Cayley Hamilton Theorem
Minimal-Characteristic Polynomials
Applications to Jordan Canonical Form

Situation: - $T \in \mathcal{L}(V)$
- V a v.s. over $\mathbb{C}$
- $\dim V = n < \infty$

Definition: The characteristic polynomial $q_T(x) \in \mathbb{C}[x]$

is
$$q_T(x) = (x-\lambda_1)^{a_1}(x-\lambda_2)^{a_2}\cdots(x-\lambda_m)^{a_m}$$
where $\lambda_1, \ldots, \lambda_m$ are the distinct eigenvalues of $a_i = \dim G_{\lambda_i}(T) = \dim(\ker(T-\lambda_i)^n)$

Thus, $\deg q_T(x) = a_1 + \cdots + a_m = n$

(note: $q_A(x) = \det(xI_n - A)$)

Definition: The minimal polynomial of T is the unique monic polynomial $m_T(x) \in \mathbb{C}[x]$ of smallest degree s.t. $m_T(T) = 0$.

"recall"

A polynomial is said to be monic if its lead coefficient is 1.

Recall: The annihilator of T zero transformatio

$$\text{ann}(T) = \{f(x) \in \mathbb{C}[x] \mid f(T) = 0\}$$

"this is an ideal in $\mathbb{C}[x]$!"

Proposition: Let I be a non-zero ideal.

Then

ⓐ $I = \langle f(x) \rangle$ for $f(x)$ the unique monic polynomial of lowest degree in I

ⓑ If $g(x) \in I$, then $f(x) \mid g(x)$

So, m_T is the generator of ⓐ.

One way to compute the minimal polynomial

Let $A \in \mathbb{C}^{n \times n}$

Consider $I, A, A^2, \ldots, A^{n^2}, \ldots$ all in $\mathbb{C}^{n \times n}$

Choose smallest m s.t. $I, A, A^2, \ldots, A^m$ of dim n^2 are LD so

$$a_0 I + a_1 A + \cdots + a_m A^m = 0.$$