PHYS 4200 Final

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1 Introduction

In biophysics, one particular area of focus is population dynamics. Population dynamics is the study of populations as a dynamical system and the environmental processes that dictate them. The factors of environmental processes include: size and age of populations, birth and death rates, and immigration/emigration. Population dynamics can then be described by competition models given that it requires different type of species to interact.

This paper will be composed of two sections: Section 1 will consist of an analysis that was made regarding some data that was retrieved and goes hand-in-hand with section 2 which consists of the Lotka-Volterra equations to describe a competition model between two species; Wildebeest and lions.

2 Section 1: Data Analysis

This section focuses on where the data comes from, what the data is, and how it was acquired.

The data that will be used comes from a case study by D. Stratton 2010 titled, "Case Studies in Ecology and Evolution: Wildebeest in the Serengeti: limits to exponential growth." At the very end of the paper, there is a table provided with data retrieved from Mduma1999. [3] that contains number of wildebeest(x1000), rainfall(mm), and years from 1959 to 1994. The case study also includes general parameters for growth rate and environmental capacity that will be implemented in the next section. Data was acquired and analyzed by the implementation of python.

Python has many useful packages for analyzing data, making it one of the most important languages out there. One of these packages is called the tabula package which allows python to read table data from a pdf file. Tabula can then save the data into pandas data frame where we can use other functions to finally analyze the data. The following figure shows the code that helped achieve this 1:

```
pd = tabula.read_pdf("2_wildebeest.pdf",encoding='utf-8', spreadsheet=True, pages = 12
```

Figure 1: Tabula-Pandas code

	Year	Number of wildebeest (x 1000)	Dry Season Rainfall (mm)
0	1959	212	NaN
1	1960	232	100.0
2	1961	263	40.0
3	1962	307	102.0
4	1963	356	104.0
5	1964	403	168.0
6	1965	439	168.0
7	1966	461	166.0
8	1967	483	78.0
9	1968	520	91.0
10	1969	570	78.0
11	1970	630	133.0

Figure 2: Data Table

The data table was retrieved 2 where an analysis on the first 19 headers of the table was made. The analysis performed was to determine if there is a correlation between the amount of rainfall and the number of wildebeest. From the graph 3, we can conclude that there is a positive correlation between rain fall and number of wildebeest. As mentioned in the case study, more rainfall leads to an increase in biomass of grass (food for wildebeest) therefore more wildebeest will be produced. The sudden drops can be described as probable predation or other factors like disease.

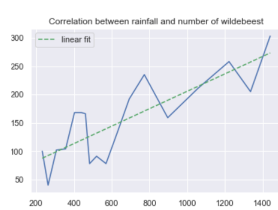

Figure 3: Correlation between wildebeest and rainfall

3 Differential Eqs.

This section describes the two equations that make a competition model possible.

The Lotka-Volterra equations successfully describe the interaction between two species [5]

$$\frac{dP_1}{dt} = r_1 * K_1\left(\frac{K_1 - P_1 - \alpha P_2}{K_1}\right) \tag{1}$$

$$\frac{dP_2}{dt} = r_2 * K_2\left(\frac{K_2 - P_2 - \beta P_1}{K_2}\right) \tag{2}$$

where P1 and P2 are the populations of wildebeest and lions respectively, r1 and r2 are the growth rates of each species, K1 and K2 are the environment carrying capacities, and alpha/beta are the interaction parameters between one another.

Given that population dynamics is a dynamical system problem, system theory suggests that we need steady state solutions. A steady state solution is a solution such that the dynamical process is unchanged in time. In other words, the derivative with respect to time is zero. Setting equations 1 and 2 to zero, we find that there are two solutions 4

$$([0, \quad \boldsymbol{K}_1 - \boldsymbol{P}_2\boldsymbol{\alpha}], \quad [0, \quad \boldsymbol{K}_2 - \boldsymbol{P}_1\boldsymbol{\beta}])$$

Figure 4: Steady State solutions

A solution where both P1 and P2 are zero and a solution where both wildebeest and lions maintain nonzero populations. Further analyzing suggests that after sometime, whatever the parameters are for each equation, the solutions should converge to the steady state solutions.

The parameters of wildebeest for growth rate, initial population, and environmental capacity were gathered from [1]. The parameters for lions were simulated using reasonable values from real life situations

4 Conclusion

Implementing information gathered from section 1 to sections 2, it was concluded the Lotka-Volterra equations for describing a prey-predator competition proved to be successful. After a time period of 100 days along with the other parameters inputted, the populations of each species converge to the steady state solutions 5

Another description of the solution is that If there were no predators for the prey, the prey would be allowed to grow exponentially. This could bring catastrophic consequences to the environment. A predator must be placed in order to keep the balance given that they will both interact with each other.

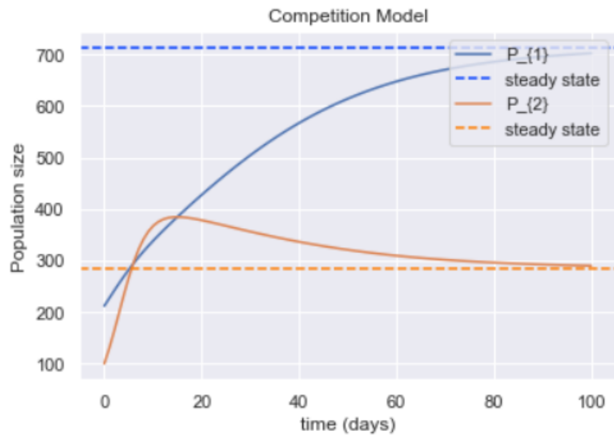

Figure 5: Populations converge to steady state solutions

References

[1] 2_wildebeest.

[2] P. Chesson and J. J. Kuang. The interaction between predation and competition. *Nature*, 456(7219):235–8, 11 2008.

[3] S. A. Mduma, A. R. Sinclair, and R. Hilborn. Food regulates the Serengeti wildebeest: A 40-year record. *Journal of Animal Ecology*, 68(6):1101–1122, 1999.

[4] S. M. Miller and P. J. Funston. Rapid Growth Rates of Lion (Panthera leo) Populations in Small, Fenced Reserves in South Africa: A Management Dilemma. *South African Journal of Wildlife Research*, 44(1):43–55, 4 2014.

[5] P. J. Wangersky. Lotka-Volterra Population Models. Technical report, 1978.