

Q4. Minimize $2x^2 + y^2 + z^2$ --- eq(1)

Subject to $2x + 2y - z \geq 1.6$ --- eq(2)

For eq(2) = $g(x, y, z)$, eq(1) = $f(x, y, z)$

$$\nabla f(x, y, z) = \lambda \nabla g(x, y, z)$$

$$\frac{df}{dx} = 4x$$

$$\frac{df}{dy} = 2y$$

$$\frac{df}{dz} = 2z$$

$$\frac{dg}{dx} = 2$$

$$\frac{dg}{dy} = 2$$

$$\frac{dg}{dz} = -1$$

$$4x = 2\lambda$$

$$2y = 2\lambda$$

$$2z = -\lambda$$

$$x = 1/2 \lambda$$

$$y = 1 \cdot \lambda$$

$$z = -1/2 \lambda$$

$$2x + 2y - z \geq 1.6$$

$$2 \times \frac{1}{2} \lambda + 2 \times \lambda - (-\frac{1}{2}) \lambda \geq 1.6$$

$$\lambda + 2\lambda + \frac{1}{2} \lambda \geq 1.6$$

$$\frac{3}{2} \lambda \geq 1.6$$

$$\lambda \geq \frac{1.2}{3}$$

$$x \geq \frac{6}{7}$$

$$y \geq \frac{12}{7}$$

$$z \geq -\frac{6}{7}$$

So to minimize $2x^2 + y^2 + z^2$

for $x = \frac{6}{7}, y = \frac{12}{7}, z = -\frac{6}{7}$

$$2x^2 + y^2 + z^2 =$$

$$= 2 \times \frac{36}{49} + \frac{144}{49} + \frac{36}{49}$$

$$= \frac{252}{49} = \frac{36}{7} = 5.142$$