Physica A 387 (2008) 5862–5867

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Properties of wealth distribution in multi-agent systems of a complex network

Mao-Bin Hu [a,b,*], Rui Jiang [a], Yong-Hong Wu [b], Ruili Wang [c], Qing-Song Wu [a]

[a] *School of Engineering Science, University of Science and Technology of China, Hefei 230026, PR China*
[b] *Department of Mathematics and Statistics, Curtin University of Technology, Perth WA6845, Australia*
[c] *Institute of Information Sciences and Technology, Massey University, New Zealand*

ARTICLE INFO

Article history:
Received 2 June 2008
Received in revised form 19 June 2008
Available online 29 June 2008

PACS:
87.23.Ge
89.75.Hc
05.10.-a

Keywords:
Complex networks
Wealth distribution
Gini coefficient
Pareto exponent

ABSTRACT

We present a simple model for examining the wealth distribution with agents playing evolutionary games (the Prisoners' Dilemma and the Snowdrift Game) on complex networks. Pareto's power law distribution of wealth (from 1897) is reproduced on a scale-free network, and the Gibbs or log-normal distribution for a low income population is reproduced on a random graph. The Pareto exponents of a scale-free network are in agreement with empirical observations. The Gini coefficient of an ER random graph shows a sudden increment with game parameters. We suggest that the social network of a high income group is scale-free, whereas it is more like a random graph for a low income group.

1. Introduction

Complex networks can describe a wide range of systems of high importance, ranging from nature to society and biological systems. Since the discovery of small-world behavior [1] and the scale-free property [2], complex networks have attracted continuous attention [3]. By representing the agents of a given population with vertices, and the contacts between agents with edges, network theory provides a natural framework for describing population structure [4]. For example, well-mixed populations can be represented by complete (fully connected, regular) networks and spatially structured populations can be associated with regular networks. Recently, much empirical evidence from real social networks has revealed that they are associated with a scale-free, power law degree distribution, $d(k) \sim k^{-\gamma}$ with $\gamma_{\text{actor}} = 2.3 \pm 0.1$ for a movie actor collaboration network [5], $\gamma_{\text{science}} = 2.1$ and 2.5 for a science collaboration graph [6], $\gamma_f = 3.5 \pm 0.2$ and $\gamma_m = 3.3 \pm 0.2$ for females and males in human sexual contacts [7], etc.

It is well known that even in developed countries, it is common for 40% of the total wealth to be owned by only 10% of the population. The distribution of wealth is often described using 'Pareto' tails (1897), which decay as a power law of the large wealth [8]. The rest of the people, with low income, follow a different distribution, either Gibbs or log-normal, [10,11]:

$$P(W) \sim \begin{cases} W^{-\upsilon}, & \text{for } (W \geq W_c), \\ \exp(-\lambda W), & \text{for } (W < W_c). \end{cases} \quad (1)$$

* Corresponding author at: School of Engineering Science, University of Science and Technology of China, Hefei 230026, PR China. Tel.: +86 551 3600127; fax: +86 551 3606459.
E-mail address: humaobin@ustc.edu.cn (M.-B. Hu).

0378-4371/$ – see front matter
doi:10.1016/j.physa.2008.06.032

where $P(W)$ is the probability of finding an agent with wealth greater than W. The exponent υ is called the Pareto exponent, and λ denotes a scaling factor. The value of the Pareto exponent was found to vary between 1 and 3 for both individual wealth and company sizes [12–17]. Studies on real data show that the high income group does indeed follow the Pareto law, with $\upsilon \approx 1.6$ for the USA [12], $\upsilon = 1.8 \sim 2.2$ for Japan [13,14], $\upsilon = 2.0 \sim 2.3$ for the UK [11], $\upsilon \approx 1.0$ for Japanese firms [18], and $\upsilon \approx 0.9$ for India [19]. The value of υ changes with the countries, the scale of the individual or company, and the statistical methods applied in the investigations.

The Gini coefficient was developed by the Italian statistician Corrado Gini in 1912 to measure the inequity of income distribution. It is defined as a ratio with values between 0 and 1: the numerator is the area between the Lorenz curve of the distribution and the uniform distribution line; the denominator is the area under the uniform distribution line [20]. Thus, a low Gini coefficient indicates more equal income or wealth distribution, while a high Gini coefficient indicates more unequal distribution. Zero corresponds to perfect equality (everyone having exactly the same income) and 1 corresponds to perfect inequality (where one person has all the income, while everyone else has zero income).

In 1968, the sociologist Robert Merton used the term "Matthew Effect" to describe the phenomenon that the rich get richer and the poor get poorer [21]. The "Matthew Effect" is also found in some areas of life such as wealth, achievement, fame, success etc [21–24]. The Matthew Effect for Countries (MEC) was also discovered [22].

In this paper, we study wealth distribution by using evolutionary games on different networks. The simulation results show the Pareto power law distribution for the wealthy population and the Gibbs or log-normal distribution for the low income group. We suggest that the social networks for the high income population and the low income group are different. The dependence of the Gini coefficient on the game parameters is also investigated.

2. The model

Previous studies of wealth distribution often adopted a kinetic exchange model in which each agent is a gas molecule and each trading is a money-conserving collision [9–11,27–29]. One can refer to [10,11] for a detailed review of historical data, empirical analysis and models of wealth distribution. These models approximate well a steady economy. However, the total wealth of the system is reserved and will not vary with time [9,12].

The evolutionary games theory has been widely used to characterize some social and biological processes [30–38]. In a typical Prisoner's Dilemma (PD) or Snowdrift Game (SG), two players simultaneously decide whether to cooperate (C) or defect (D). Each player will get a payoff in each step and then the players will choose to change or keep their strategy on the basis of some learning strategies. One can see that both games are intrinsically suitable for characterizing economic activities such as cooperation, decision, payoff and wealth accumulation [25,26].

Our simulation starts from establishing the underlying cooperation network structure. We consider two different social networks in this paper: the Erdos–Renyi random network and the scale-free network. Starting with N disconnected nodes, the ER random graphs are generated by connecting couples of randomly selected nodes, prohibiting multiple connections, until the number of edges equals $L_{\max}$. The scale-free social network is constructed according to the Barabási–Albert (BA) scale-free network model [2] with the "growth" and "preferential attachment" mechanisms. The BA model reproduces well the power law degree distribution which is in good agreement with the empirical evidence. In this model, starting from m_0 fully connected vertices, one vertex with $m \leq m_0$ edges is attached at each time step in such a way that the probability Π_i of being connected to the existing vertex i is proportional to the degree k_i of the vertex, i.e. $\Pi_i = \frac{k_i}{\Sigma_j k_j}$, where j runs over all existing vertices. In our simulation, we set $N = 10000$ for both kinds of graphs. And we set $m_0 = m = 5$ for BA networks, $L_{\max} = 49985$ for an ER random network, so that all networks have the same density of links.

In the PD or SG, each player can either 'cooperate' (invest in a common good) or 'defect' (exploit the other's investment). Initially, equal percentages of cooperators or defectors were randomly distributed among the agents (vertices) of the population. At each time step, the agents play the game with their neighbors and get payoffs according to the game rules. In the PD, a defector exploiting a cooperator gets an amount T and the exploited cooperator receives S. Two players both receive R upon mutual cooperation and P upon mutual defection, such that $T > R > P > S$. Thus in a single play of the game, each player should defect [35]. In the Snowdrift Game (SG), the order of P and S is exchanged, such that $T > R > S > P$ and thus SG is more in favor of cooperation. We rescale the games such that each depends on a single parameter [33,34]. For the PD, we choose the payoffs to have the values $T = b > 1$, $R = 1$, and $P = S = 0$, where the only parameter, $1 \leq b \leq 2$, represents the advantage of defectors over cooperators. This selection of values preserves the essentials of the evolutionary Prisoners' Dilemma. None of our findings will be qualitatively changed if one sets P to a value that is positive but significantly below 1.0 (so that $T > R > P > S$ is strictly satisfied). For the SG, we make $T = 1 + \beta$, $R = 1$, $S = 1 - \beta$, and $P = 0$ with $0 \leq \beta \leq 1$ as the only parameter.

The evolution or learning strategy is carried out by implementing the finite population analogue of replicator dynamics [30,34]. In each step, all pairs of directly connected individuals x and y engage in a single round of a given game. The total payoff of agent i for the step is stored as P_i and the accumulative payoff (wealth) of agent i since the beginning of the simulation is stored as W_i. Then the strategy of each agent (cooperate or defect) is updated in parallel according to the richest-following rule: whenever a site x is updated, a neighbor y with the most payoff P_y in this time step is drawn among all its k_x neighbors (where k_x is the connectivity or degree of site x), and then the site x will copy the strategy of the chosen neighbor y. This mechanism is adopted to reflect the common practice of agents in the economy: that they will probably learn from richest neighbors.

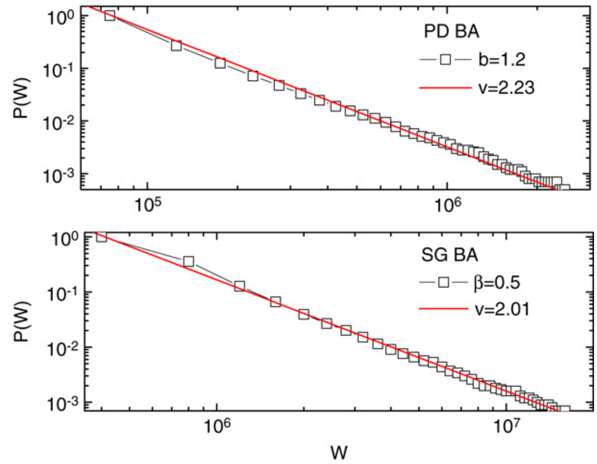

Fig. 1. (Color online.) Wealth distribution $P(W)$ for $N = 10^4$ agents playing PD with $b = 1.2$ (top) and SG with $\beta = 0.5$ (bottom) on a BA scale-free network for 10^5 steps. The frequency of cooperators is 0.10 and 0.65 respectively. The maximum personal wealth is about 5×10^6 and 5×10^7 respectively.

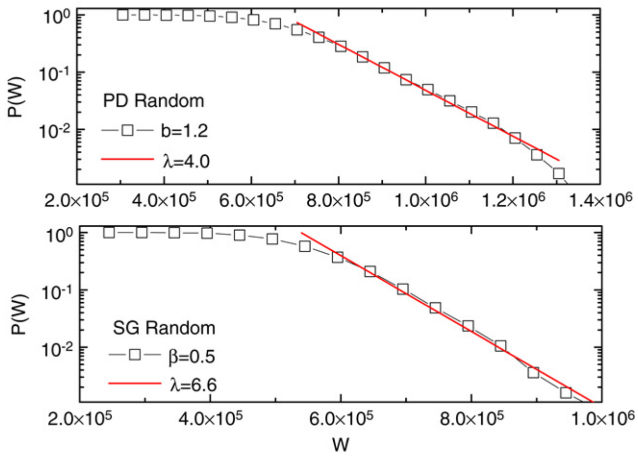

Fig. 2. (Color online.) Wealth distribution $P(W)$ for $N = 10^4$ agents playing PD with $b = 1.2$ and SG with $\beta = 0.5$ on an ER random graph for 10^5 steps. The frequency of cooperators is 0.65 and 0.31. And the maximum personal wealth is about 1.4×10^6 and 1.0×10^6 respectively.

3. Numerical simulation results

We carry out the simulation for a population of $N = 10^4$ agents occupying the vertices of a network. The distributions of wealth, the Pareto exponents and the Gini coefficients were obtained after a time period of $T = 10^5$ steps. For each value of b or β in the PD or SG, we carry out ten times iterative simulations on ten different network realizations, and the averaged results are presented.

We first examine the wealth distribution $P(W)$ of the system. Fig. 1 shows the $P(W)$ for BA scale-free social networks. One can see that both PD and SG charts show power law distribution of personal wealth which is in agreement with Pareto's law with $\upsilon = 2.23$ and $\upsilon = 2.01$ respectively. We perform different simulations by altering the values of b and β, and the results show similar wealth distributions with a robust power law. For different simulations, the exponential factor υ varies between 1.8 and 2.5, which is in agreement with the empirical values observed in the high income group [11–14]. In the simulation, the value of υ changes because of the networks' detailed structure, the game parameters and the initial distribution of C and D agents in the system. We also note that the power law persists for both high and low cooperator's frequency cases. Although the cooperation frequency changes with different parameters b or β, the model is very robust for reproducing the Pareto law of wealth distribution. And the wealth distribution is independent of the system size N and the simulation time T.

Fig. 2 shows the $P(W)$ for ER random social networks on a log-linear scale. One can see that there is an exponential decay in the wealth distribution:

$$P(W) \sim \exp(-\lambda W) \tag{2}$$

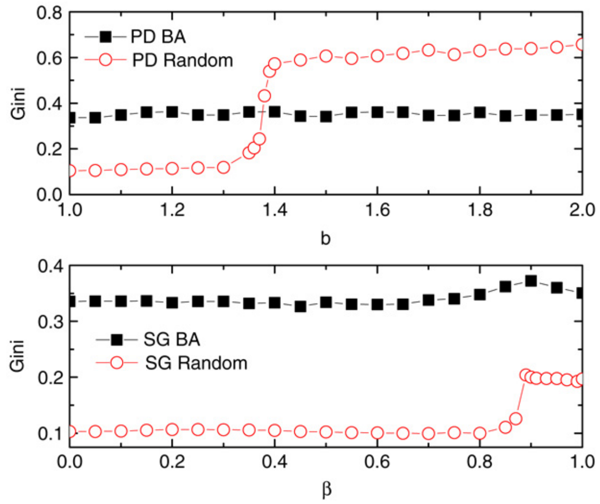

Fig. 3. (Color online.) The Gini coefficient for PD (top) and SG (bottom).

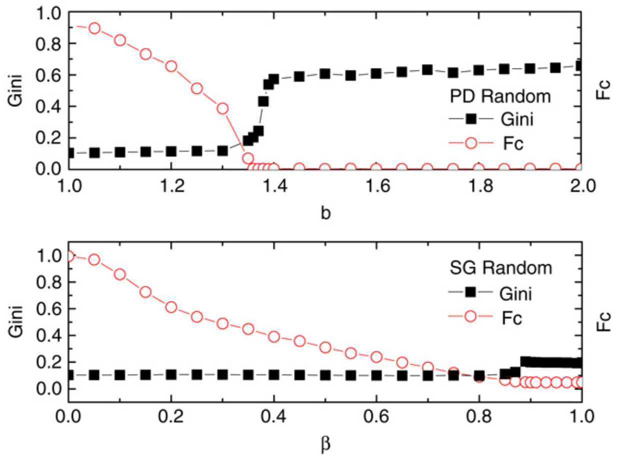

Fig. 4. (Color online.) The sudden increment of the Gini coefficient (Gini) with the decrement of the cooperation frequency (F_c) in ER random graphs for PD (top) and SG (bottom).

with λ taking different values for different gaming parameters. This distribution is in agreement with Eq. (1) for low income population.

Next we investigate the Gini coefficient for the system. Fig. 3 shows the variation of the Gini coefficient with different game parameters for PD and SG. One can see that the value remains almost constant for a BA network (Gini ≈ 0.35) in both games. This is mainly because a scale-free network is more heterogeneous in agents' connectivity and this will lead to heterogeneity of the wealth distribution. On the other hand, the value shows a sudden increment for an ER random graph in both games. For PD, there is a sharp increment around $b = 1.35$ associated with a sudden decrement in the cooperation frequency (F_c), as shown in Fig. 4. The value of the Gini coefficient becomes even larger than in the BA scale-free network when $b > 1.35$. That is, when b is large, the system become less cooperative and the social wealth will be more unequally distributed. We also note that the total social wealth for the system will decrease when $b > 1.35$. This situation is no good for the society. For SG, the increment in the Gini coefficient happens at around $\beta = 0.9$. This is because the SG payoff matrix is intrinsically more in favor of cooperation. Thus the transition is more difficult, and the value of the Gini coefficient remains below that of BA networks.

Fig. 5 shows the relation of personal wealth to its connectivity K for a BA scale-free network and an ER random graph in the case of PD and SG. One can see that in all cases the personal wealth is roughly proportional to its connectivity. That is, the agents with more contacts will simply have more wealth. This behavior will not be affected by the learning strategy or by the strategy series that the agents follow in each time step. Although everybody tries their best to gain more in the game, the final score is determined by the connectivity. Since the connectivity can also represent one's information resources, this phenomenon is in accordance with the fact that in a modern economy, agents with more information resources can gain more profit.

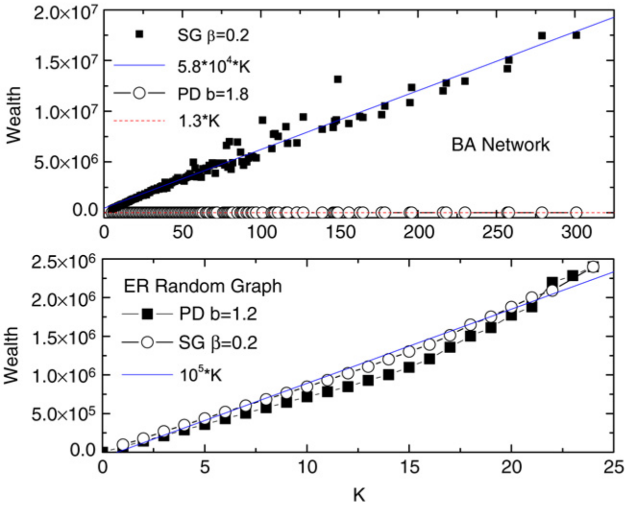

Fig. 5. (Color online.) K–wealth relation. The cooperation frequency is 0.47 (SG BA), 0.01 (PD BA), 0.59 (SG Random) and 0.67 (PD random).

The linear relation between personal wealth and its connectivity is also a possible mechanism for the emergence of the Matthew Effect in the economy. In Fig. 5, one can see that with both PD and SG, the wealth of the agent with more connectivity exceeds that of the agent with less connectivity. With the simulation time T, the agents with more partners will get richer and richer, while those with fewer partners will get relatively poorer. Thus the difference between the agents will be enlarged with time. Successful people (company, country etc) usually have more partners than ordinary ones, and the large relation network will make them even more successful.

4. Conclusion and discussions

In summary, the wealth distribution in an economy is modeled using evolutionary game theory on different social networks. The wealth distribution of the system is investigated together with the Pareto exponent and the Gini coefficient. The wealth of scale-free networks follows a power law with the exponent in agreement with empirical observations, while the wealth distribution of a random network shows an exponential decay. One can suggest that the social network takes different structures for high and low income groups. From the presented simulation results, we suggest that the network for the high income group has a heterogeneous scale-free structure, in which some people or agents have more links than others. For the low income population, the network is more like a random graph.

We also investigated the Gini coefficient for the systems. We found that the Gini coefficient of the scale-free network will remain as Gini ~ 0.35. There is a sudden increment in the value of the Gini coefficient for an ER random graph.

The simulation show that the agents' personal wealth is proportional to its number of contacts (connectivity). This mechanism can lead to the phenomenon that the rich get richer and the poor get relatively poorer with time (the Matthew Effect). Thus, one should pay more attention to the strategy of increasing partners, because although everyone tries their best in an economy, the final profit is simply determined by the connectivity of partners. This mechanism also explains why agents with more information resources can gain more profit in modern society.

Acknowledgements

This work was funded by the National Basic Research Program of China (No. 2006CB705500), the NNSFC under Key Project No. 10532060, Project Nos. 70601026, 10672160, the CAS President Foundation, and the China Postdoctoral Science Foundation (No. 20060390179). Y.-H. Wu acknowledges the support of the Australian Research Council through a Discovery Project Grant.

References

[1] D.J. Watts, S.H. Strogatz, Nature 393 (1998) 440.
[2] A.L. Barabási, R. Albert, Science 286 (1999) 509.
[3] R. Albert, A.L. Barabási, Rev. Mod. Phys. 74 (2002) 47.
[4] F.C. Santos, J.M. Pacheco, Phys. Rev. Lett. 95 (2005) 098104;
F.C. Santos, J.F. Rodrigues, J.M. Pacheco, Proc. Roy. Soc. London B 273 (2006) 51.
[5] R. Albert, A.L. Barabási, Phys. Rev. Lett. 85 (2000) 5234;
L.A.N. Amaral, A. Scala, et al., Proc. Natl. Acad. Sci. U.S.A. 97 (2000) 11149.
[6] A.L. Barabási, H. Jeong, E. Ravasz, et al., Physica A 311 (2002) 3–4.

[7] F. Liljeros, C.R. Edling, L.A.N. Amaral, et al., Nature (London) 411 (2001) 907.
[8] V. Pareto, Le Cours d'Economique Politique, Macmillan, Lausanne, Paris, 1987.
[9] A. Chatterjee, B.K. Chakrabarti, S.S. Manna, Physica A 335 (2004) 155.
[10] A. Chatterjee, S. Yarlagadda, B.K. Chakrabarti (Eds.), Econophysics of Wealth Distributions, Springer-Verlag, Milan, 2005.
[11] A. Chatterjee, B.K. Chakrabarti, Eur. Phys. J. B 60 (2007) 135.
[12] A.A. Dragulescu, V.M. Yakovenko, Physica A 299 (2001) 213.
[13] S. Moss de Oliveira, P.M.C. de Oliveira, D. Stauer, Evolution, Money, War and Computers, B.G. Tuebner, Stuttgart, Leipzig, 1999.
[14] H. Aoyama, W. Souma, Y. Nagahara, et al., Fractals 8 (2000) 293.
[15] Y. Fujiwara, W. Souma, H. Aoyama, et al., Physica A 321 (2003) 598.
[16] M. Levy, S. Solomon, Physica A 242 (1997) 90.
[17] M.H.R. Stanley, S. Buldyrev, S. Havlin, et al., Econom. Lett. 49 (1995) 453.
[18] K. Okuyama, M. Takayasu, H. Takayasu, Physica A 269 (1999) 125.
[19] S. Sinha, Physica A 359 (2006) 555.
[20] The definitions of Gini coefficient and Lorenz curve can be found in the Wikipedia- http://www.wikipedia.com.
[21] R.K. Merton, Science 159 (1968) 56; ISIS 79 (1988) 606.
[22] M. Bonitz, E. Bruckner, A. Scharnhorst, Scientometrics 40 (3) (1997) 407; Scientometrics 64 (3) (2005) 375.
[23] D.F. Brewer, Physics Today 44 (10) (1991) 154.
[24] R.H. Wade, Inter. J. Health Services 35 (4) (2005) 631.
[25] M.B. Hu, W.X. Wang, R. Jiang, et al., Eur. Phys. J. -B 53 (2006) 273.
[26] M.B. Hu, R. Jiang, Q.S. Wu, Y.H. Wu, Physica A 381 (2007) 476.
[27] A. Chakrabarti, B.K. Chakrabarti, Eur. Phys. J. -B 17 (2000) 167.
[28] A. Dragulascu, V.M. Yakovenko, Eur. Phys. J. -B 17 (2000) 723.
[29] Y. Wang, N. Ding, L. Zhang, Physica A 324 (2003) 665.
[30] H. Gintis, Game Theory Evolving, Princeton University, Princeton, NJ (2000).
[31] A.M. Colman, Game Theory and Its Applications in the Social and Biological Sciences, Butterworth-Heinemann, Oxford, 1995.
[32] G. Szabo, G. Fath, preprint:cond-mat0607344(2006).
[33] M. Nowak, K. Sigmund, Nature (London) 359 (1992) 826.
[34] C. Hauert, M. Doebeli, Nature 428 (2004) 643.
[35] J.M. McNamara, Z. Barta, A.I. Houston, Nature 428 (2004) 745.
[36] W.X. Wang, J. Ren, G. Chen, B.H. Wang, Phys. Rev. E 74 (2006) 056113.
[37] C.L. Tang, W.X. Wang, X. Wu, B.H. Wang, Eur. Phys. J. -B 53 (2006) 411.
[38] C.L. Tang, B.Y. Lin, W.X. Wang, M.B. Hu, B.H. Wang, Phys. Rev. E 75 (2007) 027101.