A Game-Theoretic Approach for Enabling Transactive Energy Frameworks among Networked Microgrids

KKT Formulation and Solving Technique for EPEC problem

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Nomenclature

Indexes

b	Index of energy storage units in set B .
g	Index of distributed generators in set G .
h	Index of transmission grid interface in set H .
i,j	Indices of nodes in set I .
l	Index of demands in set L .
m, k	Index of microgrids in set M .
r	Index of energy renewable resource units in set R .
t	Index of times in set T .
Sets	
Λ_m^B	Set of energy storage units in microgrid m .
Λ_m^L	Set of demands in microgrid m .
Λ_m^R	Set of renewable resource units in microgrid m .
Θ_i^L	Set of demands connected to node i .

 Θ_i^M Set of microgrids connected to node i.

 Θ_i^T Set of transmission grid interface connected to node i.

 Θ_m^G Set of distributed generators in microgrid m.

B(i) Set of nodes directly connected to node i.

REF Reference bus in distribution network.

Parameters

 ηb_m Efficiency of energy storage system in microgrid m (%).

 $b_{i,j}$ Susceptance of line connecting node i and j.

 C_m^E Cost of loss-of-load in microgrid m (\$/kWh).

Price of buying electricity by the distribution system operator from transmission grid at time t ($^{\text{Q}}$ / $^{\text{LWh}}$)

at time t (\$/kWh).

 Cb_m Cost of charging/discharging storage resources in microgrid m (\$/kWh).

 Cg_m Cost of power production of distributed generator in microgrid m (\$/kWh).

 $D_{m,t}$ Power demand in microgrid m at time t (kW).

 EB_m Energy capacity of installed energy storage system in microgrid m (kWh).

 $Gr_{m,t}$ Power generated by renewable resources in microgrid m at time t (kW).

 $L_{i,t}$ Power demand of electric loads at node i at time t in distribution system (kW).

 $P^{max,TM}$ Maximum power exchange in transactive market (kW).

 $P^{max,TS}$ Maximum power purchased from transmission grid (kW).

 P_m^{max} Maximum power sold/bought by microgrid m (kW).

 $P_{i,j}^{f,max}$ Maximum power flow through branch ij (kW).

 Pd_m Power rating of energy storage system of microgrid m (kW).

 Pg_m^{max} Maximum power output of the distributed generator in microgrid m (kW).

 SoC_m^{max} Maximal state of charge of energy storage system in microgrid m.

 SoC_m^{min} Minimal state of charge of energy storage system in microgrid m.

Variables

 $\mu_{m,t}^{b,D}$ Binary variable of microgrid m at time t to indicate its power purchasing status in the DSO based market, 1 means the microgrid is purchasing power in DM, 0 means otherwise.

$\mu_{m,t}^{b,T}$	Binary variable of microgrid m at time t to indicate its power purchasing status in the transactive market, 1 means the microgrid is purchasing power in TM, 0 means otherwise.
$\mu_{m,t}^{s,T}$	Binary variable of microgrid m at time t to indicate its power selling status in the transactive market, 1 means the microgrid is selling power in TM, 0 means otherwise.
$\mu_{m,t}^{s,D}$	Binary variable of microgrid m at time t to indicate its power selling status in the DSO based market, 1 means the microgrid is selling power in DM, 0 means otherwise.
$ heta_{i,t}$	Voltage phase angle of bus i at time t .
$b_{m,t}$	Bidding price of microgrid m in the transactive market at time t (kWh).
$o_{m,t}$	Offering price of microgrid m in the transactive market at time t (\$/kWh).
P_t^{TS}	Power purchased by distribution system operator from transmission system at time t (kW).
$P_{m,t}^{b,DM}$	Power bought from distribution system operator in the DSO based market by microgrid m (kW).
$P_{m,t}^{b,TM}$	Power bought in the transactive market by microgrid m (kW).
$P_{m,t}^{s,DM}$	Power sold to the distribution system operator in the DSO based market by microgrid m (kW).
$P_{m,t}^{s,TM}$	Power sold in the transactive market by microgrid m (kW).
$P_{m,t}^{sh}$	Load shedding of microgrid m at time t (kW).
$Pd_{m,t}/Pc_{m,t}$ Discharge/charge power of energy storage system in microgrid m at time t (kW).	
$Pg_{m,t}$	Power output of distributed generator in microgrid m at time t (kW).
$PL_{i,t}$	Load shedding at node i in the distribution network at time t (kW).
$Pr_{m,t}$	Power used by microgrid m at time t from renewable sources (kW).
$RC_{m,t}$	Stored energy in energy storage system in microgrid m at time t (kWh).

1 Problem description

This work is the detailed mathematical formulation of "A Game-Theoretic Approach for Enabling Transactive Energy Frameworks among Networked Microgrids".

2 System Formulation

2.1 Upper Level Problem – Microgrid Operator

2.2Lower Level Problem – Transactive Energy Operator

$$\min_{\Xi_{TEO}} \sum_{t} \sum_{m} \left(o_{m,t} P_{m,t}^{s,TM} - b_{m,t} P_{m,t}^{b,TM} \right) \tag{20}$$

s.t.
$$\sum_{m} \left(P_{m,t}^{s,TM} - P_{m,t}^{b,TM} \right) = 0 \quad (\lambda_t^{TM})$$
 $\forall t$ (21)

$$0 \le P_{m,t}^{s,TM} \le P^{max,TM} \mu_{m,t}^{s,T} \quad (\gamma_{m,t}^{sTMmin}, \gamma_{m,t}^{sTMmax}) \qquad \forall t \qquad (22)$$

$$0 \leq P_{m,t}^{s,TM} \leq P^{max,TM} \mu_{m,t}^{s,T} \quad (\gamma_{m,t}^{sTMmin}, \gamma_{m,t}^{sTMmax}) \qquad \forall t \qquad (22)$$

$$0 \leq P_{m,t}^{b,TM} \leq P^{max,TM} \mu_{m,t}^{b,T} \quad (\gamma_{m,t}^{bTMmin}, \gamma_{m,t}^{bTMmax}) \qquad \forall t \qquad (23)$$

Where $\Xi_{TEO} = \{P_{m,t}^{s,TM}, P_{m,t}^{b,TM}\}.$

2.2.1Optimally conditions for the Transactive Energy Operator

$$\sum_{m,t} \left(o_{m,t} P_{m,t}^{s,TM} - b_{m,t} P_{m,t}^{b,TM} + \gamma_{m,t}^{sTMmax} P^{max,TM} \right)$$

$$+ \gamma_{m,t}^{bTMmax} P^{max,TM} \right) = 0 \quad (\beta_{m,t}^{str})$$

$$c_{m,t} - \lambda_t^{TM} - \gamma_{m,t}^{sTMmin} + \gamma_{m,t}^{sTMmax} = 0 \quad (\beta_{m,k,t}^{oTM})$$

$$- b_{m,t} + \lambda_t^{TM} - \gamma_{m,t}^{bTMmin} + \gamma_{m,t}^{bTMmax} = 0 \quad (\beta_{m,k,t}^{bTM})$$

$$\gamma_{m,t}^{sTMmin} \ge 0 \quad (\beta_{m,k,t}^{\gamma,smin})$$

$$\gamma_{m,t}^{sTMmax} \ge 0 \quad (\beta_{m,k,t}^{\gamma,smax})$$

$$\gamma_{m,t}^{bTMmin} \ge 0 \quad (\beta_{m,k,t}^{\gamma,bmin})$$

$$\gamma_{m,t}^{bTMmin} \ge 0 \quad (\beta_{m,k,t}^{\gamma,bmin})$$

$$\gamma_{m,t}^{bTMmax} \ge 0 \quad (\beta_{m,k,t}^{\gamma,bmin})$$

$$\gamma_{m,t}^{bTMmin} \le \rho_{m,t}^{bTMmin} \beta_{m,t}^{bTMmin} \beta_{m,t}^{bTMmax}$$

$$0 \le P_{m,t}^{b,TM} \le P^{max,TM} \mu_{m,t}^{b,T} \quad (\beta_{m,k,t}^{bTMmin}, \beta_{m,k,t}^{bTMmax})$$

$$\gamma_{m,t}^{bTMmax} \ge \rho_{m,t}^{bTMmin} \beta_{m,t}^{bTMmax}$$

$$\gamma_{m,t}^{bTMmax} \le \rho_{m,t}^{bTMmin} \beta_{m,t}^{bTMmax}$$

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$$\gamma_{m,t}^{bTMmax} \le \rho_{m,t}^{bTMmax} \beta_{m,t}^{bTMmax}$$

$$\gamma_{m,t}^{bTMmax} \ge \rho_{m,t}^{bTMmax} \beta_{m,t}^{bTMmax} \beta_{m,t}^{bTMmax}$$

$$\gamma_{m,t}^{bTMmax} \ge \rho_{m,t}^{bTMmax} \beta_{m,t}^{bTMmax} \beta_{m,t}^{bTMmax} \beta_{m,t}^{bTMmax} \beta_{m,t}^{bTMmax} \beta_{m,t}^{bTMmax} \beta_{m,t}^{bTMmax} \beta_{m,t}^{bTMmax}$$

2.3 Lower Level Problem – Distribution System Operator

$$\min_{\Xi_{DSO}} \sum_{t} \left(C_{t}^{TS} P_{t}^{TS} + C_{m}^{E} P L_{i,t} \right) \tag{34}$$
s.t.
$$\sum_{\Theta_{i}^{T}} P_{t}^{TS} + \sum_{\Theta_{i}^{M}} \left(P_{m,t}^{s,TM} + P_{m,t}^{s,DM} - P_{m,t}^{b,TM} - P_{m,t}^{b,DM} \right)$$

$$+ \sum_{j \in B(i)} \left(b_{i,j} \left(\delta_{i,t} - \delta_{j,t} \right) \right) = \sum_{\Theta_{i}^{L}} \left(L_{i,t} - P L_{i,t} \right) \left(\lambda_{t}^{DM} \right) \qquad \forall i, t \quad (35)$$

$$- P_{i,j}^{fmax} \leq b_{i,j} \left(\delta_{i,t} - \delta_{j,t} \right) \leq P_{i,j}^{fmax} \left(\rho_{i,j,t}^{fmin}, \rho_{i,j,t}^{fmax} \right) \qquad \forall i, j \in B(i), \forall t \quad (36)$$

$$0 \leq P_{t}^{TS} \leq P^{max,TS} \quad \left(\rho_{i,t}^{pTSmin}, \rho_{i,t}^{bTMmax} \right) \qquad \forall i, t \quad (37)$$

$$0 \leq P L_{i,t} \leq L_{i,t} \quad \left(\rho_{i,t}^{PLmin}, \rho_{i,t}^{PLmax} \right) \qquad \forall i, t \quad (38)$$

$$\delta_{REF,t} = 0 \quad \left(\rho_{REF,t} \right) \qquad i = REF, \forall t \quad (39)$$

Where $\Xi_{TEO} = \{P_t^{TS}, PL_{i,t}, \delta_{i,t}\}.$

2.3.1 Complementarity constrains for the Distribution Network Operator

$$C_{t}^{TS} - \lambda_{t}^{DM} - \rho_{i,t}^{pTSmin} + \rho_{i,t}^{bTMmax} = 0 \qquad \forall i, t \qquad (40)$$

$$C_{m}^{E} - \lambda_{t}^{DM} - \rho_{i,t}^{pLmin} + \rho_{i,t}^{pLmax} = 0 \qquad \forall i, t \qquad (41)$$

$$C_{m}^{E} + \lambda_{t}^{DM} - \rho_{i,t}^{pLmin} + \rho_{i,t}^{pLmax} = 0 \qquad \forall i, t \qquad (42)$$

$$\sum_{j \in B(i)} \left(b_{i,j} \left(\lambda_{i,t} - \lambda_{j,t} \right) \right) - \sum_{j \in B(i)} \left(b_{i,j} \left(\rho_{i,j,t}^{fmin} - \rho_{j,i,t}^{fmin} \right) \right)$$

$$+ \sum_{j \in B(i)} \left(b_{i,j} \left(\rho_{i,j,t}^{fmax} - \rho_{j,i,t}^{fmax} \right) \right) + (\rho_{REF,t})_{i=REF} = 0 \qquad \forall i, t \qquad (43)$$

$$\left(b_{i,j} \left(\delta_{i,t} - \delta_{j,t} \right) - P_{i,j}^{fmax} \right) \perp \rho_{i,j,t}^{fmin} \qquad \forall i, j \in B(i), \forall t \qquad (44)$$

$$\left(P_{i,j}^{fmax} - b_{i,j} \left(\delta_{i,t} - \delta_{j,t} \right) \right) \perp \rho_{i,j,t}^{fmax} \qquad \forall i, j \in B(i), \forall t \qquad (45)$$

$$P_{t}^{TS} \perp \rho_{i,t}^{pTSmin} \qquad \forall i, j \in B(i), \forall t \qquad (46)$$

$$\left(P_{max,TS}^{TS} - P_{t}^{TS} \right) \perp \rho_{i,t}^{bTMmax} \qquad \forall i, j \in B(i), \forall t \qquad (47)$$

$$PL_{i,t} \perp \rho_{i,t}^{PLmin} \qquad \forall i, j \in B(i), \forall t \qquad (48)$$

$$(L_{i,t} - PL_{i,t}) \perp \rho_{i,t}^{PLmax} \qquad \forall i, j \in B(i), \forall t \quad (49)$$

$$(35) - (39)$$
 (50)

3 MPEC Formulation

Each microgrid operator solves its own bilevel problem in order to find its most beneficial decisions. The KKT conditions for each microgrid are shown in this section.

$$\begin{array}{c} Cg_{m} - \lambda_{m,t}^{MG} - \beta_{m,t}^{gmin} + \beta_{m,t}^{gmax} = 0 & \forall m,t \ \, (51) \\ C_{m}^{E} - \lambda_{m,t}^{MG} - \beta_{m,t}^{pshmin} + \beta_{m,t}^{pshmax} = 0 & \forall m,t \ \, (52) \\ Cb_{m} - \lambda_{m,t}^{MG} / \eta b_{m} - \beta_{m,t}^{pcmin} + \beta_{m,t}^{pcmax} = 0 & \forall m,t \ \, (53) \\ Cb_{m} + \lambda_{m,t}^{MG} / \eta b_{m} - \beta_{m,t}^{pcmin} + \beta_{m,t}^{pcmax} = 0 & \forall m,t \ \, (54) \\ \beta_{m,t}^{rct} - \beta_{m,t+1}^{rct} - \beta_{m,t}^{rmin} + \beta_{m,t}^{rmax} = 0 & \forall m,t \ \, (54) \\ \beta_{m,t}^{rct} - \beta_{m,t+1}^{rcmax} - \beta_{m,t}^{rmin} + \beta_{m,t}^{rmax} = 0 & \forall m,t \ \, (55) \\ \beta_{m,t}^{rct} + \beta_{m,t}^{rcmax} - \beta_{m,t}^{rmin} + \beta_{m,t}^{rmax} = 0 & \forall m,t \ \, (57) \\ - \lambda_{m,t}^{s.TM} - \beta_{m,t}^{b} + \beta_{m,t}^{prmax} = 0 & \forall m,t \ \, (57) \\ - P_{m,t}^{s.TM} - \beta_{m,t}^{b} + P_{m,t}^{str} / \beta_{m,t}^{str} + \beta_{m,m,t}^{oTM} = 0 & \forall m,t \ \, (58) \\ P_{m,t}^{s.TM} - \beta_{m,t}^{b} - P_{m,t}^{b.TM} / \beta_{m,t}^{str} + \beta_{m,m,t}^{stm} = 0 & \forall m,t \ \, (59) \\ - o_{m,t} + \lambda_{m,t}^{MG} + \beta_{m,t}^{smax} + o_{m,t} \beta_{m,m,t}^{str} + \beta_{m,m,t}^{TM} & \forall m,t \ \, (60) \\ o_{m,t} \beta_{m,t}^{str} + \beta_{m,k,t}^{TMmax} = 0 & \forall m,t \ \, (61) \\ b_{m,t} - \lambda_{m,t}^{bTMmin} + \beta_{m,k,t}^{bTMmin} + \beta_{m,k,t}^{bTMmax} = 0 & \forall m,t \ \, (62) \\ - b_{m,t} \beta_{m,t}^{str} - \beta_{m,k,t}^{TMmax} - \beta_{m,k,t}^{bTMmin} + \beta_{m,t}^{sDMmin} + \beta_{m,t}^{sDMmax} = 0 & \forall m,t \ \, (63) \\ - \sum_{e_i} \left(\lambda_{i,t}^{DM} \right) - \lambda_{m,t}^{MG} + \beta_{m,t}^{bmax} - \beta_{m,t}^{bDMmin} + \beta_{m,t}^{bDMmin} + \beta_{m,t}^{bDMmin} = 0 & \forall m,t \ \, (64) \\ \sum_{m,k,t} \left(\lambda_{i,t}^{bM} \right) - \lambda_{m,t}^{MG} + \beta_{m,t}^{bmax} - \beta_{m,t}^{bDMmin} + \beta_{m,t}^{bDMmin} + \beta_{m,t}^{bDMmin} = 0 & \forall m,t \ \, (65) \\ \beta_{m,k,t}^{oTM} - \beta_{m,k,t}^{o,smin} = 0 & \forall m,k,t \ \, (66) \\ \beta_{m,k,t}^{oTM} - \beta_{m,k,t}^{o,smin} = 0 & \forall m,k,t \ \, (68) \\ \beta_{m,k,t}^{bTM} - \beta_{m,k,t}^{o,smax} + P^{max,TM} \beta_{m,t}^{str} = 0 & \forall m,k,t \ \, (68) \\ \beta_{m,k,t}^{bTM} - \beta_{m,k,t}^{o,smax} + P^{max,TM} \beta_{m,t}^{str} = 0 & \forall m,k,t \ \, (69) \\ \end{pmatrix}$$

$$P_{m,t}^{b,TM} \perp \beta_{m,k,t}^{bTMmin} \qquad \forall m, k, t \quad (97)$$

$$\left(P^{max,TM} - P_{m,t}^{b,TM}\right) \perp \beta_{m,k,t}^{bTMmax} \qquad \forall m, k, t \quad (98)$$

$$(2) - (19), (21) - (23), (25) - (33), (35) - (49)$$
 (99)

The joint solution of each microgrid operator MPEC constitute the EPEC. Linearization techniques of the complementarity conditions that are found in this problem could be found in the following reference:

A. J. Conejo, L. Baringo, S. J. Kazempour, A. S. Siddiqui, "Investment in Electricity Generation and Transmission. Decision Making under Uncertainty" Springer: New York, New York, 2016.