

①

$$\sin z = \cos z$$

$$\frac{e^{iz} - e^{-iz}}{2i} = \frac{e^{iz} + e^{-iz}}{2}$$

$$-i(e^{iz} - e^{-iz}) = e^{iz} + e^{-iz}$$

$$i(e^{-iz} - e^{iz}) = e^{iz} + e^{-iz}$$

$$e^{iz} + ie^{iz} = ie^{-iz} - e^{-iz}$$

$$e^{iz}(1+i) = e^{-iz}(i-1)$$

$$\frac{e^{iz}}{e^{-iz}} = \frac{i-1}{(1+i)(1-i)} = \frac{i-i^2-1+i}{2}$$

$$e^{iz} = i = e^{i\left(\frac{\pi}{2} + 2\pi k\right)}$$

$$iz = \frac{\pi}{2}i + 2\pi ik, \quad k \in \mathbb{Z}$$

$$z = \frac{\pi}{4} + \pi k, \quad k \in \mathbb{Z}$$

$$\textcircled{2} \quad \phi(z) = A \ln |z| + B \quad \nabla^2 \phi = 0 \text{ for } |z| > 3$$

$$0 = A \ln(3) + B$$

$$B = -A \ln(3)$$

$$\phi(z) = A(\ln |z| - \ln(3))$$

$$= A(\ln(\sqrt{x^2 + y^2}) - \ln(3))$$

$$(3) \int_{|z|=4} \frac{z}{(z+5)(z-1)} dz$$

$$f(z) = z/(z+5)$$

$$\int_{|z|=4} \frac{f(z)}{z-1} dz = f(1) 2\pi i$$

$$= \frac{2\pi i}{6} = \frac{\pi i}{3}$$

4)

$$\sum_{k=0}^{\infty} k z^k$$

HMM

$$\frac{d}{dz}(z^k) = k z^{k-1}$$

$$z \frac{d}{dz} [z^k] = k z^k$$



Realize

$$\frac{1}{1-z} = \sum_{k=0}^{\infty} z^k$$

$$z \frac{d}{dz} \left[ \frac{1}{1-z} \right] = \boxed{\frac{z}{(1-z)^2}}$$

So,

$$\frac{z}{(1-z)^2} = \sum_{k=0}^{\infty} k z^k$$

5)

$D: |z| < 1$  (open unit-disk)

1)  $f$  is analytic IN  $D$

2)  $f$  is continuous ON  $D$  and its boundary

3)  $f(0) = i$

4)  $|f(z)| \leq 1 \quad \forall z \in D$

All  $f$  that satisfy this condition?

$f(z) = i \leftarrow$  analytic in  $D$   
continuous on  $D$

$f(0) = i$

$|f(z)| = 1 \quad \forall z \in D$

~~$f(z) = ie^{iz}$~~

~~$|z| < 1$~~

~~analytic in  $D$~~

~~continuous on  $D$~~

~~$f(0) = i$~~

~~$|f(z)| = 1 \quad \forall z \in D$~~