two special elemens $0 \neq 1$ in $\mathbb{F}$

ⓑ operation addition $(+)$: $\mathbb{F} \times \mathbb{F} \to \mathbb{F}$

$$(a, b) \mapsto a+b$$

ⓒ operation multiplication $(\cdot)$ $\mathbb{F} \times \mathbb{F} \to \mathbb{F}$

$$(a, b) \mapsto a \cdot b \text{ (or } ab)$$

$(\mathbb{F}, 0, 1, +, \cdot)$ must satisfy the following properties (axioms)

(F1) Commutativity of '+', '$\cdot$': $\forall\, a, b \in \mathbb{F}$ $\boxed{a+b = b+a}$, $\boxed{a \cdot b = b \cdot a}$

(F2) Associativity of '+', '$\cdot$': $\forall\, a, b, c \in \mathbb{F}$ $\boxed{a+(b+c) = (a+b)+c}$, $\boxed{a \cdot (bc) = (ab) \cdot c}$

(F3) Existence of identity elements of '+', '$\cdot$': $\forall\, a \in \mathbb{F}$, $\boxed{0+a = a}$, $\boxed{1 \cdot a = a}$

(F4) Existence of inverses: $\forall\, a \in \mathbb{F}$, $\forall\, b \neq 0 \in \mathbb{F}$, $\exists\, c, d \in \mathbb{F}$ s.t. $\boxed{a+c=0}$, $\boxed{b \cdot d = 1}$

(F5) Distributivity: $\forall\, a, b, c \in \mathbb{F}$ $a \cdot (b+c) = a \cdot b + a \cdot c$