Análisis de Eficiencia de Algoritmos

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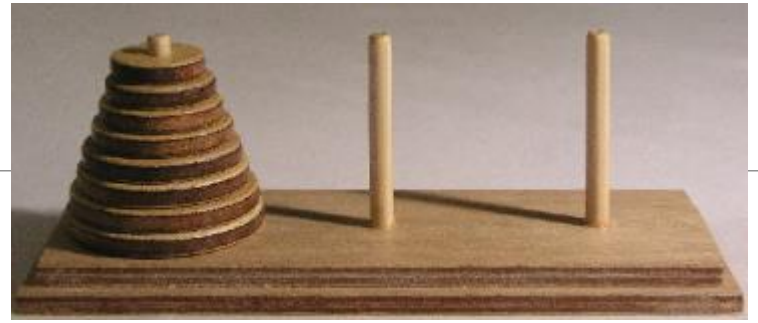
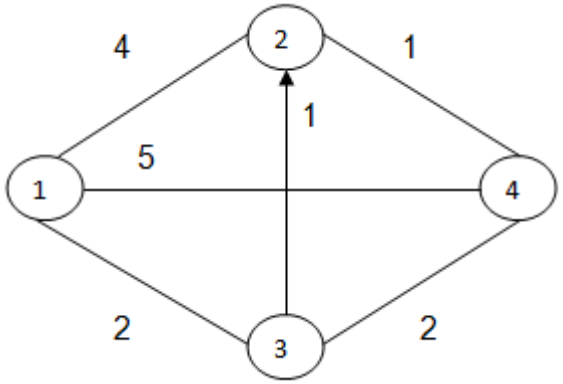
Descripción



Seleccionados:

- Floyd
- Hanoi
- Inserción
- Quicksort



Cálculos:

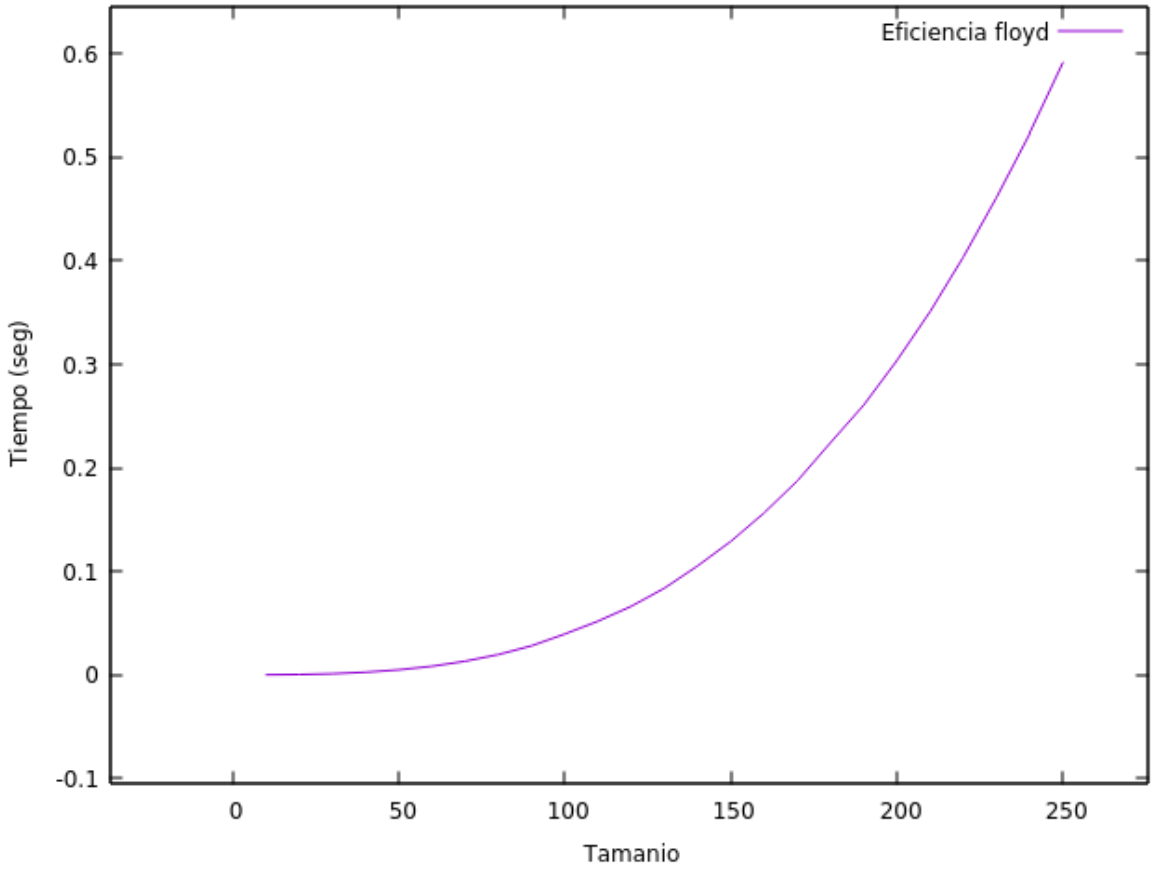
- Empírico
- Híbrido
- Constantes ocultas

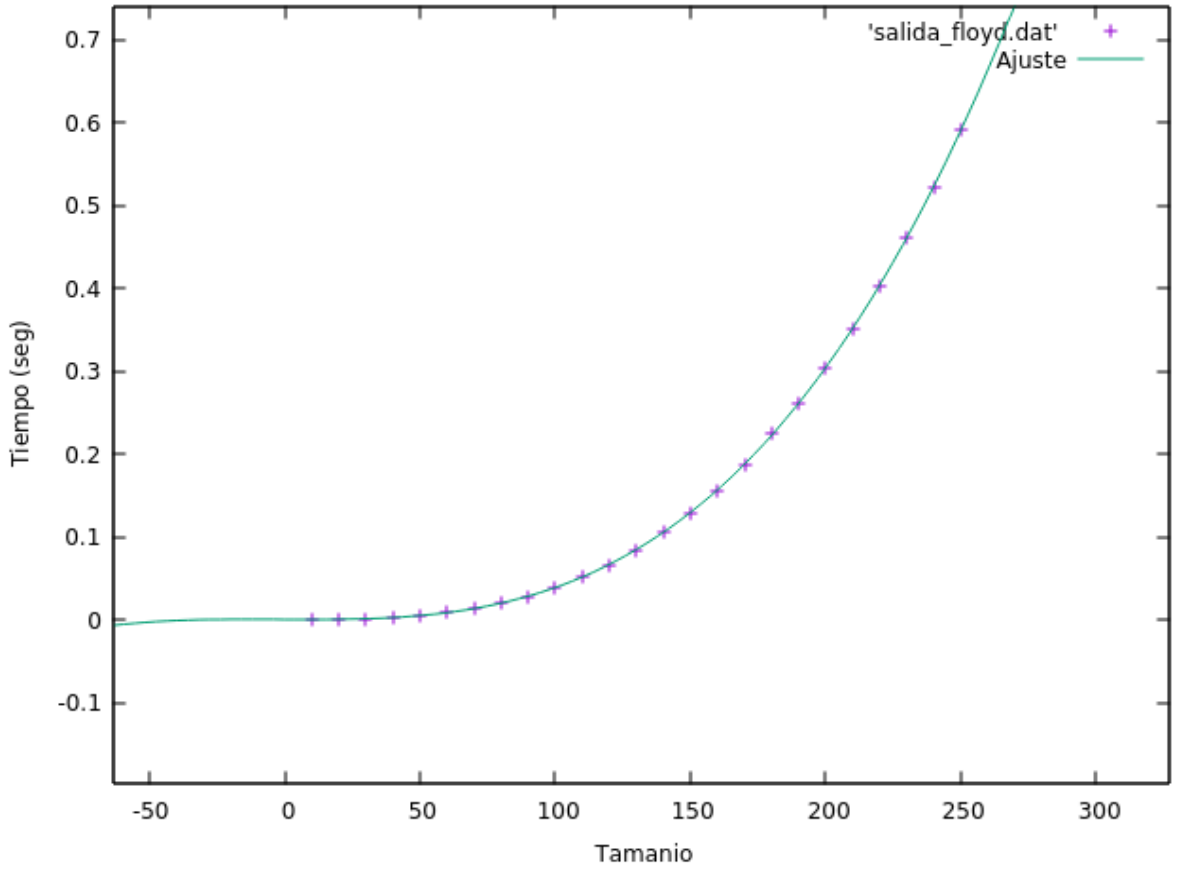
Cálculo de la eficiencia empírica

EJERCICIO 1

FLOYD

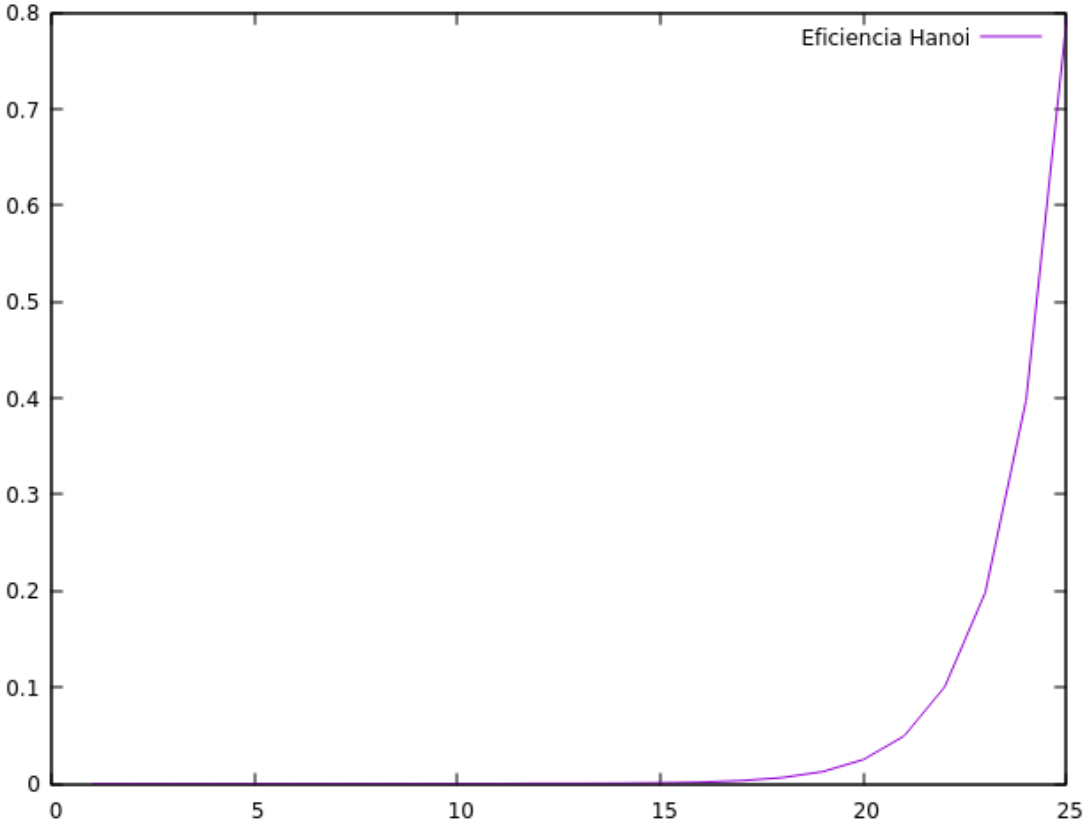
Tamaños -> 10 a 250 (incremento de 10)

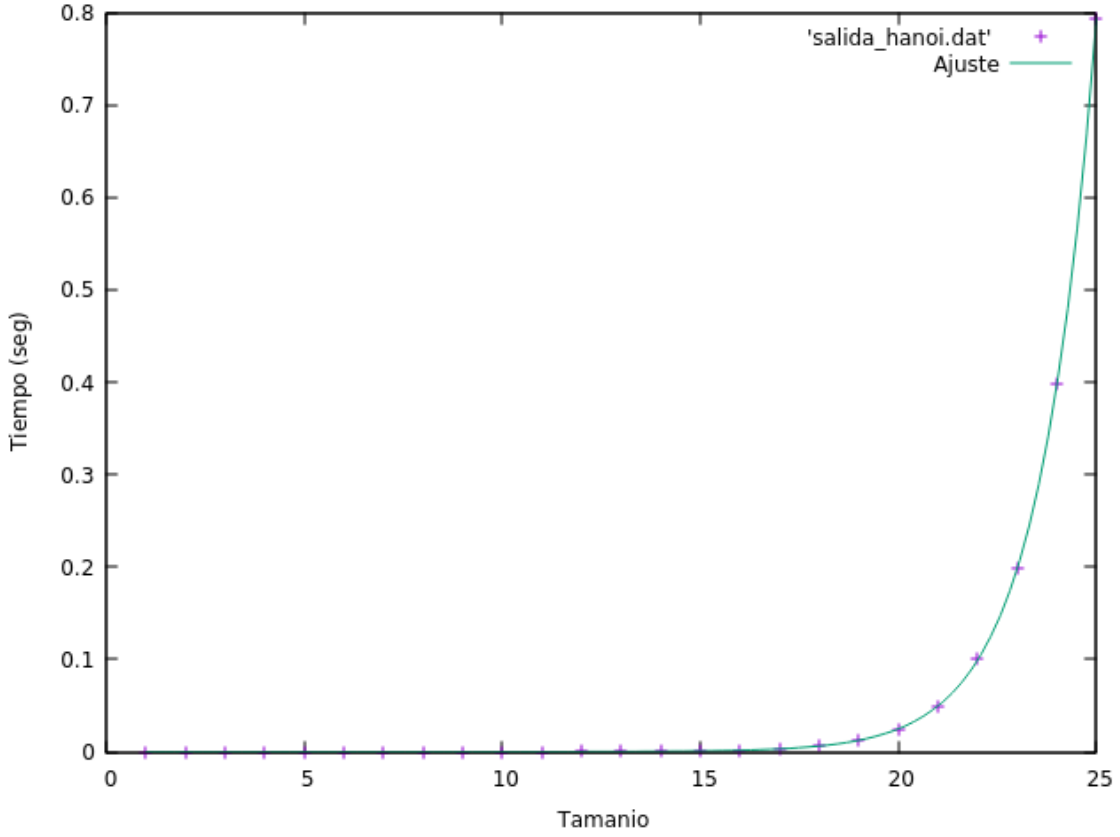




HANOI

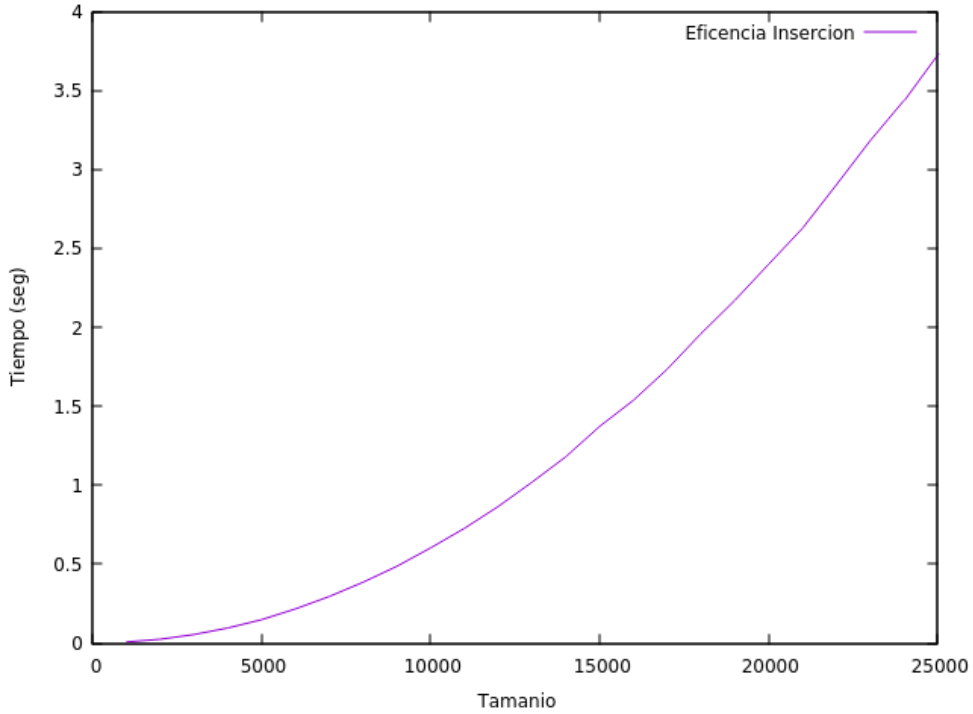
Tamaños -> 0 a 25 (incremento de 1)

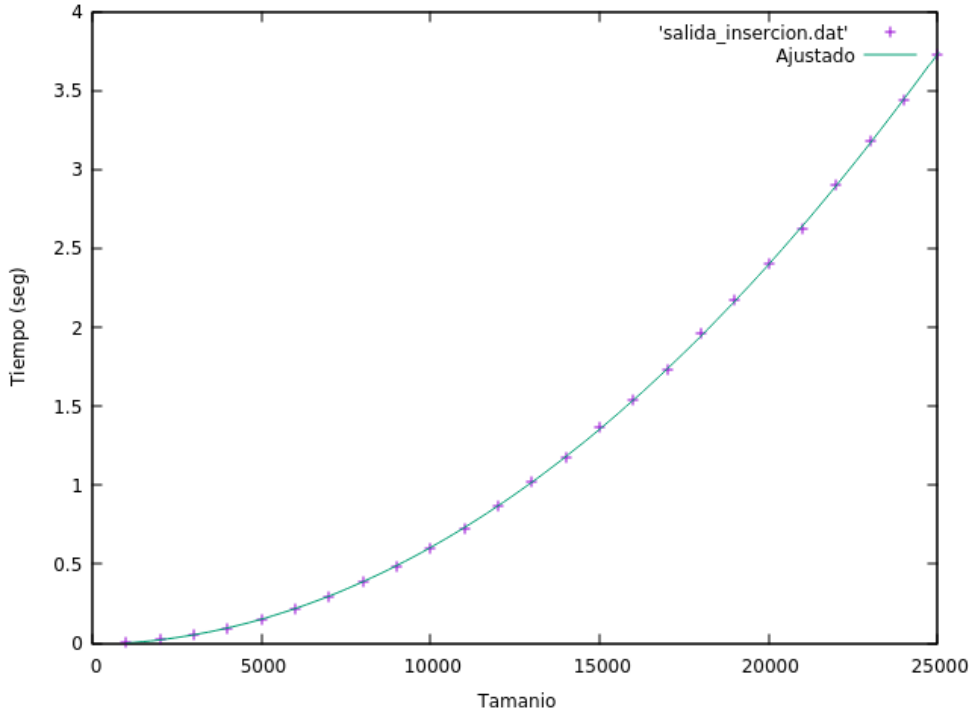




INSERCIÓN

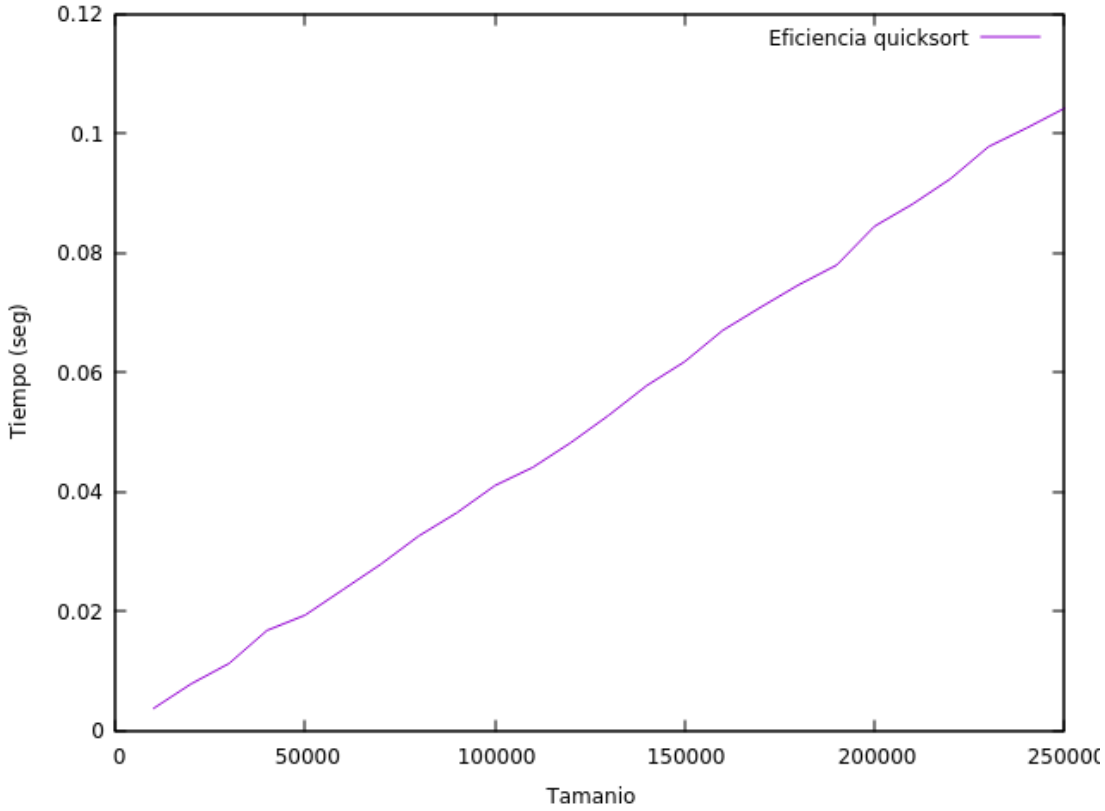
Tamaños -> 1000 a 25000 (incremento de 1000)

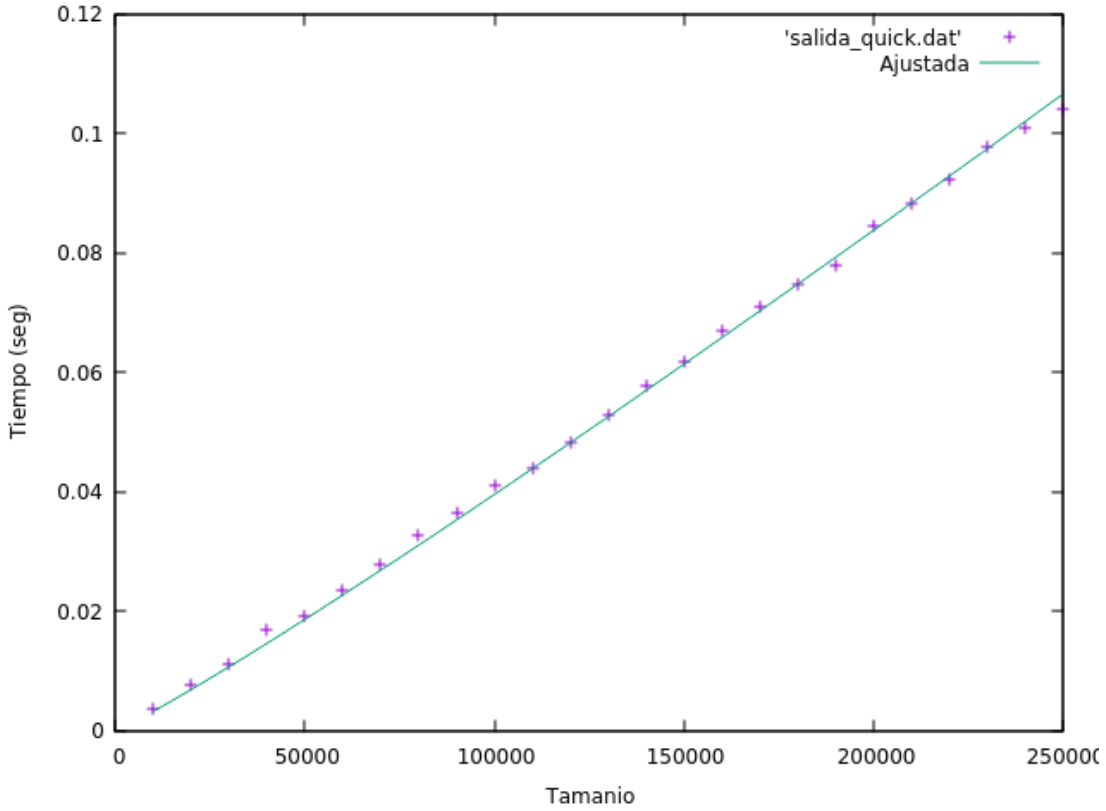




QUICKSORT

Tamaños -> 10000 a 250000 (incremento de 10000)





Eficiencia $O(n^2)$

Tamaño	burbuja	inserción	selección
1000	0,009326	0,006717	0,0039843
2000	0,036214	0,024055	0,0161128
3000	0,081513	0,05369	0,035522
4000	0,143469	0,094589	0,065098
5000	0,22451	0,148125	0,098737
6000	0,322415	0,215842	0,14236
7000	0,448451	0,294876	0,197098
8000	0,584871	0,384413	0,257235
9000	0,743189	0,485826	0,334225
10000	0,907302	0,602935	0,396813
11000	1,10214	0,726386	0,480536
12000	1,31375	0,864664	0,573581
13000	1,54938	1,01861	0,680197
14000	1,77838	1,17975	0,783223
15000	2,05705	1,37162	0,896243
16000	2,33831	1,53765	1,0198
17000	2,64727	1,73426	1,15057
18000	3,00253	1,96032	1,28965
19000	3,29181	2,17137	1,43709
20000	3,70991	2,39982	1,59251
21000	4,0858	2,6275	1,76388
22000	4,46812	2,90237	1,9482
23000	4,81371	3,18144	2,11209
24000	5,17747	3,43809	2,29516
25000	5,62167	3,72998	2,49193

Eficiencia $O(n \cdot \log(n))$

tamaño	quicksort	mergesort	heapsort
10000	0,003625	0,00665079	0,004824
20000	0,007771	0,014437	0,008362
30000	0,011211	0,018691	0,013449
40000	0,016733	0,029801	0,018422
50000	0,01928	0,04314	0,023023
60000	0,023538	0,039113	0,029465
70000	0,027832	0,049835	0,03519
80000	0,032545	0,061842	0,039763
90000	0,036439	0,074306	0,046071
100000	0,040993	0,08895	0,050819
110000	0,044036	0,070985	0,056759
120000	0,048172	0,081647	0,063013
130000	0,052775	0,09109	0,068089
140000	0,057742	0,102959	0,076176
150000	0,061734	0,114649	0,079362
160000	0,066984	0,126993	0,088945
170000	0,070871	0,139966	0,090666
180000	0,074599	0,153146	0,099272
190000	0,077911	0,167807	0,107028
200000	0,084415	0,181705	0,109911
210000	0,088137	0,138456	0,131347
220000	0,092379	0,148041	0,12323
230000	0,097764	0,158239	0,129261
240000	0,100855	0,168314	0,147278
250000	0,104177	0,179208	0,144524

Eficiencia $O(n^3)$

Tamaño	floyd
10	4,50E-05
20	0,000311
30	0,001085
40	0,002529
50	0,004823
60	0,00831
70	0,013163
80	0,019669
90	0,02811
100	0,03936
110	0,051813
120	0,066292
130	**0,083675**
140	0,105342
150	0,129086
160	0,156446
170	0,187267
180	0,224309
190	0,260606
200	0,303713
210	0,350943
220	0,403632
230	0,461212
240	0,52302
250	0,590928

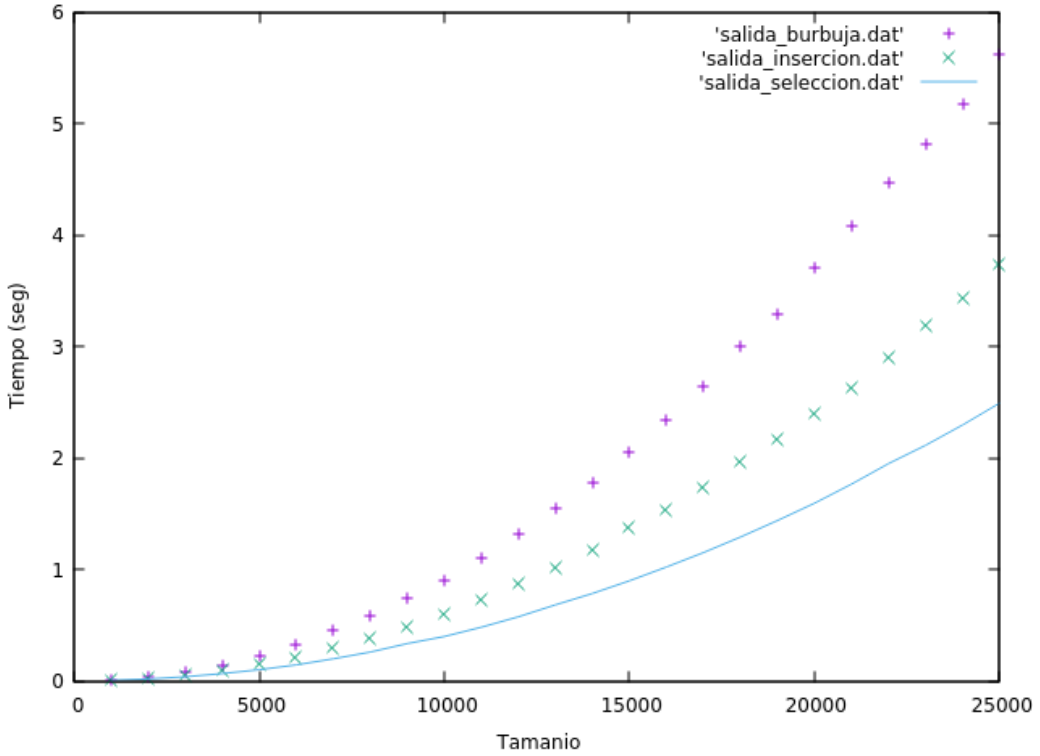
Eficiencia O(2^n)

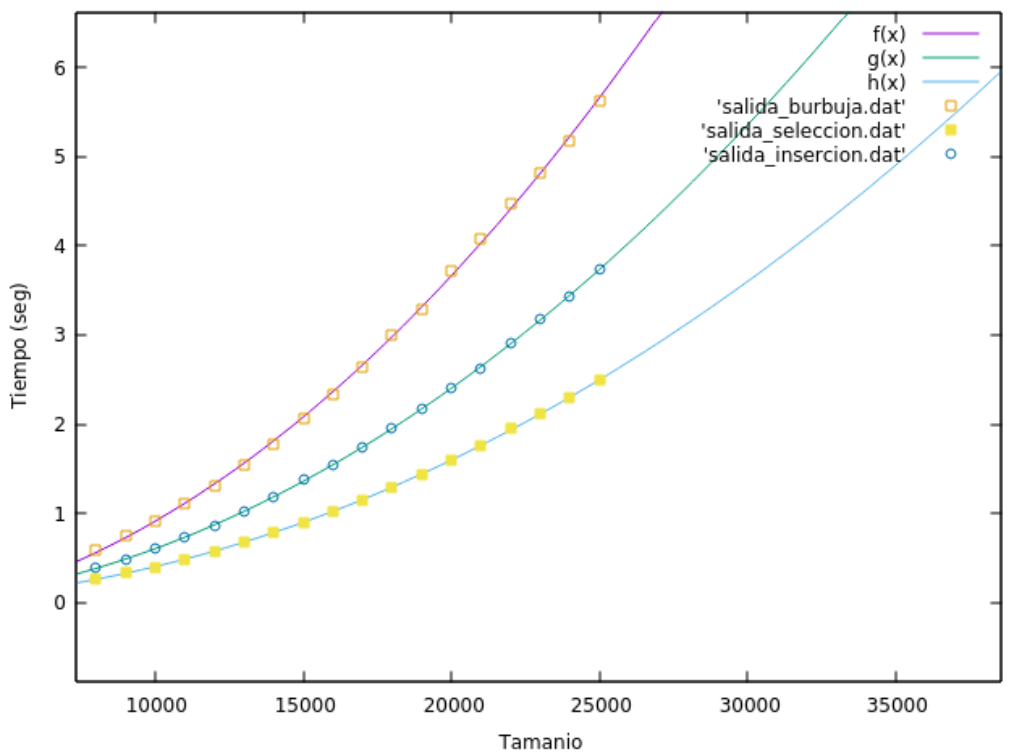
Tamaño	hanoi
1	7,00E-06
2	4,00E-06
3	4,00E-06
4	6,00E-06
5	6,00E-06
6	7,00E-06
7	8,00E-06
8	1,00E-05
9	1,70E-05
10	2,70E-05
11	5,80E-05
12	0,000103
13	**0,000203**
14	0,000382
15	0,000775
16	0,001563
17	0,003168
18	0,006212
19	0,012572
20	0,024988
21	0,049553
22	0,100354
23	0,198337
24	0,397377
25	0,792694

Gráficos de las tablas clasificadas por eficiencia

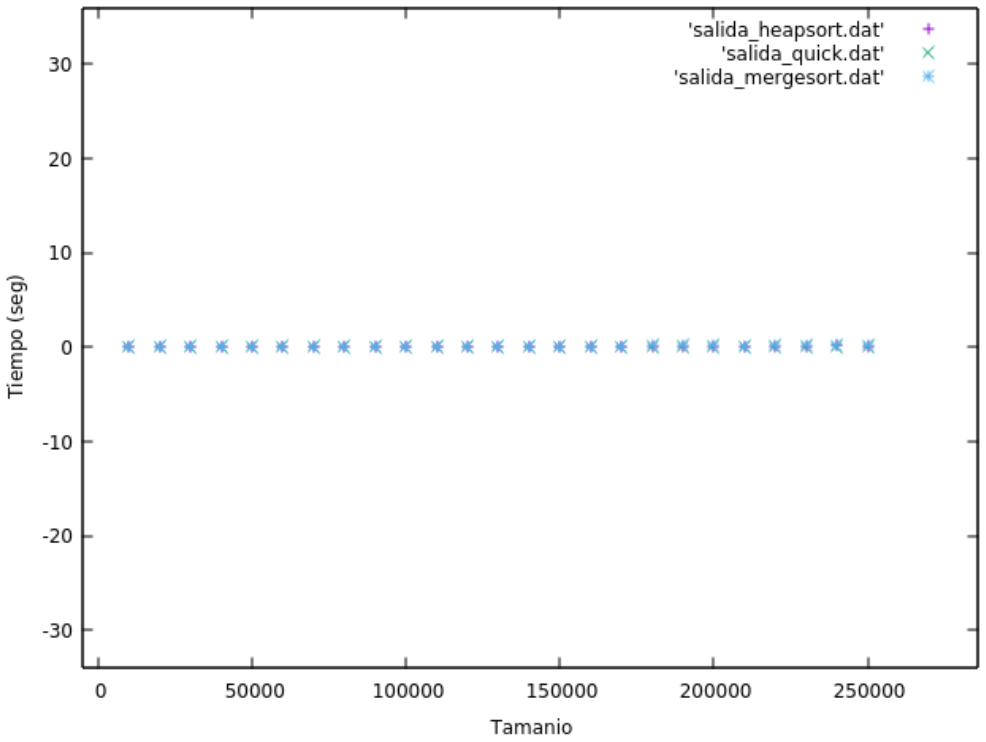
EJERCICIO 2

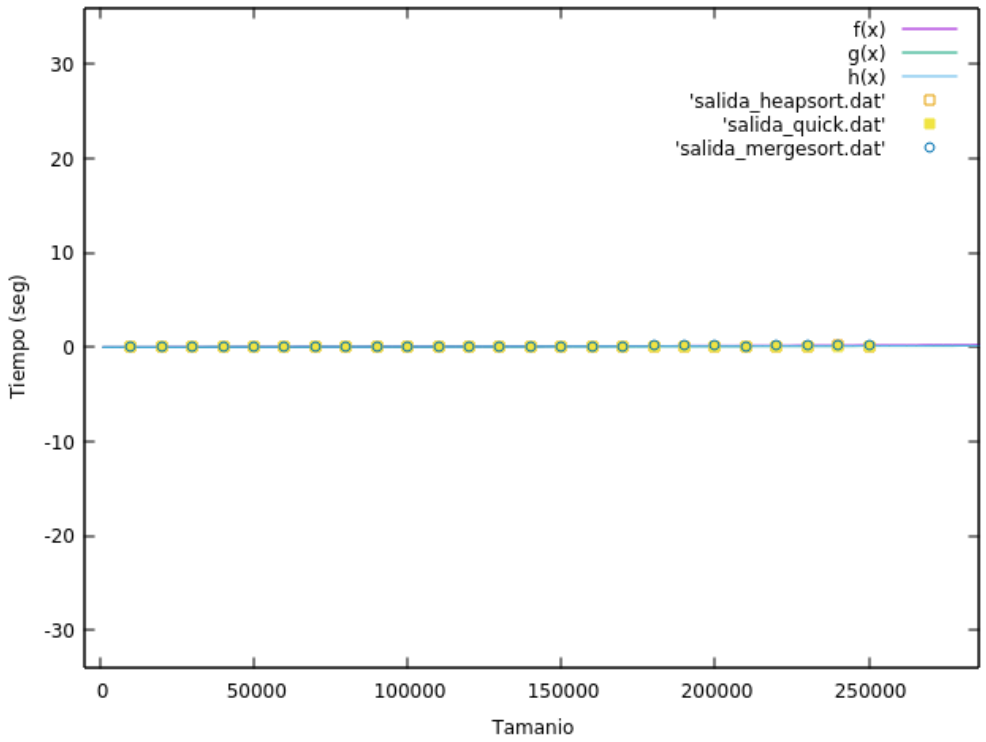
COMPARACIÓN DE ALGORITMOS CON EFICIENCIA N^2



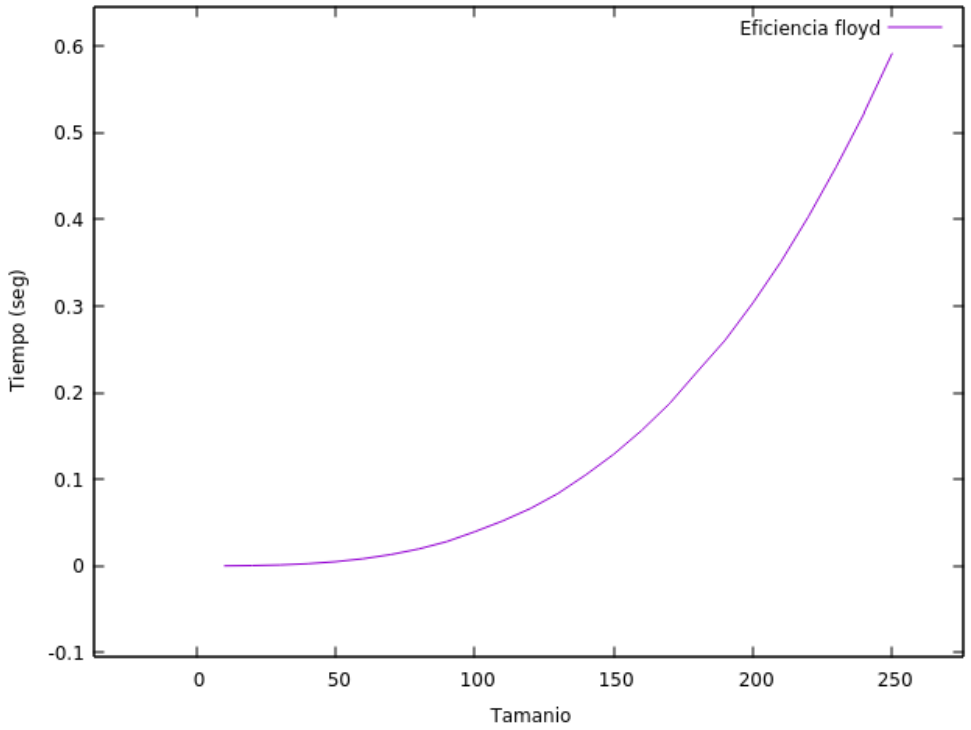


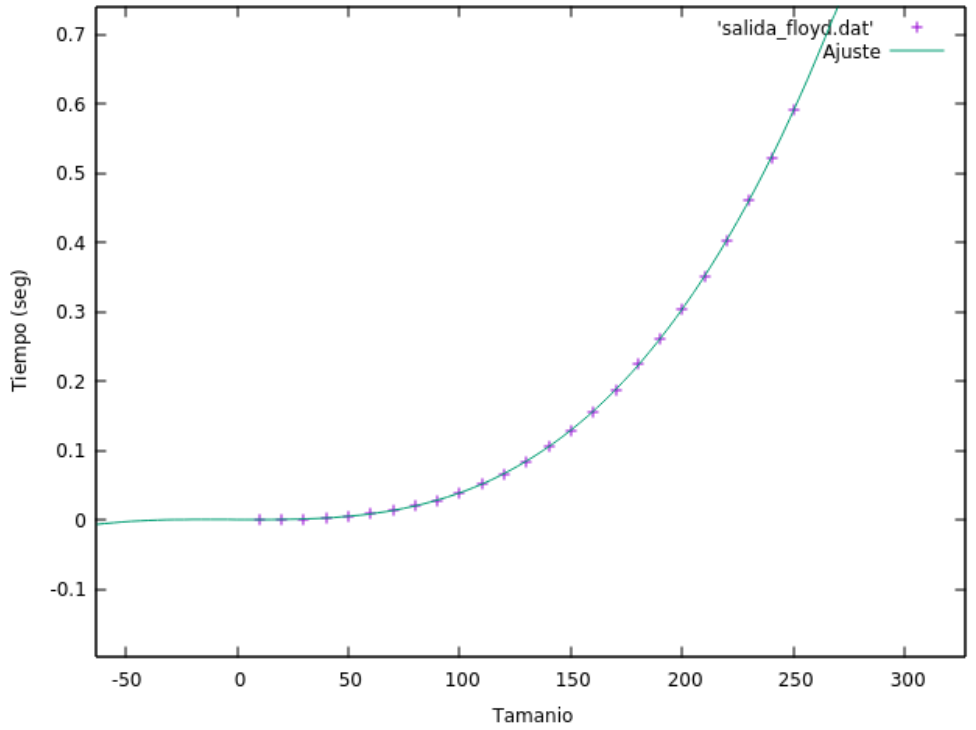
COMPARACIÓN DE ALGORITMOS CON EFICIENCIA N*LOG(N)



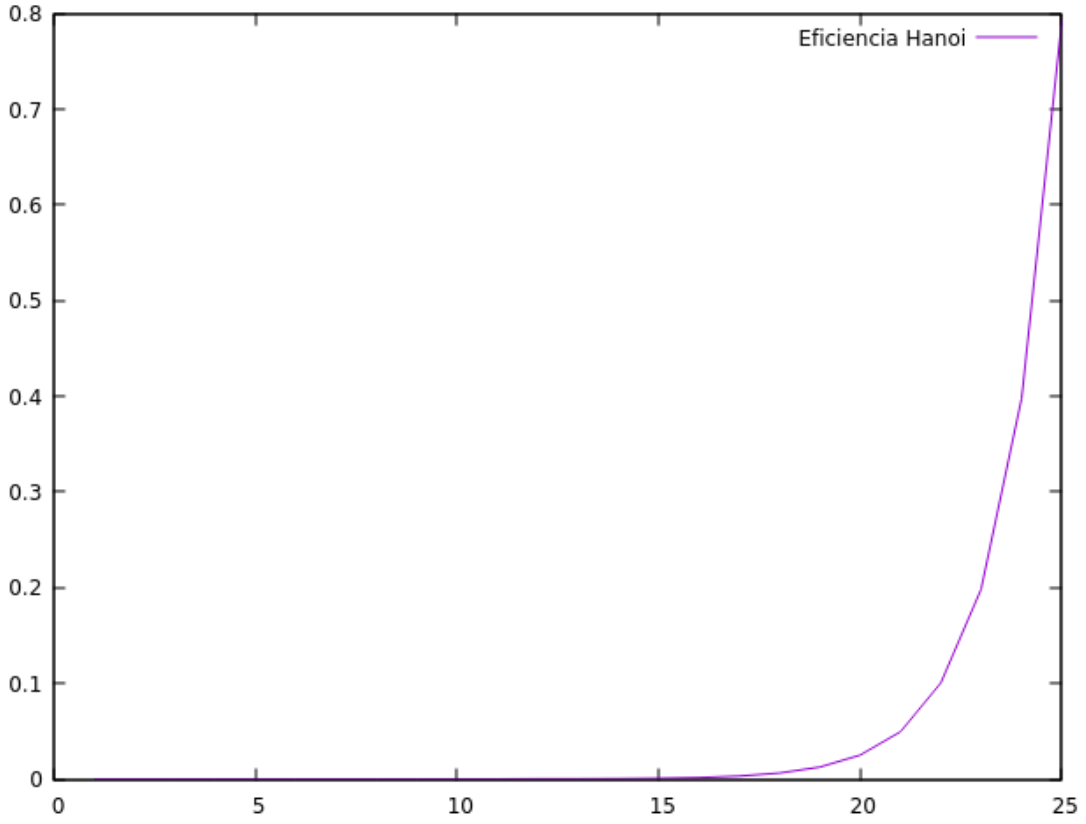


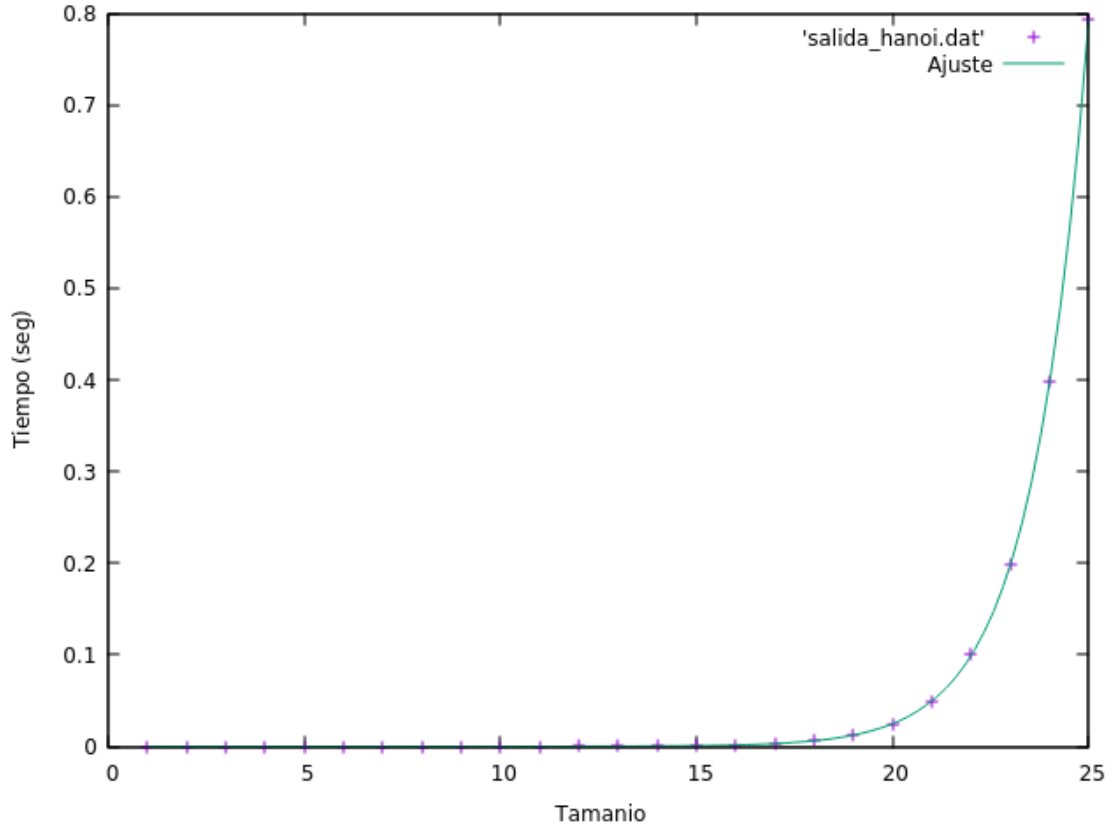
COMPARACIÓN DE ALGORITMOS CON EFICIENCIA N^3



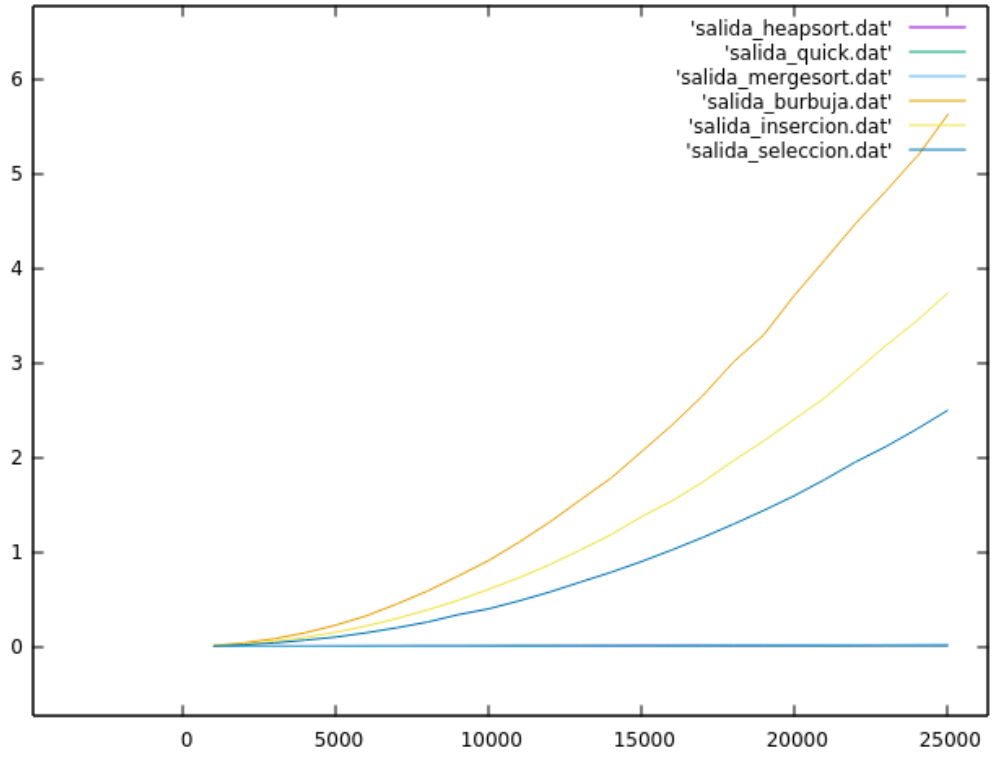


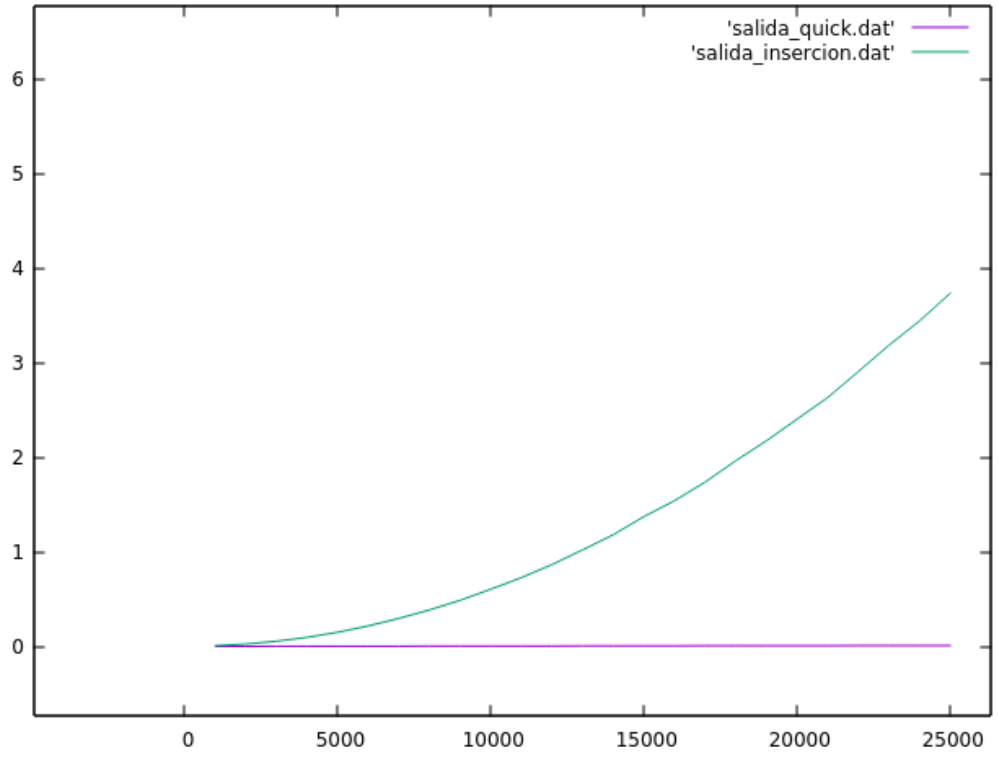
COMPARACIÓN DE ALGORITMOS CON EFICIENCIA 2^N





COMPARACIÓN DE TODOS ALGORITMOS DE ORDENACION



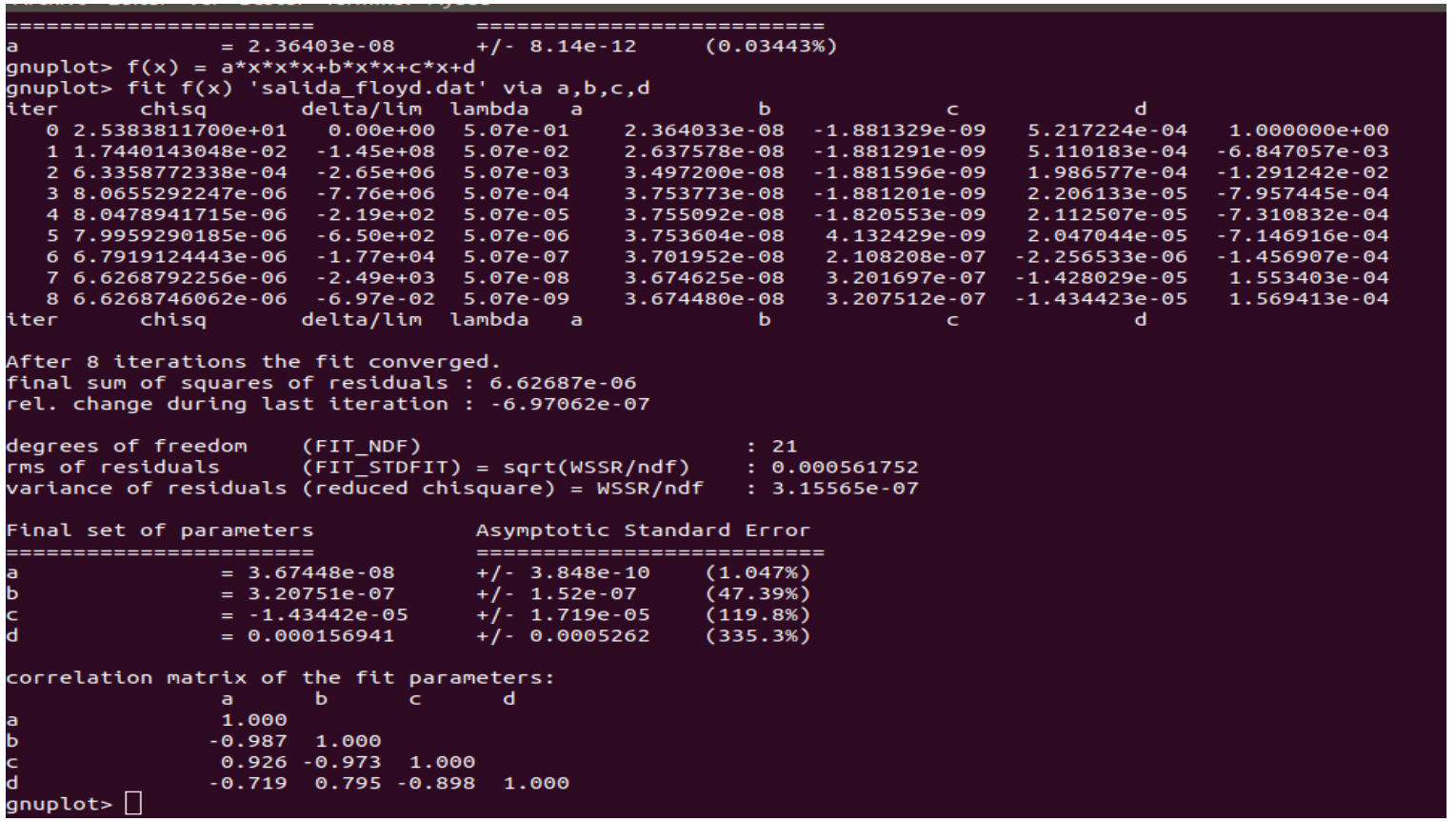


Cálculo de la eficiencia híbrida

EJERCICIO 3

PARAMETROS DEL ALGORITMO AJUSTADO DE FLOYD

a -> 3.67e-8
b-> 3.2075e-07
c-> -1.4344e5
d->0.0001569



```
======================          ==========================
a               = 2.36403e-08      +/- 8.14e-12     (0.03443%)
gnuplot> f(x) = a*x*x*x+b*x*x+c*x+d
gnuplot> fit f(x) 'salida_floyd.dat' via a,b,c,d
iter      chisq       delta/lim  lambda   a             b             c             d
   0 2.5383811700e+01   0.00e+00  5.07e-01    2.364033e-08  -1.881329e-09   5.217224e-04   1.000000e+00
   1 1.7440143048e-02  -1.45e+08  5.07e-02    2.637578e-08  -1.881291e-09   5.110183e-04  -6.847057e-03
   2 6.3358772338e-04  -2.65e+06  5.07e-03    3.497200e-08  -1.881596e-09   1.986577e-04  -1.291242e-02
   3 8.0655292247e-06  -7.76e+06  5.07e-04    3.753773e-08  -1.881201e-09   2.206133e-05  -7.957445e-04
   4 8.0478941715e-06  -2.19e+02  5.07e-05    3.755092e-08  -1.820553e-09   2.112507e-05  -7.310832e-04
   5 7.9959290185e-06  -6.50e+02  5.07e-06    3.753604e-08   4.132429e-09   2.047044e-05  -7.146916e-04
   6 6.7919124443e-06  -1.77e+04  5.07e-07    3.701952e-08   2.108208e-07  -2.256533e-06  -1.456907e-04
   7 6.6268792256e-06  -2.49e+03  5.07e-08    3.674625e-08   3.201697e-07  -1.428029e-05   1.553403e-04
   8 6.6268746062e-06  -6.97e-02  5.07e-09    3.674480e-08   3.207512e-07  -1.434423e-05   1.569413e-04
iter      chisq       delta/lim  lambda   a             b             c             d

After 8 iterations the fit converged.
final sum of squares of residuals : 6.62687e-06
rel. change during last iteration : -6.97062e-07

degrees of freedom    (FIT_NDF)                        : 21
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 0.000561752
variance of residuals (reduced chisquare) = WSSR/ndf   : 3.15565e-07

Final set of parameters            Asymptotic Standard Error
======================             ==========================
a               = 3.67448e-08      +/- 3.848e-10    (1.047%)
b               = 3.20751e-07      +/- 1.52e-07     (47.39%)
c               = -1.43442e-05     +/- 1.719e-05    (119.8%)
d               = 0.000156941      +/- 0.0005262    (335.3%)

correlation matrix of the fit parameters:
                a      b      c      d
a               1.000
b              -0.987  1.000
c               0.926 -0.973  1.000
d              -0.719  0.795 -0.898  1.000
gnuplot>
```

PARAMETROS DEL ALGORITMO AJUSTADO DE HANOI

a -> -2.10631e-8

b-> 0.9999999

```
                                        ^
         unexpected or unrecognized token

gnuplot> plot 'salida_quick.dat' with lines, 'salida_insercion' with lines
         warning: Cannot find or open file "salida_insercion"
gnuplot> plot 'salida_quick.dat' with lines, 'salida_insercion' with lines
         warning: Cannot find or open file "salida_insercion"
gnuplot> plot 'salida_quick.dat' with lines, 'salida_insercion.dat' with lines
gnuplot> Gtk-Message: 21:02:39.564: GtkDialog mapped without a transient parent. This is discouraged.

gnuplot> f(x) = a*2**x+b
gnuplot> fit f(x) 'salida_hanoi.dat' via a,b
iter      chisq       delta/lim  lambda   a             b
   0 1.5011999390e+15   0.00e+00  5.48e+06    1.000000e+00   1.000000e+00
   1 5.7716260633e+11  -2.60e+08  5.48e+05    1.960782e-02   1.000000e+00
   2 2.3099268487e+04  -2.50e+12  5.48e+04    3.899721e-06   1.000000e+00
   3 2.1996081304e+01  -1.05e+08  5.48e+03   -2.105531e-08   9.999999e-01
   4 2.1996048977e+01  -1.47e-01  5.48e+02   -2.106312e-08   9.999992e-01
iter      chisq       delta/lim  lambda   a             b

After 4 iterations the fit converged.
final sum of squares of residuals : 21.996
rel. change during last iteration : -1.46969e-06

degrees of freedom    (FIT_NDF)                        : 23
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 0.977931
variance of residuals (reduced chisquare) = WSSR/ndf   : 0.95635

Final set of parameters            Asymptotic Standard Error
=======================            ==========================
a               = -2.10631e-08     +/- 2.691e-08    (127.7%)
b               = 0.999999         +/- 0.2085       (20.85%)

correlation matrix of the fit parameters:
                a      b
a               1.000
b              -0.346  1.000
gnuplot>
```

PARAMETROS DEL ALGORITMO AJUSTADO DE INSERCION

a -> 5.90207e-09

b-> 2.27367e-6

c-> -0.00771998

```
c               = -0.0245526       +/- 0.02069      (84.26%)

correlation matrix of the fit parameters:
                a      b      c
a               1.000
b              -0.971  1.000
c               0.774 -0.884  1.000
gnuplot> f(x) = a*x*x+b*x+c
gnuplot> fit f(x) 'salida_insercion.dat' via a,b,c
iter      chisq       delta/lim  lambda   a             b             c
   0 2.1055051969e+01   0.00e+00  1.51e+00    8.879599e-09   6.192590e-06   -2.455260e-02
   1 6.1306464696e-03  -3.43e+08  1.51e-01    5.798309e-09   6.112158e-06   -2.461953e-02
   2 2.1932204070e-03  -1.80e+05  1.51e-02    5.778663e-09   5.674857e-06   -2.463640e-02
   3 1.3516727860e-03  -6.23e+04  1.51e-03    5.882826e-09   2.838234e-06   -1.101074e-02
   4 1.3253361609e-03  -1.99e+03  1.51e-04    5.902031e-09   2.274932e-06   -7.727441e-03
   5 1.3253360267e-03  -1.01e-02  1.51e-05    5.902073e-09   2.273667e-06   -7.719985e-03
iter      chisq       delta/lim  lambda   a             b             c

After 5 iterations the fit converged.
final sum of squares of residuals : 0.00132534
rel. change during last iteration : -1.01289e-07

degrees of freedom    (FIT_NDF)                        : 22
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 0.00776161
variance of residuals (reduced chisquare) = WSSR/ndf   : 6.02425e-05

Final set of parameters            Asymptotic Standard Error
=======================            ==========================
a               = 5.90207e-09      +/- 3.346e-11    (0.5669%)
b               = 2.27367e-06      +/- 8.961e-07    (39.41%)
c               = -0.00771998      +/- 0.005056     (65.49%)

correlation matrix of the fit parameters:
                a      b      c
a               1.000
b              -0.971  1.000
c               0.774 -0.884  1.000
gnuplot>
```

PARAMETROS DEL ALGORITMO AJUSTADO DE QUICKSORT

a -> 3.60584e-8

b-> 0.000163264

```
======================          ============================
a               = -2.10631e-08      +/- 2.691e-08     (127.7%)
b               = 0.999999          +/- 0.2085        (20.85%)

correlation matrix of the fit parameters:
                a      b
a               1.000
b              -0.346  1.000
gnuplot> f(x) = a*x*log(x)+b
gnuplot> fit f(x) 'salida_quick.dat' via a,b
iter      chisq       delta/lim  lambda   a             b
   0 2.4635394649e+01   0.00e+00  7.07e-01   -2.106312e-08   9.999992e-01
   1 9.9270680353e-03  -2.48e+08  7.07e-02   -2.111020e-08   2.679740e-02
   2 4.4462612503e-04  -2.13e+06  7.07e-03   -2.041287e-08   7.249628e-03
   3 8.8972523989e-05  -4.00e+05  7.07e-04    1.081145e-08   3.329670e-03
   4 1.3311696191e-07  -6.67e+07  7.07e-05    3.585593e-08   1.886608e-04
   5 1.2740123002e-07  -4.49e+03  7.07e-06    3.605842e-08   1.632656e-04
   6 1.2740122998e-07  -2.93e-05  7.07e-07    3.605843e-08   1.632636e-04
iter      chisq       delta/lim  lambda   a             b

After 6 iterations the fit converged.
final sum of squares of residuals : 1.27401e-07
rel. change during last iteration : -2.93275e-10

degrees of freedom    (FIT_NDF)                        : 23
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 7.44257e-05
variance of residuals (reduced chisquare) = WSSR/ndf   : 5.53918e-09

Final set of parameters            Asymptotic Standard Error
======================             ============================
a               = 3.60584e-08      +/- 1.993e-10     (0.5529%)
b               = 0.000163264      +/- 2.91e-05      (17.82%)

correlation matrix of the fit parameters:
                a      b
a               1.000
b              -0.859  1.000
gnuplot>
```

MAL AJUSTE DE FLOYD

$$f(x) = x * b + a * \log(x)$$

```
Final set of parameters            Asymptotic Standard Error
======================             ==========================
a               = 0.0296282        +/- 0.008025     (27.09%)
b               = -0.141038        +/- 0.0485       (34.39%)

correlation matrix of the fit parameters:
                a      b
a               1.000
b              -0.974  1.000
gnuplot> f(x) = log(x)*a+b*x
gnuplot> fit f(x) 'salida_floyd.dat' via a,b
iter      chisq       delta/lim  lambda   a             b
   0 1.1084490289e+04   0.00e+00  1.48e+01    2.962825e-02  -1.410375e-01
   1 4.7246838134e+00  -2.35e+08  1.48e+00    2.979024e-02  -2.180641e-03
   2 4.4436122707e-01  -9.63e+05  1.48e-01    2.786723e-02   6.530889e-04
   3 1.1345178248e-01  -2.92e+05  1.48e-02   -2.578218e-02   2.244920e-03
   4 8.5636521013e-02  -3.25e+04  1.48e-03   -4.640728e-02   2.856674e-03
   5 8.5636109909e-02  -4.80e-01  1.48e-04   -4.648688e-02   2.859034e-03
iter      chisq       delta/lim  lambda   a             b

After 5 iterations the fit converged.
final sum of squares of residuals : 0.0856361
rel. change during last iteration : -4.80059e-06

degrees of freedom    (FIT_NDF)                        : 23
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 0.0610189
variance of residuals (reduced chisquare) = WSSR/ndf   : 0.00372331

Final set of parameters            Asymptotic Standard Error
======================             ==========================
a               = -0.0464869       +/- 0.007575     (16.3%)
b               = 0.00285903       +/- 0.0002392    (8.367%)

correlation matrix of the fit parameters:
                a      b
a               1.000
b              -0.939  1.000
gnuplot>
```

MAL AJUSTE DE HANOI (CON 0(N)): $f(x) = x * b + a * \log(x)$

```
After 3 iterations the fit converged.
final sum of squares of residuals : 0.561478
rel. change during last iteration : -6.85233e-11

degrees of freedom    (FIT_NDF)                        : 24
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 0.152954
variance of residuals (reduced chisquare) = WSSR/ndf   : 0.0233949

Final set of parameters            Asymptotic Standard Error
=======================            ==========================
a               = 0.00799194       +/- 0.002058     (25.75%)
gnuplot> fit f(x) 'salida_hanoi.dat' via a,b
iter      chisq       delta/lim  lambda   a             b
   0 5.6147799505e-01   0.00e+00  8.48e-02    7.991945e-03  -6.782860e-03
   1 5.4870903858e-01  -2.33e+03  8.48e-03    8.979900e-03  -1.303821e-02
   2 4.2484162028e-01  -2.92e+04  8.48e-04    2.611962e-02  -1.192879e-01
   3 4.2115892127e-01  -8.74e+02  8.48e-05    2.962108e-02  -1.409931e-01
   4 4.2115890590e-01  -3.65e-03  8.48e-06    2.962825e-02  -1.410375e-01
iter      chisq       delta/lim  lambda   a             b

After 4 iterations the fit converged.
final sum of squares of residuals : 0.421159
rel. change during last iteration : -3.64916e-08

degrees of freedom    (FIT_NDF)                        : 23
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 0.135319
variance of residuals (reduced chisquare) = WSSR/ndf   : 0.0183113

Final set of parameters            Asymptotic Standard Error
=======================            ==========================
a               = 0.0296282        +/- 0.008025     (27.09%)
b               = -0.141038        +/- 0.0485       (34.39%)

correlation matrix of the fit parameters:
                a      b
a               1.000
b              -0.974  1.000
gnuplot>
```

MAL AJUSTE DE INSERCION (CON 0(N^3))

$F(x) = a*x^3+b*x^2+c*x+d$

```
correlation matrix of the fit parameters:
                a      b      c      d
a               1.000
b              -0.987  1.000
c               0.926 -0.973  1.000
d              -0.719  0.795 -0.898  1.000
gnuplot> fit f(x) 'salida_insercion.dat' via a,b,c,d
iter      chisq       delta/lim  lambda   a             b             c             d
   0 2.1060421490e+01   0.00e+00  1.68e+00   -6.297222e-14   1.133552e-08  -1.985272e-05   3.733650e-02
   1 1.0414260280e-02  -2.02e+08  1.68e-01   -6.459575e-14   8.299884e-09  -2.036574e-05   3.724227e-02
   2 4.9273731342e-03  -1.11e+05  1.68e-02   -5.352747e-14   7.970669e-09  -1.915183e-05   3.865703e-02
   3 1.1340780135e-03  -3.34e+05  1.68e-03   -1.716662e-14   6.588139e-09  -5.318723e-06   1.210510e-02
   4 1.0443539961e-03  -8.59e+03  1.68e-04   -1.149599e-14   6.350465e-09  -2.482498e-06   3.586975e-03
   5 1.0443534037e-03  -5.67e-02  1.68e-05   -1.148207e-14   6.349874e-09  -2.475319e-06   3.564597e-03
iter      chisq       delta/lim  lambda   a             b             c             d

After 5 iterations the fit converged.
final sum of squares of residuals : 0.00104435
rel. change during last iteration : -5.67238e-07

degrees of freedom    (FIT_NDF)                        : 21
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 0.00705203
variance of residuals (reduced chisquare) = WSSR/ndf   : 4.97311e-05

Final set of parameters            Asymptotic Standard Error
=======================            ==========================
a               = -1.14821e-14     +/- 4.831e-15    (42.07%)
b               = 6.34987e-09      +/- 1.908e-10    (3.005%)
c               = -2.47532e-06     +/- 2.157e-06    (87.16%)
d               = 0.0035646        +/- 0.006606     (185.3%)

correlation matrix of the fit parameters:
                a      b      c      d
a               1.000
b              -0.987  1.000
c               0.926 -0.973  1.000
d              -0.719  0.795 -0.898  1.000
gnuplot>
```

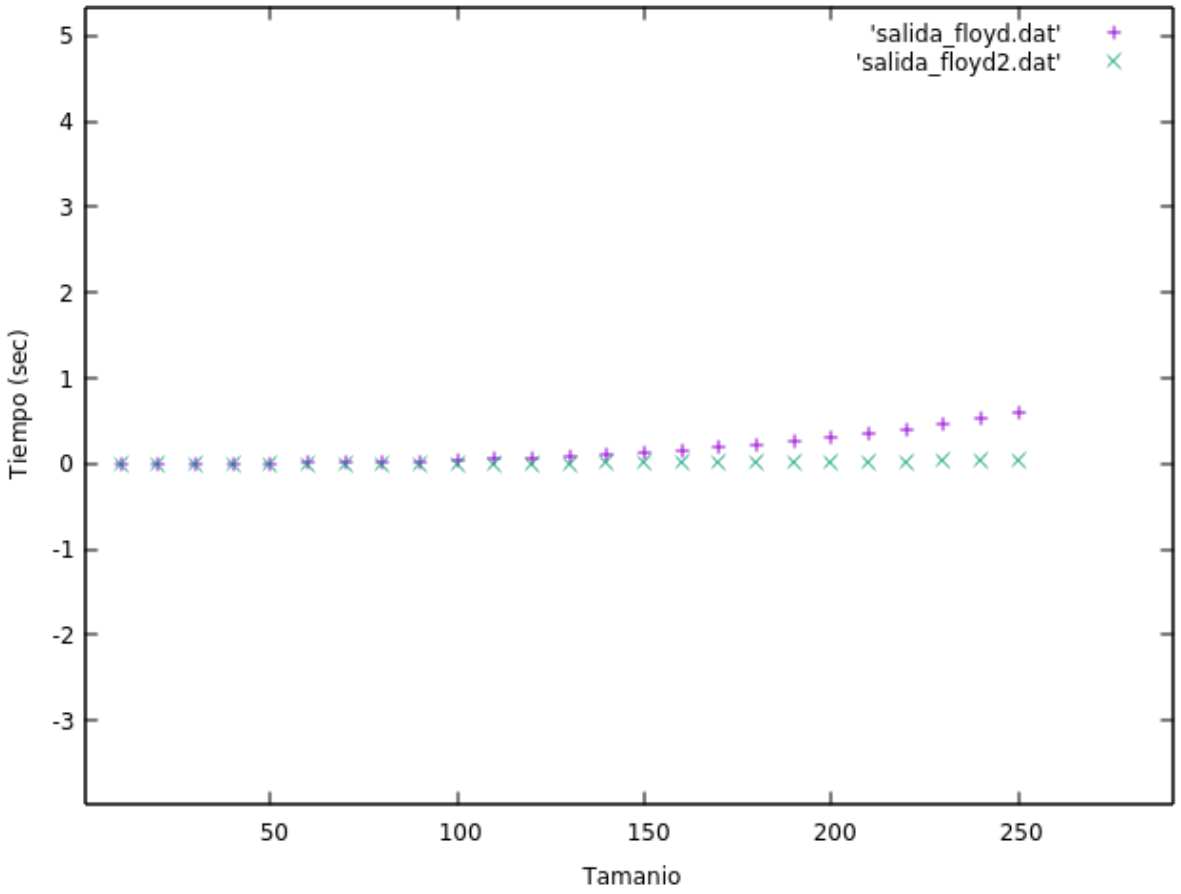
MAL AJUSTE DE QUICKSORT (CON 0(N)) F(x) = a*x+b

```
c               = 5.47544e-07      +/- 1.538e-06    (280.9%)
d               = -0.000783826     +/- 0.004709     (600.8%)

correlation matrix of the fit parameters:
                a      b      c      d
a               1.000
b              -0.987  1.000
c               0.926 -0.973  1.000
d              -0.719  0.795 -0.898  1.000
gnuplot> f(x) = a*x+b
gnuplot> fit f(x) 'salida_quick.dat' via a,b
iter      chisq       delta/lim  lambda   a             b
   0 9.5592084681e-02   0.00e+00  2.79e-09    1.328400e-15   3.939036e-09
   1 2.2119123585e-02  -3.32e+05  2.79e-10    1.290967e-08   5.099990e-02
   2 1.4334712915e-03  -1.44e+06  2.79e-11    3.205337e-07   1.203915e-02
   3 1.4145853005e-05  -1.00e+07  2.79e-12    4.246692e-07  -1.506248e-03
   4 1.4129586729e-05  -1.15e+02  2.79e-13    4.250229e-07  -1.552258e-03
   5 1.4129586729e-05  -1.33e-07  2.79e-14    4.250229e-07  -1.552260e-03
iter      chisq       delta/lim  lambda   a             b

After 5 iterations the fit converged.
final sum of squares of residuals : 1.41296e-05
rel. change during last iteration : -1.33143e-12

degrees of freedom    (FIT_NDF)                        : 23
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 0.000783792
variance of residuals (reduced chisquare) = WSSR/ndf   : 6.1433e-07

Final set of parameters            Asymptotic Standard Error
=======================            ==========================
a               = 4.25023e-07      +/- 2.174e-09    (0.5115%)
b               = -0.00155226      +/- 0.0003232    (20.82%)

correlation matrix of the fit parameters:
                a      b
a               1.000
b              -0.874  1.000
gnuplot>
```

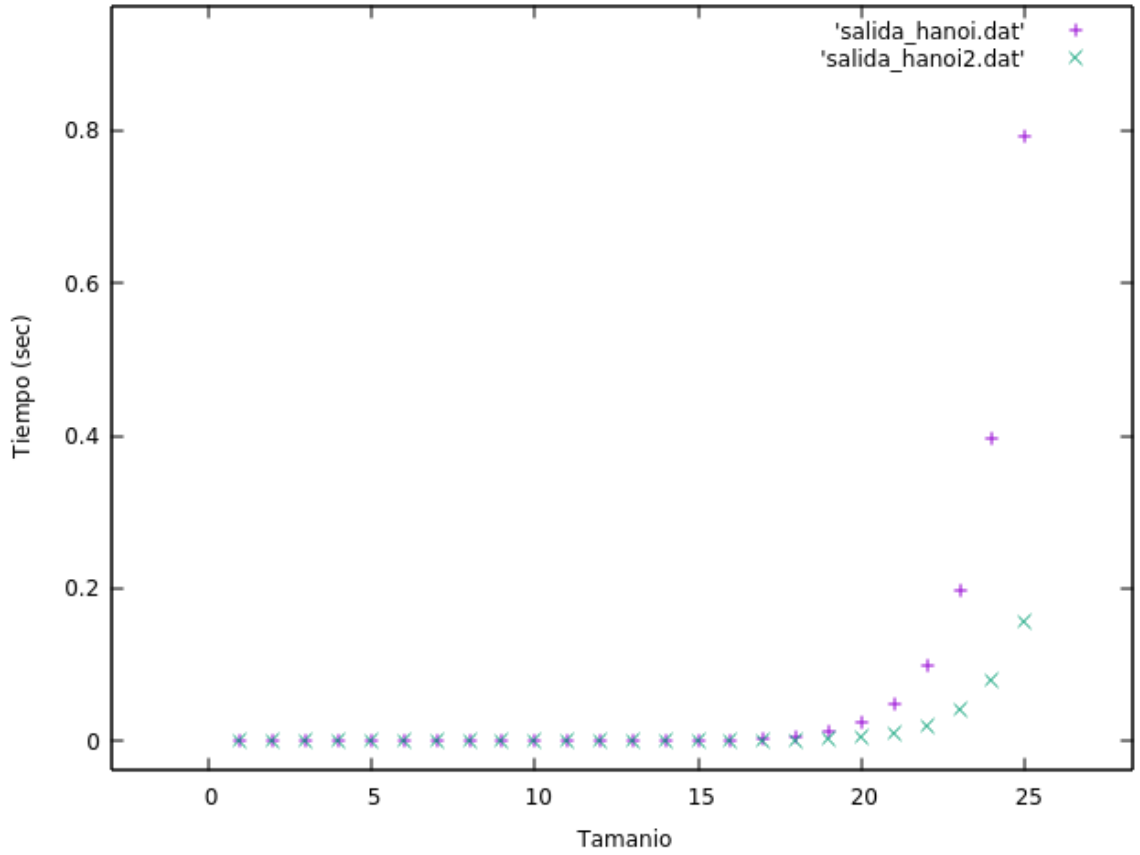
Compilación con optimización

EJERCICIO 4

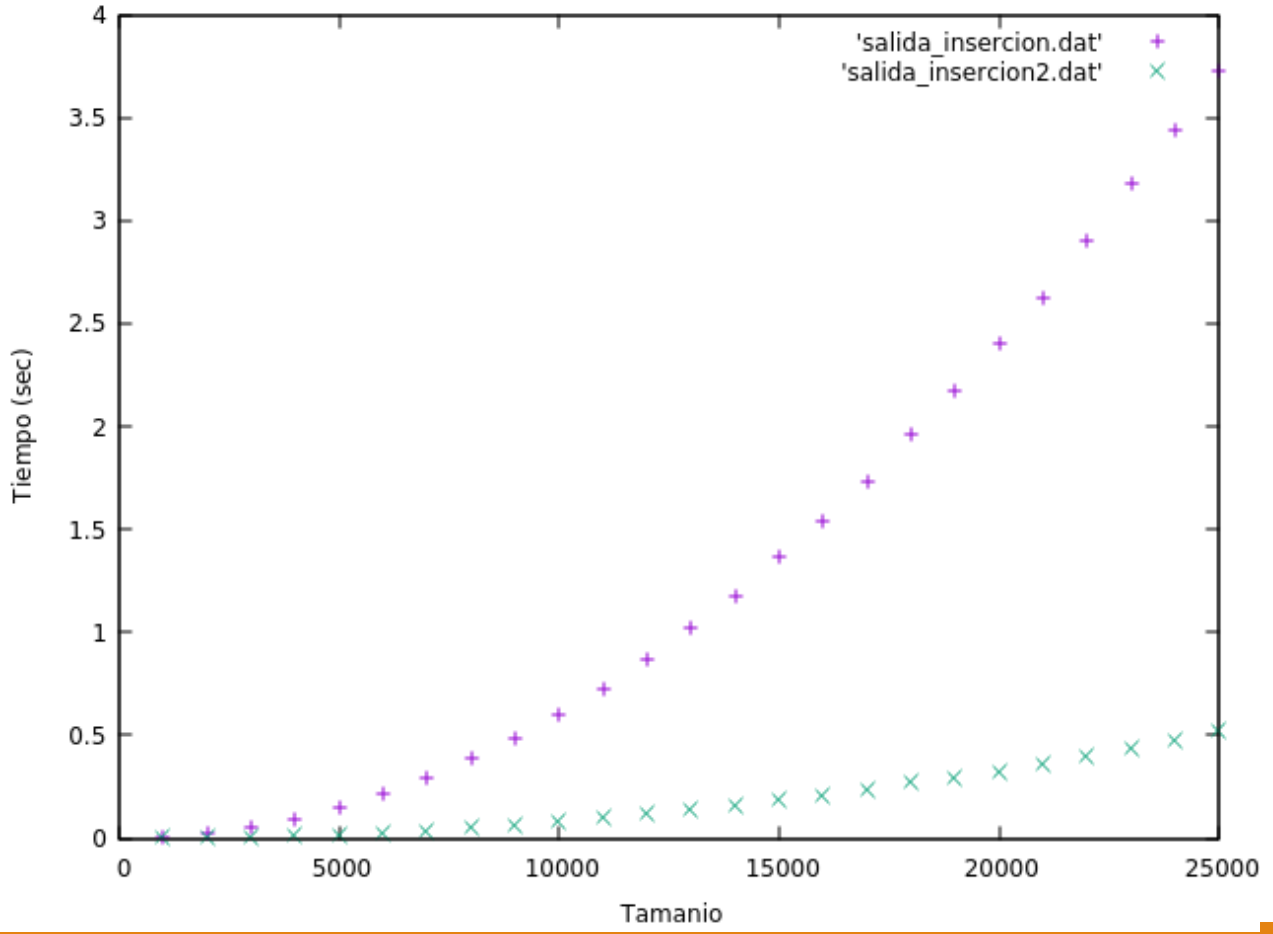
Floyd compilado con optimización (-01)



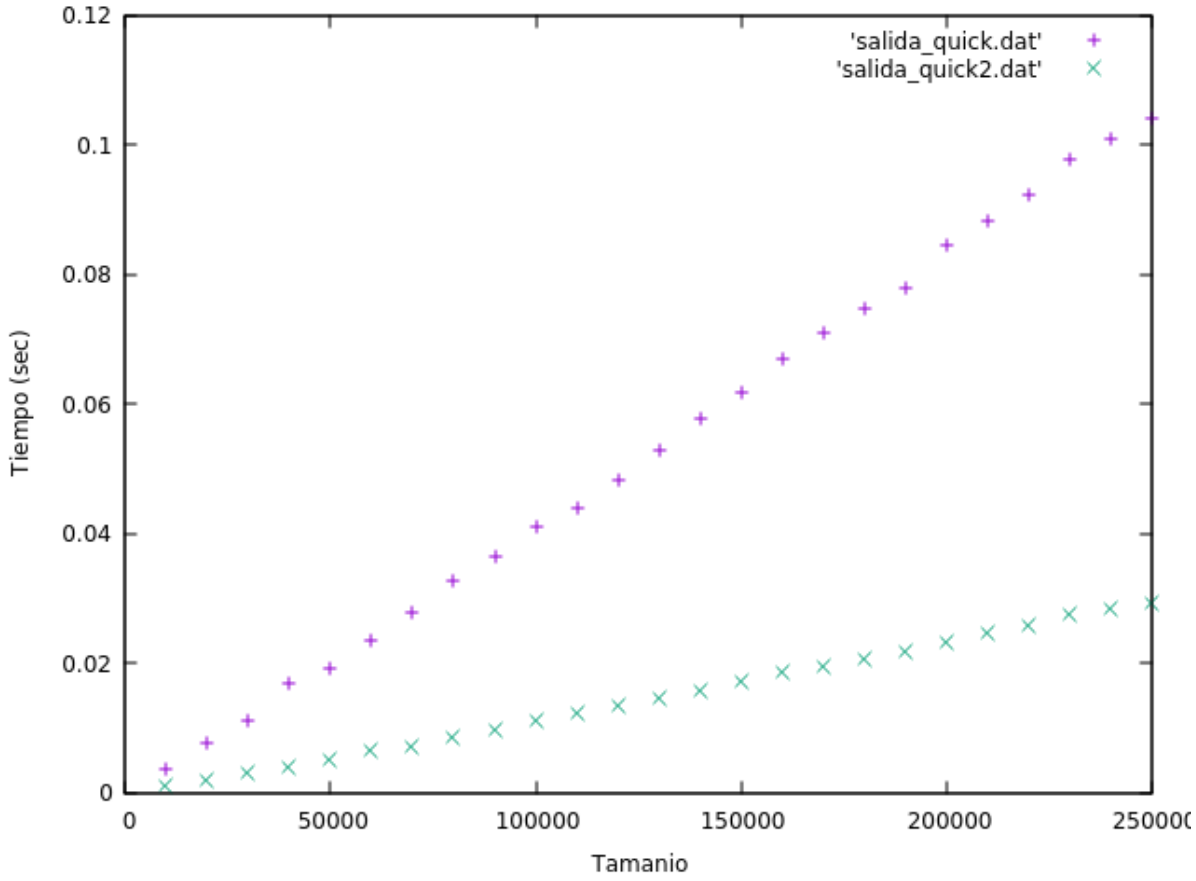
Hanoi compilado con optimización (-03)



Inserción compilado con optimización (-01)



Quicksort compilado con optimización (-02)



FIN

MUCHAS GRACIAS