Problema 1.1

- 17 $O(1)$

Como 17 es igual a $17 \cdot n^0$ y cualquier nº elevado a 0 es 1, por lo que ese es su número de orden.

- $\frac{n(n-1)}{2}$ Es $O(n^2)$ y $\Omega(n^2)$ (es decir, $\Theta(n^2)$)

Esto es que el tiempo teórico y el amortizado son iguales, n^2. Si comprobamos con $\frac{n^2}{2}$ y $\frac{n}{2}$, ambas positivas, con L'Hôpital vemos que obtenemos ∞. Por tanto n^2 es el orden más grande

$$\lim_{n\to+\infty} \frac{n^2/\cancel{2}}{n/\cancel{2}} = \lim_{n\to+\infty} n = \infty$$

- $\max(n^3, 10n^2)$ es $O(n^3)$

$$\lim_{n\to+\infty} \frac{n^3}{10n^2} = \lim_{n\to+\infty} \frac{n}{10} = \frac{\infty}{10} = \infty$$

Se aplica L'Hôpital comparando n^3 con $10n^2$ y sale ∞, n^3 es mayor por tanto

- $\log_2 n$ es $\Theta(\log_3 n)$

$$\log_2 n = \frac{\log(n)}{\log(2)} \qquad \log_3 n = \frac{\log(n)}{\log(3)}$$

Ahora hacemos L'Hôpital con base igual.

Aplicamos L'Hôpital así: $\lim_{n\to+\infty} \frac{\log(n)/\log 2}{\log(n)/\log 3} = \lim_{n\to+\infty} \frac{\log 3 \cancel{\log(n)}}{\log 2 \cancel{\log(n)}} \approx 1,58$. Entonces no sabemos

Si es cierto

Problema 1.2

① $f(n) = 13n^2 + 4n - 73$

$13n^2 + 4n - 73 = k \cdot n^2$ porque el mayor orden de la expresión es n^2

$13n^2 - kn^2 + 4n - 73 = 0$

$(13-k)n^2 + 4n - 73 = 0$ $\quad n(n(13-k)+4) = 73$

$n(13-k)+4 = 73$

$n = \frac{69}{13-k}$ $\quad k=2$, $n = 69/11$

② $f(n) = 1/(n+1)$

$\frac{1}{n+1} = k \cdot n^{-1}$

$\frac{n}{n+1} = k$ $\quad n = kn + k$

$n - kn = k$

$n(1-k) = k$

$n = \frac{k}{1-k}$ $\quad k=-1$, $n = -1/2$

3) $f(n) = 1/(n-1)$

$$\frac{1}{n-1} = k \cdot (n^{-1})$$

$$n = k(n-1)$$

$$n = kn - k$$

$$n - kn = -k$$

$$n(1-k) = -k$$

$$n = \frac{-k}{1-k}$$

$n = 1/2$

$k = -1$

4) $f(n) = (n-1)^3$

$$(n-1)^3 = k(n^3)$$

$$(n^2 - 2n + 1)(n-1) = kn^3$$

$$n^3 - 2n^2 + n - n^2 + 2n - 1 = kn^3$$

$$n^3 - 3n^2 + 3n - 1 = kn^3$$

$$n^3(1-k) - 3n^2 + 3n - 1 = 0$$

$$n^3(1-k) = 3n^2 - 3n + 1$$

$$n^3(1-k) = 3n(n-1) + 1$$

$$(1-k) = \frac{3n(n-1)+1}{n^3}$$

$. k = 3$

$$-2n^3 = 3n^2 - 3n + 1$$

$$2n^3 + 3n^2 - 3n + 1 = 0$$

2 3 -3 1

2

$$(n-1)^3 = n^3 - 3n^2 + 3n - 1$$

$$n^3 - 3n^2 + 3n - 1 = k n^3$$

$$n(n(n(1-k)-3)+3) - 1 = 0$$

$$n = 1 - \frac{9}{1-k} = 1 + \frac{9}{k-1}$$

Como $f(n) \in O(n^3)$: $k = 3$ y $n = 1 + 9/2 = 11/2$

5) $f(n) = (n^3 + 2n - 1)/(n+1)$

$$\frac{n^3+2n-1}{n+1} = k \cdot n^2$$

$$n^3+2n-1 = kn^3 + kn^2$$

$$n^3 - kn^3 - kn^2 + 2n - 1 = 0$$

$$n^3(1-k) - kn^2 + 2n - 1 = 0$$

$$n^3(1-k) = kn^2 - 2n + 1$$

$$n^3(1-k) = n(kn-2) + 1$$

$$n^3(1-k) - n(kn-2) = 1$$

$$n(n^2(1-k) - (kn-2)) = 1$$

$$n(n^2(1-k) - kn + 2) = 1$$

$$n^2(1-k) - kn + 2 = \frac{1}{n}$$

$$n^2 - kn^2 - kn + 2 = \frac{1}{n}$$

$$-kn^2 - kn = \frac{1}{2} - 2 - n^2$$

$$-k(n^2+n) = -\frac{3}{2} - n^2$$

$$-k = \frac{-3/2 - n^2}{n^2+n}$$

$$-k = \frac{\frac{-3-2n^2}{2}}{n^2+n} \Rightarrow -k = \frac{-3-2n^2}{2(n^2+n)}$$

$k = -2$

6) $f(n) = \sqrt{n^2-1}$

$$\sqrt{(n+1)(n-1)} = kn$$

$$\sqrt{(n+1)}\,\sqrt{(n-1)} = kn$$

$$\frac{\sqrt{(n+1)}\,\sqrt{(n-1)}}{n} = k \rightarrow \sqrt{\frac{n+1}{n}}\,\sqrt{\frac{n-1}{n}} = k$$

$$\sqrt{\frac{n^2-1}{n^2}} = k \qquad \frac{n^2-1}{n^2} = k^2$$

$$\frac{n^2}{n^2} - \frac{1}{n^2} = k^2$$

$$1 - \frac{1}{n^2} = k^2 \qquad 1 - k^2 = \frac{1}{n^2}$$

$$n^2 = \frac{1}{1-k^2}$$

$$n = \frac{1}{\sqrt{1-k^2}}$$

$k \geq 1$

$n = 1$

Problema 1.3

$$2000 < n\log_2\log_2(n^2) < n\log_2(n^2) < \sqrt{n} < n < n+100 < n^2 < n^3+1 < 3^{\log_2(n)} < 2^n < 3^n < 2^n + 3^{n-1} < n \cdot 2^n$$

Problema 1.4

a) Verdadero, porque ~~[illegible]~~ es una suma, pertenece a la O-grande de la más grande entre ambos si son independientes. Como la O-grande es la misma, pertenecen a $f(n)$

b) Verdadero. Como $T1(n) \in O(f(n))$, que está dentro de $f(n^2)$, se cumple.

c) Falso. Sus O-grandes no se pueden dividir entre ambos, ya sean dependientes o no.

Problema 1.5

$f_1(n) = n^2$ → $\in O(n^2)$, $\in \Omega(n^2)$

$f_2(n) = n^2 + 1000n$ → $\in O(n^2)$, $\in \Omega(n)$

$f_3(n) = \begin{cases} n & \text{si } n \text{ es impar} \\ n^3 & \text{si } n \text{ es par} \end{cases}$ → Su O-grande cambia en ese

$f_4(n) = \begin{cases} n & \text{si } n < 100 \\ n^3 & \text{si } n > 100 \end{cases}$ → $\in O(n^3)$, $\in \Omega(n)$ si $n \leq 100$

$f_1(n) \in O(f_2(n)) \notin \Omega(f_2(n))$

$f_1(n) \in O(f_4(n)) \in \Omega(f_4(n))$

$f_2(n) \in O(f_1(n)) \in \Omega(f_1(n))$

$f_2(n) \in O(f_4(n)) \in \Omega(f_4(n))$

$f_4(n) \notin O(f_1(n)) \in \Omega(f_1(n))$

$f_4(n) \notin O(f_2(n)) \in \Omega(f_2(n))$

Problema 1.6

$f(n) \in O(g(n))$ y $g(n) \in O(h(n))$ entonces $f(n) \in O(h(n))$

Con la propiedad transitiva, vemos que $g(n)$ es un subconjunto de $h(n)$ y $f(n)$ lo es de $g(n)$, entonces $f(n)$ lo es de $O(h(n))$

Problema 1.7

$O(f(n)) \in O(g(n))$ sii $f(n) \in O(g(n))$ y $g(n) \in O(f(n))$

Por la propiedad reflexiva vemos que un conjunto puede estar dentro de un conjunto que se encuentra dentro suyo, por lo que se cumple tanto que $O f(n) \in O(g(n))$ y $O(g(n)) \in O(f(n))$

Problema 1.8

Demostrar: $O(1) \subset O(\log(n)) \subset O(n) \subset O(n^2) \subset O(2^n) \subset O(n!)$

Problema 1.8

Demostrar $O(1) \subset O(\log(n)) \subset O(n) \subset O(n^2) \subset O(2^n) \subset O(n!)$

Se hace L'Hôpital:

$$\lim_{n\to\infty} \frac{1}{\log(n)} = \frac{1}{\infty} = 0$$

$$\lim_{n\to\infty} \frac{\log(n)}{n} = \lim_{n\to\infty} \frac{1/n}{1} = \frac{1}{\infty} = 0$$

$$\lim_{n\to\infty} \frac{n}{n^2} = \lim_{n\to\infty} \frac{1}{n} = 0$$

$$\lim_{n\to\infty} \frac{n^2}{2^n} = \lim_{n\to\infty} \frac{2n}{\log 2 \cdot 2^n} = \lim_{n\to\infty} \frac{2}{(\log 2)^2 \cdot 2^n} = \frac{2}{\infty} = 0$$

Problema 1.9

```
for (i=0; i<n; i++)
    for (j=0; j<n; j++){
        C[i][j]=0;  ← O(1)
        for (k=0, k<n, k++){
                                              ← O(1)
            C[i][j] += A[i][k] * B[k][j]
}
```

$$\sum_{k=0}^{n-1} 1 \quad \sum_{j=0}^{n-1} \sum_{k=1}^{n-1} 1 \quad \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} \sum_{k=1}^{n-1} 1 =$$

$$= \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} (n-j) = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} n - \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} j = \sum_{i=0}^{n-1} n^2 - \sum_{i=0}^{n-1} \frac{n(n-1)}{2} = n^3 - \frac{n^2(n-1)}{2} = n^3 - \frac{n^3 - n^2}{2} =$$

$$= \frac{2n^3 - n^3 + n^2}{2} = \frac{n^3 + n^2}{2}$$

$\in O(n^3)$

Problema 1.11

```
for(i=0; i<n; i++)
```

$\in O(n^3)$

1.10

```
void ejemplo (int n)
}
    int i, j, k;  ← O(1)

    for (i=1; i<n; i++)
        for (j=i+1; j<=n; j++)
            for(k=1; k<=j; k++)
                Global += k·i;  ← O(1)
}
```

$$\sum_{k=1}^{j} 1 \quad \sum_{j=i+1}^{n} \sum_{k=1}^{j} 1 \quad \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \sum_{k=1}^{j} 1 =$$

$$= \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} (j) = \sum_{i=1}^{n-1} \frac{n(n-1)}{2} = \sum_{i=1}^{n-1} \frac{n^2}{2} - \frac{n}{2} = \left(\frac{n^2}{2} - \frac{n}{2}\right)(n-1) =$$

$$= \left(\frac{n^2-n}{2}\right)(n-1) = \frac{n^3-n^2}{2} - \frac{n^2-n}{2} = \frac{n^3-n^2-n^2+n}{2} = \frac{n^3-2n^2+n}{2} =$$

Problema 1.11

```
for(i=0; i<n; i++)
```

$\in O(n^3)$

```
int x, cont;

cont = 0;

x = 2;

while( x < n)
```

8	2
16	3
32	4

Problema 1.13

int recursiva(int n) $\longleftarrow T(n)$

$T(n) == 1 \quad si \quad n \leq 1$

Para $n>1$:

$T(n) = 2\underbrace{T(n-1)}+1$

$T(n-1) = 2\underbrace{T(n-2)}+1$

$T(n-2) = 2\,T(n-3)+1$

$T(n) = 2T(n-k)+k$

$k<n$

$k=n-1$

$T(n) = 2T(n-n+1)+n-1 = 2T(1)+n-1 = n+1 \quad \in O(n)$

Problema 1.14

a) El valor que devuelve la función es $\log_2(n)+1$

b) int E(int n) $\longleftarrow T(n)$

$T(n) == 1 \quad si \quad n=1$

$T(n) = T(n/2)+1$

$\hookrightarrow T(n/2) = T(n/4)+1$

$T(n) = \underbrace{T(n/4)}+2$

$T(n/4) = T(n/8)+1$

$T(n) = T(n/8)+3$

$n=2^m$

$T(2^m) = T(\frac{2^m}{2})+1 = \underbrace{T(2^{m-1})}+1$

$T(2^{m-1}) = T(2^{m-2})+1$

$k<m \quad T(2^m) = T(2^{m-k})+k$

$k=m \quad T(2^m) = T(2^{m-m})+m = T(1)+m = 1+m$

$T(n) = 1+\log_2(n) \quad \in O(\log_2(n))$

Problema 1.15

$$T(n) = k \cdot \underbrace{T(n-1)} + s^2 \qquad n > 1 \qquad k, s > 1 \qquad T(1) = 1$$

$$T(n-1) = k \; T(n-2) + s^2$$

$$T(n) = k \cdot (k \; T(n-2) + s^2) + s^2 = k^2 \underbrace{T(n-2)} + ks^2 + s^2$$

$$T(n-2) = kT(n-3) + s^2$$

$$T(n) = k^2 (k \cdot T(n-3) + s^2) + ks^2 + s^2 = k^3 \underbrace{T(n-3)} + k^2 s^2 + ks^2 + s^2$$

$$T(n-3) = k \cdot T(n-4) + s^2$$

$$T(n) = k^3 (k \cdot T(n-4) + s^2) + k^2 s^2 + ks^2 + s^2 = k^4 \; T(n-4) + k^3 s^2 + k^2 s^2 + ks^2 + s^2 =$$

$$= k^4 \; T(n-4) + s^2 (k^3 + k^2 + k + 1)$$

$$T(n) = k^z \cdot T(n-z) + s^2 (k^{z-1} + k^{z-2} + \ldots + k^1 + k^0)$$

$z < n$

$z = n-1$

$$T(n) = k^{n-1} \; T(n-(n-1)) + s^2 (k^{n-2} + k^{n-3} + \ldots + k^1 + k^0) = k^{n-1} \; T(1) + s^2 (k^{n-2} + k^{n-3} + \ldots + k^1 + k^0) =$$

$$= k^{n-1} + s^2 (k^{n-2} + k^{n-3} + \ldots + k^1 + k^0) \; \in O(k^n)$$