1. (Problem 1.15 in the book) A coin is tossed twice. Frodo claims that the probability that two heads come up given that the first toss is a head is at least as large as the probability that two heads come up given that at least one head comes up. Is he right? Does the answer to that question depend on p, the probability that a head comes up on a single flip? Can you generalize Frodo's reasoning to situations other than coin flips?

2. (Problem 1.17 in the book) A quality-control inspector tests four items selected uniformly at random from a batch of 100 items. She rejects the batch if at least one tested item comes up defective. What is the probability that she rejects the batch given that it contains five defectives?

3. (Problem 1.27 in the book) Frodo tosses $n+1$ fair coins and Sam tosses n fair coins. Assume independent tosses. Use the Total Probability Theorem to find the probability that Frodo's tosses yield more heads than Sam's. (Suggestion: consider the possible states of affairs after both have tossed exactly n coins.)

4. (Problem 1.24 in the book) This problem rests on Bayes' Rule, which we haven't quite covered yet. Here's a form of Bayes' Rule sufficient to solve this problem: if A_1 and A_2 are events that partition Ω — i.e. $A_1 \cap A_2 = \phi$ and $A_1 \cup A_2 = \Omega$ — then for any event B

$$\mathbb{P}(A_k \mid B) = \frac{\mathbb{P}(B \mid A_k)\mathbb{P}(A_k)}{\mathbb{P}(B \mid A_1)\mathbb{P}(A_1) + \mathbb{P}(B \mid A_2)\mathbb{P}(A_2)} \quad \text{for } k = 1, 2 .$$

Two of three prisoners are to be released. One of the three asks a guard to tell him the identity of a prisoner other than himself who's slated for release. The guard refuses, saying, "At your present state of knowledge, your probability of being released is 2/3, but after you know my answer, your probability of being released will become 1/2, since two prisoners' fates will be unknown, including yours, and exactly one of those prisoners will be released. Do you really want me to give you information that lowers your proability of release?" Find the mistake in the guard's reasoning.