1. (Problem 2.32 in the book) $2m$ individuals form m cohabiting couples at the start of a certain time period. Each individual is alive at the end of the time period with probability p, independent of other individuals. Let A be the number of living individuals at the end of the time period and Let S be the number of couples with both partners living at the end of the time period. For any number k of survivors, find $\mathbb{E}(S \mid A = k)$.

2. In class, we proved the Total Expectation Theorem when conditioning on events, which states that when $A_1, A_2, \ldots, A_n$ are events with positive probability that partition Ω, we have

$$\mathbb{E}(X) = \sum_{k=1}^{n} \mathbb{E}(X \mid A_k)\mathbb{P}(A_k)$$

for any random variable X defined on Ω. Prove the following extension: if B is any event for which all the events $A_k \cap B$ have nonzero probability, we have for any random variable X defined on Ω

$$\mathbb{E}(X \mid B) = \sum_{k=1}^{n} \mathbb{E}(X \mid A_k \cap B)\mathbb{P}(A_k \mid B)$$

3. (Problem 2.34 in the book) A spider and a fly move along a horizontal straight line. At each second, the fly takes one unit step to the right or to the left with equal probability p and stays where it is with probability $1-2p$. At each second the spider takes one unit step toward the fly. At time zero, the insect and the arachnid sit D units apart, where D is a random variable with a given pmf that takes positive integer values. What is the expected time it takes for the spider to land on top of the fly, at which point game over? You won't be able to find a succinct closed formula for this, but please describe a procedure for computing it.

4. (Problem 2.40 in the book) An arbitrary and capricious professor grades each paper by choosing a grade uniformly at random from among A, $A-$, $B+$, B, $B-$, and $C+$, choosing a grade for each paper independent of grades for other papers. How many papers do you expect to hand in before you receive each possible grade at least once?