Análisis de Eficiencia de Algoritmos

GRUPO 6 C2

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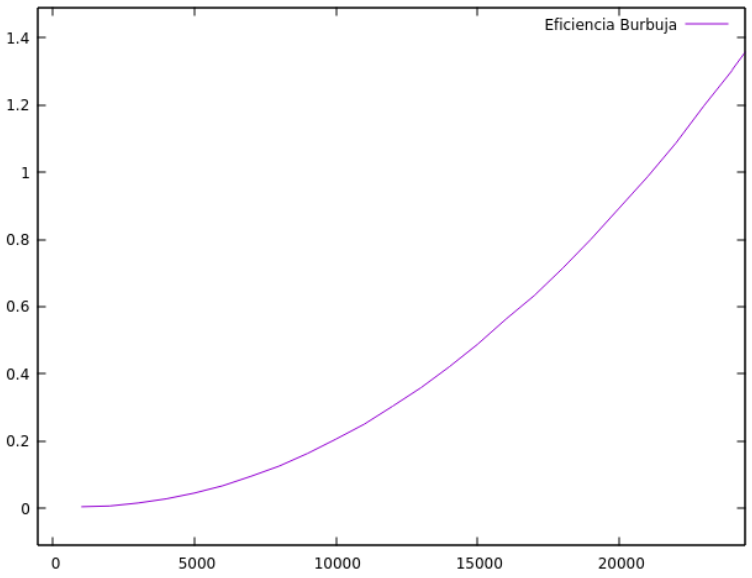
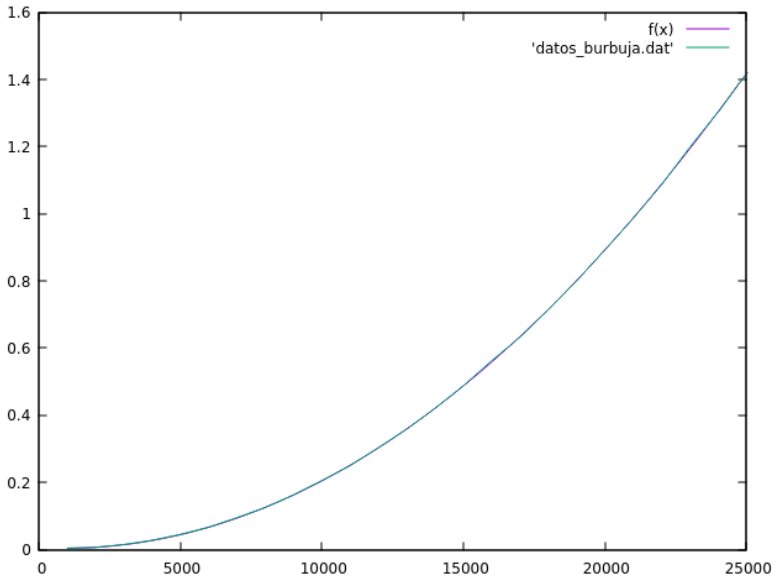
Eficiencia empírica de los algoritmos

Los algoritmos seleccionados:

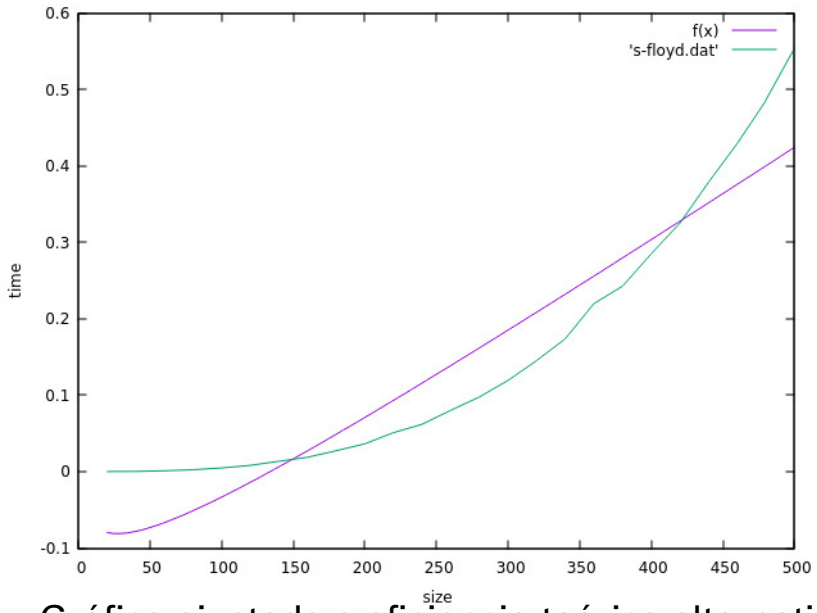
·Algoritmo de la burbuja ->O(n^2)
·Algoritmo de selección ->O(n^2)
·Algoritmo de inserción ->O(n^2)
·Algoritmo de quicksort ->O(n*log(n))
·Algoritmo de heapsort ->O(n*log(n))
·Algoritmo de mezcla -> O(n*log(n))
·Algoritmo de Floyd -> O(n^3)
·Algoritmo de Hanoi -> O(2^n)

Algoritmo de la burbuja



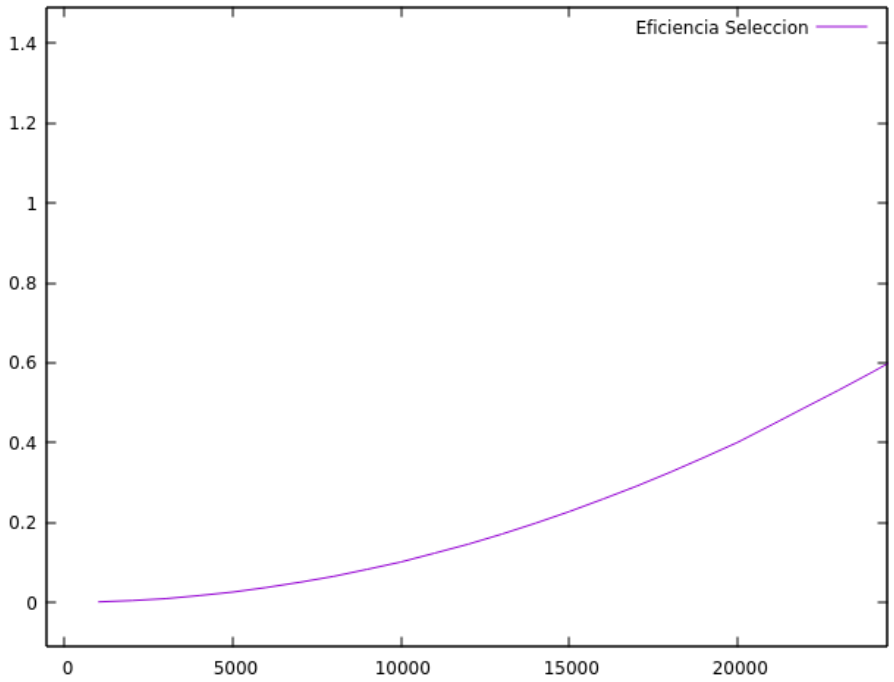


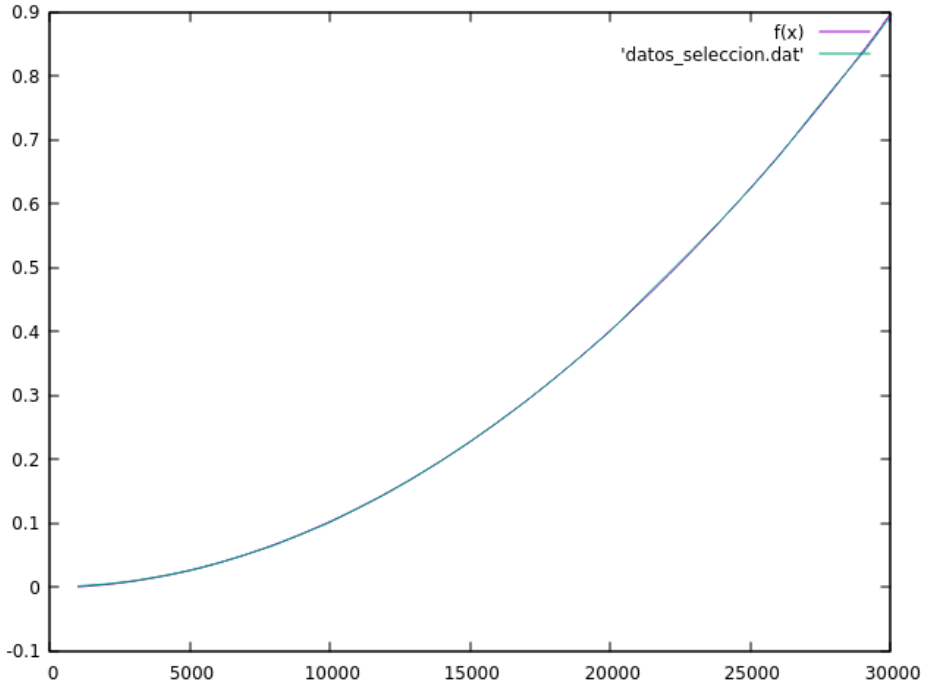
Gráfica ajustada a eficiencia teórica



Gráfica ajustada a eficiencia teórica alternativa

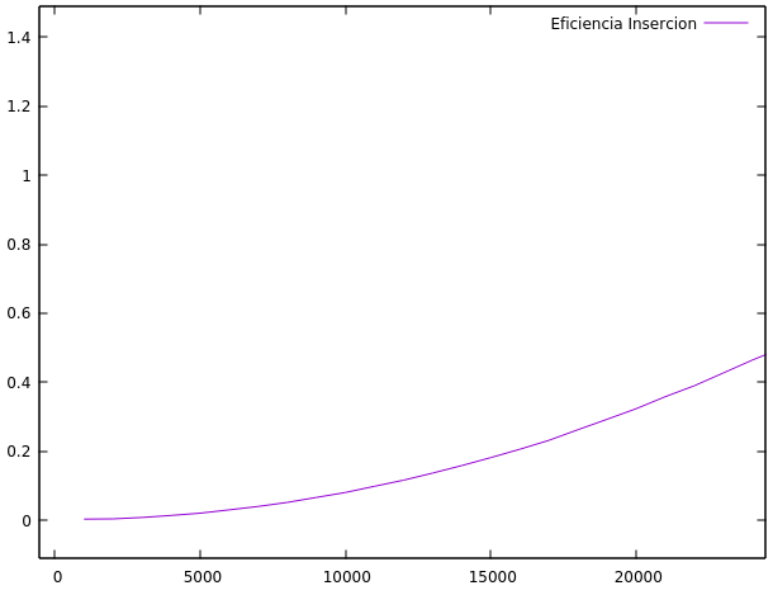
Algoritmo de selección

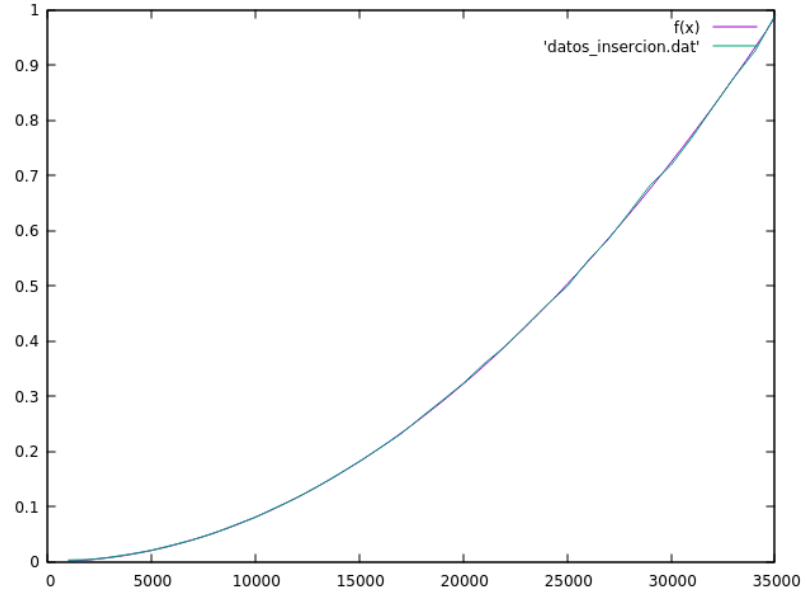




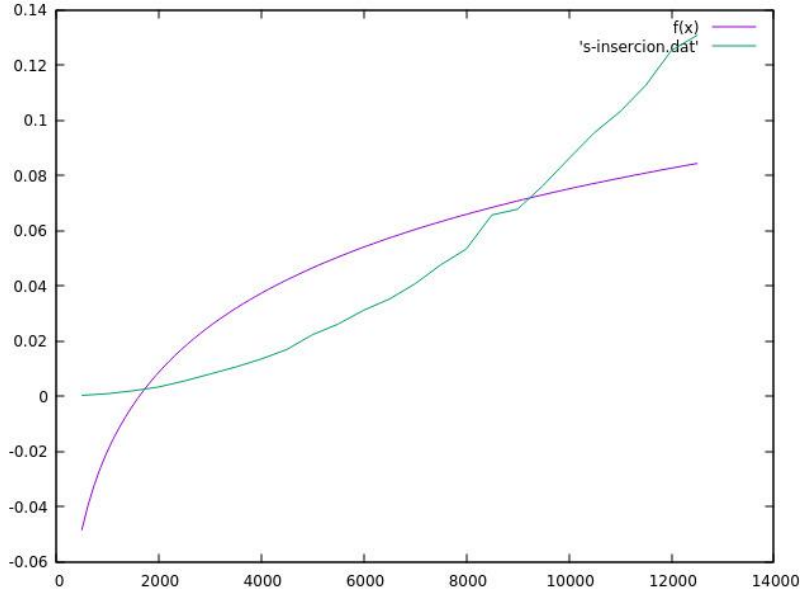
Gráfica ajustada a eficiencia teórica

Algoritmo de inserción



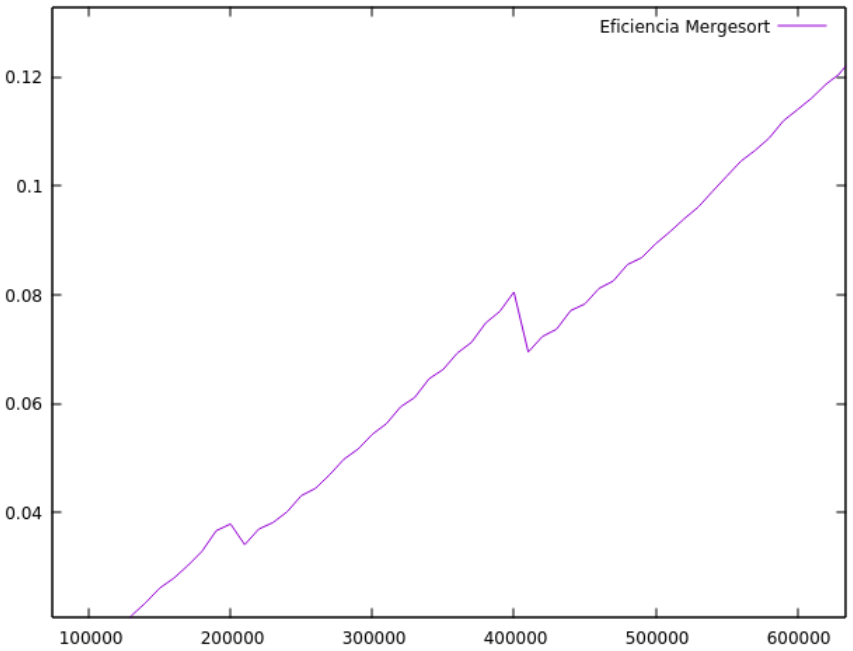


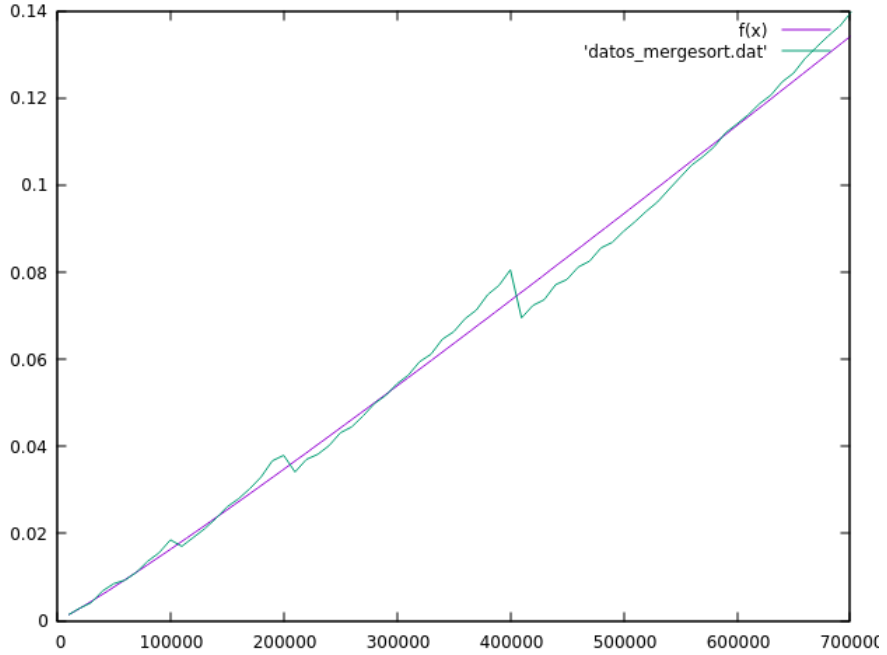
Gráfica ajustada a eficiencia teórica



Gráfica ajustada a eficiencia teórica alternativa

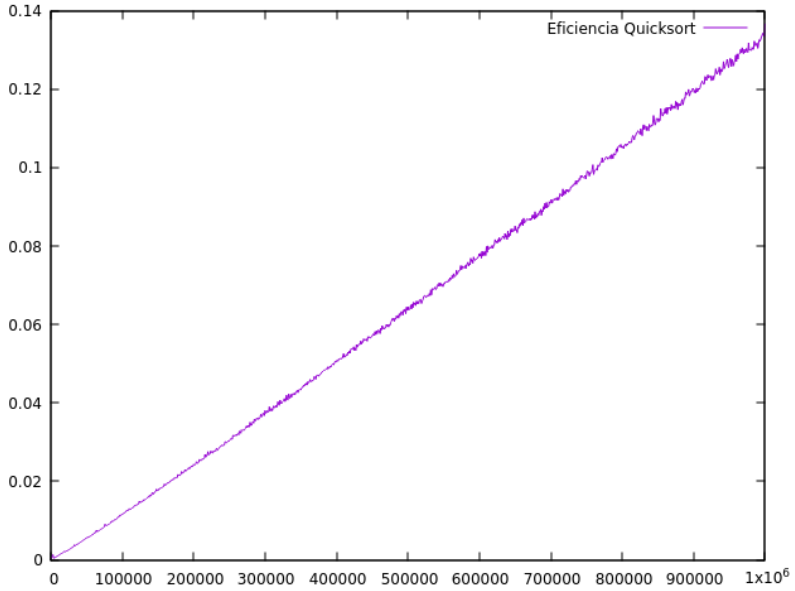
Algoritmo de mezcla (Mergesort)

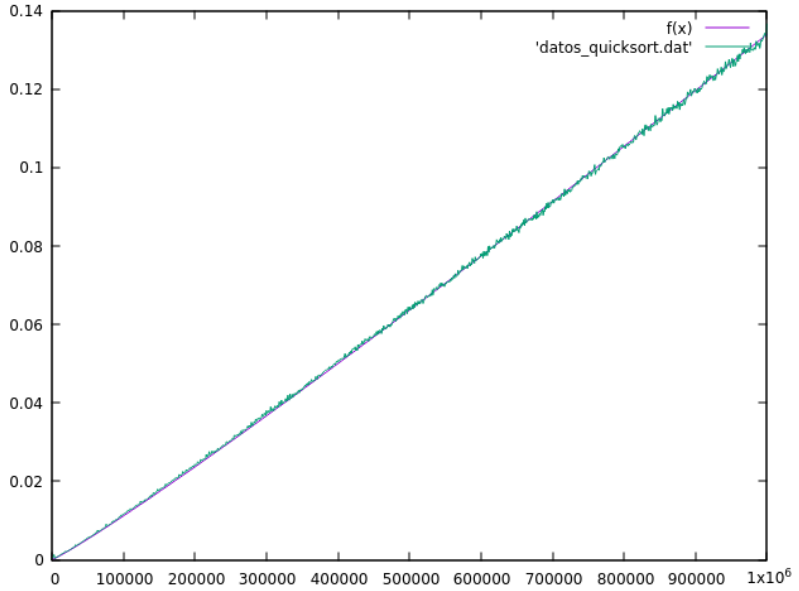




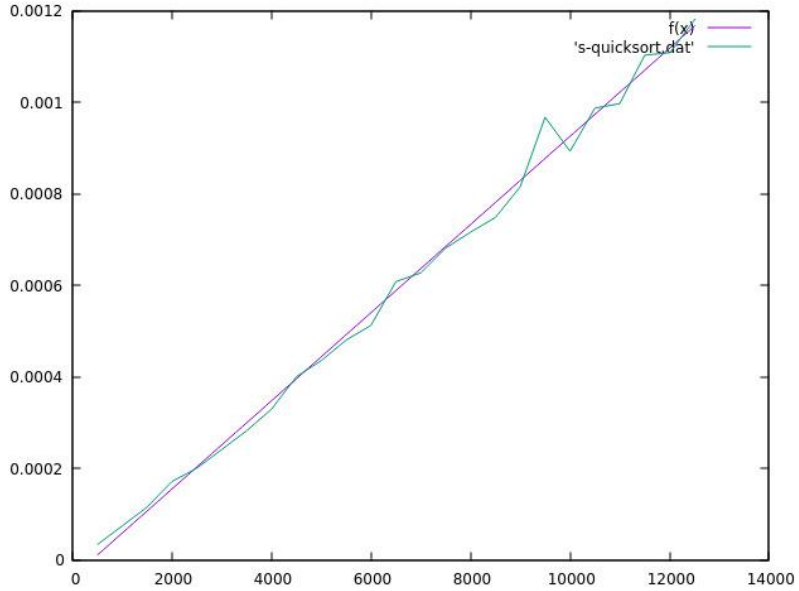
Gráfica ajustada a eficiencia teórica

Algoritmo de Quicksort



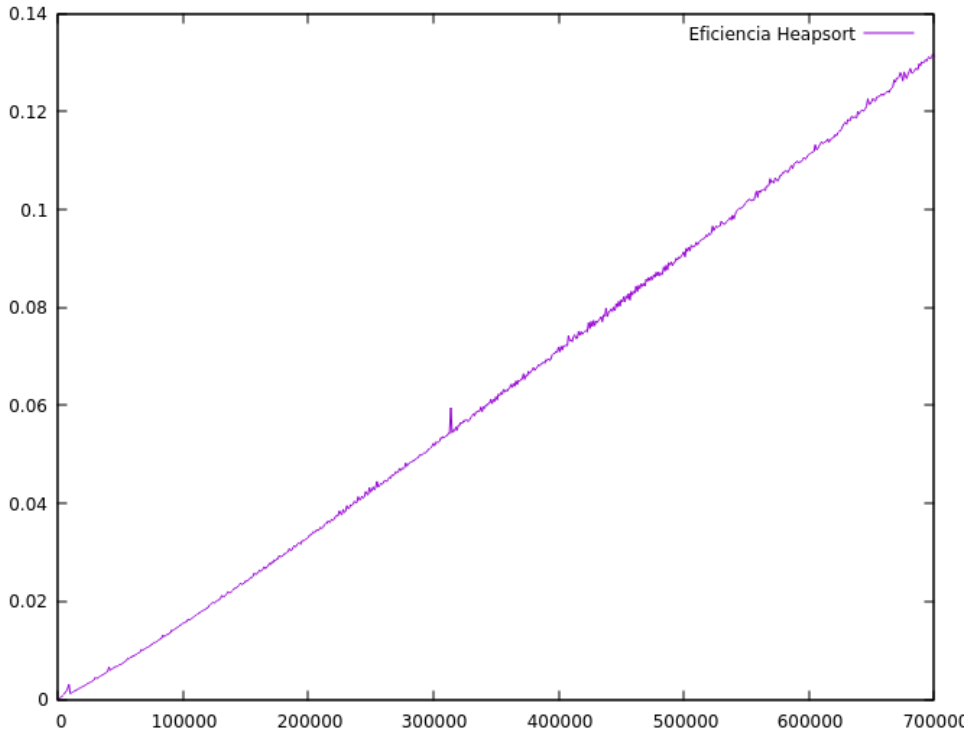


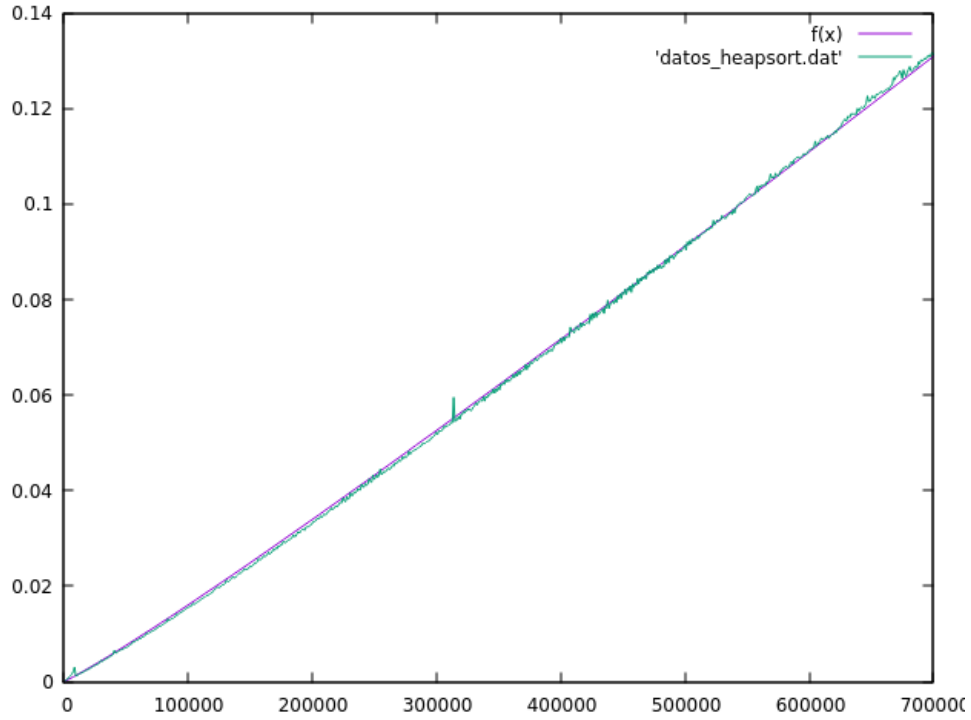
Gráfica ajustada a eficiencia teórica



Gráfica ajustada a eficiencia teórica alternativa

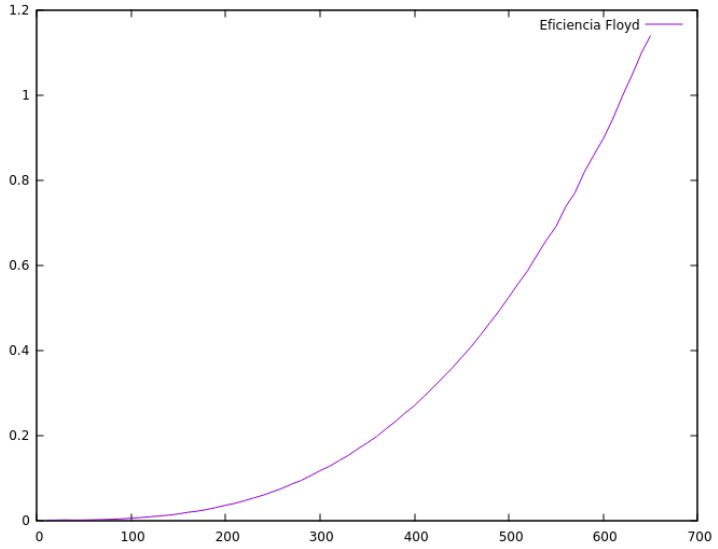
Algoritmo de Heapsort

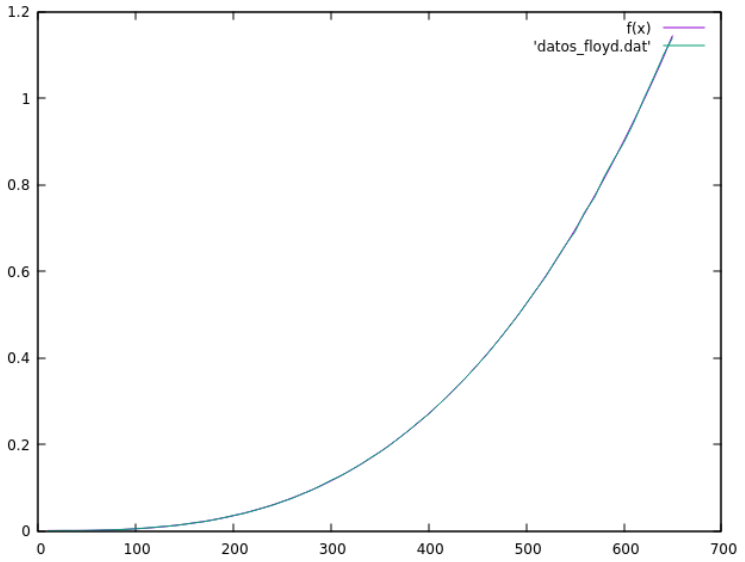




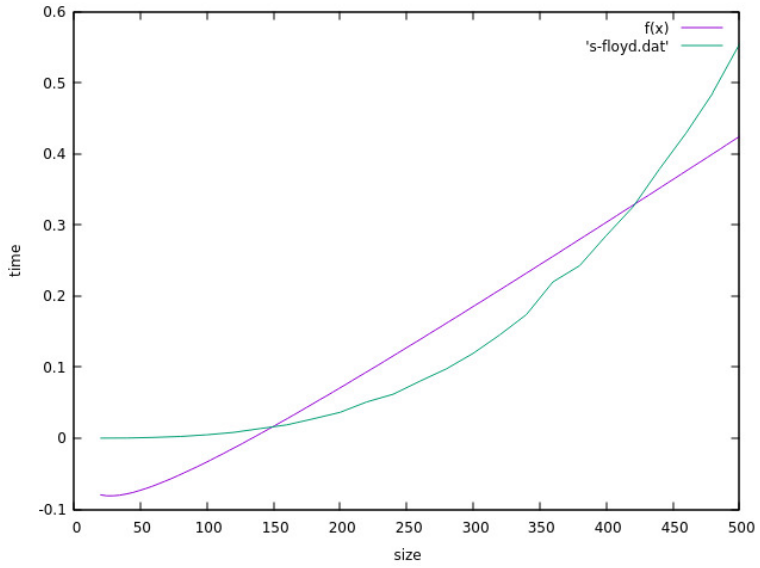
Gráfica ajustada a eficiencia teórica

Algoritmo de Floyd



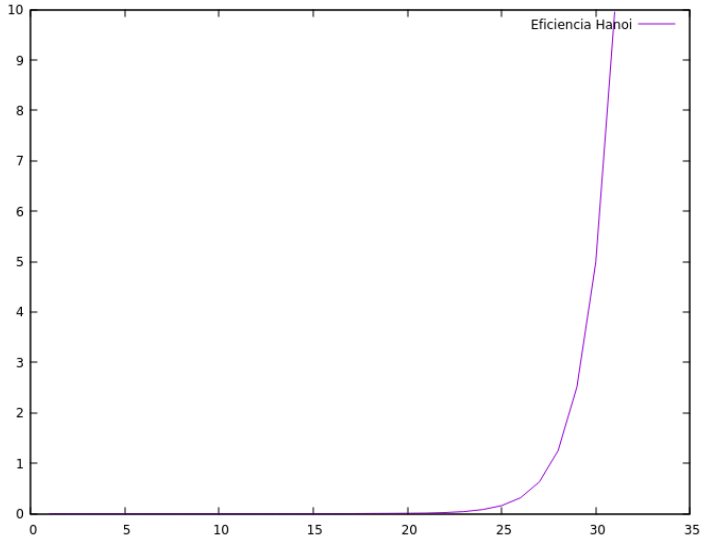


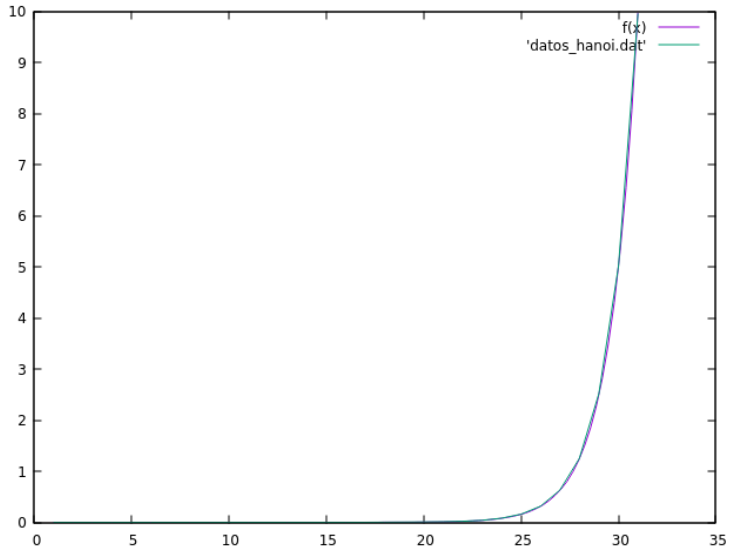
Gráfica ajustada a eficiencia teórica



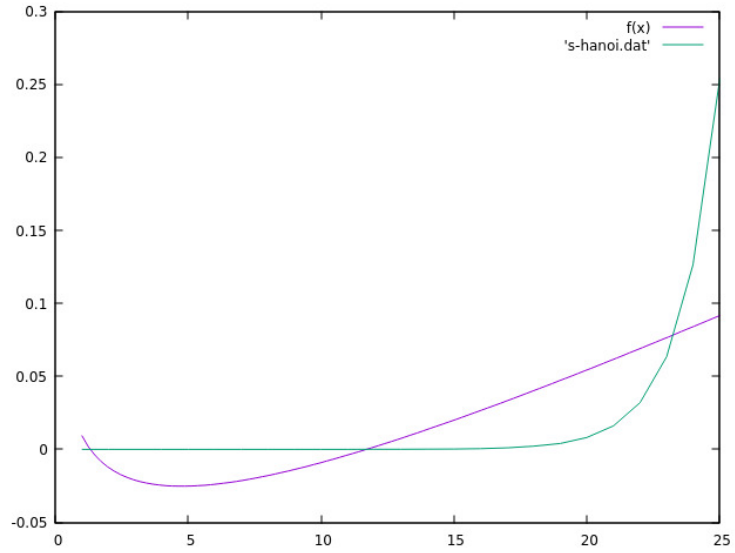
Gráfica ajustada a eficiencia teórica alternativa

Algoritmo de las torres de Hanoi



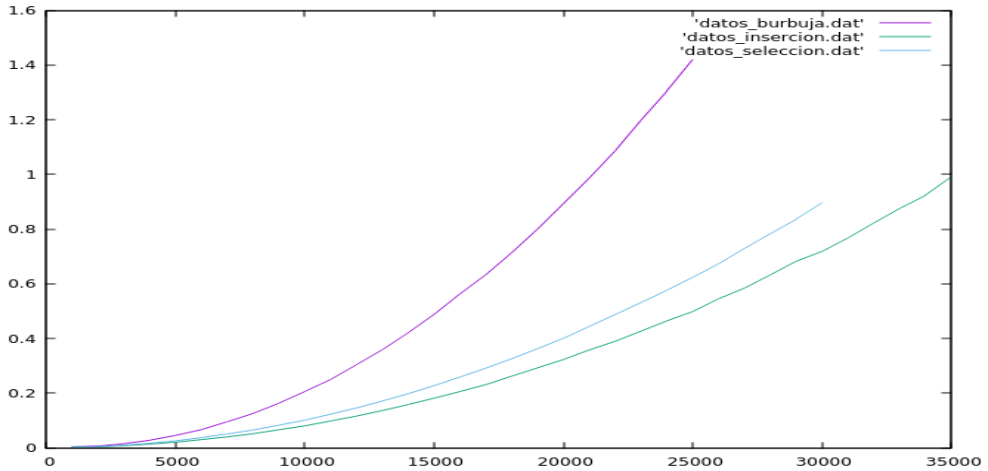


Gráfica ajustada a eficiencia teórica

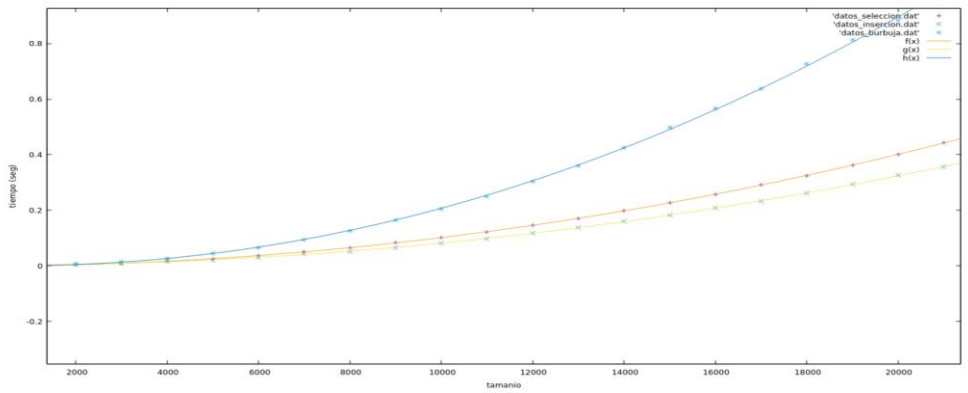


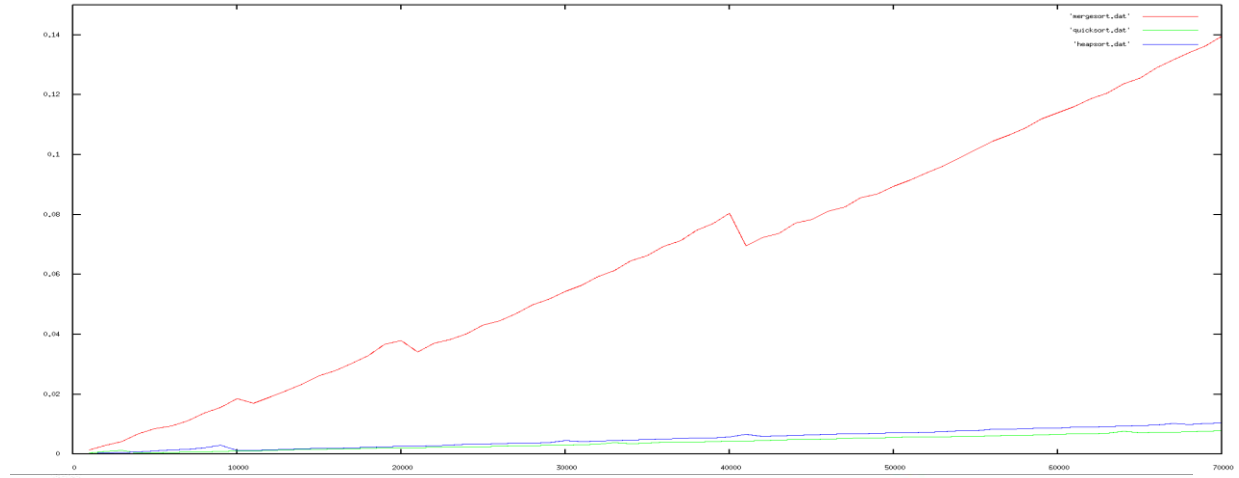
Gráfica ajustada a eficiencia teórica alternativa

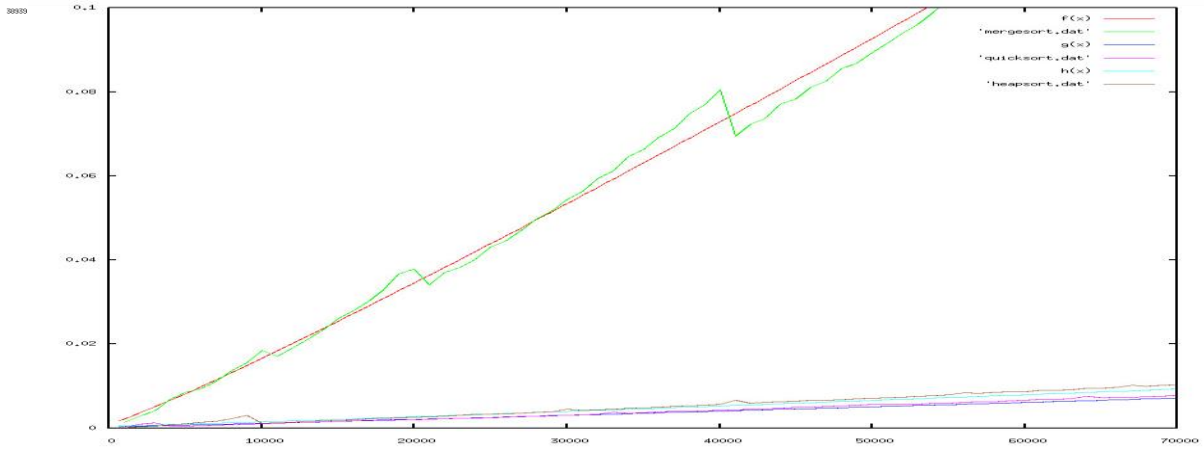
Algoritmos clasificados por su eficiencia



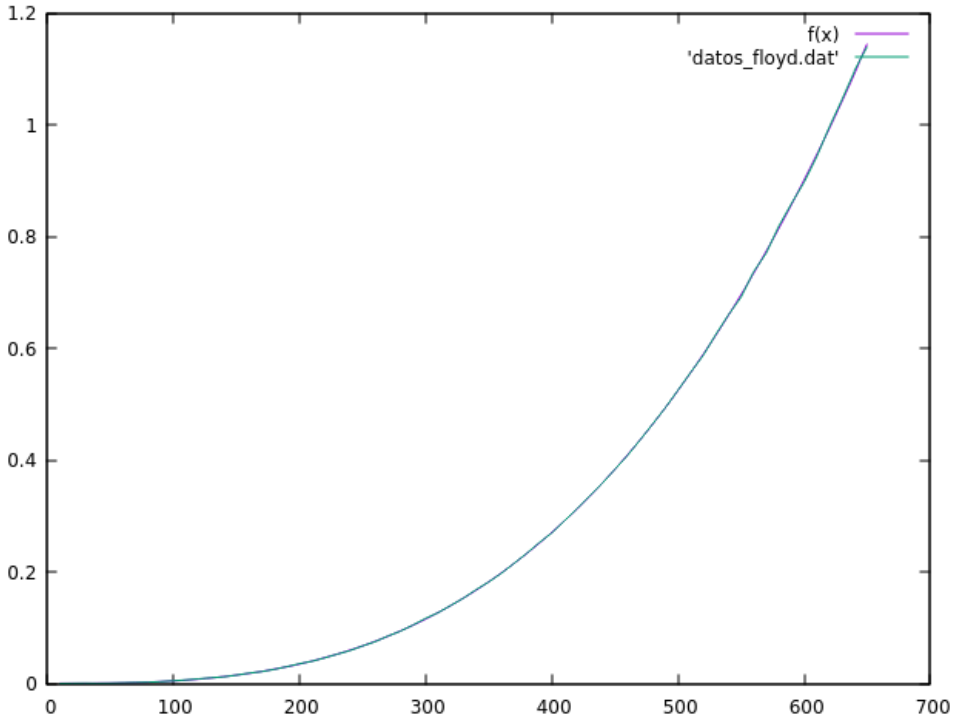
$O(n^2)$



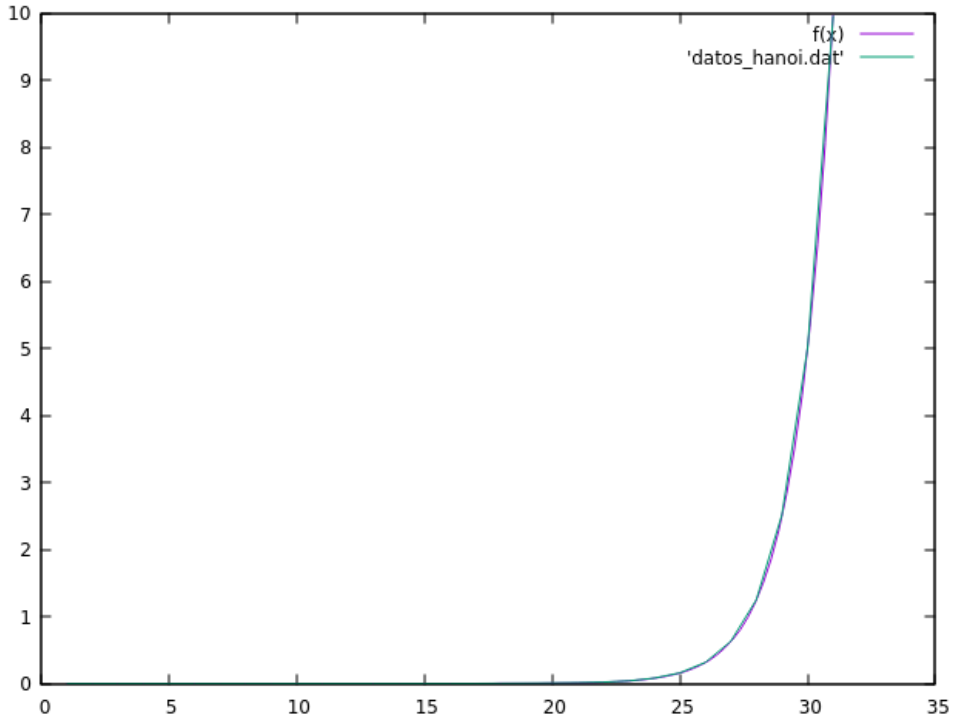




$O(n\log(n))$

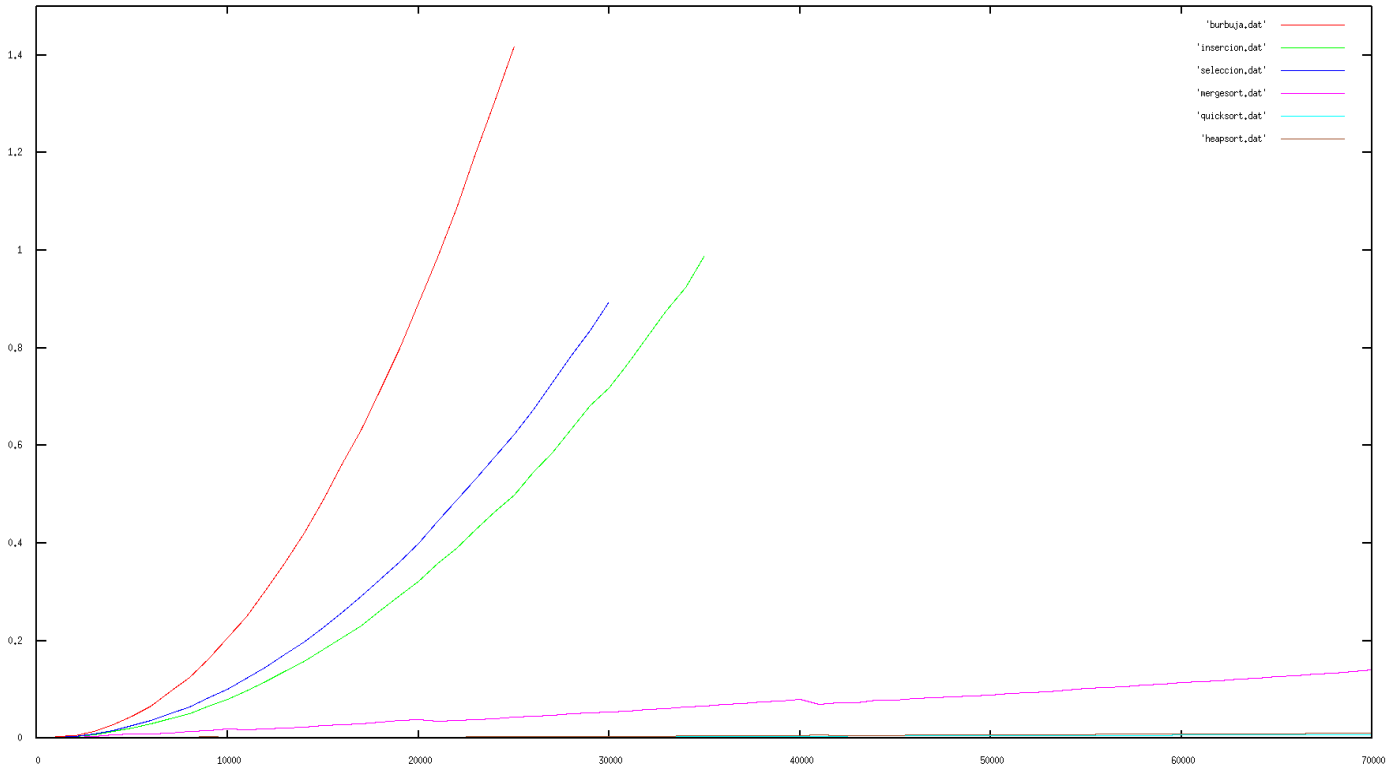


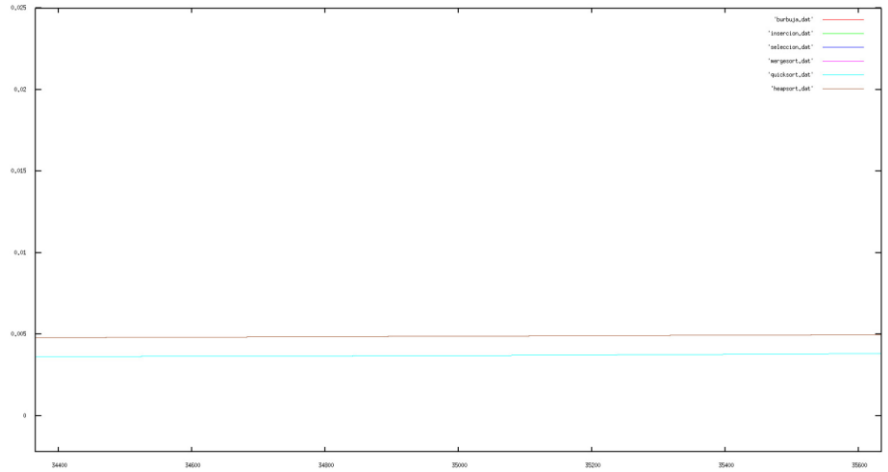
$O(an^3+bn^2+cn)$



O(2^n)

Gráfica con todos los algoritmos de ordenación juntos



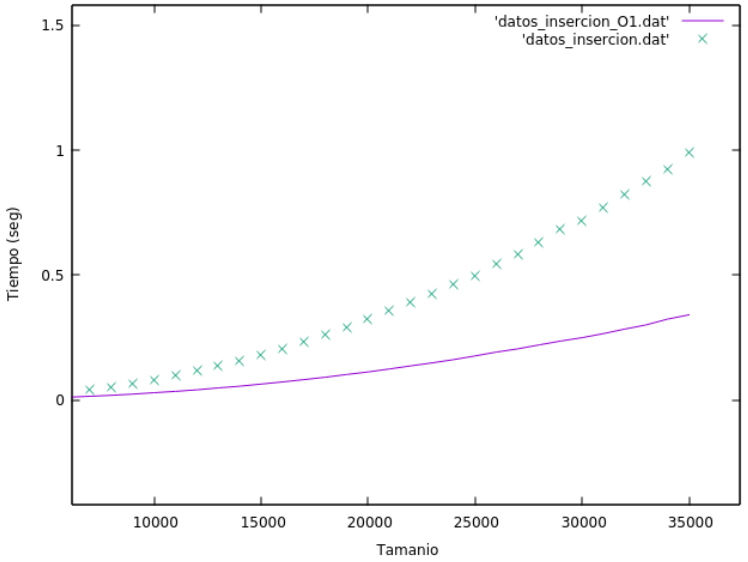


Burbuja(Rojo) Mergersort(Rosa)

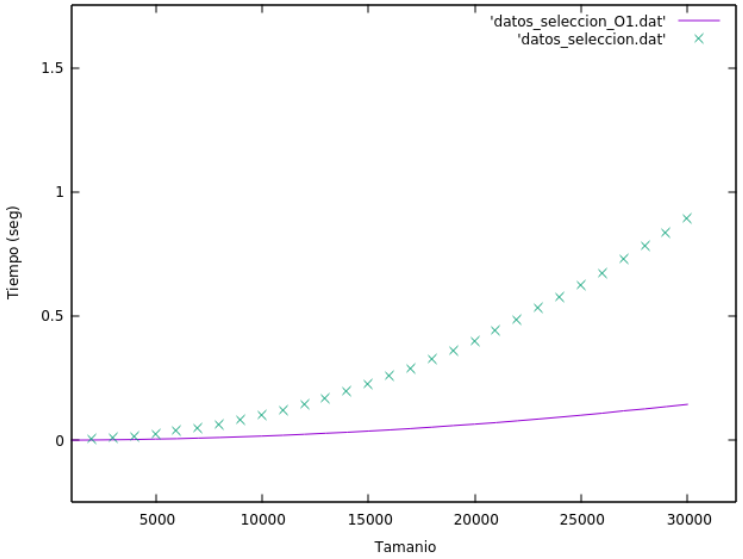
Selección(Azul) Quicksort(Azul claro)

Inserción (Verde) Heapsort(Marrón)

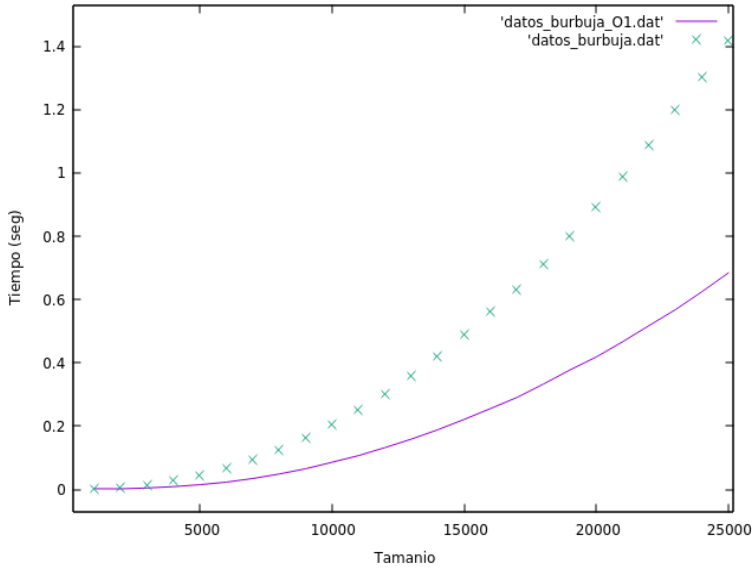
COMPILADOS CON –O1



Inserción

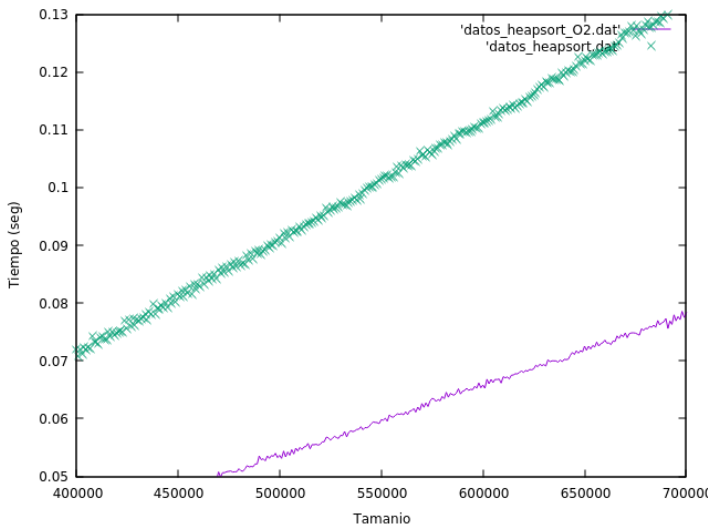


Selección

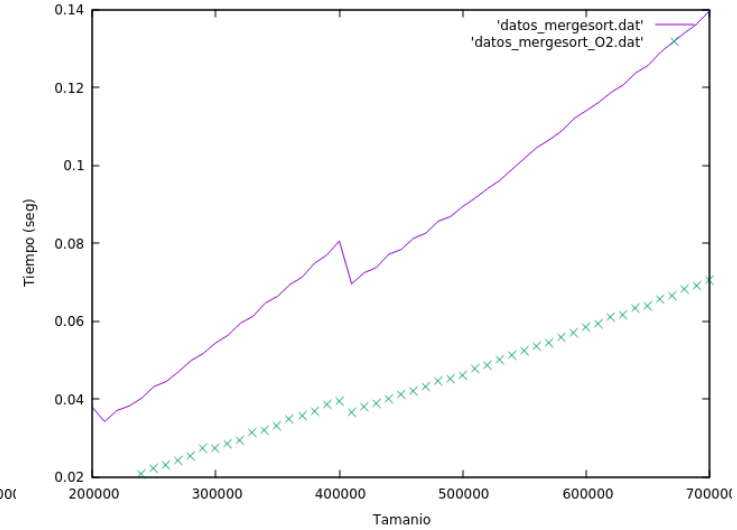


Burbuja

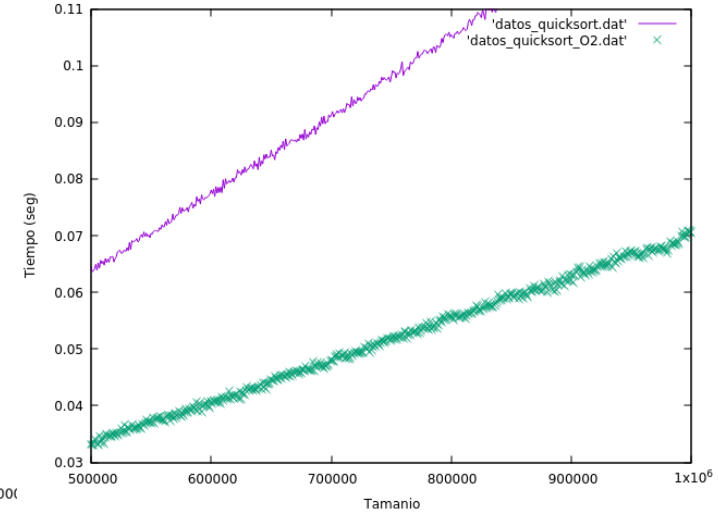
COMPILADOS CON –O2



Heapsort

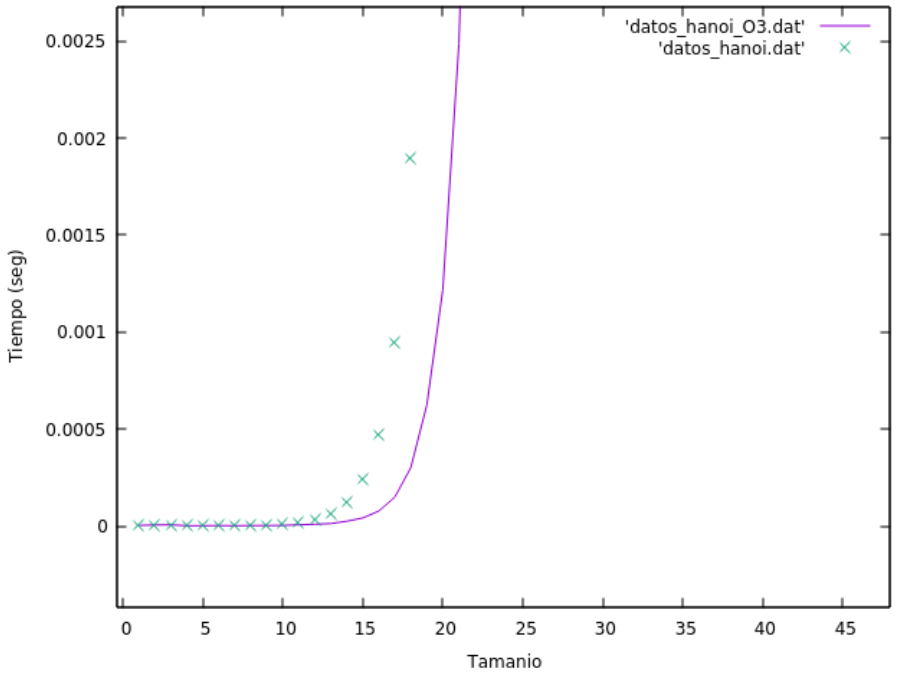


Mergesort

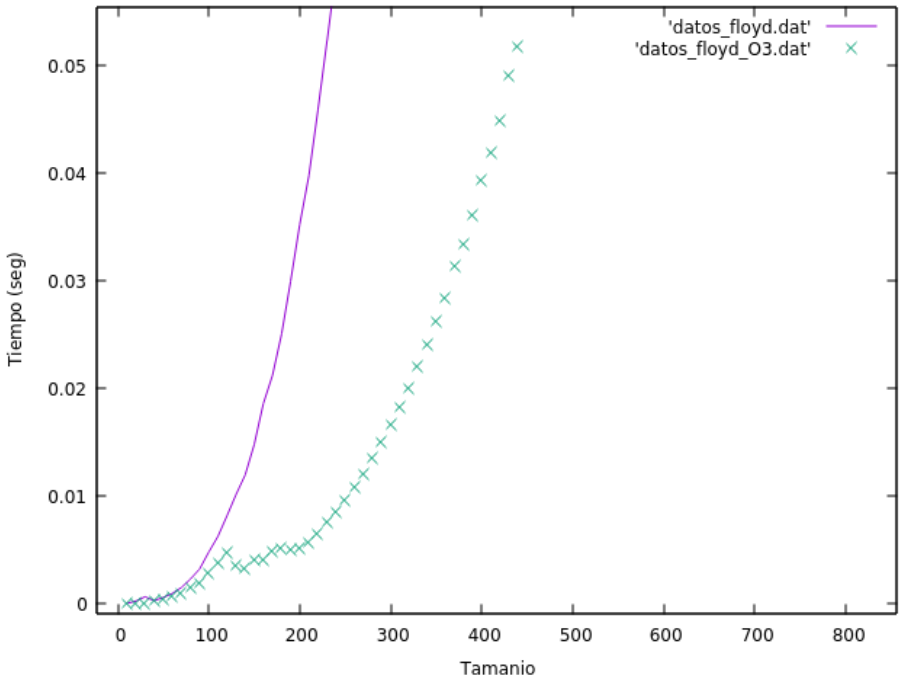


Quicksort

COMPILADOS CON –O3



Hanoi



Floyd

EJEMPLO DE PARAMETROS CON –O1

```
nuplot> fit f(x) 'datos_seleccion.dat' via a,b,c
ter      chisq       delta/lim  lambda   a             b             c
  0 3.7086597138e+00   0.00e+00  3.82e-02    1.579712e-10   6.712563e-08   3.425577e-05
  1 5.4940100028e-04  -6.75e+08  3.82e-03    9.870389e-10   7.380673e-08   3.434029e-05
  2 8.6528102582e-05  -5.35e+05  3.82e-04    9.943634e-10   1.198450e-07   3.551663e-05
  3 4.8572861830e-05  -7.81e+04  3.82e-05    9.844335e-10   3.633958e-07   5.127422e-06
  4 4.1187399112e-05  -1.79e+04  3.82e-06    9.797345e-10   5.287166e-07  -1.154464e-03
  5 4.0461902347e-05  -1.79e+03  3.82e-07    9.779495e-10   5.940001e-07  -1.651782e-03
  6 4.0461889012e-05  -3.30e-02  3.82e-08    9.779418e-10   5.942811e-07  -1.653923e-03
ter      chisq       delta/lim  lambda   a             b             c

fter 6 iterations the fit converged.
inal sum of squares of residuals : 4.04619e-05
el. change during last iteration : -3.2955e-07

egrees of freedom    (FIT_NDF)                       : 27
ms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 0.00122417
ariance of residuals (reduced chisquare) = WSSR/ndf   : 1.49859e-06

inal set of parameters            Asymptotic Standard Error
======================            ==========================
              = 9.77942e-10       +/- 3.341e-12    (0.3416%)
              = 5.94281e-07       +/- 1.067e-07    (17.96%)
              = -0.00165392       +/- 0.0007178    (43.4%)

orrelation matrix of the fit parameters:
               a      b      c
              1.000
             -0.970  1.000
              0.770 -0.882  1.000
nuplot>
```

Selección normal

```
gnuplot> fit f(x) 'datos_seleccion_O1.dat' via a,b,c
iter      chisq       delta/lim  lambda   a             b             c
   0 7.4448382017e-02   0.00e+00  6.63e-02    2.739798e-10   1.460543e-07  -2.086904e-04
   1 1.1762700744e-05  -6.33e+08  6.63e-03    1.565381e-10   1.445556e-07  -2.088405e-04
   2 2.2826614191e-06  -4.15e+05  6.63e-04    1.556496e-10   1.344716e-07  -2.108321e-04
   3 1.4918945548e-06  -5.30e+04  6.63e-05    1.572026e-10   9.455760e-08  -1.643572e-04
   4 1.3761588457e-06  -8.41e+03  6.63e-06    1.579461e-10   6.804043e-08   2.734803e-05
   5 1.3760200195e-06  -1.01e+01  6.63e-07    1.579712e-10   6.712596e-08   3.425328e-05
   6 1.3760200195e-06  -1.31e-06  6.63e-08    1.579712e-10   6.712563e-08   3.425577e-05
iter      chisq       delta/lim  lambda   a             b             c

After 6 iterations the fit converged.
final sum of squares of residuals : 1.37602e-06
rel. change during last iteration : -1.30639e-11

degrees of freedom    (FIT_NDF)                       : 27
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 0.000225751
variance of residuals (reduced chisquare) = WSSR/ndf   : 5.09637e-08

Final set of parameters            Asymptotic Standard Error
======================             ==========================
a               = 1.57971e-10      +/- 6.161e-13    (0.39%)
b               = 6.71256e-08      +/- 1.968e-08    (29.33%)
c               = 3.42558e-05      +/- 0.0001324    (386.4%)

correlation matrix of the fit parameters:
                a      b      c
a               1.000
b              -0.970  1.000
c               0.770 -0.882  1.000
gnuplot>
```

Selección optimizado

EJEMPLO DE PARAMETROS CON –O2

```
gnuplot> fit f(x) 'datos_mergesort.dat' via a,b
iter      chisq       delta/lim  lambda   a             b
   0 9.8037070736e-02   0.00e+00  2.76e-02    7.283286e-09  -3.646604e-08
   1 6.0317877867e-04  -1.62e+07  2.76e-03    1.418684e-08  -3.646602e-08
   2 5.9827767749e-04  -8.19e+02  2.76e-04    1.423615e-08  -3.646624e-08
   3 5.9827767741e-04  -1.36e-05  2.76e-05    1.423615e-08  -3.648842e-08
iter      chisq       delta/lim  lambda   a             b

After 3 iterations the fit converged.
final sum of squares of residuals : 0.000598278
rel. change during last iteration : -1.3629e-10

degrees of freedom    (FIT_NDF)                       : 68
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 0.00296618
variance of residuals (reduced chisquare) = WSSR/ndf   : 8.7982e-06

Final set of parameters            Asymptotic Standard Error
=======================            ==========================
a               = 1.42362e-08      +/- 1.285e-10    (0.9029%)
b               = -3.64884e-08     +/- 0.0006898    (1.89e+06%)

correlation matrix of the fit parameters:
                a      b
a               1.000
b              -0.858  1.000
gnuplot>
```

Mergesort normal

```
gnuplot> f(x) = a*x*log(x)+b
gnuplot> fit f(x) 'datos_mergesort_O2.dat' via a,b
iter      chisq       delta/lim  lambda   a             b
   0 8.1123222749e-02   0.00e+00  3.57e-03    9.416322e-10  -3.284030e-08
   1 6.6865422019e-05  -1.21e+08  3.57e-04    7.238310e-09  -3.283943e-08
   2 6.2788140179e-05  -6.49e+03  3.57e-05    7.283283e-09  -3.287533e-08
   3 6.2788109636e-05  -4.86e-02  3.57e-06    7.283286e-09  -3.646604e-08
iter      chisq       delta/lim  lambda   a             b

After 3 iterations the fit converged.
final sum of squares of residuals : 6.27881e-05
rel. change during last iteration : -4.86449e-07

degrees of freedom    (FIT_NDF)                       : 68
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 0.000960913
variance of residuals (reduced chisquare) = WSSR/ndf   : 9.23355e-07

Final set of parameters            Asymptotic Standard Error
=======================            ==========================
a               = 7.28329e-09      +/- 4.164e-11    (0.5717%)
b               = -3.6466e-08      +/- 0.0002235    (6.128e+05%)

correlation matrix of the fit parameters:
                a      b
a               1.000
b              -0.858  1.000
gnuplot>
```

Mergesort optimizado

EJEMPLO DE PARAMETROS CON –O3

```
   0 7.4124653901e+17   0.00e+00  5.33e+07    1.000000e+00   1.000000e+00   1.000000e+00   1.000000e+00
   1 1.0948616842e+13  -6.77e+09  5.33e+06    2.056820e-03   9.981987e-01   9.999966e-01   1.000000e+00
   2 6.6977906499e+10  -1.62e+07  5.33e+05   -1.777077e-03   9.958352e-01   9.999858e-01   1.000000e+00
   3 4.3872972567e+10  -5.27e+04  5.33e+04   -1.437450e-03   8.050995e-01   9.991117e-01   9.999966e-01
   4 7.2846962525e+07  -6.01e+07  5.33e+03   -5.383175e-05   2.839398e-02   9.955203e-01   9.999826e-01
   5 9.4274126853e+04  -7.72e+07  5.33e+02    4.862038e-06  -4.548217e-03   9.920630e-01   9.999441e-01
   6 5.3328124786e+04  -7.68e+04  5.33e+01    3.669123e-06  -3.422898e-03   7.433783e-01   9.970853e-01
   7 4.9778003114e+01  -1.07e+08  5.33e+00    8.219862e-08  -6.727347e-05   1.092799e-02   9.874475e-01
   8 2.7695336861e+00  -1.70e+06  5.33e-01   -2.315926e-08   3.064182e-05  -1.001967e-02   8.758810e-01
   9 1.4898434251e-02  -1.85e+07  5.33e-02    2.070812e-09   2.306198e-06  -7.352962e-04   6.407825e-02
  10 1.6869821890e-04  -8.73e+06  5.33e-03    4.056634e-09   7.642938e-08  -4.961598e-06   2.659492e-04
  11 1.6868911761e-04  -5.40e+00  5.33e-04    4.058196e-09   7.467529e-08  -4.387066e-06   2.157500e-04
   * 1.6868911761e-04   3.21e-10  5.33e-03    4.058196e-09   7.467527e-08  -4.387061e-06   2.157496e-04
  12 1.6868911761e-04  -9.00e-10  5.33e-04    4.058196e-09   7.467527e-08  -4.387061e-06   2.157496e-04
iter      chisq       delta/lim  lambda   a             b             c             d

After 12 iterations the fit converged.
final sum of squares of residuals : 0.000168689
rel. change during last iteration : -8.99811e-15

degrees of freedom    (FIT_NDF)                        : 61
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 0.00166295
variance of residuals (reduced chisquare) = WSSR/ndf   : 2.7654e-06

Final set of parameters            Asymptotic Standard Error
=======================            ==========================
a               = 4.0582e-09       +/- 3.981e-11    (0.981%)
b               = 7.46753e-08      +/- 3.995e-08    (53.5%)
c               = -4.38706e-06     +/- 1.139e-05    (259.7%)
d               = 0.00021575       +/- 0.000875     (405.6%)

correlation matrix of the fit parameters:
                a      b      c      d
a               1.000
b              -0.986  1.000
c               0.920 -0.970  1.000
d              -0.684  0.765 -0.879  1.000
gnuplot>
```

Floyd normal

```
gnuplot> fit f(x) 'datos_floyd_O3.dat' via a,b,c,d
iter      chisq       delta/lim  lambda   a             b             c             d
   0 9.4410692646e+00   0.00e+00  2.16e-01    4.058196e-09   7.467527e-08  -4.387061e-06   2.157496e-04
   1 3.1195230885e-04  -3.03e+09  2.16e-02    5.011773e-10   7.219608e-08  -4.405253e-06   2.156578e-04
   2 9.1375847127e-05  -2.41e+05  2.16e-03    5.204694e-10   5.408346e-08  -4.625662e-06   2.149907e-04
   3 4.1877098785e-05  -1.18e+05  2.16e-04    5.822706e-10   1.183293e-08  -7.485716e-07   3.448877e-04
   4 2.8495956333e-05  -4.70e+04  2.16e-05    6.313003e-10  -3.207144e-08   7.663218e-06   6.218449e-04
   5 2.8490432536e-05  -1.94e+01  2.16e-06    6.321982e-10  -3.285702e-08   7.800238e-06   6.305567e-04
   6 2.8490432536e-05  -2.66e-06  2.16e-07    6.321980e-10  -3.285674e-08   7.800127e-06   6.305697e-04
iter      chisq       delta/lim  lambda   a             b             c             d

After 6 iterations the fit converged.
final sum of squares of residuals : 2.84904e-05
rel. change during last iteration : -2.66125e-11

degrees of freedom    (FIT_NDF)                        : 61
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 0.000683415
variance of residuals (reduced chisquare) = WSSR/ndf   : 4.67056e-07

Final set of parameters            Asymptotic Standard Error
=======================            ==========================
a               = 6.32198e-10      +/- 1.636e-11    (2.588%)
b               = -3.28567e-08     +/- 1.642e-08    (49.97%)
c               = 7.80013e-06      +/- 4.683e-06    (60.03%)
d               = 0.00063057       +/- 0.0003596    (57.03%)

correlation matrix of the fit parameters:
                a      b      c      d
a               1.000
b              -0.986  1.000
c               0.920 -0.970  1.000
d              -0.684  0.765 -0.879  1.000
gnuplot>
```

Floyd optimizado

Variación en calidad de ajustes

```
correlation matrix of the fit parameters:
                a      b      c
a               1.000
b              -0.971  1.000
c               0.774 -0.884  1.000
```

$T(6) = 4{,}07 * 10^{-3}$

Algoritmo de la Burbuja

```
Final set of parameters            Asymptotic Standard Error
=======================            ==========================
a                = 5.93674e-05     +/- 3.287e-06    (5.537%)
b                = -0.281823       +/- 0.04886      (17.34%)

correlation matrix of the fit parameters:
                a      b
a               1.000
b              -0.874  1.000
```

$T(6) = -0{,}2796$

Recopilación final

$O(n^2)$	BURBUJA	INSERCION	SELECCIÓN
Tamaño			
1000	0.003725	0.002859	0.00108605
2000	0.006237	0.003452	0.00411013
3000	0.014832	0.007514	0.009112
4000	0.027287	0.013465	0.016582
5000	0.044807	0.020175	0.025423
6000	0.066308	0.029558	0.036851
7000	0.094743	0.039514	0.050307
8000	0.125221	0.051306	0.064776
9000	0.162785	0.065847	0.082499
10000	0.205539	0.079912	0.101053
11000	0.249845	0.098017	0.122921
12000	0.303364	0.115706	0.145535
13000	0.358567	0.136202	0.17079
14000	0.42073	0.158119	0.198004
15000	0.487893	0.181173	0.227203
16000	0.562743	0.205654	0.258552
17000	0.632581	0.231057	0.290887
18000	0.71375	0.262041	0.325722
19000	0.80031	0.29224	0.362522
20000	0.893149	0.322848	0.400376
21000	0.986542	0.357905	0.444063
22000	1.08645	0.389459	0.487566
23000	1.19856	0.426719	0.531152
24000	1.30425	0.464225	0.576454
25000	1.4192	0.49827	0.623446
26000		0.545143	0.673326
27000		0.584223	0.729135
28000		0.632708	0.783412
29000		0.682245	0.835421
30000		0.718393	0.894877
31000		0.767888	
32000		0.822539	
33000		0.875339	
34000		0.923252	
35000		0.990398	

$O(2^n)$	HANOI
Tamaño	
1	1,00E-06
2	1,00E-06
3	1,00E-06
4	2,00E-06
5	1,00E-06
6	2,00E-06
7	2,00E-06
8	3,00E-06
9	5,00E-06
10	9,00E-06
11	1.5e-05
12	3.2e-05
13	6.2e-05
14	0.000121
15	0.000239
16	0.000475
17	0.000946
18	0.001897
19	0.003841
20	0.007353
21	0.011324
22	0.020701
23	0.040178
24	0.080462
25	0.158489
26	0.315409
27	0.628951
28	1.25188
29	2.51728
30	5.00059
31	9.94644

$O(n^3)$	FLOYD
Tamaño	
10	3.4e-05
20	0.000268
30	0.000659
40	0.000296
50	0.000587
60	0.000971
70	0.001518
80	0.002325
90	0.003238
100	0.004807
110	0.006263
120	0.008157
130	0.010148
140	0.011979
150	0.014787
160	0.018596
170	0.021205
180	0.025059
190	0.029963
200	0.035242
210	0.039764
220	0.045828
230	0.052292
240	0.05891
250	0.067004
260	0.074894
270	0.08475
280	0.093455
290	0.104588
300	0.116475
310	0.126596
320	0.139711
330	0.152502
340	0.167651
350	0.181532
360	0.1971
370	0.214649
380	0.232571
390	0.251855
400	0.269403
410	0.290881

$O(nlog(n))$	QUICKSORT	HEAPSORT	MERGESORT
Tamaño			
1000	0.000366	0.000127	0.00134151
2000	0.000868	0.000309	0.00288
3000	0.001292	0.000482	0.004131
4000	0.000393	0.000759	0.006818
5000	0.000446	0.001009	0.008514
6000	0.000564	0.001368	0.009313
7000	0.000656	0.001673	0.0111
8000	0.000812	0.002185	0.01364
9000	0.000925	0.003046	0.01558
10000	0.00103	0.001181	0.018529
11000	0.001162	0.001312	0.017051
12000	0.001243	0.001461	0.019053
13000	0.001375	0.00163	0.021068
14000	0.001508	0.001765	0.023425
15000	0.001609	0.001853	0.026135
16000	0.001717	0.001999	0.027959
17000	0.001838	0.002182	0.030299
18000	0.001981	0.002309	0.03295
19000	0.002107	0.002454	0.036694
20000	0.002034	0.002602	0.037946
21000	0.002154	0.002708	0.034124
22000	0.002252	0.002872	0.037008
23000	0.002371	0.003026	0.03819
24000	0.002395	0.003263	0.040167
25000	0.002505	0.003334	0.043132
26000	0.002626	0.003454	0.04448
27000	0.002808	0.00359	0.046989
28000	0.002859	0.00374	0.049827
29000	0.003032	0.00389	0.051695
30000	0.003012	0.004454	0.054386
31000	0.00321	0.004204	0.056358
32000	0.003353	0.004377	0.059447
33000	0.003792	0.004533	0.061166
34000	0.003566	0.004725	0.064593
35000	0.003672	0.004874	0.066304
36000	0.003906	0.005001	0.069316
37000	0.003962	0.005142	0.071288
38000	0.004027	0.005336	0.074837
39000	0.004157	0.005478	0.076965
40000	0.004297	0.005658	0.080537
41000	0.004418	0.00658	0.069507

Análisis de Eficiencia de Algoritmos

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