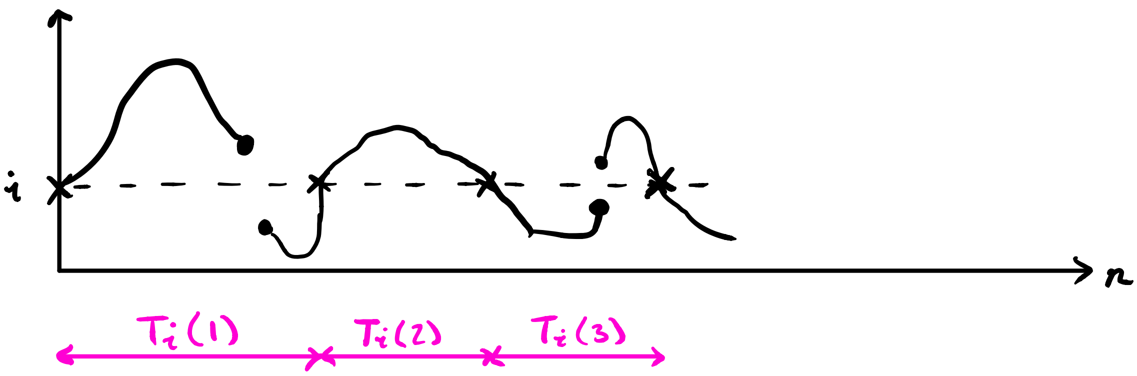
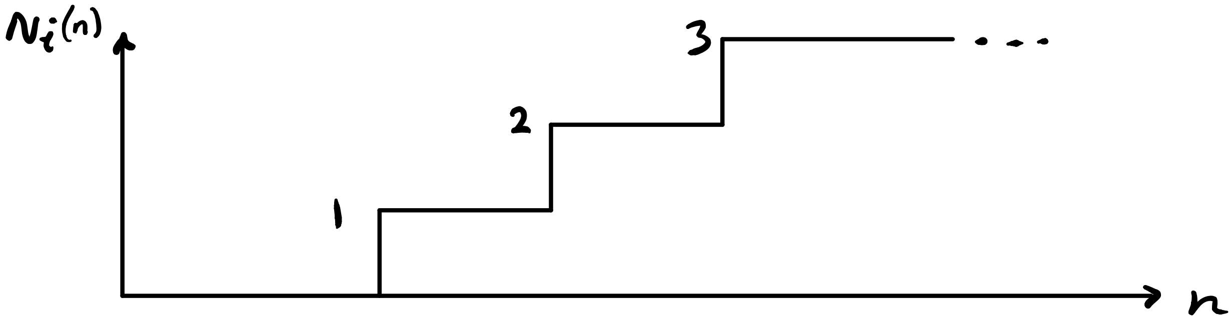


regenerative cycle



i transient : $\Pr[T_i < \infty] < 1$

i recurrent : $\Pr[T_i < \infty] = 1$

Two notions of recurrence

Null	Positive
$\lim_{n\to\infty} \frac{N_i(n)}{n} = 0$	$\lim_{n\to\infty} \frac{N_i(n)}{n} > 0$

equivalent statement

$\mathbb{E}[T_i] = \infty$	$\mathbb{E}[T_i] < \infty$

Occupancy rate of i.

$$r_i = \frac{\lim_{n\to\infty} \sum_{m=1}^{n} \mathbb{1}[X_m = i \mid X_0 = i]}{n}$$

(kinda like arrival rate)

If i transient,

$$r_i = 0$$

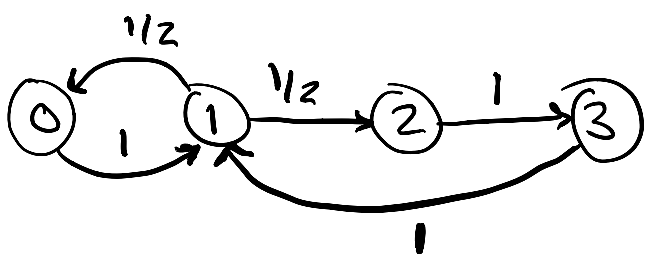
If i is NULL recurrent,

$$r_i = 0$$

If i positive recurrent,

$$r_i = \frac{1}{\mathbb{E}[T_i]}$$

Example



State 0 :

$$T_0 = \begin{cases} 2 & \text{w.p. } 1/2 \\ 4 & \text{w.p. } 1/2 \end{cases}$$

$$\mathbb{E}[T_0] = 2 \cdot \tfrac{1}{2} + 4 \cdot \tfrac{1}{2} = 3$$

$$r_0 = \frac{1}{3}$$

"$\frac{1}{3}$ of the time, the state is @ state 0"

Recurrent Class

A communication class that is recurrent $\begin{pmatrix}\text{NULL} \\ \text{or} \\ \text{POSITIVE}\end{pmatrix}$

$$\boxed{\text{Every recurrent class is closed.}}$$

Cononical Decomposition

$$\mathcal{X} = \mathcal{T} \cup C_1 \cup C_2 \cup C_3 \cup \cdots$$

last labels

States $\{1, 2, \ldots, m_1\}$ States $\{m_1+1, \ldots, m_1+m_2\}$

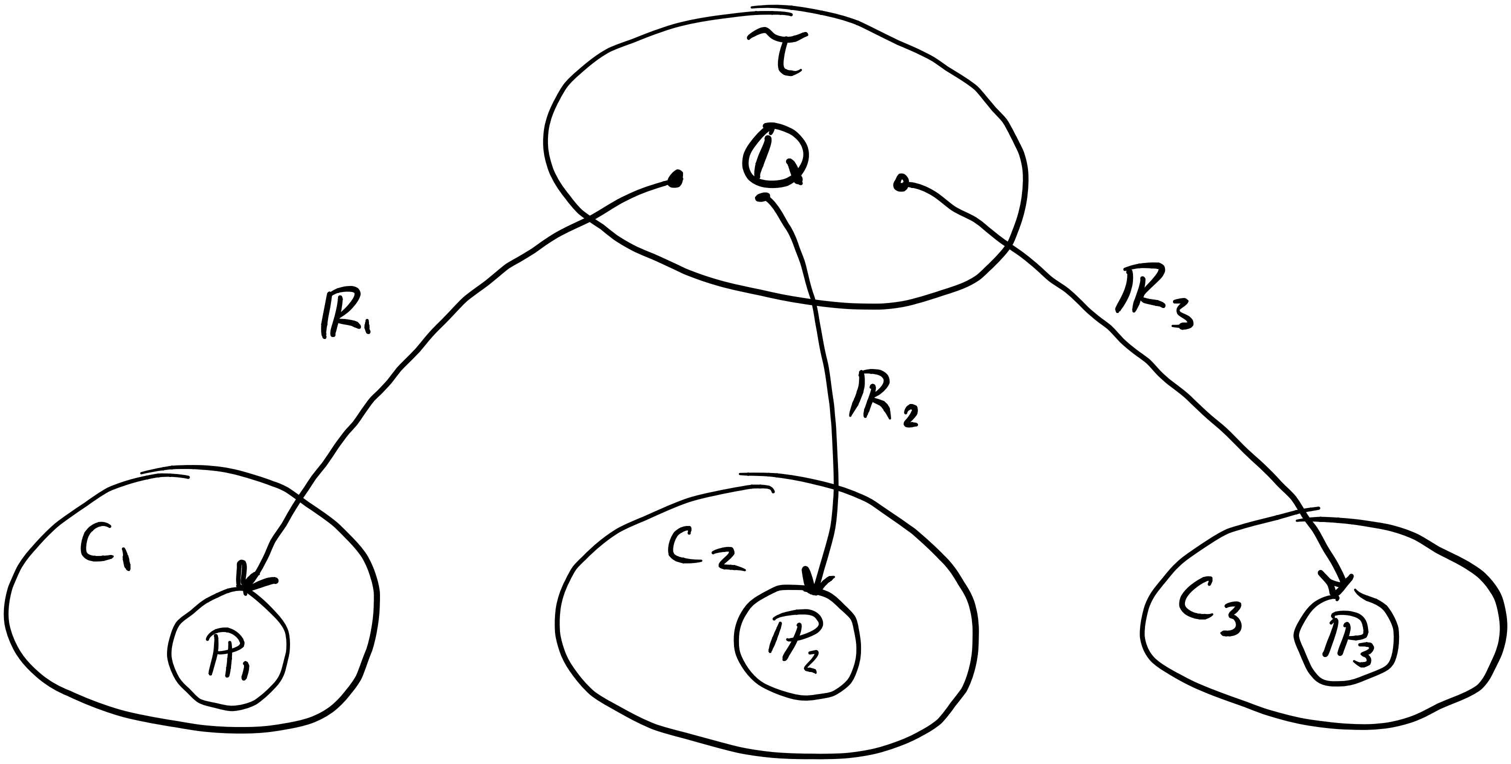
Expect (for C_1, C_2, C_3)

transition matrix b/t states of C_1

$$\mathbb{P} = \begin{array}{c} 1 \\ \vdots \\ m_1 \\ m_1+1 \\ \vdots \\ m_1+m_2 \\ m_1+m_2+1 \\ \vdots \\ m_1+m_2+m_3 \\ \tau \end{array} \overset{\begin{array}{c} 1 \cdots m_1 \end{array}}{\begin{bmatrix} \text{NON ZERO } \mathbb{P}_1 & 0 & 0 & 0 \\ 0 & \text{Non-zero } \mathbb{P}_2 & 0 & 0 \\ 0 & 0 & \mathbb{P}_3 & 0 \\ \mathbb{R}_1 & \mathbb{R}_2 & \mathbb{R}_3 & Q \end{bmatrix}}$$

transition matrix b/t states of C_2

transition matrix b/t states of C_3



Existence of Stationary Distribution

$$\overset{\bar{\pi}}{\begin{bmatrix} \pi_1 & \pi_2 & \cdots & \pi_n \end{bmatrix}} \; \mathbb{P} \; = \; \overset{\bar{\pi}}{\begin{bmatrix} \pi_1 & \pi_2 & \cdots & \pi_n \end{bmatrix}}$$

$$\sum_{i \in \mathcal{X}} \pi_i = 1$$

- π_i : occupancy rate $= r_i = \frac{1}{\mathbb{E}[T_i]}$

① If all states transient OR null recurrent,

$$r_i = 0 \quad \forall i \in \mathcal{X}$$

then this chain has 0 stationary distribution.

② If the chain has a single positive recurrent classes (no NULL recurrent) then $\exists$ unique stationary distribution

③ If the chain has multiple positive recurrent classes, then $\exists$ infinitely many stationary distributions

use

$$\boxed{\pi_1 \ \pi_2 \ \cdots \ \pi_{m_1}} \ \boxed{\mathbb{P}_1}_{m_1 \times m_1} = \boxed{\pi_1 \ \pi_2 \ \cdots \ \pi_n}$$

For $\mathbb{P}$:

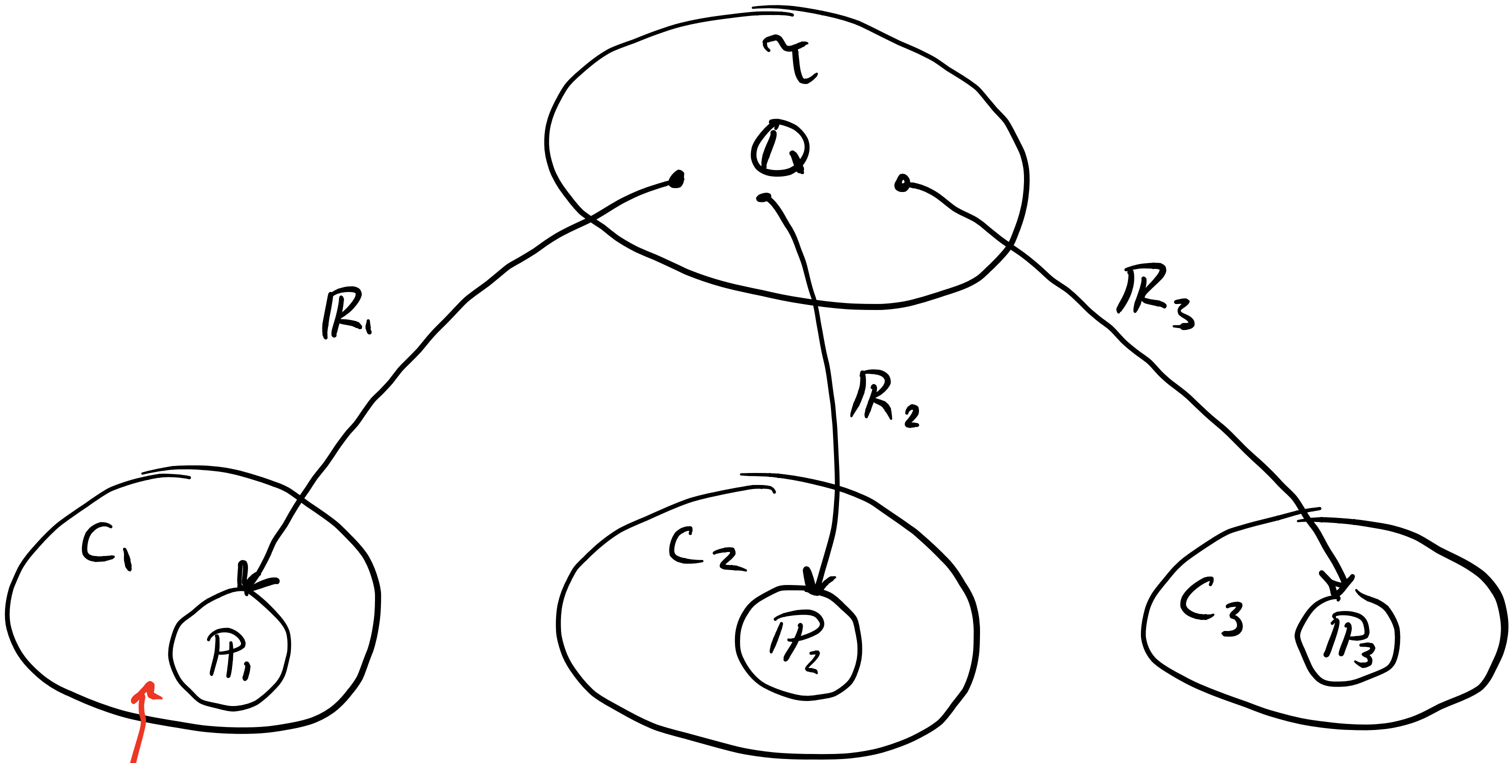
$$\alpha_1 \ \boxed{\pi_1 \ \pi_2 \cdots \pi_{m_1} \ 0 \ \cdots \ 0 \ \cdots \ 0} \quad (1)$$

$$+ \ \alpha_2 \ \boxed{0 \ 0 \ \cdots \ 0 \ \pi_{m_1+1} \ \cdots \ \pi_{m_1+m_2} \ 0 \cdots 0} \quad (2)$$

$$+ \ \alpha_3 \ \boxed{0 \ 0 \ \cdots \ 0 \ 0 \ \cdots \ 0 \ \pi_{m_1+m_2+1} \cdots \pi_{m_1+m_2+m_3}} \quad (3)$$

infinite b/c can choose $\alpha_1(1) + \alpha_2(2) + \alpha_3(3)$ s.t. $\alpha_1+\alpha_2+\alpha_3=1$ where $\alpha_i \in \mathbb{R}$ and the linear combination of (1), (2), (3) is a valid $\bar{\pi}$ satisfying our criteria

"like a change of basi!"



Probability of being absorbed here?